Economic Determinants of Default Risks and Their Impacts on the Pricing of Credit Derivatives

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Introduction

- The credit crunch has depressed the global economy since the burst of the housing bubble in 2007.

- Investors made poor decisions, investment banks underestimated the systemic risk generated from contagious defaults, and rating agencies also failed to recognize the worst-case scenario represented by the subprime mortgage mess.
Introduction

- Our Objective

  ✓ Address the joint behavior of economic conditions and the default risk in order to get the clues of the recent subprime mortgage meltdown.
  ✓ Provide an insight into the fundamental economic determinants of default intensities.
  ✓ Build a reduced-form model with economic fundamentals for the valuation of credit derivatives.
Prior Research

- Default intensity can be explained by macroeconomic performance
  - Das et al. (2007)
  - McDonald and Van de Gucht (1999)
  - Couderc and Renault (2004)
  - Duffie et al. (2005)
Prior Research

- Defaults have clustered and the explanation of default arrivals can be improved by some macroeconomic indicators
  - Hull (2007)
  - Das et al. (2007)
  - Bhansali et al. (2008)
  - Longstaff and Rajan (2008)
Methodology

1. Extract factor dynamics from economic and financial variables
2. Set default intensity with economic factors
3. Build default intensity processes with no arbitrage constraint
4. Estimate the link between default intensities and economic factors
1. Extracting factor dynamics from economic and financial variables

- We use a dynamic factor model to compress useful information.
- *Three fundamental groups* of various economic and financial data are identified.
  - *The real activities* in economic environment
  - *The nominal measures* of the economy
  - Various financial *variables correlated to housing market.*
- The subprime mortgage crisis shows that the housing market plays an important role in the economy.
- After the subprime mortgage crisis was unfolded, the crisis spreads out the effects from lending investment and financial institutions to the global economy.
1. Extracting factor dynamics from economic and financial variables

- **Advantages**
  - Collect information from a large number of economic and financial variables to make a useful compression of information
  - The real activities and inflation variables are classified into different groups to identify their particular effects on default intensities
  - Quantify the effect of housing depression on default intensities
1. Extracting factor dynamics from economic and financial variables

- The dynamics of economic factors in physical measure

\[ dN_t = -\varphi N_t dt + dB_t^P \]

Rewrite it as a VAR(1) process

\[ N_t = \Phi N_{t-1} + \nu_t \]

The measurement equation makes the observable economic data series be an affine function of economic factors

\[ M_t = CN_t + \varepsilon_t^M \]
1. Extracting factor dynamics from economic and financial variables

- Using Kalman filter

✓ One-step ahead prediction error $e_t$ and variance $\Sigma_t$

$$e_t = M_t - C\hat{N}_{t|t-1} = C\left(N_t - \hat{N}_{t|t-1}\right) + \varepsilon_t^M,$$

$$\Sigma_t = CP_{t|t-1}C' + \sigma^2,$$

✓ Assume that the prediction errors follow a normal distribution

✓ We obtain the parameters that maximize the sum of the log likelihood values of prediction errors over all sample times.
2. Setting default intensity with economic factors

- To clarify how the credit environment evolves with different economic and financial conditions.
- Assume that the default intensity $\lambda_t$ is an affine function of the three economic factors:
  \[ \lambda_t(N_t) = \alpha_{\lambda} + \beta_{\lambda}^T N_t. \]
- By Kalman filter, we have the measurement equation for default intensity:
  \[ \lambda_t = \alpha_{\lambda} + \beta_{\lambda}^T N_t + \epsilon_{\lambda}. \]
3. Build default intensity processes with no arbitrage constraint

- To clarify the impact of economic shock to credit risk, we calibrate the market quotes of CDX to get the default intensities with no-arbitrage.

- CDX
  - A standard credit default index to facilitate trading and improve the liquidity of credit default swaps (CDSs).
  - In the case of default event, the default entities are removed from the index and the swap premium payments are continuously made by a reduced notional amount until maturity.
3. Build default intensity processes with no arbitrage constraint

- Dynamic CDX pricing model
- Under risk neutral measure

✓ Investors receive the payments at each payment time from $t_1$ to $t_T$

$$PL_1 = s \sum_{c=1}^{T} (t_c - t_{c-1}) E(t_c) D(t_c)$$

✓ The present value of the accrual payments in default

$$PL_2 = s \left[ 0.5 \sum_{c=1}^{T} (t_c - t_{c-1})(E(t_{c-1}) - E(t_c)) D(t_c^d) \right]$$
3. Build default intensity processes with no arbitrage constraint

- Present value of default leg is the discounted default payoffs

\[ DL = \sum_{c=1}^{T} \left( E(t_{c-1}) - E(t_c) \right) D(t_c^d) \]

