# Dependence Evolution in International Equity Markets*

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**Abstract**

This paper investigates the dynamics of dependence in international equity markets. To this end, we develop a multiple-regime smooth-transition copula GARCH model and address several important questions, including the number of regimes and the existence of asymmetry in dependence evolution. Our results suggest that two or three regimes are enough to describe dependence evolution in international equity markets over the last 35 years with a significant asymmetric increase. In addition, the implied time-series of three dependence measures shows a wide variety of dynamics, demonstrating the usefulness of our framework to describe the dynamics of dependence in international equity markets. Finally, we evaluate the economic significance of our empirical finding based on the 99% value at risk and expected shortfall. Our result indicates that both risk measures have increased approximately 20% over the last 35 years due to almost vanishing international diversification effects.

JFL classification: C32, C51, G15

Key Words: Smooth transition model; Copula; Spearman's rho, Tail dependence

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# 1 Introduction

The study of time-varying dependence structures in international equity markets has recently attracted increasing attention among theorists, empirical researchers, and practitioners for numerous reasons. For instance, investors need to assess the degree of comovement among international stock returns accurately to construct a well diversified portfolio. In addition, to evaluate risk measures, such as the Value at Risk (VaR) and expected shortfall (ES), risk managers should take into account interdependence in international equity markets. Ignoring an increase in dependence could lead considerable under-evaluation of those risk measures. Policy makers also have to pay close attention to the contagion, which is caused by dependence between extreme negative shocks or lower tail dependence across international financial markets. If contagion effects became unnegligible, a financial crisis occurring in one country would have substantial effects on other countries, amplifying concerns for policy makers as well as market investors.

Another reason for a growing number of studies on dynamics of dependence in international equity markets is associated with financial market integration. Over the last three decades, the circumstances of the world financial markets have changed dramatically. Examples of this include an increase of world economic relation, competition and globalization, development of the world transportation system, reduction of trade barriers, evolution in information technology, and improvement of monetary policy design and implementation. In addition, according to the IMF's Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER),[1] many industrial countries have experienced a rapid increase in their degree of financial openness since the mid-1980s. One natural consequence of these changes is a promotion of financial market integration. Indeed, the de facto measures recently constructed by Lane and Milesi-Ferretti (2006) indicate that financial integration in industrial countries was promoted gradually throughout the 1970s and 1980s, and accelerated throughout the mid-1990s.[2] It is not unreasonable to assume that the promotion of financial market integration would affect comovements among international financial markets. It is, therefore, very instructive to examine the evolution of dependence in international equity markets.

The main contribution of this paper is to investigate the dynamics of dependence in interna-

[1]AREAER reports a set of de jure measures of legal restrictions on cross-border capital flows, and is widely used to measure financial openness.

[2]See Kose et al. (2006) for details of financial integration and related measures.

tional equity markets systematically. In particular, the paper addresses the following questions: (i) Is the multivariate normal (MVN) model appropriate to describe the dependence evolution in international equity markets? (ii) How many dependence regimes are enough to characterize dependence evolution over the last 35 years? (iii) Did dependence in international equity markets change? (iv) Is there any asymmetry in evolution between upper and lower tail dependences? (v) When did important dependence changes occur? (vi) Which tail dependence contributed more to these changes?

The paper is related to voluminous previous studies investigating time-varying dependence in international equity markets. For instance, Von Furstenberg and Jeon (1989) estimate a VAR model consisting of daily stock returns for four major markets (Japan, Germany, UK, and US) and detect an increase in correlations since the crash of 1987. Koch and Koch (1991) reach a similar conclusion based on the daily returns across eight different national equity markets. On the other hand, King, Sentana and Wadhwani (1994) claim that their finding of increasing dependence only reflected a transitory increase caused by the 1987 crash. To test an increase in correlation more precisely, Longin and Solnik (1995) estimate a bivariate GARCH model with a trend term in correlations between the US and other G7 countries. They find the significant increase in correlations for four pairs out of six. Berben and Jansen (2005) model the dynamics of correlation with a smooth transition model and show that correlations among the German, UK, and US stock markets have doubled, whereas Japanese correlations have remained the same. Finally, Bekaert, Hodrick, and Zhang (2009) establish that there is no evidence for an upward trend in international stock return correlations, except for the European stock markets based on parsimonious risk-based factor models. Thus, whereas there has been much empirical work in this area, it is fair to say that there is no definitive evidence that dependence in international equity markets is significantly and permanently increasing. Notwithstanding, this paper will provide clear evidence of asymmetric increasing dependence in international equity markets.

This paper develops a novel multiple-regime smooth-transition copula-GARCH (STCG) model, differing from these previous studies in several ways. First, the paper employs a notion of copula to examine dependence structures explicitly and flexibly. As a result, we can choose more suitable dependence structures than previous literature based on elliptical distributions, including multivariate

Normal and $t$ distributions.[3] Second, we use three copula-based dependence measures, specifically, Spearman's rho, and upper and lower tail dependences to evaluate dependence from three aspects, hence in a more appropriate way than using only one non-copula based measure, particularly correlation. In fact, Embrechts, McNeil, and Straumann (2002), and Embrechts, Lindskog, and McNeil (2003) emphasize that a correlation is not a good measure of dependence for non-elliptical distributions. Three, the paper considers several number of dependence regimes ranging from one to four. To our best knowledge, none of the previous literature examines a possibility of more than three dependence regimes. It is, however, very questionable whether dependence evolution in international equity markets over the last 35 years can be characterized by two regimes, given the many factors affecting the financial market integration mentioned above. Fourth, the paper investigates the possible different evolutions between upper and lower tail dependence. Given an asymmetry in dependence structures in international equity markets found by a number of studies,[4] this could be a very important extension.

We apply the multiple-regime STCG model to four of the largest equity markets, namely France (FR), Germany (GE), U.K. (UK), and U.S. (US) markets. The results of our empirical analysis are summarized as follows. First, the paper confirms the importance of capturing the fat-tailness to characterize the stock returns, showing the inappropriateness of the MVN model employed by the most of previous studies, such as those of Longin and Solnik (1995) and Berben and and Jansen (2005). In addition, the results suggest that the symmetrized HR copula model dominates the normal and symmetrized JC copula model. Second, the analysis demonstrates that three dependence regimes are enough to describe time evolution of dependence in international equity markets over the last 35 years. Third, the results indicate a significant increase in both upper and lower tail dependences. Fourth, the results provide clear evidence of the asymmetric evolution in upper and lower tail dependences.

Following these empirical findings, we calculate three copula-based dependence measures to see when the important increases occurred and which tail contributed more to the increases. The results demonstrate that FR-GE and FR-UK pairs experienced a rapid increase in dependence between 1986 and 1991, and 2000 and 2004, with lower tail dependence playing more important

[3] Kumar and Okimoto (2010) propose the similar STCG model to examine correlation dynamics in the international bond markets, but they just use two-regime normal copula model.

[4] See Okimoto (2008) and the reference therein for the asymmetric dependence in international equity markets.

role in these increases. On the other hand, GE-US and FR-US pairs underwent a gradual increase from 1987 onward, whereas GE-UK and UK-US pairs' dependence increased almost linearly over the entire sample. Furthermore, the results show that for these four pairs upper tail dependence contributed to increasing dependence more than lower tail dependence.

Lastly, we investigate the economic significance of our empirical findings from a risk management point of view based on the 99% VaR and ES. Our results indicate that both 99% VaR and ES in 2008 are larger by about 20% compared to those in 1973. In addition, the benefit from international diversification has almost vanished in recent years. It is, therefore, critical to recognize the paper's finding of increasing dependence with possible asymmetry in international equity markets.

The remainder of the paper is organized as follows. Section 2 provides preliminary analysis for the correlation dynamics in international equity markets. Section 3 introduces the model and the idea behind our methodology, while Section 4 provides the empirical results. Lastly, section 5 concludes.

## 2 Preliminary Analysis

This study is based on weekly total market price index data in US dollars of four of the largest countries, namely France, Germany, the U.S., and the U.K. The data are obtained from Datastream with the sample period lasting from January, 1973 to June, 2008. To achieve our goal of examining dependence evolutions in international equity markets, the investigated markets should be representative of total markets and should be reasonably integrated during our sample period, which are arguably the case for these four markets.

A preliminary look at the data gives an indication of increase in correlations among international equity markets. To get a flavor of this increase, Table 1 summarizes correlations across international stock returns, defined as 100 times the change in the natural logarithm of each country's stock index, for five subsamples. As can be seen, the correlations are almost monotonically increasing over the last 35 years for all pairs. In particular, most of pairs have experienced significant increase in correlation during the third and five subsamples.

