Economic Determinants of Default Risks and Their Impacts on the Pricing of Credit Derivatives

Szu-Lang Liao* and Jui-Chen Chang**

Abstract

We examine the correlations between default intensity processes and the extracted fundamental risk factors based on certain constrains of no-arbitrage conditions. Thus we quantify the impacts of economic determinants on the credit risk. Since the economic environment changes stochastically with the release of new informations, in order to summarize the noisy economic system with continuously updated observations we extract dynamic economic factors from a large number of variables. Although default risks respond differently to distinct economic factors, all parameters are statistically significant. Moreover, we propose a reduced-form model with economic fundamentals to price credit derivatives. We find that the economic environment already signaled for the credit crisis at the end of 2006.

Key Words: Default intensity, Economic observations, Subprime mortgage crisis, Reduced-form model, Kalman filter.

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1. Introduction

The credit crunch has depressed the global economy since the burst of the housing bubble in 2007. It induced a serious financial crises sweeping from the stock markets to the overall economy. Many investors made poor decisions without fully considering the risk coming with such highly complicated financial derivatives and investment banks underestimated the systemic risk generated from contagious defaults. Moreover, rating agencies also failed to recognize the worst-case scenario represented by the subprime mortgage mess. This article addresses the joint behavior of economic conditions and the default risk in order to get the clues of the recent subprime mortgage meltdown.

This paper provides an insight into the fundamental economic determinants of default intensities. Through the no-arbitrage dynamic factor model, we incorporate a large array of economic and financial information into default intensities to capture the evolution of credit environments and also enrich the intensity model with economic underpinnings. To measure how the default intensities vary with economic conditions, we divide different economic and financial variables into three groups: the real activity in macroeconomic variables, the nominal measures of the economy, and the financial variables correlated to real estate. By imposing the no arbitrage restrictions, we construct the default intensity processes as an affine model of these fundamental economic dynamic factors. Then the default intensity model contains the economic rationale for defining default processes and quantifies the impact of various economic and financial factors on the pricing of derivatives.

In order to identify the link between economic environment and the credit evolution in default intensity, we include macroeconomic variables and financial data series in housing market. At first, the macroeconomic variables are included and decomposed into the real and nominal activities as previous studies (Andrew and Piazzesi (2003), and Ang et al. (2004)). Because firms are exposed to the same macroeconomic conditions, systematic factors, and financial markets, their contagious default intensities make defaults clustered in time. Many researchers suggested that the default intensity can be explained by macroeconomic performance (Das et al. (2007), Lo 1986, Lennox (1999), McDonald and Van de Gucht (1999), Couderc and Renault...
Moreover, Bhansali et al. (2008) supposed that credit spread can be decomposed into three types of credit risk, namely the idiosyncratic defaults, sector-wide defaults, and economy-wide defaults. Then, Longstaff and Rajan (2008) showed that, compared to the other two components, the economy-wide credit risk has been rising dramatically after 2007 and was the major factor of the increase in credit spreads during 2007. Macroeconomic credit problems have generated great impact on the total credit risk. To quantify the connection between default intensity processes and the subprime mortgage problems, we include the third financial factor which summarizes the information from real estate markets. As we known, the US housing market was the first market affected by current credit crunch. Many researches indicated that the overexpansion of subprime mortgage in US had led to the recession of the whole economy and credit derivatives brought the credit mess to the rest of the world. To capture the environmental evolution of US housing market, such as subprime mortgage problem, the boosting delinquency and foreclosure risk, and housing boom, the third dynamic economic factor in our paper summarizes the information from various variables in US housing market. Finally, we examine the correlations between default intensity processes and the three fundamental risk factors based on certain constrains of no-arbitrage conditions. The default intensity processes are linked to the dynamics of various macroeconomic variables.

Also, there are researches found that the defaults has clustered and the explanation of default arrivals can be improved by some macroeconomic indicators, such as gross domestic product (GDP) (Hull (2007), and Das et al. (2007)). In this paper, instead of using some specific macroeconomic indicators, we extract dynamic economic factors from a large number of variables to represent the complex economic system.

Our estimation results show that the parameters of all economic factors are statistically significant and fairly coincide with the nature of these economic factors. The inflation factor and housing-related factor both have strong positive effects on default intensities. In contrast, the real economic factor has significantly negative influence. After quantifying the default intensities responding to economic factors, we price the CDX index through the default intensity process with economic fundamentals. The credit spreads resulting from the model with economic fundamentals show that the economic environment already reveals the credit mess. Examining the economic and financial data series and quantifying the linkage
between the economic factors and default risk can help us to foresee the subprime mortgage crisis at the end of 2006.

