

# Nurturing an Infant Industry by Markovian Subsidy Schemes

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November 30, 2017

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## Abstract

We model a small open economy with an infant industry facing competition from imports. We derive the welfare maximizing output path and knowledge path for an infant industry under the central planner who can dictate the industry output. We next show how the social planner's optimal path can be achieved when the infant industry is in private hand, focusing on two cases: the case where the infant industry consists of a monopoly, and the case where it is a duopoly. In the case of a monopoly we show that free trade can induce the monopoly to choose the socially optimal production path. Contrary to conventional wisdom, we show that the volume of imports is large when the stock of knowledge is small, and gradually declines as this stock grows. In the case of a duopoly with knowledge spillovers we derive a subsidy scheme inducing a Markov Perfect Nash Equilibrium that replicates the social optimum. When the subsidy rule is linear affine in the state variable, we show that the subsidy rate per unit of output must be an increasing function of the stock. The underlying intuition is that the government should put domestic firms under a tough competition in their infancy with a promise to make their life easier as their knowledge grows.

## 1. Introduction

The study of trade policies under imperfect competition has traditionally been set in a static framework, though many authors do acknowledge that important policy issues need dynamic considerations (Dixit, 1984; Brander and Spencer, 1986; Eaton and Grossman, 1986; Long and Stähler, 2009; see Long, 2010, for a survey). Dockner and Haug (1990, 1991) have contributed much to the modelling of trade policy issues involving oligopolies in a dynamic setting. They extended the literature on international duopoly (with one firm in each country) to the case of dynamic international duopoly, and compared the equilibrium outcomes under open-loop and closed-loop informational

structures.<sup>1</sup> In this paper, we honor Engelbert Dockner’s contribution to dynamic trade issues by formulating a model of infant industry protection involving domestic learning-by-doing duopolists in a small open economy facing given world prices.

The traditional literature on infant industry protection assumes that firms are so small that they take the aggregate industry output as exogenously given. This literature began with the work of John Stuart Mill (1848, pp. 918-919). Kemp (1960) was the first to offer a formal model of infant industry consisting of price-taking firms that benefit from the industry’s learning. Further extensions to the basic infant industry model were made by Clemhout and Wan (1970), Bardhan (1971), Succar (1987), Young (1991), Melitz (2005), and Ederington and McCalman (2011), to deal with issues such as learning-by-doing with spillovers across industries and technology adoption. In all these dynamic models, each firm is assumed to be of infinitesimal size, and thus they have no strategic interactions. The empirical literature (e.g. Head, 1994; Das 1995; Lee, 1997; Das and Srinivasan, 1997; Irwin, 2000) documented a variety of policy measures that various governments have used to support infant industries. Firms in an infant industry are encouraged to expand outputs because they are being “pulled” by explicit or implicit production subsidies, or “pushed” by the threat of possible withdrawals of support if they fail to achieve production target. According to Lahiri and Ono (1988), in post-war Japan, weak firms were subject to the threat of being “weeded out” by the Japanese Ministry of International Trade and Industry, MITI.

The main theoretical arguments for infant industry protection are nicely expounded Corden (1997). The main point is that, in the case where domestic firms are price takers facing foreign competition, there is a need for intervention due to the market failures. A major source of market failure arises from imperfections in the capital market. In their early learning stage, the unit production cost of domestic firms is typically higher than the world-market price of the same product. In

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<sup>1</sup>In their models, the state variable is the “sticky price” that adjusts slowly over time.

the absence of tariff protection, or domestic production subsidies, in their infancy domestic firms do not have sufficient revenues to pay for their input costs. In a world with a perfect capital market, these short-term losses can be financed by borrowing from financial institutions as long as the expected future profits are sufficient to pay back the loans, with interests. When the capital market is imperfect, due to informational asymmetry creating moral hazard or adverse selection problems, the market outcomes might involve credit rationing, and might not be efficient. Under these conditions, there might be a case for government loan subsidies, or production subsidies<sup>2</sup>. A second source of market failure is the spillover effect of learning-by-doing. When one firm learns, other firms may benefit from it via “learning-by-watching.” The learning firm cannot capture this spillover effect, and hence its incentives to learn are weak relative to what would be in a social optimum. As a result, endogenous growth may be hampered (Young, 1991; see also Long and Wong (1997) for a survey of trade theory with endogenous growth).

A major source of difficulty is how to design policies that corrects for dynamic externalities when domestic firms are sufficiently large that they do not take domestic prices, import quotas or subsidy rates as non-manipulable. For example, if a domestic firm has market power because foreign supplies are banned or restricted by quotas, it will have an incentive to restrict output to raise price above the competitive level. Furthermore, a government may not be able to commit to a time-path of actions (such as a time path of subsidy rates, or of import quotas). As shown by Kydland and Prescott (1977), discretionary policies are in general time inconsistent: it is likely that as time passes, the government will find it optimal to renege from its previously announced subsidy path<sup>3</sup>.

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<sup>2</sup>Many economists are not convinced that government intervention can improve welfare in the presence of moral hazard or adverse selection. See, for example, Dixit (1987).

<sup>3</sup>The time-inconsistency property explained in Kydland and Prescott (1977) has been given fuller treatment in several books and articles. See, for example,

One way of overcoming the time-inconsistency problem is to look for a time-invariant rule which prescribes the subsidy rate at any point of time on the basis of the currently observed values of the states of the system. Such a rule would be optimal if it can be shown that the firms reacting to it actually replicate what a hypothetical central planner with full control over outputs would produce. In this paper, we seek optimal time-consistent policies for the case of an infant industry.