The table seems to indicate clear evidence of recent increase in correlation in international equity markets. We, however, should interpret the result with some care. For instance, Boyer, Gibson, and

Loretan (1999) show that changes in correlations over time cannot be detected reliably by splitting a sample based on the realized values of the data. Thus, it could be misleading to conclude an increase in correlations based on the behavior of subsample estimates of correlations. Instead, we should consider some models with possible structural changes, and decide whether the estimated parameters imply changing correlations as emphasized by Boyer, Gibson, and Loretan (1999) and Berben and Jansen (2005). In addition, the correlation may not be a good measure of dependence in international equity markets as discussed by Embrechts, McNeil, and Straumann (2002), and Embrechts, Lindskog, and McNeil (2003). It is, therefore, very important to seek the appropriate methodologies and models to describe and assess changes in dependence. By identifying suitable models, this paper tries to provide a new evidence of dependence evolution in international equity markets. More specifically, the paper seeks answers for the important empirical questions stated in the previous section.

## 3 Model and Estimation

The main purpose of this paper is to investigate the evolution of dependence in international equity markets. For this purpose, it is desirable to model a dependence structure as well as evolution process in a flexible way. To this end, we propose using a multiple-regime smooth-transition copula-GARCH (STCG) model. The basic idea behind the model is to employ copula theory to model a dependence structure and to specify the dynamics of dependence, or copula parameter(s) with a multiple-regime smooth-transition model.

According to the copula theory based on Sklar's (1959) theorem,[5] the joint distribution $H$ of two random variables $X_1$ and $X_2$ can be decomposed into two parts; marginal distributions $F_1$ and $F_2$, describing the marginal behavior of $X_1$ and $X_2$, and a copula $C$, representing the dependence structure between $X_1$ and $X_2$. More specifically, $H$ can be written as follows:

$$H(x_{,1} x_2; \theta) = C\left(F_1(x_1; \theta_1), F_2(x_2; \theta_2); \theta_C\right). \tag{1}$$

Here $\theta_C$ is a parameter vector for the copula, $\theta_1$ and $\theta_2$ are parameter vectors for each marginal distribution, and $\theta = (\theta_1', \theta_2', \theta_C')'$ is a parameter vector for the joint distribution. As a consequence, marginal distributions and dependence structure can be specified independently with great

[5]See Joe (1997) and Nelsen (2006) for details of the copula theory.

flexibility. In addition, we model copula parameters using the multiple-regime smooth-transition model. By doing so, we can capture dominant, long-run trends of dependence with possible regime changes. In the following subsections, we describe the models for marginal distribution, copula, and dynamics of dependence successively. Then we derive the log-likelihood function to estimate the models via the maximum likelihood estimation (MLE).

### 3.1 Marginal distributions

For marginal distributions we use the GARCH(1,1) model with Normal or Student's $t$-disturbance. Specifically, the model for margins can be expressed as

$$\begin{aligned} x_{it} &= c_i + \varepsilon_{it}, \\ \varepsilon_{it} &= \sqrt{h_{it}} v_{it}, \\ h_{it} &= \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, \end{aligned}$$

for $t = 1, \ldots, T$ and $i = 1, 2$, where $T$ is a sample size. We assume $v_{it}$ follows standard Normal distribution or Student's $t$-distribution with mean 0, variance 1, and the degree of freedom $\nu_i$. This specification is motivated by the well-known fact that the GARCH(1,1) model (with Student's $t$-disturbance) is able to capture many features of financial data, such as volatility clustering and fat-tailness. In addition, since no strong serial correlation is observed in each series, we simply model the conditional expectation as constant without an AR term.

Note that our copula framework allows to use the $t$-disturbance to capture fat-tailness. This is of great importance, since the tail behavior of the international stock returns may not be characterized by the Normal GARCH model very well as we will confirm in the next section. With an extra parameter for the tail-fatness, the $t$-GARCH model contains five parameters; $\boldsymbol{\theta}_i = (c_i, \omega_i, \alpha_i, \beta_i, \nu_i)'$, while the Normal GARCH model contains four parameters; $\boldsymbol{\theta}_i = (c_i, \omega_i, \alpha_i, \beta_i)'$.

### 3.2 Copula

For a copula, or a dependence structure, we use the Normal copula as a bench mark. The Normal copula is a copula for multivariate Normal distribution and given as follows:

$$C^{NOR}(u, v; \delta_1) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\delta_1^2}} \exp\left\{ -\frac{s^2 - 2\delta_1 st + t^2}{2(1-\delta_1^2)} \right\} dsdt, \;\; \delta_1 \in (-1, 1).$$

Here $\Phi(\cdot)$ is a cumulative distribution function (CDF) of the standard Normal distribution. Notice that joint distributions with a Normal copula and Normal GARCH model reduce to the multivariate Normal GARCH model, which is used in the most of previous literature concerning comovement of stock returns. Note also that $\delta_1$ is equivalent to the usual linear correlation between two variables. For this reason the Normal copula can capture only linear and symmetric dependence. In addition, it can be shown that the Normal copula exhibit no tail dependence.[6] To investigate the possible asymmetric evolution of tail dependences we also employ two other copulas; the symmetrized Joe-Clayton (SJC) copula and the symmetrized Hüsler-Reiss (SHR) copula.

The SJC copula is proposed by Patton (2006) to model asymmetric exchange rate dependence and based on the Joe-Clayton (JC) copula given by

$$C^{JC}(u,v;\delta_1,\delta_2) = 1 - \left(1 - \left\{[1-(1-u)^\kappa]^{-\gamma} + [1-(1-v)^\kappa]^{-\gamma} - 1\right\}^{-1/\gamma}\right)^{1/\kappa},\ \delta_1,\delta_2 \in (0,1)$$

where $\kappa = 1/\log_2(2-\delta_1)$ and $\gamma = 1/\log_2(\delta_2)$. With this specification, two copula dependence parameters, $\delta_1$ and $\delta_2$, for the JC copula are coincident with upper and lower tail dependences, respectively. Patton (2006) points out that even when the two copula dependence parameters are equal, there is still some (slight) asymmetry in the JC copula, and symmetrizes it as

$$C^{SJC}(u,v;\delta_1,\delta_2) = 0.5 \cdot \left\{C_{JC}(u,v;\delta_1,\delta_2) + C_{JC}(1-u,1-v;\delta_2,\delta_1) + u + v - 1\right\}.$$

Thus the SJC copula is a mixture of JC copula and the survival JC copula.[7] By construction, the SJC copula nests symmetry as a special case, making it a more interesting specification than the JC copula from empirical perspective to examine the existence of asymmetric dependence.

The SHR copula is based on the Hüsler-Reiss (HR) copula, whose survival copula is identified as the best copula to describe the bear regime in international equity markets by Okimoto (2008).[8] The HR copula is expressed as

$$\begin{aligned} C^{HR}(u,v;\delta) \quad &= \quad \exp\left\{\log u \cdot \Phi\left(\delta^{-1} + \frac{\delta}{2}\log\left(\frac{\log u}{\log v}\right)\right)\right. \\ &\qquad \left. + \log v \cdot \Phi\left(\delta^{-1} - \frac{\delta}{2}\log\left(\frac{\log u}{\log v}\right)\right)\right\},\ \delta \in (0,\infty). \end{aligned}$$

[6]For a proof of this statement see Embrechts, McNeil, and Straumann (2003).

[7]A survival copula $C'$ of a copula C is defined using a survival function as

$$C'(u,v) = u + v - 1 + C(1-u,1-v).$$

[8]Okimoto (2008) calls the survival HR copula as simply the HR copula.

Note that the HR copula has only one parameter, which is monotonically related with the upper tail dependence.[9] In addition, it is known that the HR copula exhibits no lower tail dependence. In other words, the HR copula is incapable of capturing lower tail dependence and symmetric dependence for any parameter values, which is undesirable from empirical perspective. Therefore, following the idea of Patton (2006), we symmetrize it as

$$C^{SHR}(u, v; \delta_1, \delta_2) = 0.5 \cdot \left\{ C^{HR}(u, v; \delta_1) + C^{HR}(1-u, 1-v; \delta_2) + u + v - 1 \right\}.$$

As a consequence, the SHR copula has two dependence parameters, $\delta_1$ and $\delta_2$. While $\delta_1$ characterizes the upper tail dependence, $\delta_2$ captures the lower tail dependence.

Although both the SHR and SJC copulas can describe asymmetric dependence, there is one notable difference between the SHR and SJC copulas, which is the maximum degree of tail dependence. By construction, the maximum values of upper and lower tail dependences for the SHR copula is 0.5, while those of the SJC copula is 1. Recall also that the normal copula has no tail dependence. Thus, our model specification can capture very wide variety of tail dependences.