2. Methodology

The credit default swaps are the most popular instruments in credit derivative market. In order to facilitate trading, standard credit default indices are created for credit risk benchmark indices, namely, CDX indices. To reveal the connection between economic conditions and credit market environments, we need to generally summarize economic information and value credit derivatives efficiently. The major models for pricing portfolio credit derivatives are the structured model, the reduced-form model and the simple one-factor model for default time such as Copula method. Due to the difficulties of calibrating the specific dynamic model to each credit entity in structured models and the disadvantage of determining the default environment with single distribution in one-factor model, we propose a reduced-form model with economic underpinnings.

The economic environment changes stochastically with the release of new informations. In order to summarize the noisy economic system with continuously updated observations, we extract dynamic economic factors from a large number of variables by Kalman filter. Then the economic environment is represented by these dynamic economic factors instead of variables chosen by specific purpose and each dynamic economic factor is updated as new observations are available.

2.1 Extracting factor dynamics from economic and financial variables

In order to extract the information from economic and financial data series and to suppress the noises in the data, we use a dynamic factor model to compress useful information. Three fundamental groups of various economic and financial data are identified. The first group captures the real activities in economic environment, the second group comprises the nominal measures of the economy, and the third group represents the credit crunch from subprime mortgage to overall economy, including various financial variables correlated to housing market. After the subprime mortgage crisis was unfolded, the crisis spreads out the effects from lending investment and financial institutions to the global economy. The crisis is no only a subprime mortgage crisis, but also shows that the housing market plays an important role in the economy.

We decompose the economy and financial variables into three dynamic factors for
the following reasons. First, instead of examining the potential role of economic variables for default intensities by regression with specific choices of the explanatory economic variables, we collect information from a large number of economic and financial variables to make a useful compression of information. Second, the real activities and inflation variables are classified into different groups to identify their particular effects on default intensities. Third, in order to determine the correlation between the housing bubble and the credit crunch, we use the housing market variables to quantify the effect of housing depression on default intensities. The current credit crunch was caused by subprime mortgage crisis, and the crisis was triggered by the downturn in housing market, loosing lending standards and too complex structured credit derivatives. Thus, it is important to quantify the magnitude of the performance of housing market for risk management.

We set the dynamic economic factors to follow first order vector autoregressive processes, i.e. VAR(1), and the transition equation is as follows:

$$N_t = \Phi N_{t-1} + \nu_t.$$  
(1)

The vector $N_t$ denotes the economic factor with dimension $n$ by 1, $N \in \mathbb{R}^n$. The autoregressive covariance matrix $\Phi$ is a $n \times n$ matrix, and $\nu_t$ is a $n \times 1$ vector of errors with mean zero, variance $\mathbb{E}(\nu_t \nu_t')$, and no correlation across time. Since we decompose economic and financial variables into three economic factors, the dimension $n = 3$. Then, the dynamics of economic factors in physical measure $\mathbb{P}$ can be written as

$$dN_t = -\varphi N_t dt + dB_t^\rho,$$  
(2)

where the $\varphi$ is the $n \times n$ transition matrix, and $B_t^\rho$ is a standard Brownian motion vector under physical measure $\mathbb{P}$. We can rewrite equation (1) as follows:

$$N_t = e^{-\varphi} \left( N_{t-1} + \int_1^t e^{\varphi u} dB_u^\rho \right)$$

$$= e^{-\varphi} N_{t-1} + e^{-\varphi} \int_1^t e^{\varphi u} dB_u^\rho.$$  

In addition, we set the following measurement equation which makes the observable economic and financial data series be affine functions of dynamic economic factors $N_t$:

$$M_t = CN_t + \epsilon_t^M,$$  
(3)
where $M_t$, denotes the economic and financial variables with dimension $m$ by 1, $M \in \mathbb{R}^m$. $C$ is the coefficient matrix with dimension $m$ by $n$, and the disturbance $\varepsilon_t^M$ is a $m$ by 1 vector with mean zero and covariance $\sigma^2$. This disturbance can be seen as the distinct effect not measured by the economic system factors.