One of our rather surprising results is that the optimal time path of imports can be a decreasing function of the stock of knowledge: as the domestic industry gradually matures, the import volume falls. In the case of a domestic duopoly, we obtain a striking result: When the efficiency-inducing subsidy rule is linear affine in the state variable, we show that the subsidy rate per unit of output must be an increasing function of the stock of knowledge. The underlying intuition is that the government should put the domestic firms under a tough competition in their infancy with a promise to make their life easier as their knowledge grows. This is exactly the opposite of the policy followed by many governments that strongly protect infant industries in their infancy and reduce the protection as the domestic industry grows. In fact the tough competition for the infant industry in its early stage turns the domestic firms into “industrious infants.”<sup>4</sup>

The model is described in section 2. In section 3 we offer a preliminary analysis of the trade-offs in supporting an infant industry. In section 4 we determine the optimal path of output, imports, and knowledge accumulation if the economy is directly controlled by a central planner. Section 5 shows how the social optimum can be replicated by a domestic monopoly facing the discipline of competition from imports. Section 6 turns to the case of a domestic duopoly with

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Karp and Livernois (1992), Benchekroun and Long (1998), and Dockner et al. (2000).

<sup>4</sup>We thank J. Peter Neary for the pun. He pointed out that our results indicate that there is a parallel between our result for infant industries and the educational experience of “industrious infants.” Typically, children that turned out to be genius were pushed by their parents. For example, Frank Gehry acknowledged that his mother would push him when he was a child (Lacayo, 2000).

knowledge spillovers, and shows that there is a family of Markovian subsidy rules that lead the duopolists to achieve the social optimum. In particular, one of such subsidy rules are linear affine in the state variable. In that case, we prove that the subsidy rate is increasing in the stock of knowledge.

## 2. The Basic Model

We consider a small open economy that can produce two goods: a numeraire good (say food) produced by labor alone, under constant returns to scale, and a manufactured good with marginal cost that depends on the industry's stock of knowledge,  $K(t)$ , which results from learning-by-doing. The economy has a constant endowment of labour,  $\bar{L}$ , which is the same for all  $t$ .

The economy, being small, faces a constant and exogenous world price  $p^*$  of the manufacturing good (in terms of the numeraire good). The price  $p^*$  includes the transport cost of a unit of the manufactured good from the foreign countries to our small open economy. If our small open economy exports the manufactured good, it can only receive  $\lambda p^*$  per unit, where  $0 < \lambda < 1$ , because it has to incur the “transport cost” (inclusive of other barriers to exports) to foreign destinations.

The economy is populated by identical individuals. The utility function is quasi-linear: it is strictly concave in the consumption of the manufactured good,  $D_M$ , and linear in the consumption of the numeraire good,  $D_N$  :

$$U(D_M, D_N) = u(D_M) + D_N$$

where  $u' > 0$  and  $u'' < 0$ . Quasi-linearity is a common assumption in trade models, especially when authors address issues such as market powers and oligopoly; see, for example, Grossman and Helpman (1994), Feenstra (2016), Long et al. (2011).

The production function for the numeraire good is simple: each unit of the numeraire good requires one unit of labour. Thus the wage

rate is  $w = 1$ . Let  $q(t)$  denote the country's output of the manufactured good at time  $t$ . The amount of labour required to produce  $q(t)$  depends on both the output level  $q(t)$  and the level of knowledge,  $K(t)$ , specific to the infant industry.

The stock of knowledge, denoted by  $K(t)$ , evolves according to the equation

$$\dot{K}(t) = q(t) - \delta K(t)$$

where  $\delta > 0$  is the rate of depreciation of  $K$ . In particular, we assume that there exists a level  $\bar{K} > 0$  such that if  $K < \bar{K}$ , then the amount of labour required in manufacturing is decreasing in  $K$ ,

$$C(q(t), K(t)) = [c + \gamma \max(0, \bar{K} - K(t))] q(t) + \frac{\beta}{2} q(t)^2$$

where  $c > 0$ ,  $\beta > 0$  and  $\gamma > 0$ . The marginal cost function, denoted by  $MC$ , is linear affine in  $q(t)$ :

$$MC(q(t), K(t)) = [c + \gamma \max(0, \bar{K} - K(t))] + \beta q(t)$$

The parameter  $\gamma$  is a measure of the benefit of learning-by-doing. Our specification of the function  $C(q, K)$  implies that if  $K < \bar{K}$ , then a marginal increase in  $K$  will shift the marginal cost curve downwards. Once  $K$  has reached the level  $\bar{K}$ , any further increase in  $K$  will have no effect on the marginal cost. The lowest possible marginal cost intercept (at  $q = 0$ ) is attained when  $K = \bar{K}$

$$MC(0, K = \bar{K}) = c$$

At the other extreme, when  $K = 0$ , the marginal cost at  $q = 0$  is

$$MC(0, K = 0) = c + \gamma \bar{K} > c$$

We make the following assumption concerning the parameter  $c$ :

**Assumption A1:**

$$\lambda p^* < c < p^* \tag{1}$$

The first inequality in Assumption A1 implies that our small open economy would never find it optimal to export the manufactured good, because the net price received from exporting,  $\lambda p^*$ , is smaller than the marginal production cost. This assumption is made in order to focus on the domestic market.<sup>5</sup> The second inequality in Assumption A1 implies that if the knowledge level  $\bar{K}$  has been reached then there is a range of output levels such that the marginal cost of production is smaller than the price of the imported good. However, if the industry wants to sustain the knowledge level  $\bar{K}$  as a steady-state knowledge level, it would need to produce a constant flow of output  $\bar{q} \equiv \delta \bar{K}$ , and this may be quite costly, if the marginal cost of producing  $\bar{q}$  is much higher than the price  $p^*$  of the imported good.<sup>6</sup>

We are interested in the case of a small economy which would still require imports even under  $q = \bar{q}$ . We therefore make the following assumptions concerning costs, imported price, and demand conditions.

**Assumption A2:** The marginal cost of producing the quantity  $\bar{q} \equiv \delta \bar{K}$  is (weakly) greater than the world price  $p^*$  :

$$c + \beta \bar{q} \equiv c + \beta \delta \bar{K} \geq p^* \quad (2)$$

**Assumption A3:** If the domestic price is set at  $p = c + \beta \bar{q}$ , the quantity demanded by domestic consumers is greater than  $\bar{q}$ .

$$D_M(c + \beta \bar{q}) > \bar{q} \quad (3)$$

Note that Assumptions A2 and A3 imply that

$$D_M(p^*) > \bar{q} \quad (4)$$

i.e., if there is complete free trade (i.e., the domestic price  $p$  is equal to the world price  $p^*$ ), the quantity demanded by domestic consumers is greater than  $\bar{q}$ .

