Large Sample Properties for Exponential Family Time Series Models

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Abstract

The primary aim of this talk is review what is known about the sample path behaviour and asymptotic properties of two classes of times series models for discrete data. It will also identify areas in which future work is required. Let Y_t be the observation variable at time t as a function of the state variable, w_t. The distribution of the observation, Y_t, at time t conditional upon the past and present states and the past observations, depends only on the present state w_t and is a member of the exponential family:

$$f(y_t|w_t) = \exp[y_t w_t - b(w_t))/a(\phi) + c(y_t, \phi)]$$

The state equation is $w_t = x_t^{\mathrm{T}}\beta + \alpha_t$ which depends upon a p-dimensional vector of covariates x_t through a linear regression of the form $x_t^{\mathrm{T}}\beta$ as well as a random process $\{a_t\}$. We consider two specifications of $\{\alpha_t\}$. In the 'parameter driven' specification, $\{\alpha_t\}$ is a stationary Gaussian time series with mean $\mu_\alpha(\theta)$, variance $\sigma_\alpha^2(\theta)$ and covariance function $\{\gamma_\alpha(\theta)(h) : h = 0, \pm 1, \pm 2, \ldots\}$. In the 'observation driven' specification

$$\alpha_t = h(y^{(t-1)}, x^{(t-1)}) = \sum_{i=1}^{q} \gamma_i(\theta) e_{t-i}$$

in which the "innovations" are suitably defined such as Pearson type residuals of the form

$$e_t = \frac{y_t - \dot{b}(w_t)}{\ddot{b}(w_t)^{0.5}}.$$

Primary interest is in efficient estimation of the regression parameters β as well as obtaining correct standard errors for these estimates which allow for the serial dependence present in the above two classes of models.

Some brief comments will also be made about other classes of models for discrete time series such as the integer valued GARCH process.