Using Kalman filter, we obtain the one-step ahead prediction error $e_t$ and variance $\Sigma_t$ as follows:

$$e_t = M_t - CN_{t-1} = C(N_t - \hat{N}_{t-1}) + \varepsilon_t^M,$$

$$\Sigma_t = CP_{t-1}C' + \sigma^2,$$

where $\hat{N}_{t-1}$ denotes the prediction of $N_t$ at time $t$ conditional on the information given at time $t-1$, and its covariance at time $t$ conditional on the information given at time $t-1$ is given by

$$P_{t|t-1} = \Phi P_{t-1|t-1} \Phi' + \text{Diag}(v_t, v_t')$$

Thus, the conditional expectation of $M_t$ at time $t-1$ is $CN_{t-1}$. All estimates are improved as additional observations become available. By assuming that the prediction errors follow a normal distribution, we obtain the parameters that maximize the sum of the log likelihood values of prediction errors over all sample times.

2.2 The default intensity with economic factors

In order to clarify whether the default intensity is affected by economic environments and how the credit environment evolves with different economic and financial conditions, we assume that the default intensity is an affine function of the three dynamic economic factors which we extracted from various economic and financial series data. Thus, we specify the default intensity $\lambda_t$ as

$$\lambda_t(N_t) = \alpha_{\lambda} + \beta_{\lambda}^T N_t.$$  

(5)

The coefficient matrix $\beta_{\lambda}^T$ represents the simultaneous effects of the default intensity from the changes of the economic factors, the dynamics of default intensity can be linked to the shocks on a large number of economic variables. By Kalman filter, we have the measurement equation for default intensity:

$$\lambda_t = \alpha_{\lambda} + \beta_{\lambda}^T N_t + \varepsilon_{\lambda_t},$$

(6)
where the measurement error $\varepsilon_i^t$ is identified as the disturbances that are not measured by the economic factors. Then, $\varepsilon_i^t$ is independent to each economic factor $N_i$.

**2.3 The default intensity processes with no arbitrage constraint**

Since we use the data of the Dow Jones CDX North America Investment Grade (DJ CDX NA IG) index to reveal the linkage between default intensities and economic conditions for clarifying the impact of economic shock to credit risk, we need to calibrate the market quotes of CDX to get the default intensities with no-arbitrage. The DJ CDX NA IG index is a standard credit default index to facilitate trading and improve the liquidity of credit default swaps (CDSs). Therefore the valuation for CDX index contracts is slightly different from single-name CDSs. In the case of default event, the swap premium payment ceases in single-name CDS contract. In contrast, for the CDX, the default entities are removed from the index and the swap premium payments are continuously made by a reduced notional amount until maturity. We provide a dynamic CDX pricing model as follows.

We assume that under risk neutral measure $\mathbb{Q}$ investors receive the payments at each payment time from $t_i$ to $t_f$ and the present value of these regular payments is denoted as the first part of premium leg:

$$PL_1 = s \sum_{c=1}^{T} (t_c - t_{c-1}) E(t_c) D(t_c),$$

where $s$ is the spread, $c$ counts the payment times, $E(t_c)$ is the expected principal at time $t_c$, and $D(t_c)$ denotes the discount factor. If we assume in average defaults occur in the middle of payments, the present value of the accrual payments in default is the other part of premium leg:

$$PL_2 = s \left[ 0.5 \sum_{c=1}^{T} (t_c - t_{c-1}) (E(t_{c-1}) - E(t_c)) D(t_c^d) \right],$$

where $t_c^d = 0.5(t_{c-1} + t_c)$. The following present value of default leg is the discounted default payoffs:

$$DL = \sum_{c=1}^{T} (E(t_{c-1}) - E(t_c)) D(t_c^d).$$

Then, the CDX index is the breakeven spread that makes the present value of default
leg equal to the premium leg and leaves no arbitrage opportunities.

\[
s = \frac{\sum_{i=1}^{T} (E(t_{i-1}) - E(t_i))D(t_i)}{\sum_{i=1}^{T} (t_{i} - t_{i-1})E(t_i)D(t_i) + 0.5\sum_{i=1}^{T} (t_{i} - t_{i-1})(E(t_{i-1}) - E(t_i))D(t_i)}.
\]

Without loss of generality, we assume the principal is \( V = 1 \). The expectative value at each payment time is

\[
E(t_i) = VE[S(t_i)] = E[S(t_i)],
\]

where \( S(t_i) \) is the cumulative survival probability at time \( t_i \). We denote cumulative survival probabilities from time \( t \) to time \( t + \tau \) as a function of default intensities:

\[
S(\lambda, \tau) = E\left[\exp\left(-\int_{t}^{t+\tau} \lambda_s ds\right)\big|\mathcal{F}_t\right]
\]

