Reexamination of Strategic Public Policies*

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Abstract

This paper attempts to reinterpret the familiar approach to strategic public policies from the viewpoint of inefficiencies involved in oligopoly where firms engage in Cournot competition. To this end, we introduce tools called "quasi-reaction functions" and "quasi-supply curves". These tools allow us to conduct analyses by using the standard partial-equilibrium diagram, i.e., the quantity-price plane. We can directly find the relationship between prices and quantities and hence easily deal with inefficiencies and policies to correct them. We specifically reexamine public policies related to mixed-oligopoly, excess entry, technology choices with free entry and exit, and foreign oligopoly.

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1 Introduction

The late twentieth century may be called the age of "strategic public policies", i.e., government intervention under strategic interactions among various types of private and public (or semi-public) agents. von Neuman and Morgenstern's seminal book (1944) and Nash (1951) provided us with theoretical tools to analyze strategic interactions among firms and governments. However, it is the concept of Selten's (1975) "subgame perfection" which really burst the later rapid development of the theory for strategic public policies. Spence (1977) and Dixit (1979,1980) urged economists to reorganize the traditional industrial organization and oligopoly theories with game theory, and Brander and Spencer (1984,1985) made trade theorists rush towards the so-called "strategic trade policies" under international oligopoly.

We have found that game theory is the most effective in exploring government intervention in oligopoly. On the other hand, however, we often face (often too) many difficulties in analysis. In fact, many of the recent papers on strategic public policies start with specific game-theoretic models, but end up with simulations using Mathematica and the like instead of completing the qualitative analysis in more general frameworks. Some pieces of "economic" intuition are provided, but they are mostly based on strategic aspects of interactions among players, such as "strategic competition for rents", often lacking perspective of the traditional economic theory.

When we begin studying economics, we learn three concepts of Pareto-efficiency in (i) consumption, (ii) production, and (iii) product-mix. Consumption efficiency means that the private marginal benefits of consumption should be equal across households. Production efficiency means that the private marginal costs should be equal across firms. And product-mix efficiency means that the social marginal benefits of consumption should be equal to the social marginal cost. Whenever a certain allocation has any inefficiency, a comparison with another allocation enables us to decompose the associated welfare gains and losses (at least conceptually) into greater or smaller inefficiency of each type. In this sense, the welfare effects of any policy including those towards oligopoly can and should be expressed in terms of any of the three types of inefficiency.

Furthermore, when we review the literature of strategic public policies, it is not often the case that Pigovian taxes and subsidies are fully available to the policy authorities. A government may be able to just regulate entry and exit of firms. There underlies an idea that the market cannot fully elicit socially desirable firms, which should be another type of inefficiency in the market.

In this paper, we attempt to reinterpret the familiar approach to strategic public policies from the viewpoint of inefficiencies involved in oligopoly where firms compete à la Cournot. Although the results themselves are not necessary novel, we specifically use tools called "quasi-reaction functions" (instead of the standard reaction functions) and "quasi-supply curves". These tools allow us to conduct analyses by using the standard partial-equilibrium diagram, i.e., the quantity-price plane. We can directly deal with the relationship between prices and quantities and hence easily handle economic surpluses. We specifically reexamine public policies related to mixed-oligopoly, excess entry, technology choices with free entry and exit, and foreign oligopoly.

We should mention that our quasi-reaction and quasi-supply function approach is the same in its essence with what Kiyono (1988) once called the "quasi-supply curve" in the quasi-Cournot oligopoly market or the so-called "Cournot-Ikema curve" devised by Ikema (1991) and further elaborated by Ishikawa (1996,1997).[1] In the present study, we explicitly illustrate production and product-mix inefficiencies in the standard partial-equilibrium diagrams.

The rest of the paper is organized as follows. Section 2 defines quasi-reaction functions and quasi-supply curves and examines Cournot-Nash equilibrium in the short run. We show that there are both production and product-mix inefficiencies with the aid of quasi-supply curves. Section 3 considers Cournot-Nash equilibrium in the long run (or, with free entry and exit). We first reexamine the excess entry theorem established by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) and then free entry equilibrium with multiple technologies. Section 4 applies quasi-supply curves to strategic trade policy under international oligopoly. We specifically consider tariffs. Section 5 concludes.