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<sup>5</sup>In India, the computer hardware industry is in its infancy and supplies only the domestic market.

<sup>6</sup>To grasp a first understanding of the nature of the trade-offs involved, in Section 3 we will conduct a preliminary analysis of the choice problem, under a very special assumption: the social discount rate is zero. Then in Section 4, we will return to the main model, where the social discount rate is strictly positive.

Moreover inequalities (2) and (3) imply

$$u'(\bar{q}) > p^*. \quad (5)$$

We further assume that

**Assumption A4:** The marginal cost at output  $q = 0$  when  $K = 0$  is lower than the marginal cost at output  $\bar{q}$  when  $K = \bar{K}$  :

$$c + \gamma\bar{K} < c + \beta\delta\bar{K} \quad (6)$$

Assumption A4 is equivalent to the following parameter restriction:

$$\beta\delta > \gamma \quad (7)$$

Clearly, due to Assumption A4, the marginal cost curve  $MC(q; K = \bar{K})$  intersects the horizontal line  $c + \gamma\bar{K}$  at the unique output level  $\tilde{q}$ , where

$$\tilde{q} = \frac{\gamma\bar{K}}{\beta} < \bar{q} \quad (8)$$

If the quantity of the manufactured good  $q(t)$  at time  $t$  is produced, the remaining amount of labour,  $\bar{L} - C(q(t), K(t))$ , is used to produce the numeraire good. Let  $m(t)$  denote the imports of manufactured goods. We assume that trade balance is required in every period: the country must pay for its imports,  $p^*m(t)$ , by exporting the numeraire good, i.e., the number of units of the numeraire good to be exported is exactly  $p^*m(t)$ . Then the economy's consumption of the numeraire good is

$$D_N(t) = \bar{L} - C(q(t), K(t)) - p^*m(t)$$

We assume that  $\bar{L}$  is sufficiently large, so that  $D_N(t)$  is positive at all  $t$ .

The flow of utility at time  $t$  is then

$$u(q(t) + m(t)) + [\bar{L} - C(q(t), K(t)) - p^*m(t)]$$

Social welfare is defined as the integral of the discounted flow of utility:

$$\int_0^\infty e^{-\rho t} \{u(q(t) + m(t)) + [\bar{L} - C(q(t), K(t)) - p^*m(t)]\} dt$$

where  $\rho$  is the rate of discount.



### 3. A preliminary analysis

In this section (and only in this section), we assume that the social discount rate is zero. This allows us to focus on comparing two feasible steady states (among many). One steady state is  $K^\infty = \bar{K}$ , with  $q^\infty = \bar{q} = \delta\bar{K}$  (we call this “the Scenario A”), and the other steady state is  $K^\infty = 0$ , with  $q = 0$ , i.e. there is no domestic industry (we call this “the Scenario B”).

The question we seek to address is whether a social planner would prefer to have a domestic industry with conditions as in Scenario A, or no industry at all and just resort to imports.

For expositional reason, let us begin with the knife-edge case where the world price  $p^*$  happens to be equal to  $c + \gamma\bar{K}$ . (Please refer to Figure 1). Let us compare (a) the steady-state welfare level if the planner maintains  $K$  at level  $\bar{K}$  by producing output level  $\bar{q} \equiv \delta\bar{K}$ , and consumption is set at  $D_M(p^*)$ , with (b) the steady-state welfare level if the planner maintains  $K$  at level 0 and domestic output is zero, and consumption is set at  $D_M(p^*)$ . Clearly, given Assumptions A1 to A4, the welfare under scenario (a) is lower than the welfare under scenario (b) if and only if the area of the triangle  $T_1$  in Figure 1 is greater than the area of the triangle  $T_2$ . Note that, when  $K = \bar{K}$ , the area  $T_2$  is the cost-saving obtained by producing the quantity  $\tilde{q}$  rather than importing that quantity, while the area  $T_1$  is the excess cost incurred by society for producing the additional amount  $(\bar{q} - \tilde{q})$  instead of importing it. Now, the condition  $T_1 > T_2$  is equivalent to

$$\frac{1}{2} \times (\beta\delta\bar{K} - \gamma\bar{K}) \times \left( \delta\bar{K} - \frac{\gamma}{\beta}\bar{K} \right) > \frac{1}{2} \times (\gamma\bar{K}) \times \left( \frac{\gamma}{\beta}\bar{K} \right)$$

which is in turn equivalent to the condition

$$\beta\delta > 2\gamma \tag{9}$$

Thus, if condition  $\beta\delta > 2\gamma$  holds, then in the knife-edge case where  $p^* = c + \gamma\bar{K}$ , a planner with a zero discount rate would judge that the society is strictly better off by having no industry producing the

manufacturing good at all (i.e.  $K = 0$ ) than by maintaining the industry at knowledge level  $\bar{K}$ . A similar reasoning shows that this ranking of steady states also holds if  $p^* \leq c + \gamma\bar{K}$  (as the cost-saving triangle will be even smaller, and the excess cost triangle will be even bigger).

By a similar argument, we can show that if condition  $\beta\delta > 2\gamma$  holds, then when  $p^* = c + \gamma\bar{K}$ , society is strictly better off by having no industry producing the manufacturing good at all (i.e.  $K = 0$ ) than by maintaining the industry at any knowledge level  $\hat{K}$  where  $0 < \hat{K} \leq \bar{K}$ .