Note that we allow the copula parameters to be time dependent to describe the time-varying dependence. The model for the dynamics of dependence is discussed in detail in the next subsection.

### 3.3 Dynamics of dependence

To examine the evolution of dependence in international equity market, we need to specify a model for the dynamics of copula parameters. To this end, we adopt the multiple-regime smooth-transition model. The smooth-transition model is formally analyzed by Teräsvirta (1994) in a autoregressive model framework and recently applied by Berben and Jansen (2005) and Kumar and Okimoto (2010) to examine correlation dynamics in international equity and bond markets. In what follows, we will explain the three-regime smooth-transition model as an example. With this three-regime smooth-transition framework we can model each copula parameter $\delta_i$ ($i = 1$ for the normal copula and $i = 1, 2$ for the SJC and SHR copulas) as

$$\delta_{it}(s_t; \boldsymbol{\theta}_{ci}) = \delta_i^{(1)} + (\delta_i^{(2)} - \delta_i^{(1)}) G^{(1)}(s_t; \gamma_i^{(1)}, c_i^{(1)}) + (\delta_i^{(3)} - \delta_i^{(2)}) G^{(2)}(s_t; \gamma_i^{(2)}, c_i^{(2)}).$$

[9] Since no analytical representation for the upper tail dependence is available for the HR copula, we cannot parametrize the HR copula using the upper tail dependence as the JC copula.

Here, $\boldsymbol{\theta}_{ci} = (\delta_i^{(1)}, \delta_i^{(2)}, \delta_i^{(3)}, \gamma_i^{(1)}, \gamma_i^{(2)}, c_i^{(1)}, c_i^{(2)})'$, and $G(\cdot)$ is a transition function and modeled by the logistic function as

$$G(s_t; \gamma, c) = \frac{1}{1 + \exp(-\gamma(s_t - c))}, \quad \gamma > 0, \tag{2}$$

where $s_t$ is a transition variable, and $\gamma$ and $c$ are smoothness and location parameters, respectively. Note that for the SHR and SJC copula models, we assume that two copula parameters share the same transition function. This assumption is necessary to conduct the hypothesis test for the equality of one of copula parameters across regimes without identification problems. Nonetheless, this assumption is innocuous for the purposes of the paper, since dynamics of two copula parameters can be very different by allowing them to take different values in the same regimes.

For a transition variable, following Lin and Teräsvirta (1994) and Berben and Jansen (2005), we use a linear time trend, specifically, we set $s_t = t/T$. In addition, we assume $0.01 \leq c_i^{(1)} < c_i^{(2)} \leq 0.99$ so that we can detect the dependence transition within the sample period. Under this assumption, each copula parameter, $\delta_{it}$, changes smoothly from $\delta_i^{(1)}$ via $\delta_i^{(2)}$ to $\delta_i^{(3)}$ with time, as first the function $G^{(1)}$ changes from 0 to 1, followed by a similar change of $G^{(2)}$. As a consequence, we can capture dominant, long-run trends of dependence in international equity markets with possible regime changes over the last 35 years. Another attractive feature of this framework is that data can choose the best pattern for transition of copula parameters. The change is abrupt for large values of $\gamma$, while the transition is gradual for small values of $\gamma$. In addition, the location parameter $c$ can adjust the location of the reflection points.

### 3.4 Likelihood Function

Since we have fully specified the model, we can derive the log-likelihood function $l(\boldsymbol{\theta})$ to implement the MLE, where $\boldsymbol{\theta}$ is a vector of parameters to estimate. Differentiating the joint distribution (1) with respect to each $x_i$, we can get the joint density $h$ as

$$\begin{aligned}
h(x_1, x_2; \boldsymbol{\theta}) &= \frac{H(x_1, x_2; \boldsymbol{\theta})}{\partial x_1 \partial x_2} \\
&= \frac{C\left(F_1(x_1; \boldsymbol{\theta}_1), F_2(x_2; \boldsymbol{\theta}_2); \boldsymbol{\theta}_C\right)}{\partial x_1 \partial x_2} \\
&= c\left(F_1(x_1; \boldsymbol{\theta}_1), F_2(x_2; \boldsymbol{\theta}_2); \boldsymbol{\theta}_C\right) \cdot f_1(x_1; \boldsymbol{\theta}_{X_1}) \cdot f_2(x_2; \boldsymbol{\theta}_2),
\end{aligned}$$

where $f_i$ for $i = 1, 2$ is the marginal density of $X_i$, and $c$ is the density of a copula defined as

$$c(u, v; \boldsymbol{\theta}_C) = \frac{C(u, v; \boldsymbol{\theta}_C)}{\partial u \partial v}.$$

With the use of these two equations, the (conditional) log-likelihood function $l(\boldsymbol{\theta})$ can be obtained as

$$l(\boldsymbol{\theta}) = \sum_{t=1}^{T} \ln c(F_1(x_{1t}), F_2(x_{2t}); s_t, \boldsymbol{\theta}_C) + \sum_{t=1}^{T} \ln f_1(x_{1t}; \boldsymbol{\theta}_1) + \sum_{t=1}^{T} \ln f_2(x_{2t}; \boldsymbol{\theta}_2),$$

where $\boldsymbol{\theta}_C = \boldsymbol{\theta}_{c1}$ for the normal copula and $\boldsymbol{\theta}_C = (\boldsymbol{\theta}'_{c1}, \boldsymbol{\theta}'_{c2})'$ for the SJC and SHR copulas. By maximizing it with respect to $\boldsymbol{\theta} = (\boldsymbol{\theta}'_1, \boldsymbol{\theta}'_2, \boldsymbol{\theta}'_C)'$, we can get the MLE of $\boldsymbol{\theta}$ and calculate the Akaike information criterion (AIC) for model comparison.

# 4 Empirical Results

In this section, we document the empirical results. To make a comprehensive comparison, we consider the following four STCG models: Multivariate normal model (MVN model), Normal copula model with Student-$t$ margins (NC model), SHR copula model with Student-$t$ margins (SHRC model), and SJC copula model with Student-$t$ margins (SJCC model). Note that outperformance of $t$-GARCH model over the normal GARCH model is evident, as we will see below, we use the normal GARCH specification only for the MVN model as a bench mark. We set the number of regimes from one to four for each model, generating sixteen models as a total. By estimating these models, we can compare a wide variety of evolution and choose empirically the best dynamics for the international stock return data, which is particularly attractive for the purpose of this paper. More specifically, we try to answer the following questions: (i) Is the MVN model appropriate to model the dependence evolution? (ii) How many dependence regimes are enough to characterize dependence evolution in international equity markets over the last 35 years? (iii) Did dependence in international equity markets change? (iv) Is there any asymmetry in evolution between upper and lower tail dependences? (v) When did important dependence changes occur? (vi) Which tail dependence contributed more to these changes?

## 4.1 Model comparison

In this section, we compare the estimation results of four STCG models. We fit these models to the pair of stock returns taken from FR, GE, UK, and US equity markets. In other words, all models are estimated for six different country-pairs. In addition, we allow each model to have multiple regimes ranging from one to four. Thus, we estimate sixteen models for six pairs.

Table 2 reports the best model based on the AIC and its AIC value for each STCG model and each pair. Note that the best model among entire models is indicated by bold face. There are a number of observations that should be emphasized. First, AICs of the MVN model with normal GARCH margins are much larger than those of other models with $t$-GARCH margins for all pairs. This result suggests the significance of capturing fat-tailness of marginal distributions, providing a definite answer for the first question.

Second, the results indicate that two and three regime models are chosen as the entire best model for three pairs out of six, respectively. In other words, neither one- nor four- regime models are the entire best model for all country-pairs, meaning two or three regimes are suitable to describe the dynamics of dependence in international equity market over the last thirty-five years. This result gives us a clear answer for the second question.

Lastly, the table also provides a strong indication of the dominance of the SHRC model over the other three models. The SHRC model is chosen as the entire best model for five pairs out of six. This arguably indicates the importance of capturing asymmetric evolution of tail dependence as we will confirm formally in the next subsection. Although the SJCC model can also describe the asymmetric tail-dependence evolution, it seems to have too much tail dependence, worsening the fit compared to the SHRC model for all cases.[10]

In sum, the conclusions of our comprehensive model comparison seem to be clear: Capturing fat-tailness of marginal distributions is indispensable to model international stock returns. In addition, the three dependence regimes are enough to describe dependence evolution in international equity markets over the last 35 years. Furthermore the SHRC model outperforms over the other models. Given these observations, in the following subsections, we will use the best SHRC model, specifically the three-regime model for the FR-GE and FR-UK pairs and the two-regime model for the rest of pairs, to investigate the time evolution of dependence in international equity market in detail, seeking answers for the remaining questions.