Let \( \varepsilon_+^{\lambda} \) be the disturbance part which is not explained by the economic factors. Then the other parts of cumulative survival probabilities can be explained by the economic factors, and is derived as

\[
S(N, \tau) = E^Q\left[\exp\left(-\int_{t}^{t+\tau} \lambda(N_s) ds\right)\big|\mathcal{F}_t\right]
\]

Since the spreads of CDX are priced under risk neutral measure \( Q \), we change the dynamics of the three economic factors to the same measure to obtain the expectation survival probability under measure \( Q \). Using Girsanov’s theorem, we have the Radon-Nikodým derivative of \( Q \) with respect to the physical measure \( P \) as

\[
\eta = \frac{dQ}{dP}.
\]

Because \( \eta \) represents the price of market risk, without loss of generality, we specify \( \eta \) being an affine model of economic factors:

\[
\eta_t = \alpha_\eta + \beta_\eta N_t.
\]

Then the dynamics of economic factors under risk neutral measure is

\[
dN_t^Q = \left[-\alpha_\eta - (\varphi + \beta_\eta)N_t\right] dt + dB_t^Q
\]

\[
= (\varphi + \beta_\eta)^{-1}\left[-\alpha_\eta (\varphi + \beta_\eta) - N_t\right] dt + dB_t^Q.
\]
We can rewrite the survival probability as

$$S(N, \tau) = \exp\left(-\alpha_\lambda(\tau) - \beta_\lambda'(\tau)N_i^G\right),$$

and with the two ordinary differential equations

$$d\alpha_\lambda(\tau) = \alpha_\lambda - \beta_\lambda'(\tau)\left[\alpha_\eta \right] - \frac{1}{2} \beta_\lambda'(\tau)\beta_\lambda(\tau),$$

$$d\beta_\lambda(\tau) = \beta_\lambda - \left(\varphi + \beta_\eta\right)^{-1}\beta_\lambda(\tau).$$

Since $\alpha_\lambda(\tau)$ and $\beta_\lambda(\tau)$ solve the ordinary differential equations with initial conditions $\alpha_\lambda(0) = 0$ and $\beta_\lambda(0) = 0$, we then obtain the coefficients $\alpha_\lambda$ and $\beta_\lambda$ of $\hat{\lambda}_\lambda(N_i)$ in Equation (5).

### 2.4 Estimating the link between default intensities and economic factors

As we mention in section 2.1, we derive three dynamic economic factors from a large number of economic variables by measurement Equation (3) using Kalman filter approach. In this section, we set another measurement equation to derive the link between default intensities and these economic factors:

$$MM_t = \left[\text{CDX}(N, j)\right] + \epsilon_t^{MM},$$

where $MM_t$ is the observable market CDX index spread at time t, $\text{CDX}(N, j)$ is the model spread determined by functions of survival probabilities and economic factors, j is the maturity of CDX, and $\epsilon_t^{MM}$ denotes normally distributed measurement errors. By applying Kalman filter approach again, we have the theoretical CDX index spreads.
3. Data

3.1 Credit Spreads

We use the spreads of DJ CDX NA IG index to derive the credit default intensities and to quantify the correlation between default intensities and economic conditions. CDX indices are credit risk benchmark indices created to improve liquidity and ease trading of credit default swaps (CDSs) which are the most actively traded form of credit derivatives. A CDX index consists of multiple CDS entities and covers default risk on the pool of reference entities in it. Thus, CDX indices closely reflect the broad credit market. The two primary indices for CDSs are the DJ CDX NA IG index for the U.S and the Dow Jones iTraxx Europe (DJ iTraxx Europe) index. Because the current credit crunch started with the overexpansion of credit in the US housing market, our estimation is based on the index of DJ CDX NA IG instead of iTraxx.