**Remark 1:** Under condition  $\beta\delta > 2\gamma$ , there exists a unique level of world price,  $p^{**} > c + \gamma\bar{K}$ , such that, given that the discount rate is zero, society is indifferent between (i) having no industry producing the manufacturing good at all (i.e.  $K = 0$ ), and (ii) maintaining the industry at knowledge level  $\bar{K}$  by producing  $\bar{q}$ . In fact, let us represent  $p^{**}$  by

$$p^{**} = c + \gamma\bar{K} + \varepsilon\bar{K}$$

where  $\varepsilon > 0$ . Then the the cost-saving triangle and the excess cost triangle will be of equal size iff

$$(\gamma + \varepsilon)^2 = (\beta\delta - (\gamma + \varepsilon))^2$$

i.e.

$$\varepsilon = \frac{\beta\delta - 2\gamma}{2}$$

i.e.,

$$p^{**} = c + \gamma\bar{K} + \left(\frac{\beta\delta - 2\gamma}{2}\right)\bar{K}$$

This indicates that if the world price  $p^*$  is sufficiently high, it is worthwhile to support the infant industry.

**Remark 2:** When there is positive discounting,  $\rho > 0$ , we establish below that the counterpart of condition (9) is

$$\beta\delta > \left(\frac{2\delta + \rho}{\delta + \rho}\right)\gamma. \quad (10)$$

This inequality reduces to  $\beta\delta > 2\gamma$  as  $\rho$  tends to zero.

#### 4. The command and control scenario

We now return to our main model, where the social planner has a strictly positive discount rate,  $\rho > 0$ . The social planner chooses the time paths of  $q(t) \geq 0$  and  $m(t) \geq 0$  to maximize the discounted stream of utility of the representative consumer:

$$\int_0^{\infty} e^{-\rho t} \{u(q(t) + m(t)) + [\bar{L} - C(q(t), K(t)) - p^*m(t)]\} dt \quad (11)$$

subject to

$$\dot{K}(t) = q(t) - \delta K(t)$$

with  $K(0) = K_0 \geq 0$ .

Let  $\psi(t)$  denote the co-state variable. The Hamiltonian is

$$H = u(q + m) + \bar{L} - C(q, K) - p^*m + \psi(q - \delta K)$$

And the Lagrangian is

$$L = H + \mu m + \pi q$$

where  $\mu(t) \geq 0$  and  $\pi(t) \geq 0$  are the Kuhn-Tucker multipliers associated with the constraints  $m(t) \geq 0$  and  $q(t) \geq 0$ . The optimality conditions are

$$u' - p^* + \mu = 0, \text{ with } \mu \geq 0, m \geq 0, \mu m = 0 \quad (12)$$

$$u' - C_q + \psi + \pi = 0, \text{ with } \pi \geq 0, q \geq 0, \pi q = 0 \quad (13)$$

$$\dot{\psi} = (\delta + \rho)\psi + \frac{\partial C}{\partial K} = (\delta + \rho)\psi - \gamma q \quad (14)$$

$$\dot{K} = q - \delta K. \quad (15)$$

4.1. *Steady state with imports and domestic production*

We now consider the case where at a steady state both domestic output,  $q^\infty$ , and the volume of imports,  $m^\infty$ , are strictly positive.

**Lemma 1:** *Define  $\tilde{q} = \frac{p^* - c - \gamma\bar{K}}{\beta - \frac{2\delta + \rho}{\delta + \rho} \frac{\gamma}{\delta}}$ . Suppose that  $0 < \tilde{q} < \delta\bar{K}$  and that  $\tilde{q} < u'^{-1}(p^*)$ . Then the planner's problem (11) has a unique steady state  $(q^\infty, K^\infty)$  with the following properties:*

$$q^\infty = \tilde{q} \text{ and } K^\infty = \frac{\tilde{q}}{\delta} < \bar{K}$$

and

$$\psi^\infty = \frac{\gamma\tilde{q}}{\delta + \rho}.$$

The steady state level of imports is  $m^\infty$  such that

$$u'(m^\infty + \tilde{q}) = p^*.$$

**Proof:**

At a steady-state with  $m > 0$  we have from the Maximum Principle

$$u' - p^* = 0$$

and

$$u' - C_q + \psi = 0.$$

This implies

$$C_q = p^* + \psi^\infty$$

or

$$c + \gamma[\bar{K} - K^\infty] + \beta q^\infty = p^* + \psi^\infty$$

that is

$$c + \gamma(\bar{K} - K^\infty) + \beta\delta K^\infty = p^* + \frac{\gamma\delta K^\infty}{\delta + \rho}$$

which yields after simplification

$$q^\infty = \delta K^\infty = \frac{p^* - c - \gamma\bar{K}}{\beta - \frac{2\delta + \rho}{\delta + \rho} \frac{\gamma}{\delta}}.$$

The steady state level of imports is  $m^\infty$  such that

$$u'(m^\infty + q^\infty) = p^*.$$

Indeed since  $q^\infty < u'^{-1}(p^*)$  the above condition implies  $m^\infty = D_M - q^\infty > 0$  where  $D_M = u'^{-1}(p^*) = \text{constant} = \bar{D}_M$ . ■

**Remark 3:** A sufficient condition for the steady state level of capital to be positive is that

$$\beta - \frac{2\delta + \rho\gamma}{\delta + \rho} > 0 \text{ and } p^* - c - \gamma\bar{K} > 0$$

Note that  $p^* - c - \gamma\bar{K} > 0$  implies that the marginal cost of producing the first unit when the stock of capital is 0 is below the import price. Note also that as  $p^*$  falls towards the level  $c + \gamma\bar{K}$  we have  $K^\infty$  falls towards 0, and so does domestic production. This is intuitively plausible: if the import price is small enough it becomes less attractive to have a domestic industry.

**Lemma 2:** Under Assumptions A2 and A3, it is not possible for the planner's problem (11) to have a steady state with  $K^\infty > 0$  and  $m^\infty = 0$ .

**Proof:**

The steady-state capital stock can never exceed  $\bar{K}$ . Suppose that there were a steady state  $(q^\infty, K^\infty)$  with  $m^\infty = 0$  and  $0 < K^\infty \leq \bar{K}$ . Then

$$q^\infty = \delta K^\infty \leq \delta\bar{K} = \bar{q}$$

Then, due to  $u'' < 0$ , we have

$$u'(m^\infty + q^\infty) = u'(q^\infty) \geq u'(\bar{q}) > p^*$$

where the last inequality follows from (5). This violates the necessary condition (12). ■

Other cases imply zero domestic production and are therefore of no interest. They are omitted.