[10] Recall that by construction, the maximum values of upper and lower tail dependences for the SHR copula is 0.5, while those of the SJC copula is 1.

## 4.2 Hypothesis tests

In this subsection we conduct two hypothesis tests to answer the questions (iii) and (iv). The first hypothesis test examines whether there is a significant change in dependence in international equity markets over the last 35 years. To this end, we conduct the Wald test for equivalence of the upper and lower tail copula parameters between the first and last regimes. Thus the null hypothesis can be expressed as $H_0 : \delta_i^{(1)} = \delta_i^{(2)}$ for the two regime model or $H_0 : \delta_i^{(1)} = \delta_i^{(3)}$ for the three regime model. Correspondingly, the alternative hypothesis can be written as $H_1 : \delta_i^{(1)} \neq \delta_i^{(2)}$ for the two regime model or $H_1 : \delta_i^{(1)} \neq \delta_i^{(3)}$ for the three regime model.

The second test concerns the asymmetric dependence evolution. We conduct the Wald test for equivalence between the upper and lower tail copula parameters across each regime. In other words, the null hypothesis of the test is $H_0 : \delta_1^{(j)} = \delta_2^{(j)}$, while the corresponding alternative hypothesis is $H_1 : \delta_1^{(j)} \neq \delta_2^{(j)}$.

Table 3 reports the estimates of the SHR copula parameters for each regime and the $p$-values of the two tests stated above. As can be seen from the table, the estimation results indicate that the latter regime always has larger copula parameters for all pairs. For instance, for the FR-GE pair, the upper tail copula parameter is estimated as 1.05 for the first regime, 2.37 for the second regime, 5.30 for the third regime, while the lower tail copula parameter of each regime is estimated as 0.92, 1.47, and 6.56, respectively. Thus, the estimation results suggest that international equity markets have become more interdependent in recent years. This statement is formally tested by the first test. The $p$-values of the tests can be found in the last column of Table 3. As can be seen, all $p$-values are less than 0.05, meaning there is a significant increase in dependence in both tail for all country pairs. Here we can obtain a clear answer for the question (iii).

This result seems to be a contrast to the recent finding of no significant upward trend in cross-country correlations by Bekaert, Hodrick, and Zhang (2009). However, their estimate of trend in correlation for G7 countries indicate that correlation in G7 countries increase at least 0.18 during the last 26 years, which is fairly consistent with our result. Therefore, the difference most likely comes from the low power of their tests due to nonparametric framework and relatively small sample with 52 observations.

Regarding the asymmetric evolution, the evidence is less clear, but the results still suggest the importance of accommodating asymmetric dependence. As can be seen from the $p$-values of the second tests shown in the last row of each pair's result of Table 3, five country pairs out of six demonstrate some asymmetry in at least one regime at the 5% significance level. In particular, four pairs out of six indicate asymmetric dependence in either the first or middle regime. Note also that these asymmetries have become negligible in the latter regime. As a consequence asymmetric dependence is insignificant in the most recent regime for four pairs out of six. We, therefore, conclude that allowing asymmetric dependence is important, particularly in earlier period, to describe the international equity market dependence evolution, which could answer the question (iv).

### 4.3 Dynamics of dependence

In the last subsection, we showed that there has been a significant increase in dependence in international equity markets and dependence evolution could be asymmetric between upper and lower tails of the joint distribution. In this subsection, we investigate the dynamics of dependence in international equity markets over the last 35 years to see when the important increases occurred and which tail contributed more to the increases. To this end, we calculate three copula-based dependence measures, Spearman's rho, and upper and lower tail dependences, at each time based on the estimation results of the best SHRC model. By doing so, we can evaluate dependence from three aspects, hence in more appropriate way than using only one non-copula-based measure, namely linear correlation. As emphasized by Embrechts, McNeil, and Straumann (2002), and Embrechts, Lindskog, and McNeil (2003), a linear correlation is not a good measure of dependence for non-elliptical distribution model such as our SHRC model.

The Spearman's rho is sometimes called rank correlation, since it can be interpreted as the linear correlation between some "ranks" of the data. Unlike linear correlation, the Spearman's rho satisfies all conditions for a measure of concordance proposed by Scarsini (1984). It is, therefore, a reasonable alternative to linear correlation as a measure of global dependence for non-elliptical distributions.

Spearman's rho is defined to be proportional to the probability of concordance minus the probability of discordance for the two vectors $(X_1, Y_1)$ and $(X_2, Y_3)$, i.e., a pair of vectors with the same margins, but one vector has distribution function $H$, while the components of the other are independent. It can also be considered as linear correlation between $F_X(X)$ and $F_Y(Y)$, which can be calculated from a copula as follows:

$$\begin{aligned}
\rho_S &= 3\{\text{Prob}[(X_1 - X_2)(Y_1 - Y_3) > 0] - \text{Prob}[(X_1 - X_2)(Y_1 - Y_3) < 0]\} \\
&= \frac{\text{Cov}(F_X(X), F_Y(Y))}{\sqrt{\text{Var}(F_X(X) \cdot \text{Var}(F_Y(Y))}} \\
&= 12 \int_0^1 \int_0^1 C(u,v) du dv - 3.
\end{aligned}$$

Tail dependence measures the dependence in the upper-right-quadrant or lower-left-quadrant tail of a bivariate distribution. It is a concept that is relevant to dependence in extreme values. The definition of upper (lower) tail dependence is the probability that one variable takes an extremely large positive (negative) value, given that the other variable took an extremely large positive (negative) value. The upper tail dependence can be equivalently defined in terms of copulas as follows:

$$\begin{aligned}
\lambda_U &= \lim_{u \uparrow 1} \text{Prob}[X \geq F_X^{-1}(u) | Y \geq F_Y^{-1}(u)] \\
&= \lim_{u \uparrow 1} \text{Prob}[Y \geq F_Y^{-1}(u) | X \geq F_X^{-1}(u)] \\
&= \lim_{u \uparrow 1} \frac{1 - 2u + C(u,u)}{1-u}
\end{aligned}$$

provided the limit exists. Similarly, the lower tail dependence can be defined as

$$\begin{aligned}
\lambda_L &= \lim_{u \downarrow 0} \text{Prob}[X \leq F_X^{-1}(u) | Y \leq F_Y^{-1}(u)] \\
&= \lim_{u \downarrow 1} \text{Prob}[Y \leq F_Y^{-1}(u) | X \leq F_X^{-1}(u)] \\
&= \lim_{u \downarrow 1} \frac{C(u,u)}{u}
\end{aligned}$$

. A bivariate copula $C$ has upper (lower) tail dependence if $\lambda_U \in (0,1]$ ($\lambda_L \in (0,1]$) and no upper (lower ) tail dependence if $\lambda_U = 0$ ($\lambda_L = 0$).

Using these three copula-based measures we can examine the degree of dependence from three aspects, namely, general dependence among stock returns measured by the Spearman's rho, and dependence between the joint extreme positive (negative) returns evaluated by the upper (lower) tail dependence. In addition, our smooth-transition copula framework allows us to evaluate these three measures at each time of the sample period. As a consequence we can investigate the dynamics of dependence and understand the sources of changes in dependence in detail.

Figure 1 plots the time series of three dependence measures implied by the best SHRC model. As can be seen from the figure, there seem to be three groups sharing the similar pattern of dependence evolution. The first group consists of the FR-GE and FR-UK pairs, whose entire best model is the three-regime SHRC model. This group experienced rapid increase in dependence between 1986 and 1991, and 2000 and 2004. It is very clear that the lower tail dependence played more important role in the second increase for both pairs. However, there is some difference in the first increase between two pairs. For the FR-GE pair the upper tail dependence contributed slightly more than the lower tail dependence, whereas for the FR-UK pair the lower tail dependence was the main cause.

The second group includes the FR-US and GE-US pairs. These pairs underwent gradual increase in dependence from 1987 with the larger increase in the upper tail dependence than the lower tail dependence. For the FR-US pair the upper tail dependence at the beginning of the sample period was 0.06, which was less than half of the lower tail dependence. However, the upper tail dependence rose to 0.33 at the end of the sample, making it slightly higher than the end-of-sample lower tail dependence of 0.31. The GE-US pair shows more remarkable increase in the upper tail dependence; the upper tail dependence was 0 at the beginning, but it went up to 0.35 at the end of sample, passing the lower tail dependence by 0.10.