To estimate the default intensities, we use the spreads for the DJ CDX NA IG index of maturities 5 and 10 years from September 28, 2004 to July 14, 2008. The standard maturities for the CDX are 1, 2, 3, 4, 5, 7, and 10 years. The most trading volume is the 5-year index, followed by 10-year index. Concerning the liquidity, in this paper we include the most liquid 5-year index and the longest 10-year index to consider the complete yield curve. DJ CDX NA IG index is comprised of 125 most liquid investment grade CDS entities traded in North America with equal weights. That is, each reference entity is weighted 0.8% in the index. The composition is chosen by member banks. New series of DJ CDX NA IG is recreated every six months (March 20 and September 20) and the underlying CDS entities are reconstituted. Similarly, our data roll every 6 months and the roll date is the start date of a new version index. Figure 1 provides plots of the spreads of 5-year and 10-year indices.
3.2 Economic and Financial Variables

We extract the dynamic economic factors from 11 monthly and quarterly economic variables from March 1998 to July 2008. All data are from the DataStream database except the house price index which is from Standard and Poor’s Case-Shiller Home Price Indices. The economic and financial variables are sorted in three components which stand for fundamental economic factors. The three economic factors are the real economic factor, inflation factor, and the housing-related factor. The variables contained in each factor are defined as follows.

The real economic component includes four variables, the inflation component contains two variables, and the housing-related component has the other five variables. The four variables included in the real economic component are gross domestic product (GDP), industrial production (IP), unemployment, and personal income (PI). We compute the year-over-year percentage change of each variable and standardize it by sample mean and sample standard deviation.

The two variables contained in the inflation component are consumption price index (CPI) and producer price index (PPI). The two variables are also converted into year-over-year percentage changes as the variables in the real economic component and then standardized.

The five variables for the housing-related factor, an indicator for the credit crunch from subprime mortgage, are house price index (HPI), the delinquency of all US real
estate mortgage loans (Delinquency-All), the delinquency of US real estate subprime mortgage loans (Delinquency-Subprime), the foreclosure of all US real estate mortgage loans (Foreclosure-All), and the foreclosure of US real estate subprime mortgage loans (Foreclosure-Subprime). Since the data of subprime mortgage loans are available until March 1998, our sample period for extracting the dynamics of economic factors is from March 1998 to July 2004. In this component, only house price index is converted into year-over-year percentage changes and standardized because other variables are already scaled for changes.

The statistics of all variables are summarized in Table 1, and the dynamics of each variable in the sample period are illustrated in Appendix A.

### Table 1. Summary statistics of data.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Frequency</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>gross domestic product (GDP)</td>
<td>quarterly</td>
<td>5.1875</td>
<td>1.2696</td>
</tr>
<tr>
<td>industrial production (IP)</td>
<td>monthly</td>
<td>2.0196</td>
<td>2.7072</td>
</tr>
<tr>
<td>personal income (PI)</td>
<td>monthly</td>
<td>4.3883</td>
<td>1.5105</td>
</tr>
<tr>
<td>consumption price index (CPI)</td>
<td>monthly</td>
<td>2.6943</td>
<td>0.9117</td>
</tr>
<tr>
<td>producer price index (PPI)</td>
<td>monthly</td>
<td>3.6746</td>
<td>4.4835</td>
</tr>
<tr>
<td>unemployment</td>
<td>monthly</td>
<td>2.2982</td>
<td>13.247</td>
</tr>
<tr>
<td>house price index (HPI)</td>
<td>monthly</td>
<td>9.1465</td>
<td>8.6683</td>
</tr>
<tr>
<td>Delinquency-All</td>
<td>quarterly</td>
<td>1.9364</td>
<td>0.5483</td>
</tr>
<tr>
<td>Delinquency-Subprime</td>
<td>quarterly</td>
<td>12.694</td>
<td>2.3400</td>
</tr>
<tr>
<td>Foreclosure-All</td>
<td>quarterly</td>
<td>0.4714</td>
<td>0.1690</td>
</tr>
<tr>
<td>Foreclosure-Subprime</td>
<td>quarterly</td>
<td>2.0074</td>
<td>0.7547</td>
</tr>
</tbody>
</table>

### 4. Results

#### 4.1 The dynamic economic factors and their compositions

The estimation results of extracting dynamic economic factors are reported in Table 2. The coefficients, $C_1$, $C_2$, and $C_3$, represent the correlations of economic variables between real economic factor, inflation factor, and housing-related factor, respectively. The absolute t-statistic values of these estimations are reported in parentheses. The last column shows the prediction error variance. For each economic variable, the lower prediction error variance indicates that the dynamic economic
factor provides the better prediction performance for this economic variable. It also means that this economic variable is helpful to extract an economic factor from complex economic system.