For the rest of the paper we focus on the case where inequality  $p^* - c - \gamma\bar{K} > 0$  holds, and add the following assumption:

**Assumption A5:** We have

$$\beta - \frac{2\delta + \rho}{\delta} \frac{\gamma}{\delta} > 0.$$

#### 4.2. Optimal transition dynamics

We characterize here the optimal domestic production path chosen by a planner.

**Proposition 1:**

Assume A1- A5, and  $p^* - c - \gamma\bar{K} > 0$ . Let  $\tilde{q} = \frac{p^* - c - \gamma\bar{K}}{\beta - \frac{2\delta + \rho}{\delta} \frac{\gamma}{\delta}}$ . Suppose that  $0 < \tilde{q} < \delta\bar{K}$  and  $\tilde{q} < u'^{-1}(p^*)$ . Then the optimal path of capital, domestic production and imports are given by

$$K^{so}(t) = (K_0 - K^\infty) e^{\lambda_1 t} + K^\infty$$

$$q^{so}(t) = \frac{K(0) - K^\infty}{Z} e^{\lambda_1 t} + q^\infty$$

$$m^{so}(t) = u'^{-1}(p^*) - q(t)$$

where

$$\lambda_1 = \frac{1}{2} \left( \rho - (2\delta + \rho) \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right) < 0$$

and

$$Z = \frac{\beta}{2\gamma} \left( 1 + \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right) > 0.$$

**Proof:**

The necessary conditions from the Maximum Principle are:

$$u' - C_q + \psi = 0$$

$$\dot{\psi} = (\delta + \rho)\psi + \frac{\partial C}{\partial K} = (\delta + \rho)\psi - \gamma q$$

$$\dot{K} = q - \delta K$$

Replace  $u'$  by  $p^*$  yields

$$C_q - p^* = \psi$$

and

$$C_{qq}\dot{q} + C_{qK}\dot{K} = \dot{\psi}$$

Therefore

$$C_{qq}\dot{q} + C_{qK}\dot{K} = (\delta + \rho)\psi - \gamma q$$

or

$$C_{qq}\dot{q} = (\delta + \rho)(C_q - p^*) - \gamma q - C_{qK}(q - \delta K)$$

The transition path is therefore solution to the following system of differential equations:

$$\begin{aligned} C_{qq}\dot{q} &= (\delta + \rho)(C_q - p^*) - \gamma q - C_{qK}(q - \delta K) \\ \dot{K} &= q - \delta K \end{aligned}$$

Substituting  $C$  gives

$$\begin{aligned} \beta\dot{q} &= (\delta + \rho)\beta q - (\rho + 2\delta)\gamma K + (\delta + \rho)(c + \gamma\bar{K} - p^*) \\ \dot{K} &= q - \delta K \end{aligned}$$

or

$$\begin{aligned} \dot{K} &= q - \delta K \\ \dot{q} &= (\delta + \rho)q - \frac{(\rho + 2\delta)\gamma}{\beta}K + \frac{(\delta + \rho)}{\beta}(c + \gamma\bar{K} - p^*) \end{aligned}$$

$$\begin{pmatrix} \dot{K} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} -\delta & 1 \\ -\frac{\gamma(\rho+2\delta)}{\beta} & (\rho + \delta) \end{pmatrix} \begin{pmatrix} K \\ q \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{(\delta+\rho)}{\beta}(c + \gamma\bar{K} - p^*) \end{pmatrix} \quad (16)$$

One can check that when

$$q^\infty = \delta K^\infty = \frac{p^* - c - \gamma\bar{K}}{\beta - \frac{2\delta+\rho}{\delta+\rho}\frac{\gamma}{\delta}}$$

we have the following equality

$$(\delta + \rho)q^\infty - \frac{(\rho + 2\delta)\gamma}{\beta}K^\infty + \frac{(\delta + \rho)}{\beta}(c + \gamma\bar{K} - p^*) = 0$$

Indeed

$$(\delta + \rho)\frac{p^* - c - \gamma\bar{K}}{\beta - \frac{2\delta + \rho}{\delta}\gamma} - \frac{(\rho + 2\delta)\gamma}{\beta\delta}\frac{p^* - c - \gamma\bar{K}}{\beta - \frac{2\delta + \rho}{\delta}\gamma} + \frac{(\delta + \rho)}{\beta}(c + \gamma\bar{K} - p^*) = 0.$$

The system (16) can thus be rewritten as

$$\begin{pmatrix} \dot{K} \\ \dot{q} \end{pmatrix} = A \begin{pmatrix} K - K^\infty \\ q - q^\infty \end{pmatrix}$$

where

$$A = \begin{pmatrix} -\delta & 1 \\ -\frac{\gamma(\rho + 2\delta)}{\beta} & (\rho + \delta) \end{pmatrix}$$

The matrix  $A$  has two eigenvalues

$$\lambda_{1,2} = \frac{1}{2}\rho \mp \frac{(2\delta + \rho)}{2} \sqrt{1 - \frac{4\gamma}{(2\delta + \rho)\beta}} \leq 0$$

Note that Assumption A5 implies  $1 - \frac{4\gamma}{(2\delta + \rho)\beta} > 0$  and therefore both  $Z$  and  $\lambda_{1,2}$  are real. Indeed Assumption A5 can be rewritten as

$$\frac{\delta + \rho}{2\delta + \rho}\delta > \frac{\gamma}{\beta}$$

and therefore

$$\frac{4\gamma}{(2\delta + \rho)\beta} < \frac{4\delta(\delta + \rho)}{(2\delta + \rho)^2} < 1.$$