2 Cournot Oligopoly in a Closed Economy

Let us start with an oligopoly market in a closed economy with a set of active firms $\mathcal{N} = \{1, 2, \dots, n\}$ where the price of the good in question is denoted by p and the total output by X. Let $U(X)$ express the gross benefit or utility from consuming the good as much as X, and assume that the marginal benefit $U'(X)$ is decreasing in the consumption, i.e., $U''(X) < 0$. The inverse market demand function is given by $p = P_D(X)$. Since the demand price $P_D(X)$ is equal to the marginal benefit of consumption $U'(X)$, the law of decreasing marginal benefit ensures $P'_D(X) < 0$. The consumer surplus is then expressed by $S(X) = U(X) - P_D(X)X$.

Let us denote the output of firm $i (\in \mathcal{N})$ by x_i and its total cost function

[1] Kiyono (1988) discusses the pass-through, or, the change in the import and export prices associated with a change in the exchange rate. Ikema (1991) shows how to illustrate Cournot equilibrium on the quantity-price plane. Ishikawa (1997) deals with Stackelberg and Bertrand equilibria as well as Cournot equilibrium. Ishikawa (1996) depicts Cournot oligopsony on the quantity-price plane.

by $C_i(x_i)$. The profit is then expressed by

$$\tilde{\pi}^i(x_i, X_{-i}, t_i) = P_D(x_i + X_{-i})x_i - C_i(x_i) - t_i x_i, \tag{1}$$

where X_{-i} represents the aggregate output other than firm i, and t_i the specific tax on firm i's output. We impose the following assumption.

Assumption 1 *The marginal cost of all active firms is non-decreasing in the output, i.e., $C_i''(x_i) \geq 0$ for all $x_i \geq 0$ for $i \in \mathcal{N}$.*

2.1 Short-Run Cournot Equilibrium

In this subsection, we consider the short-run equilibrium where the number of active firms is fixed. We ignore the production tax for the moment. We specifically introduce new tools, the quasi-reaction function and the quasi-supply function, to explore Cournot equilibrium.

The short-run Cournot equilibrium requires each active firm to maximize its profit given the outputs chosen by the other firms, which implies the following first-order condition for profit maximization to hold.

$$0 = \frac{\partial \tilde{\pi}^i(x_i, X_{-i})}{\partial x_i} = P_D(X) + x_i P_D'(X) - C_i'(x_i), \tag{2}$$

where $X = x_i + X_{-i}$.

When we express the right-hand side with the function $\psi(x_i, X)$, Assumption 1 implies $\psi_{x_i}(x_i, X) = P_D'(X) - C_i''(x_i) < 0$, so that the implicit function theorem ensures that the best-response output of firm i is uniquely determined. It gives rise to what we may call the "generalized Cournot reaction function" or the ***quasi-reaction function,*** $r^i(X)$, which shows firm i's profit maximizing output against the total output X. It satisfies

$$r^{i\prime}(X) = -\frac{P_D'(X) + r^i(X)P_D''(X)}{P_D'(X) - C_i''(r^i(X))}.$$

Note that the firm's output is a strategic substitute to the others' in the usual sense if and only if $r^{i\prime}(X) < 0$.[2]

Then the equilibrium total output in the Cournot-Nash equilibrium, X^e, is a solution to

$$X^e = \sum_{i \in \mathcal{N}} r^i(X^e).$$

[2] When we express firm i's reaction function in terms of the aggregate output by the other firms with $\gamma^i(X_{-i})$, it is a solution to $\partial \pi^i\left(\gamma^i(X_{-i}), X_{-i}\right)/\partial x_i = 0$. This reaction function is well-defined when the profit function is strictly concave in the own output, which we assume here. Then firm i's output is a strategic substitute to the others' if and only if $\gamma^{i\prime}(X_{-i}) < 0$. The condition is equivalent to $P_D'\left(\gamma^i(X_{-i}) + X_{-i}\right) + \gamma^i(X_{-i})P_D''\left(\gamma^i(X_{-i}, t_i) + X_{-i}\right) < 0$, which is further equivalent to $r^{i\prime}(X) < 0$ with Assumption 1.

The associated equilibrium output by firm i, which we express with x_i^e, is determined by its quasi-reaction function, i.e.,

$$x_i^e = r^i\left(X^e\right).$$

There is an alternative approach to capture the Cournot equilibrium, which we call the ***quasi-supply curve*** approach. The first-order condition for profit maximization, (2), shows the price (net of taxes) required for the firm to produce a certain assigned (equilibrium) output, which we call the ***quasi-supply price*** of the firm, i.e., $C_i'(x_i) - x_i P_D'(X)$. Since it depends not only on its output but also on the total output, we express it by $v^i(x_i, X)$, i.e.,

$$v^i(x_i, X) = C_i'(x_i) - x_i P_D'(X).$$

One should note that the difference between the market price and the quasi-supply price is equal to the specific production tax.