- The CDX index is the breakeven spread that makes the present value of default leg equal to the premium leg and leaves no arbitrage opportunities.

\[
S = \frac{\sum_{c=1}^{T} \left( E(t_{c-1}) - E(t_c) \right) D(t_c^d)}{\sum_{c=1}^{T} (t_c - t_{c-1}) E(t_c) D(t_c) + 0.5 \sum_{c=1}^{T} (t_c - t_{c-1}) \left( E(t_{c-1}) - E(t_c) \right) D(t_c^d)}
\]
3. Build default intensity processes with no arbitrage constraint

- Without loss of generality, we assume the principal \( V = 1 \). The expected value at each payment time is

\[
E(t_c) = VE[S(t_c)] = E[S(t_c)]
\]

- \( S(t_c) \) is the cumulative survival probability at time \( t_c \), and is a function of default intensities

\[
S(\lambda_t, \tau) = E\left[ \exp \left( -\int_t^{t+\tau} \lambda_s ds \right) | F_t \right] = E\left[ \exp \left( -\int_t^{t+\tau} \lambda(N_s) + \varepsilon_s ds \right) | F_t \right].
\]
3. Build default intensity processes with no arbitrage constraint

✓ Since the spreads of CDX are priced under risk neutral measure \( \mathbb{Q} \), we change the dynamics of the three economic factors to the same measure to obtain the expected survival probability under measure \( \mathbb{Q} \).

✓ Using Girsanov’s theorem, we have the Radon-Nikodým derivative \( \eta \), which represents the price of market risk and is assumed as

\[
\eta_t = \alpha_\eta + \beta_\eta N_t
\]

✓ Then, the survival probability is

\[
S(N_t, \tau) = \exp \left( -\alpha_\lambda (\tau) - \beta_\lambda^* (\tau) N_t^Q \right)
\]
3. Build default intensity processes with no arbitrage constraint

From ordinary differential equations

\[ d\alpha_{\lambda}(\tau) = \alpha_{\lambda} - \beta'_{\lambda}(\tau) \left[ \alpha_{\eta} \right] - \frac{1}{2} \beta_{\lambda}'(\tau) \beta_{\lambda}(\tau), \]

\[ d\beta_{\lambda}(\tau) = \beta_{\lambda} - \left( \phi + \beta_{\eta} \right)^{-1} \beta_{\lambda}(\tau). \]

and initial conditions \( \alpha_{\lambda}(0) = 0 \) and \( \beta_{\lambda}(0) = 0 \)

We obtain the coefficients \( \alpha_{\lambda} \) and \( \beta_{\lambda} \), which represents the simultaneous effects of the default intensity from the changes of the economic factors.

\[ \lambda_{t}(N_{t}) = \alpha_{\lambda} + \beta_{\lambda}^{T} N_{t}. \]
4. Estimating the link between default intensities and economic factors

- Using Kalman filter again

We set another measurement equation to derive the link between default intensities and these economic factors:

\[ MM_t = \left[ \text{CDX}(N_t, j) \right] + \varepsilon_{t}^{MM} \]

- \(MM_t\) is the observable market CDX index spread at time \(t\), \(\text{CDX}(N_t, j)\) is the model spread determined by functions of survival probabilities and economic factors, \(j\) is the maturity of CDX, and \(\varepsilon_{t}^{MM}\) denotes normally distributed measurement errors.
Data

- **Credit Spreads**
  - We use the data of the Dow Jones CDX North America Investment Grade (DJ CDX NA IG) Index.

*Figure 1. The DJ CDX NA IG index spreads.*
Data

- **Economic and Financial Variables**
- The real economic component
gross domestic product (GDP), industrial production (IP), unemployment, and personal income (PI).
- The inflation component
consumption price index (CPI), and producer price index (PPI).
- The housing-related component
house price index (HPI), the delinquency of all US real estate mortgage loans (Delinquency-All), Delinquency-Subprime, the foreclosure of all US real estate mortgage loans (Foreclosure-All), and Foreclosure-Subprime.
# Data

<table>
<thead>
<tr>
<th>Variables</th>
<th>Frequency</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
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<tr>
<td>gross domestic product (GDP)</td>
<td>quarterly</td>
<td>5.1875</td>
<td>1.2696</td>
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<td>industrial production (IP)</td>
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<td>2.7072</td>
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<td>personal income (PI)</td>
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<td>unemployment</td>
<td>monthly</td>
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<td>consumption price index (CPI)</td>
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<td>producer price index (PPI)</td>
<td>monthly</td>
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<td>house price index (HPI)</td>
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<td>Delinquency-All</td>
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<td>0.5483</td>
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<tr>
<td>Delinquency-Subprime</td>
<td>quarterly</td>
<td>12.694</td>
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<td>Foreclosure-All</td>
<td>quarterly</td>
<td>0.4714</td>
<td>0.1690</td>
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<td>Foreclosure-Subprime</td>
<td>quarterly</td>
<td>2.0074</td>
<td>0.7547</td>
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</table>