The remaining two pairs compose the last group. The GE-UK and UK-US pairs' dependence increased almost linearly over the entire sample. Like the second group, the upper tail dependence played more active role for this group. In particular, for the GE-UK pair the upper tail dependence always exceeded the lower tail dependence with extending the difference from 0.02 to 0.1 through the sample period. For the UK-US pair, the upper and lower tail dependence were 0.05 and 0.13, respectively, at the beginning of the sample period, while both became about 0.3 at the end of the sample.

In sum, our analysis shows that there are three types of evolution in international equity markets, confirming usefulness of our framework to describe a wide variety of dependence dynamics in international equity markets. For the FR-GE and FR-UK pairs the lower tail dependence contributed more for the significant dependence increase, while the remaining pairs the upper tail dependence was the main cause of increasing dependence. This is somewhat surprising, given recent growing number of contagion literature.

### 4.4 Effects of currency on dynamics of dependence

Our empirical results are based on weekly total market price index data in US dollars for FR, GE, UK, and US. This is relevant because the US investors considering international asset allocation or risk management problems normally care the stock returns in US dollars. However, a natural question arising from the use of US dollar denominated returns is that our result of increasing dependence in international equity markets could be an artifact produced by the currency unification. For instance, when US dollar appreciates, stock returns in US dollars for countries other than the US tend to be small, generating positive correlations between stock returns in US dollars. If this tendency becomes stronger as development of exchange rate markets, the results would be increasing dependence as we found in previous sections. It is, therefore, very instructive to check the robustness of our results against the currency used in the analysis.

To examine the effects of currency on our results, we estimate the same models as the best SHRC models used in previous subsections for each pair using stock returns in local currency. Table 4 reports the estimates of the SHR copula parameters for each regime and the $p$-values of the tests discussed in subsection 4.2. As can be seen, the estimation results indicate the same story as before. The latter regime always has larger copula parameters for all pairs and these increases are statistically significant at the 5% significance level. In addition, all country pairs except UK-US pair demonstrate asymmetric dependence in either the first or middle regime at the 10% significance level. However, only GE-UK and GE-US pairs show significant asymmetric dependence in the most recent regime.

We also calculate three copula-based dependence measures at each time based on the estimation results of the SHRC model using international stock returns in local currency and plot them in Figure 2. Again these graphs are fairly consistent with those in Figure 1. FR-GE and FR-UK pairs experienced rapid increases in dependence between 1986 and 1991, and 2000 and 2004 with larger increases in the lower tail dependence. On the contrary, other country pairs underwent either gradual increase from 1987 or almost linear increase in dependence with the upper tail dependence playing more important role.

In sum, the results based on international stock returns in local currency indicate that our findings of increasing dependence and asymmetric dependence are not driven by currency unification, but is an intrinsic property in international equity markets.

# 5 Implications to international diversification and risk management

In previous section, we document clear evidence of asymmetric increasing dependence in international equity markets. In this section, we evaluate economic significance of our empirical results in terms of international diversification and risk management.

Following Guidolin and Timmermann (2006) and Okimoto (2008), we assess the economic significance based on VaR and ES ratios. This is relevant because comovement of international stock returns play a crucial role in evaluation of the risk measures, such as $100 \cdot \alpha\%$ VaR, VaR($\alpha$), which is defined as the $100 \cdot \alpha$ percentile point of a portfolio loss distribution, and $100 \cdot \alpha\%$ ES, ES($\alpha$), which is defined as the expected loss conditional on the loss exceeding the VaR($\alpha$). Therefore, increasing dependence should influence considerably the calculation of VaR and ES.

Specifically, we calculate the portfolio minimizing VaR(0.99) at the end of June every 5 years based on the estimation results from the best SHRC model for each pair.[11] Then we evaluate the ratio of VaR(0.99) in every 5 years to VaR(0.99) in 1973, the first year of the sample, to investigate the evolution of risk in international equity markets over the last 35 years. In addition, we calculate the VaR(0.99) ratio of the portfolio minimizing VaR(0.99) to the portfolio investing all money to the less risky country.[12] By doing so, we can examine how the benefit from international diversification changed over time. Furthermore, we conduct the same analysis using the ES(0.99) as another measure of risk.

To calculate the portfolio minimizing VaR(0.99) and ES(0.99), we generate 100,000 data using the estimated best SHRC model for international stock returns in US dollars and numerically solve the minimization problem. For this purpose, we have to generate a random vector $(U, V)'$ from the HR and survival HR copulas, since the SHR copula is a mixture of these two copulas. This can be done using conditional copula, which can be calculated as a partial derivative of $C$ with respect to

[11] For this calculation, we only consider the portfolios with nonnegative weight on each stock. In other words, we do not allow taking short position.

[12] The less risky countries are France, UK, US, UK, US, and US for FR-GE, FR-UK, FR-US, GE-UK, GE-US, and UK-US pairs, respectively.

the first variable:

$$C_{2|1}(v|u) = \frac{C(u,v)}{\partial u}.$$

For the HR copula, this can be expressed as

$$C_{2|1}^{HR}(u|v;\delta) = C^{HR}(u,v;\delta) \cdot u^{-1}\Phi\left(\delta^{-1} + 0.5\delta \log\left(\frac{\log u}{\log v}\right)\right),$$

from which the conditional copula of survival HR copula can be easily calculated. By definition, if $U \sim U(0,1)$ and $V \sim C_{2|1}(\,\cdot\,|U)$, then $(U,V) \sim C$. Hence, if $U$ and $Q$ are independent $U(0,1)$ random variables and define $V$ so that $C_{2|1}(V|U) = Q$, then $(U,V) \sim C$. Once we obtain $(U,V)'$ from the SHR copula, the desired data can be produced by transforming $U$ and $V$ with the inverse of CDF of each marginal distribution. Note that for a marginal distribution we use the estimated unconditional distribution all the time to capture only the effect of dependence evolution.

Figure 3 plots the VaR and ES ratios of 99% VaR and ES of every 5 years to those of 1973. As can be seen, both ratios are generally increasing over the last 35 years for all country pairs, meaning the 99% VaR and ES have become larger in more recent years. In particular, the VaR ratio in 2008 ranges from 1.11 to 1.29 with mean 1.20, while the ES ratio in 2008 is between 1.08 and 1.28 and 1.19 on average. Thus the VaR (ES) of 2008 is larger than that of 1973 by 11% (8%) to 29% (28%).

The VaR and ES ratios of the minimized 99% VaR and ES to those of the less risky country are depicted in Figure 4. Similar to Figure 3, both ratios are mostly increasing for all country pairs, indicating that the benefit from the international diversification is diminishing over the sample period. For instance, if we allocate our assets optimally to France and Germany in the sense of minimizing 99% VaR (ES), we can reduce the 99% VAR by 21% (19%) compared with investing all money to Germany in 1973, but only 2% (3%) in 2008.

Given the fact that the 99% VaR and ES are most widely used risk measures, these changes are nonnegligible. If a risk manager for a bank overlooked the changes in these risk measures, he/she would substantially underestimate the risk, causing a serious problem for the bank with nontrivial probability. It is, therefore, very critical to recognize the paper's finding of increasing dependence with possible asymmetry in international equity markets.

## 6 Conclusion

In this paper, we investigated the dynamics of dependence in international equity market over the last 35 years. In particular, the paper addressed the several important empirical questions including the number of dependence regimes and existence of asymmetry in dependence evolution. To find answers for these questions, we developed a multiple-regime smooth-transition copula-GARCH (STCG) model. More specifically, we estimated the Multivariate normal model (MVN model), Normal copula model with Student-$t$ margins (NC model), SHR copula model with Student-$t$ margins (SHRC model), and SJC copula model with Student-$t$ margins (SJCC model). Then we compared them to find the suitable evolution of dependence in international equity markets. Through the comprehensive comparison of these competing specifications, we selected the SHRC model as the best model to describe the dependence evolution in international equity markets over the last 35 years with several important implications. First, the fat-tailness of marginal distributions is indispensable, indicating the inappropriateness of the MVN model. Second, two or three dependence regimes are sufficient to describe the dynamics of dependence in international equity markets. Third, capturing the asymmetric evolution between upper and lower tails is substantial, suggesting the importance of use of non-elliptical copula, such as the SHR copula.

Based on the best SHRC model, the paper conducted two hypothesis tests to formally examine increase and existence of asymmetry in dependence evolution. The results indicated that both upper and lower tail dependences increased significantly during the last 35 years. In addition, our statistical tests confirmed that at the early period of the sample there is some asymmetry in the comovement between lower and upper tails for the most of country pairs, but they became about the same degree in 2008.