Also note that this quasi-supply price is fully compatible with the standard concept of the supply price in perfect competition, once we notice that the supply price of a price-taking firm is just equal to its marginal cost and the firm does not demand any rent over the marginal cost.[3] In the Cournot oligopoly, there is an additional second term which shows the average rent demanded by the firm to produce the output in question. Therefore, the market power as a cause for each firm to earn oligopoly rents raises each firm's supply price relative to perfect competition.

Only for the technical reason to define the quasi-supply price for $X = 0$, we assume

Assumption 2 $\lim_{X \to +0} \left|P_D'(X)\right| < +\infty.$

When the demand function is iso-elastic in the form of $P_D(X) = X^{-\frac{1}{\varepsilon}}$, there holds $\lim_{X\to+0} P_D'(X) = -\infty$ which causes a problem to define the quasi-supply price. Since we are interested in illustrating distortions under oligopoly, we impose Assumption 2. In view of this assumption, whenever we speak of iso-elastic demand functions, we mean that the demand is iso-elastic only over the prices relevant for the analysis.

Now let us find out the Cournot equilibrium. Assuming away taxes and subsidies, each firm's supply price should be equalized in equilibrium. Let

[3] Let λ_i denote the (quasi-)conjectural variations of firm i which shows the increase in the total output expected when it increases its own output by one unit. Then the relevant first-order condition for profit maximization is given by

$$0 = P_D(X) + \lambda_i x_i P_D'(X) - C_i'(x_i) - t_i,$$

which implies that firm i's quasi-supply price v^i is equal to $C_i'(x_i) - \lambda_i x_i P_D'(X)$. Since $\lambda_i = 0$ for a price-taking firm and $\lambda_i = 1$ for a Cournot oligopolist, the quasi-supply price for a price-taker is obtained by setting $\lambda_i = 0$.

v denote the equalized supply price. Then we can solve $v = v^i(x_i, X)$ with respect to the individual firm's output, i.e., $x_i = S^i(v, X)$, and call it firm i's ***quasi-supply function***. It satisfies

$$S^i_v(v, X) = \frac{1}{C''_i(x_i) - P'_D(X)}, \tag{3}$$

$$S^i_X(v, X) = \frac{x_i P''_D(X)}{C''_i(x_i) - P'_D(X)}. \tag{4}$$

The output decisions by the firms should be consistent in the industry as a whole, i.e.,

$$X = \sum_{i \in \mathcal{N}} S^i(v, X). \tag{5}$$

Solve this for v as a function of X, which we express with $v_S(X)$. It gives the supply price for the industry to produce the total output X, and we call it the ***industry quasi-supply price*** function, which is now a function of only the total output.[4] Its property is easily captured by summing the quasi-supply prices over the industry, i.e.,

$$N_\nu = \sum_{i \in \mathcal{N}} C'_i\left(S^i(v, X)\right) - XP'_D(X).$$

Then the implicit function theorem implies

$$v'_S(X) = \frac{1 - \sum_{i\in\mathcal{N}} \frac{x_i P''_D(X)}{C''_i(x_i) - P'_D(X)}}{\sum_{i\in\mathcal{N}} \frac{1}{C''_i(x_i) - P'_D(X)}} = \frac{1 - \sum_{i\in\mathcal{N}} r^{i\prime}(X)}{\sum_{i\in\mathcal{N}} \frac{1}{C''_i(x_i) - P'_D(X)}} + P'_D(X). \tag{6}$$

The associated quasi-supply price curve can be either upward-sloping or downward-sloping. In fact, it is straightforward to establish

Lemma 1 *Suppose that the marginal cost is constant for all the active firms. Then the following hold:*

1. $v'_S(X) = 1/N > 0$ *if the demand function is linear with a form of* $P_D(X) = \bar{p} - X$.

2. $v'_S(X) = P'_D(X)/\varepsilon N < 0$ *if the demand is iso-elastic with a form of* $P_D(X) = X^{-\frac{1}{\varepsilon}}$ *where* ε *is a positive constant.*[5]

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[4] In Kiyono (1988), a function $P_S(X) = P_D\left(\sum_{i\in\mathcal{N}} r^i(X)\right)$ is called the quasi-supply function.

[5] Note $v'_S(X) = \{-P'_D(X)/N\}\{1 + XP''_D(X)/P'_D(X)\} = P'_D(X)/\varepsilon N < 0$, for $d\ln|P'_D(X)|/d\ln X = -1/\varepsilon - 1$.