We can now compute the optimal path of production and the associated path of the stock of knowledge. Convergence to the steady state implies that

$$\begin{pmatrix} K(t) - K^\infty \\ q(t) - q^\infty \end{pmatrix} = \alpha \begin{pmatrix} Z \\ 1 \end{pmatrix} e^{\lambda_1 t}$$



where  $\alpha = \frac{K(0) - K^\infty}{Z}$  and where  $Z$  is such that  $\begin{pmatrix} Z \\ 1 \end{pmatrix}$  is an eigenvector associated with  $\lambda_1$ ; we have

$$Z = \frac{\beta}{2\gamma} \left( 1 + \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right) \blacksquare$$

Note that the optimal production and import paths can be written in a feedback form with  $q^{so}(t) = \phi^{so}(K(t))$  and  $m^{so}(t) = \eta^{so}(K(t))$  where

$$\phi^{so}(K) = \frac{K - K^\infty}{Z} + q^\infty$$

and

$$\eta^{so}(K) = D_M(p^*) - q(K)$$

with  $Z > 0$ . Note that

$$\psi = -(p^* - C_q(q, K))$$

and therefore it can be given the feedback representation

$$\psi^{so}(K) = -(p^* - C_q(q(K), K)).$$

## 5. Implementing the planner's first best by trade policy

Suppose the domestic industry consists of a single firm, a domestic monopoly. In this case, the planner can achieve the social optimum by declaring free trade (i.e. consumers are free to import as much as they like at the price  $p^*$ ), where  $q^\infty < u'^{-1}(p^*)$ . Indeed, then, the monopolist is free to maximize profit under the constraint that she cannot charge a price greater than  $p^*$ .

We argue that the monopolist's optimal program corresponds to  $q^{so}(t)$  for all  $t \geq 0$ . We first note the following facts. First, the monopolist cannot charge a price greater than  $p^*$ . Second, at the price  $p^*$ , consumers are willing to buy from the domestic firm any quantity

$q$  that it supplies, as long as  $q < D_M(p^*)$ . Third, if the firm wants to sell more than  $D_M(p^*)$ , it will have to set a price lower than  $p^*$ . Fourth, the firm has no interest in selling at a price less than  $p^*$ , since its marginal cost of producing any quantity greater than  $D_M(p^*)$  is higher than  $p^*$ . From these considerations, the optimization problem of the monopolist becomes

$$\max_{q \geq 0} \int_0^{\infty} e^{-\rho t} [p^* q(t) - C(q(t), K(t))] dt$$

subject to

$$\dot{K} = q - \delta K$$

where  $K(0) = K_0$ . The Hamiltonian is

$$H = p^* q(t) - C(q(t), K(t)) + \psi(q - \delta K)$$

And the necessary conditions include

$$p^* - C_q(q, K) + \psi \leq 0 \quad (= 0 \text{ if } q < 0)$$

$$\dot{\psi} = (\delta + \rho)\psi + C_K$$

These are exactly the same as in the social planner's problem. So the solution is exactly the same as the planner's.

While this replication of the social optimum by decentralised market is a positive result, it is not straightforward to extend it to the case where the industry is oligopolistic. Indeed, in that case, we would have a differential game between oligopolists facing a joint constraint. In the next section we examine an alternate policy: unitary production incentive, e.g. per unit subsidy to production. Indeed intervention can be needed in the case of an oligopoly even more so when there are knowledge spillovers, for example when the accumulated knowledge is a public good. For ease of exposition we will consider the case of a duopoly, however the results easily extend to the case of an oligopoly with  $n > 2$  firms.

## 6. The case of a duopoly

The domestic industry consists of two firms, and  $K$  is a public good, i.e.,

$$\dot{K} = q_1 + q_2 - \delta K$$

Assume that

$$C_1(q_1, K) = [c + \gamma \max(0, \bar{K} - K(t))] q_1(t) + \beta q_1(t)^2$$

$$C_2(q_2, K) = [c + \gamma \max(0, \bar{K} - K(t))] q_2(t) + \beta q_2(t)^2$$

This assumption ensures that the industry's cost of producing the aggregate quantity  $q$  is the same as under the preceding sections. Indeed,

$$\begin{aligned} C(q, K) &= \min_{q_1 \in [0, q]} [C_1(q_1, K) + C_2(q - q_1, K)] \\ &= [c + \gamma \max(0, \bar{K} - K(t))] q + \frac{\beta}{2} q^2 \end{aligned}$$

i.e., the industry's marginal cost curve is the horizontal sum of the firms' marginal cost curves. Given  $K$ , to produce any quantity  $q$  at the lowest cost, each firm must produce the quantity  $q_i = q/2$ , and the marginal cost of each firm at that output level is

$$[c + \gamma \max(0, \bar{K} - K(t))] + 2\beta q_i = [c + \gamma \max(0, \bar{K} - K(t))] + \beta q$$

where  $q_i = q/2$ .

Now, because of the public-good nature of  $K$ , the government must give the duopolists an output subsidy, at the rate  $s(K)q_i$ . Then firm  $i$  maximizes

$$\max \int_0^\infty e^{-\rho t} [(p^* + s(K))q_i(t) - C_i(q_i(t), K(t))] dt \quad (17)$$

subject to

$$\dot{K} = q_i + q_j - \delta K \quad (18)$$

and  $K(0) = K_0$ . We argue below that there exists a continuum of feedback subsidy scheme  $s(K)$  (per unit of output) such that at each moment the production of each firm corresponds to the socially optimal production .

**Proposition 2:**

*A subsidy scheme  $s(K)$  that solves the following differential equation*

$$s'(K) \left( \frac{\phi^{so}}{2} - \delta K \right) - \left( \rho + \delta - \frac{\phi^{so'}}{2} \right) s(K) = \psi^{so}(K) \left( \frac{\phi^{so'}}{2} - \delta \right) \quad (19)$$

*leads to a successful decentralization of the social optimum as a Markov Perfect Nash Equilibrium of the duopoly game between the domestic producers facing imports at a price  $p^*$ .*

**Proof:**

Firm  $i$  chooses a strategy  $\phi_i(K)$  that is  $q_i(t) = \phi_i(K(t))$  for all  $t \geq 0$ . Given a subsidy scheme  $s(K)$  and firm  $j$ 's strategy  $\phi_j$  firm  $i$  chooses  $\phi_i$  to maximize (17) subject to (18).