We sort 11 financial and economic variables into three dynamic economic factors. Table 2 shows that the estimates on all variables are statistically significant. The estimates of four variables in the real economic factor are all significantly positive except the standardized year over-year unemployment rate. It really corresponds with the actual economic phenomenon. In recession, the output and personal income decrease while the unemployment increased in contrast. The variables comprise the real economic factors are informative with low prediction error variance except the personal income variable. However, the estimate of this variable is significantly positive and thus some useful information is provided from this variable.

The inflation factor contains CPI and PPI. Both of them are significantly positive with low prediction error variance, therefore instead of ad hoc choice, it is appropriate to use them both to capture the useful information about the nominal economic activities.

Except the house price index, the other financial series of the housing-related factor are significantly positive with low prediction error variances. The lowest prediction error variance is provided by Delinquency-Subprime followed by Foreclosure-Subprime, Foreclosure-All, and Delinquency-All. We find that the subprime mortgage related series have comparatively lower than others. Thus the delinquency and foreclosure data of the subprime mortgages are quite informative about the state of the US housing market. Moreover, the series about all US real estate mortgage loans also have low prediction error variance, and thus they are informative about the housing market too. The largest prediction error variance comes from the house price index. Nevertheless, the estimate of the house price index is significantly negative. The house-related factor negatively responds to the shocks of the house price index.

According to the parameters estimated by using Kalman filter, we derive the three dynamic economic factors from 11 economic and financial series. Figure 2 illustrates graphs the series of these economic factors. The real economic factor is characterized in solid line and denoted as factor 1 in Figure 2. The real economic activities decreased dramatically after 2000 and began to creep upward until hit lowest point in 2002. Then, the real economic factor slids again at the end of 2006. The first recession
period corresponds to the burst of dot-com bubble and the second depression coincides with the serious current credit crunch. The dashed line represents the nominal economic factor, the factor 2, and appears relatively smooth. However its steep slope after 2007 coincided with the crude oil price settled at a record high in 2007. The other line characterizes the housing-related factor, the factor 3. It broke record high in 2007 as the house price index drops and multiple subprime loans boosted delinquency and foreclosure risks even higher.

Table 2. The statistics of variable extraction.

<table>
<thead>
<tr>
<th>Economic Variables</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$\epsilon_i^M$</th>
<th>$\epsilon_i^M$</th>
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<tr>
<td>gross domestic product (GDP)</td>
<td>0.5137</td>
<td>—</td>
<td>—</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(19.4739)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>industrial production (IP)</td>
<td>0.4956</td>
<td>—</td>
<td>—</td>
<td>0.26</td>
<td></td>
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<tr>
<td></td>
<td>(19.2850)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>personal income (PI)</td>
<td>0.227</td>
<td>—</td>
<td>—</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(14.7297)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unemployment</td>
<td>-0.5569</td>
<td>—</td>
<td>—</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(19.5841)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>consumption price index (CPI)</td>
<td>—</td>
<td>0.9133</td>
<td>—</td>
<td>0.10</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(26.0722)</td>
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<tr>
<td>producer price index (PPI)</td>
<td>—</td>
<td>0.9171</td>
<td>—</td>
<td>0.10</td>
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<td></td>
<td></td>
<td>(26.1870)</td>
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<td>house price index (HPI)</td>
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<td>-0.5268</td>
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<td></td>
<td></td>
<td></td>
<td>(22.0331)</td>
<td></td>
<td></td>
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<tr>
<td>Delinquency-All</td>
<td>—</td>
<td>—</td>
<td>0.5399</td>
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<td></td>
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<tr>
<td>Delinquency-Subprime</td>
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<td>—</td>
<td>0.5769</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(22.4362)</td>
<td></td>
<td></td>
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<tr>
<td>Foreclosure-All</td>
<td>—</td>
<td>—</td>
<td>0.5801</td>
<td>0.16</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(22.4512)</td>
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<td></td>
</tr>
<tr>
<td>Foreclosure-Subprime</td>
<td>—</td>
<td>—</td>
<td>0.5921</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(22.4294)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The parameters are estimated by the measure equation $M_i = CN_i + \epsilon_i^M$ with Kalman filter approach, and the dynamic of economic factor is $N_i = \Phi N_{i-1} + \nu_i$. 
4.2 Credit environment responses to economic conditions

We estimate the parameters of the impact on default intensities from dynamic economic factors by using maximum likelihood method and Kalman filter approach again. The estimation results, reported in Table 3, show the impacts on default intensities from economic factors. Although different impacts come from distinct economic factors, the parameters of all economic factors are statistically significant. The inflation factor and housing-related factor both have strong positive effects on default intensities. In contrast, the real economic factor has significant negative influence. In Table 4, we find our estimation results fairly coincide with the nature of these economic factors. The real economic factor has negative correlation with other economic factors and CDX index across different maturities. This negative correlation corresponds to our estimation results that the real economic factor has negative impact on the default intensities and then the spreads of CDX indices rise with the depression in real economic activities.