The equilibrium total output X^e should then satisfy

$$P_D(X^e) = v_S(X^e).$$

We can delineate the equilibrium by using Figure 1. In the figure, the industry quasi-supply price curve is shown by the upward-sloping curve $v_S v'_S$,[6] which cuts the market demand curve DD' at point E. This point E represents the equilibrium with the total output X^e and the market price p^e. When we assume that the industry consists of two firms, L and H. Firm i's quasi-supply price, $v^i(x_i, X)$, depends not only on its own output but also on the total output.[7] Given the equilibrium total output, its quasi-supply price curve as a function of its own output is shown by the curve $c_i v_i (i = L, H)$ where firm L's output is measured rightward from the origin $O_L(= O)$ and firm H's leftward from the origin $O_H(= X^e)$. Since the quasi-supply prices should be equal for the two firms at the equilibrium, the two quasi-supply price curves cross each other at point E' the height of which is just equal to the market price p^e. Firm L produces x_L^e and firm H x_H^e.

2.2 Short-Run Stability of Cournot Equilibria

It is standard to assume that the short-run output adjustment is subject to the following process:

$$\dot{x}_i = r^i(X) - x_i,$$

where $r^i(X) = r^i(X, 0)$ represents firm i's quasi-reaction function with $t_i = 0$. Summing these output adjustments over the industry, we obtain

$$\dot{X} = \sum_{i \in \mathcal{N}} r^i(X) - X.$$

The Cournot equilibrium with the total output X^e satisfying $X^e = \sum_{i \in \mathcal{N}} r^i(X^e)$, if it ever exists, is then stable if there holds

Assumption 3 $1 > \sum_{i \in \mathcal{N}} r^{i\prime}(X)$ *for all* $X \geq 0$.

What is the counterpart for this stability condition in our quasi-supply curve approach? In fact, given Assumption 1 we can show the equilibrium expressed with the demand and quasi-supply curves is stable in the Marshallian sense, i.e., $v'_S(X) > P'_D(X)$ if and only if it is stable in the sense of

[6] The same argument also applies to the case in which the industry quasi-supply curve is downward-sloping as far as the equilibrium is stable in the sense of Cournot, as is suggested by the discussion below.

[7] The firm quasi-supply price curve corresponds to Cournot-Ikema curve in Ikema (1991) and the monopoly-equilibrium curve in Ishikawa (1997). The industry quasi-supply price curve corresponds to the oligopoly-equilibrium curve in Ishikawa (1997).

Cournot.[8] In fact, (6) implies

$$v_S'(X) - P_D'(X) = \frac{1 - \sum_{i\in\mathcal{N}} r^{i\prime}(X)}{\sum_{i\in\mathcal{N}} \frac{1}{C_i''(x_i) - P_D'(X)}}, \tag{7}$$

so that in view of Assumption 1 $v_S'(X) > P_D'(X)$ if and only $1 > \sum_{i\in\mathcal{N}} r^{i\prime}(X)$.

Lemma 2 *The short-run Cournot equilibrium is stable in the Marshallian sense, i.e.,* $v_S'(X) > P_D'(X)$ *if and only if it is stable in the Cournot sense, i.e.,* $1 > \sum_{i\in\mathcal{N}} r^{i\prime}(X)$.

There is one remark in order here as regards the so-called "Cournot-Ikema" curve developed by Ikema (1991) and Ishikawa (1996,1997) or what Kiyono (1988) once called the "quasi-supply curve" in the quasi-Cournot oligopoly market. They are basically defined by $P_S(X) = P_D\left(\sum_{i\in\mathcal{N}} r^i(X)\right)$. Even with this formulation, the Marshallian output adjustment process is expressed by

$$\dot{X} = P_D(X) - P_S(X).$$

The associated stability condition is given by

$$0 > P_D'(X) - P_S'(X) = P_D'(X)\left\{1 - \sum_{i\in\mathcal{N}} r^{i\prime}(X)\right\},$$

which is equivalent to Cournot stability condition $1 > \sum_{i\in\mathcal{N}} r^{i\prime}(X)$. However, Ikema (1991) and Ishikawa (1997) do not elucidate the equivalence between the Marshallian and Cournot adjustment processes, especially for the case of the downward-sloping "original" quasi-supply curve.[9] Furthermore, as we see below, the present approach is more useful for demonstrating the production inefficiency in particular.