Next, we provided the implied time series of three copula-based dependence measures from the best SHRC model to identify when the important dependence increases occurred and which tail dependence contributed more for these increases. The results showed that FR-GE and FR-UK pairs experienced rapid increase in dependence between 1986 and 1991, and 2000 and 2004 with lower tail dependence played more important role in these increases. On the other hand, GE-US and FR-US pairs underwent gradual increase from 1987, whereas GE-UK and UK-US pairs' dependence increased almost linearly over the entire sample. Furthermore, the results suggested that for these pairs upper tail dependence contributed more than lower tail dependence in increasing dependence. Interestingly enough, these results demonstrated that the upper tail dependence played a more important role in increasing dependence than the lower tail dependence for four pairs out of six. Thus, when we analyze the international stock returns, we should take into account the interdependence between extreme positive shocks as well as contagion effects caused by the lower tail dependence. This point is often overlooked in the growing number of contagion literature.

As a final contribution of the paper we investigated the economic significance of our empirical findings from a risk management point of view based on the 99% Value at Risk and expected shortfall. Our results indicated that both risk measures have increased about 20% over the last 35 years due to diminishing international diversification effects. In particular, the benefit from international diversification has almost vanished by 2008. Thus, the paper's finding of increasing dependence with possible asymmetry in international equity markets has significant effects on international asset allocation and risk management.

These conclusions also raise several topics for future investigation. One of these is to pursue the economic factors behind the dependence evolution in international equity markets. Our results showed there was a marked increase in dependence around 1987 for most of countries, suggesting the crash of 1987 could play some role in the increase as suggested by Von Furstenberg and Jeon (1989) and Koch and Koch (1991). If that's the case, the recent simultaneous global drop in stocks could cause another increase in dependence in international equity markets. Examining this possibility by clarifying the economic factors contributing the dependence evolution is challenging future work. A recent study by Engle and Rangel (2008) investigates the global macroeconomic causes of volatility fluctuation using the implied volatility time series from the Spline GARCH model. Similar methodology could be used to identify the economic factors of dependence evolution.

The final topic is to develop the multi-dimensional framework to investigate dependence evolution in international markets. This paper employed the pairwise analysis as Longin and Solnik (2001) and Okimoto (2008), which is satisfactory for the first investigation. However, to analyze the dynamics of dependence more precisely, the multi-dimensional model may be desirable. There are at least two problems associated with the multivariate extension. The first problem is that the number of parameters gets larger quickly as the number of variable increases. For instance, the fourvariate three-regime MVN model has more than 44 parameters to estimate. In other words, we have to maximize the log-likelihood function with respect to 44 parameters, which is not easy task to do. One possibility to mitigate this problem is to develop a MCMC algorithm for the multiple-regime STCG model. Another problem is that no flexible multivariate non-elliptical copula is available. As we confirmed, accommodating the asymmetric dependence could be very important to analyze the dependence evolution in international equity markets. To our best knowledge, however, it is very hard to construct a multi-variate non-elliptical copula with sufficient flexibility. Solving these problems and examining dependence evolution across several countries is a fruitful endeavor.

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Table 1: Correlations for five subsamples

| | Sample period | | | | |
|---|---|---|---|---|---|
| Pair | 1973–1979 | 1980–1986 | 1987–1993 | 1994–2000 | 2001–2008 |
| FR–GE | 0.351 | 0.424 | 0.726 | 0.752 | 0.929 |
| FR–UK | 0.354 | 0.403 | 0.564 | 0.653 | 0.878 |
| FR–US | 0.270 | 0.252 | 0.465 | 0.501 | 0.736 |
| GE–UK | 0.165 | 0.431 | 0.523 | 0.636 | 0.726 |
| GE–US | 0.103 | 0.307 | 0.393 | 0.522 | 0.830 |
| UK–US | 0.290 | 0.404 | 0.466 | 0.584 | 0.705 |

Note: The table reports the sampe correlation coefficients between stock returns in US dollars across G4 countries for five subsamples.

Table 2: Result of model comparison

| Model | | FR–GE | FR–UK | FR–US | GE–UK | GE–US | UK–US |
|---|---|---|---|---|---|---|---|
| MVN | AIC | 15943.7 | 16635.0 | 16382.1 | 16295.2 | 15910.6 | 15995.4 |
| | Best | 3 regimes | 4 regimes | 3 regimes | 2 regimes | 3 regimes | 2 regimes |
| Normal copula | AIC | 15859.9 | 16521.1 | **16289.5** | 16186.0 | 15806.1 | 15919.2 |
| | Best | 3 regimes | 4 regimes | 3 regimes | 2 regimes | 3 regimes | 2 regimes |
| SHR copula | AIC | **15849.3** | **16515.1** | 16291.8 | **16160.4** | **15800.8** | **15912.1** |
| | Best | 3 regimes | 3 regimes | 2 regimes | 2 regimes | 2 regimes | 2 regimes |
| SJC copula | AIC | 15882.6 | 16550.4 | 16300.9 | 16187.1 | 15827.0 | 15924.9 |
| | Best | 4 regimes | 3 regimes | 3 regimes | 4 regimes | 4 regimes | 2 regimes |

Note: The table reports the best model based on the AIC and its AIC value for each STCG model and each country pair. The best model among entire models is indicated by bold face.

Table 3: Results of hypothesis testing (US dollars)

| | | $\delta_i^{(1)}$ | $\delta_i^{(2)}$ | $\delta_i^{(3)}$ | $p$ −value for $\delta_i^{(1)} = \delta_i^{(2)}$ or $\delta_i^{(1)} = \delta_i^{(3)}$ |
|---|---|---|---|---|---|
| FR−GE | upper−tail parameter | 1.052 | 2.367 | 5.299 | 0.000 |
| | lower−tail parameter | 0.920 | 1.467 | 6.559 | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.662 | 0.010 | 0.276 | |
| FR−UK | upper−tail parameter | 1.320 | 2.021 | 4.427 | 0.000 |
| | lower−tail parameter | 0.635 | 1.644 | 3.788 | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.001 | 0.488 | 0.624 | |
| FR−US | upper−tail parameter | 0.626 | 2.765 | | 0.001 |
| | lower−tail parameter | 0.912 | 2.371 | | 0.015 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.202 | 0.661 | | |
| GE−UK | upper−tail parameter | 0.340 | 8.055 | | 0.000 |
| | lower−tail parameter | 0.420 | 3.640 | | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.713 | 0.000 | | |
| GE−US | upper−tail parameter | 0.000 | 3.393 | | 0.000 |
| | lower−tail parameter | 0.906 | 1.706 | | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.000 | 0.000 | | |
| UK−US | upper−tail parameter | 0.506 | 2.674 | | 0.000 |
| | lower−tail parameter | 0.803 | 2.430 | | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.007 | 0.493 | | |