Table 1. Summary statistics of data.
Results

→ 1. The dynamic economic factors and their compositions
→ 2. Credit environment responses to economic conditions
→ 3. Pricing credit derivatives with the economic factors
1. The dynamic economic factors and their compositions

- Table 2 shows that the estimates on all variables are statistically significant.
- Corresponding to the actual economic phenomenon, the estimates of four variables in the real economic factor are all significantly positive except the unemployment rate.
- Both CPI and PPI are significantly positive to the inflation factor with low prediction error variance.
- Except the house price index, the other financial series of the housing-related factor are significantly positive with low prediction error variances.
<table>
<thead>
<tr>
<th>Economic Variables</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$\varepsilon_n^M, \varepsilon_{it}^M$</th>
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<tr>
<td>gross domestic product (GDP)</td>
<td>0.5137</td>
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<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(19.4739)</td>
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<td>industrial production (IP)</td>
<td>0.4956</td>
<td></td>
<td></td>
<td>0.26</td>
</tr>
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<td></td>
<td>(19.2850)</td>
<td></td>
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<td>personal income (PI)</td>
<td>0.227</td>
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<td>(14.7297)</td>
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<tr>
<td>Foreclosure-Subprime</td>
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<td>0.5921</td>
<td>0.12</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(22.4294)</td>
<td></td>
</tr>
</tbody>
</table>
1. The dynamic economic factors and their compositions

Figure 2. The economic factors
1. The dynamic economic factors and their compositions

- The real economic activities decreased dramatically after 2000 and slid again at the end of 2006. The first recession period corresponds to the burst of dot-com bubble and the second depression coincides with the serious current credit crunch.

- The dashed line represents the nominal economic factor, the factor 2, and appears relatively smooth. However its steep slope after 2007 coincided with the crude oil price settled at a record high in 2007.

- The third line characterizes the housing-related factor, the factor 3. It broke record high in 2007 as the house price index drops and multiple subprime loans boosted delinquency and foreclosure risks even higher.
2. Credit environment responses to economic conditions

- Equation (5), \( \lambda_t(N_t) = \alpha_\lambda + \beta_\lambda'' N_t \)
- Table 3. The impact on default intensities from economic factors

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>((\varphi + \beta_\eta))^{-1}</th>
<th>(-\alpha_\eta)</th>
<th>(\alpha_\lambda)</th>
<th>(\beta_\lambda)</th>
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<td>0.1447</td>
<td>0.25357</td>
<td>2.63357</td>
<td>0.01057</td>
<td>-0.00234</td>
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<td>(0.8198)</td>
<td>(10.4403)</td>
<td>(14.8991)</td>
<td>(22.5800)</td>
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<tr>
<td>-0.2055</td>
<td>-0.99203</td>
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<td>0.00210</td>
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<td>(0.6943)</td>
<td>(28.2810)</td>
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<td>(10.3047)</td>
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<td>-0.4003</td>
<td>-0.03276</td>
<td>1.02679</td>
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</tr>
</tbody>
</table>
2. Credit environment responses to economic conditions

- All economic factors are statistically significant.
- The inflation factor and housing-related factor both have strong positive effects on default intensities. In contrast, the real economic factor has significant negative influence.
  - Since the real economic factor has negative impact (-0.00234) on default intensities, the default intensities simultaneously rise as the real economic depresses.
  - The shock of GDP has negative impact on default intensities, because GDP has positive coefficient with the real economic factor and this factor has negative effect on default intensities.
  - On the contrary, the unemployment has positive impact on default intensities.
We use the estimation parameters in Table 3 to price the spreads of CDX at the two most popular trading maturities (5 and 10 years).

The sample period is from September 2004 to July 2008.
3. Pricing credit derivatives with the economic factors

- The model spreads show that the economic environment already revealed the aggravation of the credit conditions at the end of 2006.
- As the fundamental economic environment does not get better, our model shows that the default panic has not receded.
- Because the economic indicators are monthly or quarterly data, the model spreads for CDX is smooth relative to market quotes.
Conclusions

- We provide an insight into the fundamental economic determinants of default risks through the no-arbitrage dynamic factor model.
- Price the credit derivatives by using these economic and financial factors with the reduced-form model.
- The joint behavior of economic conditions and the default risk reveal the clues of the recent subprime mortgage meltdown.
- The reduced-form models are reinforced with fundamental economic underpinnings.