From equation (5), $\lambda_t(N_t) = \alpha_2 + \beta_2^T N_t$, the parameter $\alpha_2$ and $\beta_2$ determinants
the impacts on default intensities of economic factors. Since the top row of $\beta_\lambda$ is -0.00234, the real economic factor has negative impact on default intensities. That means the default intensities simultaneously rise as the real economic depresses. Take the GDP and unemployment variables for instance. The shock of GDP has negative impact on default intensities, because GDP has positive coefficient with the real economic factor and this factor has negative effect on default intensities. On the contrary, the unemployment has positive impact on default intensities. Since the unemployment has negative coefficient with real economic factor and the default intensities respond positively to this factor, the reaction of default intensities to unemployment impulse is positive. Similarly, as we consider the financial series contained in the housing-related factor, the house price index has negative impact on default risk. Although the parameter determines the impact of the housing-related factor on default risk is positive, the house price index has negative coefficient with housing-related factor. Moreover, because the CPI and PPI both have positive coefficients with inflation factor and default risk positively respond to inflation pressure, both inflation series have positive effects on default risk.

### Table 3. The estimation results and the impact on default intensities from economic factors.
This table shows the parameters that represent the impact on default intensities from dynamic economic factors. The parameters are derived from the dynamics of economic factors and the historical data of CDX indices by using maximum likelihood method and Kalman filter approach. The absolute t-statistic values of these estimations are reported in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>$\varphi$</th>
<th>$\left( \varphi + \beta_\eta \right)^{-1}$</th>
<th>$-\alpha_\eta$</th>
<th>$\alpha_\lambda$</th>
<th>$\beta_\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1447</td>
<td>-</td>
<td>0.25357</td>
<td>2.63357</td>
<td>0.25357</td>
<td>-0.00234</td>
</tr>
<tr>
<td>(0.8198)</td>
<td></td>
<td>(10.4403)</td>
<td>(14.8991)</td>
<td></td>
<td>(35.7111)</td>
</tr>
<tr>
<td>-</td>
<td>0.2055</td>
<td>-</td>
<td>0.49077</td>
<td>0.99203</td>
<td>0.01057</td>
</tr>
<tr>
<td>(0.6943)</td>
<td></td>
<td>(28.2810)</td>
<td>(1.9390)</td>
<td></td>
<td>(10.3047)</td>
</tr>
<tr>
<td>-</td>
<td>-0.4003</td>
<td>-</td>
<td>0.03276</td>
<td>0.03276</td>
<td>0.00210</td>
</tr>
<tr>
<td>(2.0487)</td>
<td></td>
<td>(0.4666)</td>
<td>(3.1019)</td>
<td></td>
<td>(55.6481)</td>
</tr>
</tbody>
</table>
Table 4. The correlation of all variables and indices.

<table>
<thead>
<tr>
<th></th>
<th>$N_{real}$</th>
<th>$N_{nominal}$</th>
<th>$N_{housing-related}$</th>
<th>CDX-5 year</th>
<th>CDX-10 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{real}$</td>
<td>1</td>
<td>-0.3101</td>
<td>-0.9572</td>
<td>-0.71</td>
<td>-0.6463</td>
</tr>
<tr>
<td>$N_{nominal}$</td>
<td>-0.3101</td>
<td>1</td>
<td>0.3061</td>
<td>0.5601</td>
<td>0.5298</td>
</tr>
<tr>
<td>$N_{housing-related}$</td>
<td>-0.9572</td>
<td>0.3061</td>
<td>1</td>
<td>0.7686</td>
<td>0.6929</td>
</tr>
<tr>
<td>CDX-5 year</td>
<td>-0.71</td>
<td>0.5601</td>
<td>0.7686</td>
<td>1</td>
<td>0.9828</td>
</tr>
<tr>
<td>CDX-10 year</td>
<td>-0.6463</td>
<td>0.5298</td>
<td>0.6929</td>
<td>0.9828</td>
<td>1</td>
</tr>
</tbody>
</table>

4.3 Pricing credit derivatives with the economic factors

Our estimation results coincide with the characteristics of credit market. The default intensities across different maturities simultaneously rise with stressed markets. We use the estimation parameters in Table 3 to price the spreads of CDX at the two most popular trading maturities (5 and 10 years). The market spreads and our model estimation spreads of CDX at selected maturities from September 2004 to July 2008 are plotted in Figure 3. The market credit spreads began to increase substantially in the middle of 2007 and peaked in the early 2008. These credit spread jumps are correspond to the recently default events in credit market.