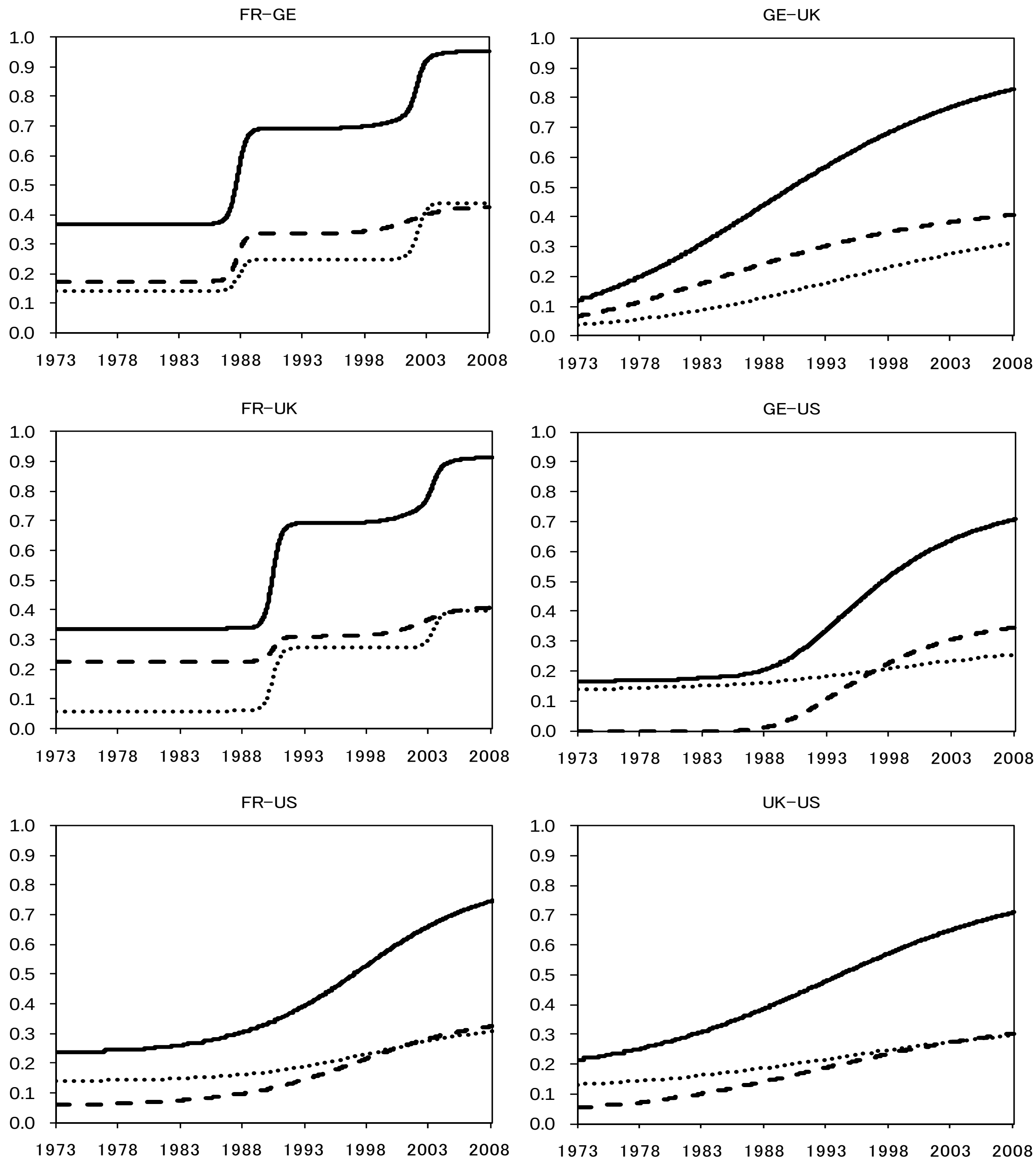
Note: The table reports the estimates of the SHR copula parameters for each regime and the $p$-values of the two hypothesis tests. The first test is about equivalence of the upper and lower tail copula parameters between the first and last regimes. The second test is about equivalence between the upper and lower tail copula parameters across each regime. The results are based on international stock returns in US dollars.

Table 4: Results of hypothesis testing (local currency)

| | | $\delta_i^{(1)}$ | $\delta_i^{(2)}$ | $\delta_i^{(3)}$ | $p$ −value for $\delta_i^{(1)} = \delta_i^{(2)}$ or $\delta_i^{(1)} = \delta_i^{(3)}$ |
|---|---|---|---|---|---|
| FR−GE | upper−tail parameter | 0.000 | 1.928 | 4.633 | 0.000 |
| | lower−tail parameter | 0.945 | 1.375 | 4.384 | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.000 | 0.048 | 0.849 | |
| FR−UK | upper−tail parameter | 0.865 | 2.105 | 3.990 | 0.000 |
| | lower−tail parameter | 0.692 | 1.480 | 3.612 | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.487 | 0.084 | 0.695 | |
| FR−US | upper−tail parameter | 0.537 | 3.031 | | 0.018 |
| | lower−tail parameter | 0.924 | 2.925 | | 0.013 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.087 | 0.859 | | |
| GE−UK | upper−tail parameter | 0.000 | 5.450 | | 0.000 |
| | lower−tail parameter | 0.615 | 2.809 | | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.000 | 0.000 | | |
| GE−US | upper−tail parameter | 0.000 | 2.727 | | 0.017 |
| | lower−tail parameter | 0.885 | 1.802 | | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.000 | 0.000 | | |
| UK−US | upper−tail parameter | 0.389 | 2.613 | | 0.003 |
| | lower−tail parameter | 0.491 | 2.212 | | 0.000 |
| | $p$ −value for $\delta_1^{(j)} = \delta_2^{(j)}$ | 0.509 | 0.396 | | 0.000 |

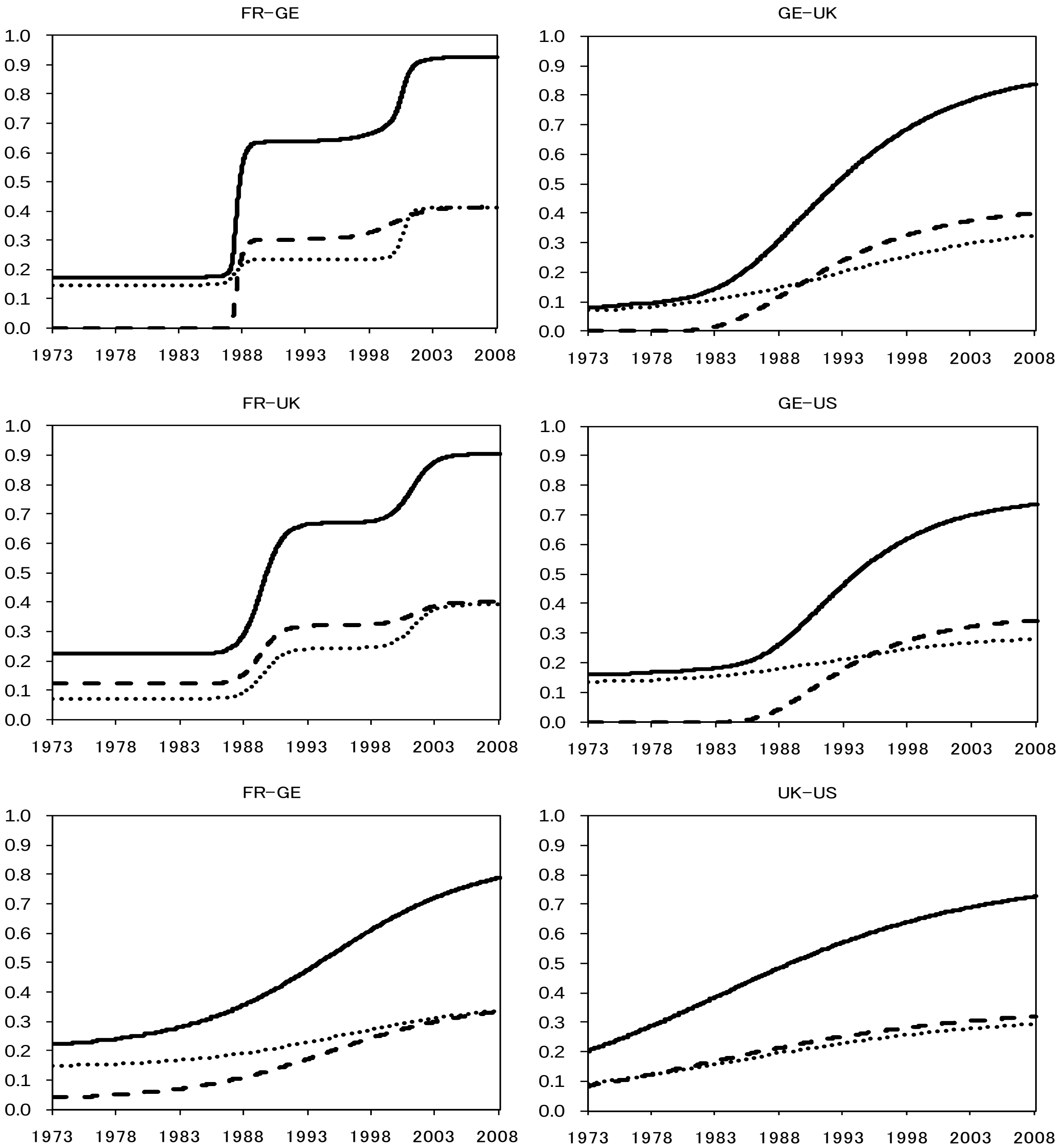
Note: The table reports the estimates of the SHR copula parameters for each regime and the $p$-values of the two hypothesis tests. The first test is about equivalence of the upper and lower tail copula parameters between the first and last regimes. The second test is about equivalence between the upper and lower tail copula parameters across each regime. The results are based on international stock returns in local currency.