We look for a  $s(K)$  such that  $q_i$  corresponds to the planner's production path, which satisfies

$$\begin{aligned} p^* - C_q(q, K) + \psi &= 0 \\ \dot{\psi} &= (\delta + \rho)\psi + C_K \end{aligned}$$

The Hamilton Jacobi Bellman (HJB) equation associated with firm  $i$ 's problem is

$$\rho V_i(K) = \text{Max}_{q_i} \{ (p^* + s(K))q_i - C_i(q_i, K) \} + V_i'(K) (q_i + \phi_j - \delta K)$$

The first order condition gives

$$p^* + s(K) - C_{iq_i} + V_i'(K) = 0 \quad (20)$$

Substituting  $q_i$  by  $\phi_i$  solution to (20) into the HJB and using symmetry we have

$$\rho V(K) = (p^* + s(K))\phi - C_i(\phi, K) + V'(K) (2\phi - \delta K) \quad (21)$$

where  $V_i(K) = V(K)$  and  $\phi_i = \phi$  for  $i = 1, 2$ .

Our goal is to design  $s(K)$  such that the solution  $V$  to (21) results in  $\phi_i$  being a solution to (20) that replicates the socially optimal production path. Observe that  $C_{iq_i}(q_i(t), K(t)) = C_q(q, K)$  at the first best and that  $C_{iK} = -\gamma q_i$  and  $C_K = -\gamma q$ .

Therefore from equation (20)  $s(K)$  induces the social optimum if

$$(p^* + s(K)) - C_q(\phi^{so}, K) + V'(K) = 0 \quad (22)$$

where  $\phi^{so}(K)$  is the socially optimal industry's production policy. Recall that at the social optimum the industry's production satisfies

$$\begin{aligned} p^* - C_q(q, K) + \psi &= 0 \\ \dot{\psi} &= (\delta + \rho)\psi + C_K \end{aligned}$$

Using the feedback form of the shadow price, condition (22) is satisfied iff

$$s(K) = \psi(K) - V'(K). \quad (23)$$

This condition is intuitive: the subsidy must correct for the difference between the social and the private shadow value of the stock of knowledge.

Requiring that  $\frac{\phi^{so}}{2}$  is the solution to the maximization problem of each firm yields

$$\rho V(K) = (p^* + s(K)) \frac{\phi^{so}}{2} - C_i\left(\frac{\phi^{so}}{2}, K\right) + V'(K) \left(2 \frac{\phi^{so}}{2} - \delta K\right)$$

Taking the derivative with respect to  $K$  and using the envelop theorem gives

$$\rho V'(K) = s'(K) \frac{\phi^{so}}{2} - C_{iK}\left(\frac{\phi^{so}}{2}, K\right) + V'(K) \left(\frac{\phi^{so}}{2} - \delta\right) + V''(K) \left(2 \frac{\phi^{so}}{2} - \delta K\right)$$

Using the fact that  $C_{iK}\left(\frac{\phi^{so}}{2}, K\right) = C_K(\phi^{so}, K)$ , we obtain

$$\rho V'(K) = s'(K) \frac{\phi^{so}}{2} - C_K(\phi^{so}, K) + V'(K) \left(\frac{\phi^{so}}{2} - \delta\right) + V''(K) \left(2 \frac{\phi^{so}}{2} - \delta K\right)$$

At the social optimum we have

$$\dot{\psi} = (\delta + \rho)\psi + C_K$$

that is

$$C_K = \psi^{so'}(K) \dot{K} - (\delta + \rho)\psi^{so}$$

$$\rho V'(K) = s'(K) \frac{\phi^{so}}{2} - \left( \psi^{so'}(K) \dot{K} - (\delta + \rho)\psi^{so} \right) + V'(K) \left( \frac{\phi^{so'}}{2} - \delta \right) + V''(K) \left( 2 \frac{\phi^{so}}{2} - \delta K \right)$$

Using  $\dot{K} = 2 \frac{\phi^{so}}{2} - \delta K$ , we get

$$0 = s'(K) \frac{\phi^{so}}{2} + (\rho + \delta) (\psi - V'(K)) + V'(K) \left( \frac{\phi^{so'}}{2} \right) + (V''(K) - \psi'(K)) \left( 2 \frac{\phi^{so}}{2} - \delta K \right)$$

Using (23)

$$0 = s'(K) \frac{\phi^{so}}{2} + (\rho + \delta) s(K) + (\psi^{so}(K) - s(K)) \left( \frac{\phi^{so'}}{2} \right) - s'(K) \left( 2 \frac{\phi^{so}}{2} - \delta K \right)$$

Then

$$0 = -s'(K) \left( \frac{\phi^{so}}{2} - \delta K \right) + \left( \rho + \delta - \frac{\phi^{so'}}{2} \right) s(K) + \psi^{so}(K) \left( \frac{\phi^{so'}}{2} \right) \blacksquare$$

While Proposition 2 describes a family of efficiency-inducing subsidy policies, it is useful to focus on one of them. We now show that a member of this family is a (per unit) subsidy rule that is linear affine in the state variable,  $K$ . Recall that

$$\phi^{so}(K) = \frac{K - K^\infty}{Z} + q^\infty$$

and that

$$\psi^{so}(K) = -(p^* - C_q(\phi^{so}(K), K))$$

that is

$$\psi^{so}(K) = -(p^* - \gamma(\bar{K} - K) - \beta\phi^{so}(K))$$

where

$$Z = \frac{\beta}{2\gamma} \left( 1 + \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right).$$

Therefore there exists an affine subsidy scheme that induces the social optimum. We denote it by

$$s_a(K) = \sigma K + \alpha.$$

Clearly, we must determine the parameters  $\sigma$  and  $\alpha$ . Interestingly, we can show that  $\sigma$  is strictly positive. This implies that, under the efficiency-inducing linear affine subsidy rule, firms will receive more subsidy per unit of output as the industry's knowledge stock grows. We state this as Corollary 2.1.