After quantifying the default intensity responses to economic factors, we price the CDX index through the default intensity processes with economic fundamentals. The model estimation of credit spreads of CDX index is plotted in the bottom panel of Figure 3. We find that the credit spread began to rise in December of 2006. In 2007, the credit spread rose, stayed, and then rose dramatically until the end of our sample period. The model spreads with economic fundamentals show that the economic environment already reveals the credit mess. As shown in Figure 2, at the end of 2006, the real economic activity decreased and the high delinquency and foreclosure rates made the slope of the house-related factor even steep. After the subprime mortgage crisis unfolded, the market lost its liquidity, and borrowing money become much more expensive. Then the house-related factor kept on rising and broke the record high. As the fundamental economic environment does not get better, our model shows that the default panic is not recede. Because the economic indicators are monthly or quarterly data, the model spreads for CDX is smooth relative to market quotes. Although, the
joint behavior of economic conditions and the default risk reveal the clues of the recent subprime mortgage meltdown. Examining the economic and financial data series and quantifying the linkage between the economic factors and default risk can help us to foresee the subprime mortgage crisis at the end of 2006.

Figure 3. The market and model estimation credit spreads of the DJ CDX NA IG index.
The top panel is the market quotes of the DJ CDX NA IG index, and the bottom panel is the theoretical spreads derived from our model. The sample period covers September 2004 to July 2008.
5. Conclusion

In this paper we provide an insight into the fundamental economic determinants of default risks through the no-arbitrage dynamic factor model and price the credit derivatives by using these economic and financial factors with the reduced-form model. As we define the default intensities as affine functions of dynamic economic factors, the reduced-form models are reinforced with fundamental economic underpinnings.

We summarize the noise information with continuously updated observations from many economic and financial variables, and sort these variables into three components. The extracted factors are the real economic factor, nominal economic factor, and housing-related factor. Because the subprime mortgage crisis spillovered and led to the current credit crunch, the evolution of housing market becomes an important sector of the whole economy. Thus, except the real economic part and inflation part, we extract the housing-related factor from various financial series. Then the economy environment is represented by these dynamic economic factors instead of variables chosen by specific purpose and each dynamic economic factor is updated as new observations arrived. By imposing the no-arbitrage restriction, we construct the default intensity processes as an affine function of these fundamental economic factors. Our estimation results indicate that the parameters of all economic factors are statistically significant. The inflation factor and housing-related factor both have strong positive effect on default risks, and the real economic factor has significant negative influences.

In addition, we use the estimated parameters to value the CDX spreads across maturities with dynamic portfolio pricing model. The model spreads show that the economic environment already reveals the aggravation of the credit conditions at the end of 2006. Quantifying the linkage between the economic factors and default risk and capturing the environment evolution of real activities, inflation and housing market can help us to foresee the current credit crunch. Moreover, in a down housing market, since homeowners found it difficult to refinance loans or sell houses and much more subprime mortgage clients cannot afford their house payment and have to default on their loans, the delinquencies and foreclosures are accelerated. As the fundamental economic environment does not get better, our model shows that the default panic is not recede in the near future.
Since the fundamental economic and financial observations are released monthly or quarterly, the times series of CDX spreads obtained from our model is relatively smooth. We suggest that the CDX pricing model could be improved by adding some jump processes to default intensities.
References


Appendix A

Time series of GDP in year-over-year changes

Time series of PI in year-over-year changes

Time series of the unemployment in year-over-year changes
Time series of the personal income in year-over-year changes

Time series of CPI in year-over-year changes

Time series of PPI in year-over-year changes
Time series of house price index in year-over-year changes

Time series of the delinquency rates of all mortgage loans and subprime mortgage loans

Time series of the foreclosure rates of all mortgage loans and subprime mortgage loans