Figure 1: Dynamics of three dependence measures (US dollars)



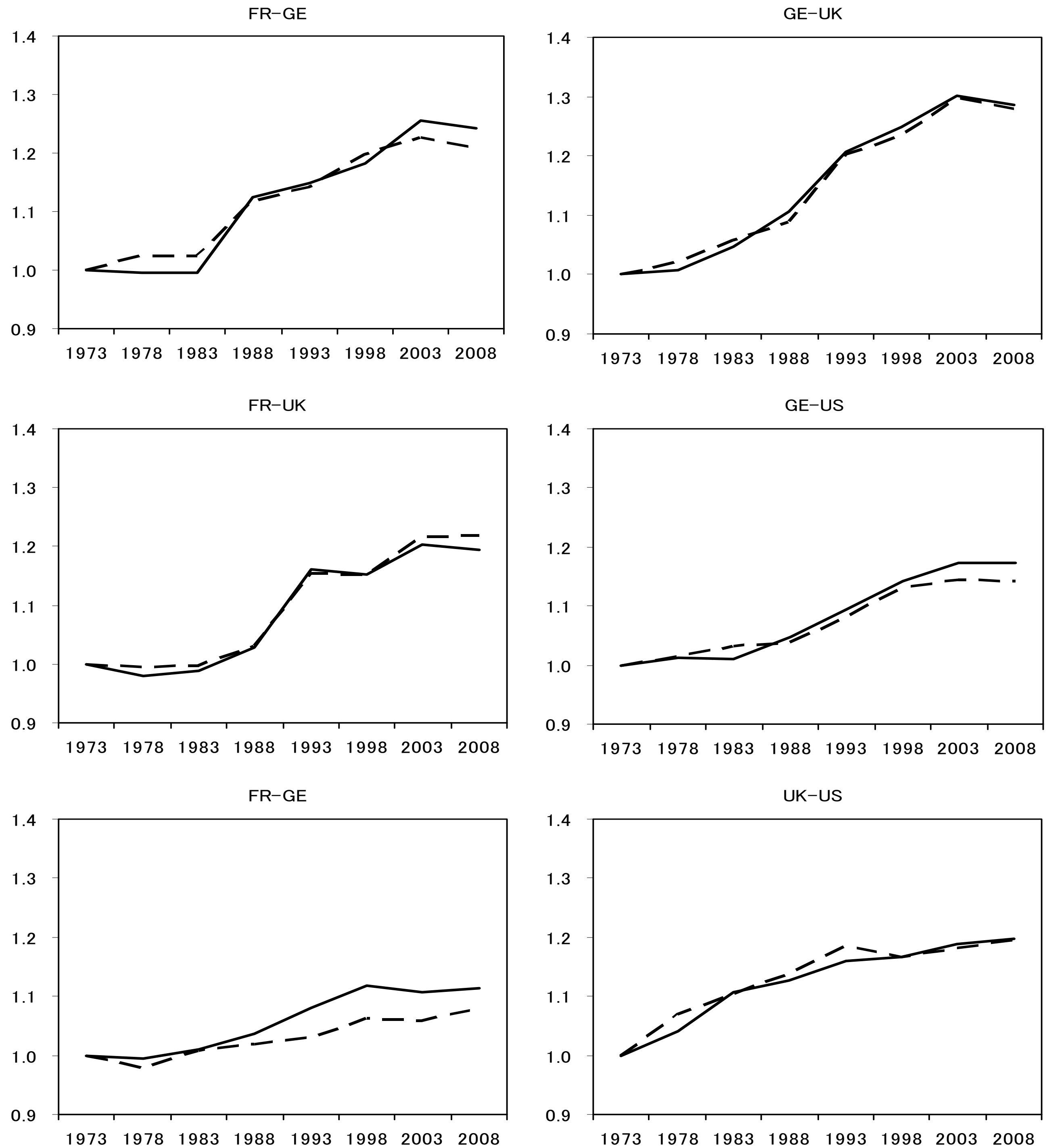

Note: The figure plots the time series of three dependence measures (Spearman's rho, upper and lower dependences) between each country pair's stock returns in US dollars implied by the best smooth-transition SHRC model. The solid line shows the implied spearman's rho. The broken line shows the implied upper tail dependence. The dotted line shows the implied lower dependence.

Figure 2: Dynamics of three dependence measures (local currency)



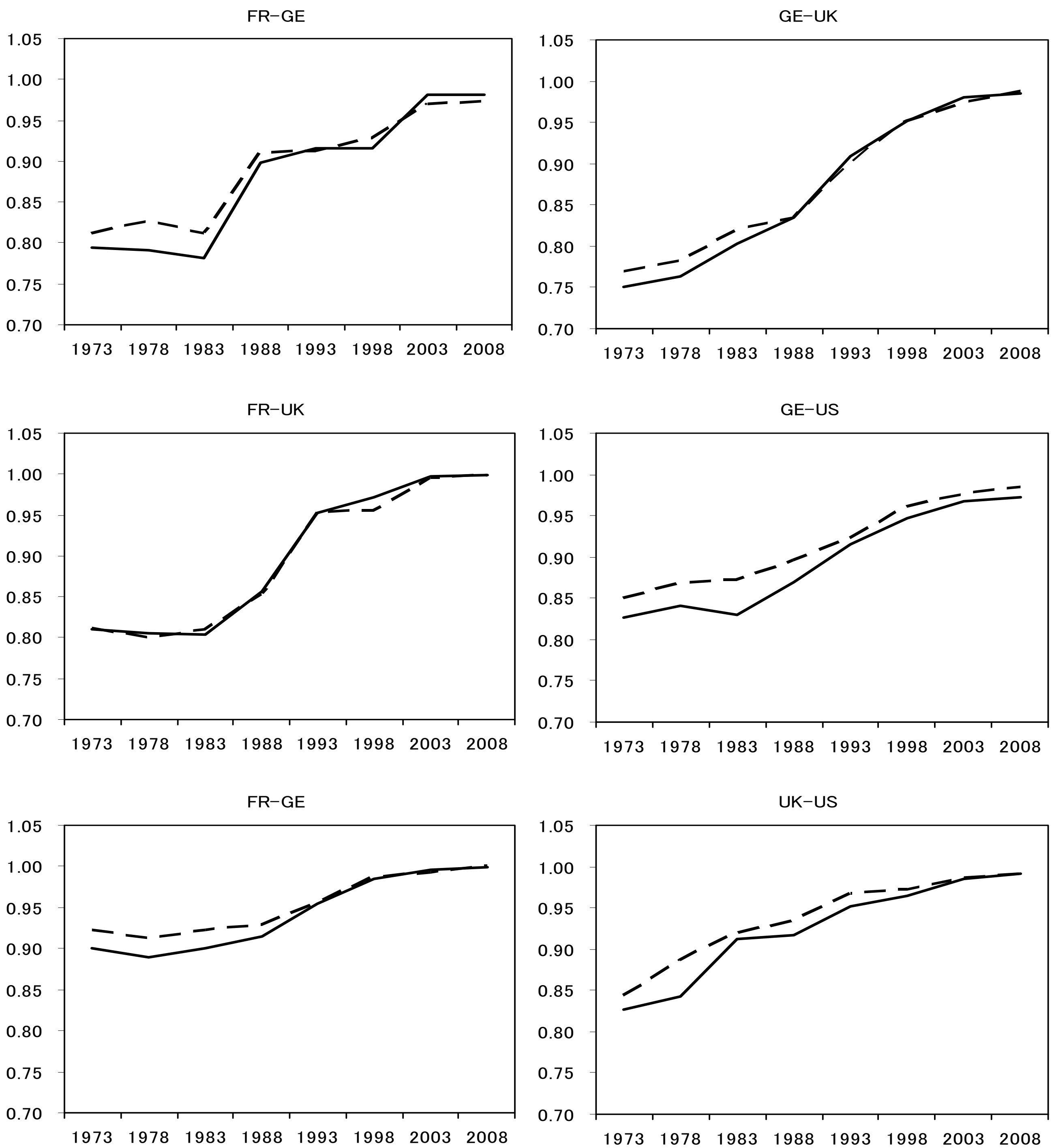

Note: The figure plots the time series of three dependence measures (Spearman's rho, upper and lower dependences) between each country pair's stock returns in local currency implied by the best smooth-transition SHRC model. The solid line shows the implied spearman's rho. The broken line shows the implied upper tail dependence. The dotted line shows the implied lower dependence.

Figure 3: Dynamics of VaR and ES ratios of 99% VaR and ES of every 5 years to those of 1973




Note: The figure plots the VaR and ES ratios of 99% VaR and ES of every 5 years to those of 1973. The calculation is based on the results from the estimated best SHRC model for international stock returns in US dollars. The solid line shows the VaR ratio. The broken line shows the ES ratio.

Figure 4: Dynamics of VaR and ES ratios of the minimized 99% VaR and ES to those of the less risky country




Note: The figure plots the VaR and ES ratios of the minimized 99% VaR and ES to those of the less risky country. The calculation is based on the results from the estimated best SHRC model for international stock returns in US dollars. The solid line shows the VaR ratio. The broken line shows the ES ratio.