**Corollary 2.1:** *The efficiency inducing subsidy rule  $s_a(K)$  is such that  $s'_a(K) = \sigma > 0$ : the subsidy increases as the industry grows. The incentive for firms to increase their production is the larger subsidy they will enjoy as knowledge capital accumulates.*

**Proof:**

We have from (19)

$$\sigma \left( \frac{\phi^{so}}{2} - \delta K \right) - \left( \rho + \delta - \frac{\phi^{sol}}{2} \right) (\sigma K + \alpha) = \psi(K) \left( \frac{1}{2Z} - \delta \right).$$

For this to hold for all  $K$  we must have

$$\sigma \left( \frac{1}{2Z} - \delta \right) - \sigma \left( \rho + \delta - \frac{1}{2Z} \right) = \left( -\gamma + \frac{\beta}{Z} \right) \left( \frac{1}{2Z} - \delta \right)$$

thus

$$\sigma \left[ \frac{1}{Z} - (2\delta + \rho) \right] = \gamma \left( \frac{\beta}{\gamma Z} - 1 \right) \left( \frac{1}{2Z} - \delta \right). \quad (24)$$

Recall that

$$Z = \frac{\beta}{2\gamma} \left( 1 + \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right)$$

or

$$\frac{1}{Z} = \frac{(2\delta + \rho)}{2} \left( 1 - \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right).$$

Thus

$$\frac{1}{Z} - (2\delta + \rho) = -\frac{(2\delta + \rho)}{2} \left( 1 + \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right) < 0 \quad (25)$$

Moreover, we also have  $\left(\frac{\beta}{\gamma Z} - 1\right) > 0$ , because

$$\frac{\beta}{\gamma Z} = 2 \left( 1 + \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right) > 1. \quad (26)$$

It follows from (24), along with inequalities (25) and (26), that  $\sigma > 0$  iff

$$\frac{1}{2Z} - \delta < 0$$

i.e. iff

$$\frac{1}{2Z} - \delta = \frac{(2\delta + \rho)}{4} \left( 1 - \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right) - \delta < 0 \quad (27)$$

Inequality (27) holds iff

$$\left( 1 - \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \right) < \frac{4\delta}{2\delta + \rho}$$

iff

$$1 - \frac{4\delta}{2\delta + \rho} < \sqrt{\left( 1 - \frac{4\gamma}{(2\delta + \rho)\beta} \right)} \quad (28)$$

Clearly, if  $\rho \leq 2\delta$ , then the left-hand side (LHS) of (28) is negative and (27) holds. We now show that the LHS of (28) is negative also



in the case where  $\rho > 2\delta$ . In this case, without loss of generality, let  $\rho = k\delta$  where  $k > 2$ . Then (28) holds iff

$$1 - \frac{4}{2+k} < \sqrt{\left(1 - \frac{4\gamma}{(2+k)\delta\beta}\right)} \text{ for all } k > 2$$

iff

$$\left(1 - \frac{4}{2+k}\right)^2 < \left(1 - \frac{4\gamma}{(2+k)\delta\beta}\right) \equiv \frac{(2+k)\delta\beta - 4\gamma}{(2+k)\delta\beta} \text{ for all } k > 2$$

iff

$$1 + \frac{16}{(2+k)^2} - \frac{8(2+k)}{(2+k)^2} < \frac{(2+k)\delta\beta - 4\gamma}{(2+k)\delta\beta} \text{ for all } k > 2. \quad (29)$$

From Assumption A5 and  $\rho = k\delta$ , we have

$$\beta\delta > \frac{(2+k)}{(1+k)}\gamma$$

or

$$\frac{(1+k)\beta\delta}{(2+k)} > \gamma$$

This implies that the RHS of (29) satisfies

$$\frac{(2+k)\delta\beta - 4\gamma}{(2+k)\delta\beta} > \frac{(2+k)\delta\beta - 4\frac{(1+k)}{(2+k)}\beta\delta}{(2+k)\delta\beta} = 1 - \frac{4(1+k)}{(2+k)^2}.$$

Therefore, for (29) to hold it is sufficient that

$$1 + \left(\frac{16}{(2+k)^2}\right) - \frac{8(2+k)}{(2+k)^2} < 1 - \frac{4(1+k)}{(2+k)^2} \text{ for all } k > 2.$$

This inequality holds for because

$$16 < 8(2+k) - 4(1+k) = 12 + 4k \text{ for all } k > 2.$$

This completes the proof. ■

The subsidy rate  $s_a(K)$  is an increasing function of the stock of knowledge. The scheme implies that the industry starts with a smaller subsidy in its infancy and gradually gets higher subsidy as the industry matures. This is reminiscent of an import rule that is a decreasing function of the stock of knowledge in the case of a monopoly. The result runs against the conventional wisdom of providing greater support to domestic industries while in their infancy and gradually reducing the support as they mature. The intuition behind the result is that since knowledge is a public good, the government must subsidize output to counter the tendency for underproduction of knowledge, and in the absence of a subsidy, firms' incentive to undercontribute to this public good is very strong at high levels of knowledge.

## 7. Conclusion

We have examined the nurturing of an infant industry that experiences downward shifts in the marginal cost curve as the country's stock of knowledge increases. In the case of a small open economy where the domestic industry faces the competition of a mature foreign market we have characterized the socially optimal production path. It turns out that it is optimal to start with large imports and reduce the level of imports as the industry's knowledge accumulated. We have examined the game between a regulator and a decentralized industry. When the industry consists of a monopoly, the planner can induce the socially optimal domestic production path by adopting free trade. When the industry is oligopolistic and there are knowledge spillovers, we have characterized a set of per unit subsidy schemes that induce firms to replicate the socially optimal production policy as outcome of a Markov perfect Nash equilibrium.

**Acknowledgements:** We would like to thank Murray C. Kemp and J. Peter Neary for comments and discussions. This research is supported by SSHRC and FRQSC.

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