2.3 Social Optimum

Social welfare, $\widetilde{W}(\boldsymbol{x})$, is the sum of the consumer, producer, and government surpluses, i.e., $S(X) + \sum_{i\in\mathcal{N}} \tilde{\pi}^i(x_i, X_{-i}, t_i) + G(\boldsymbol{t}, \boldsymbol{x})$. The government surplus is given by $G(\boldsymbol{t}, \boldsymbol{x}) = \sum_{i\in\mathcal{N}} t_i x_i$, where $\boldsymbol{t} = (t_1, \ldots, t_n)$ denotes the tax vector, and $\boldsymbol{x} = (x_1, \ldots, x_n)$ the output vector:

$$\widetilde{W}(\boldsymbol{x}) = U\left(\sum_{i\in\mathcal{N}} x_i\right) - \sum_{i\in\mathcal{N}} C_i(x_i).$$

[8] The Marshallian adjustment process is given by $\dot{X} = P_D(X) - v_S(X)$ within the present framework. Then the equilibrium, if it ever exists, is stable if $P_D'(X) < v_S'(X)$.

[9] Ishikawa (1997) deals with the stability of a Cournot equilibrium but does not investigate the equivalence between the Cournot stability and the Marshallian stability. Kiyono (1988) discusses such equivalence to a limited extent.

Given the set of active firms $\mathcal{N}$, the socially optimal output vector $\boldsymbol{x}^*$ $(= (x_1^*, \ldots, x_n^*))$ should maximize social welfare, and thus satisfies

$$0 = \frac{\partial \widetilde{W}(\boldsymbol{x}^*)}{\partial x_i} = P_D(X^*) - C_i'(x_i^*), \tag{8}$$

where use was made of $P_D(X^*) = U'(X^*)$ and $X^* \equiv \sum_{i\in\mathcal{N}} x_i^*$. For this condition to actually ensure the socially optimum, the following condition is sufficient.

$$P_D'(X) - C_i''(x_i^*) < 0 \text{ for all } X \geq 0,$$

for all firms in $\mathcal{N}$, which are viable at the first-best state. This second-order condition holds owing to Assumption 1.

The first-order condition in (8) elucidates the following two types of efficiency.

1. ***Production efficiency*** The marginal costs should be equal among firms in the industry.

2. ***Product-mix efficiency*** The market price should be equal to the social marginal cost, i.e., the equalized marginal costs in the industry.

Oligopoly jeopardizes these two efficiencies achieved in perfect competition due to the market power perceived by each active firm. Let us delineate the social costs caused by those inefficiencies below.

2.4 Short-Run Inefficiency in an Oligopoly Market

The Cournot oligopoly involves two familiar types of inefficiency, production inefficiency and product-mix inefficiency. With quasi-supply price curves, we can delineate these inefficiencies in Cournot oligopoly in Figure 1 where the market is served by two firms, L and H.

First, production inefficiency arises from the failure to minimize the total production costs in the industry given the total output. In the figure, the curve c_ic_i' represents the marginal cost of firm $i(\in \{L, H\})$. Equality of the quasi-supply prices between the firms implies

$$C_L'(x_L^e) - C_H'(x_H^e) = (x_L^e - x_H^e)\, P_D'(X^e),$$

so that the firm with the lower marginal cost should produce more than the other firm. The figures shows that the case in which firm L's marginal cost, $E''L$, is lower than firm H's, $E''H$. Then reshuffling the outputs between the two firms with taxes and subsidies realizes the gains from minimizing the total production costs as much as the area HLC. This is the loss from the production inefficiency at the Cournot equilibrium.[10]

Second, even after eliminating the production inefficiency, there still remains the product-mix inefficiency. Note that in the figure the curve $c_L J c_S$ the social marginal cost which measures the marginal costs equalized over the industry to produce each amount of the total output. Since the social marginal cost $X^e K$ is lower than the marginal benefit of consumption $X^e E$ measured by the height of the demand curve, an increase in the total output further enhances social welfare till they become equal at point B^*. The welfare increases as much as the area EKB^*, which gives the loss from the product-mix inefficiency in the Cournot equilibrium.

Proposition 1 *Given the distribution of firms, there are dual inefficiencies in the Cournot equilibrium, i.e., production inefficiency due to unequal marginal costs within the industry and product-mix inefficiency due to the market power effects of the firms, both of which can be eliminated by taxes and subsidies on the active firms.*

2.5 Inefficiency in Mixed Oligopoly

The discussion above can be applied to more general modes of output competition once we introduce (quasi-)conjectural variations. Let λ_i denote the conjectural variation of firm i showing how much the firm expects the total output to increase when it increases its own output by one unit. Assume that these conjectural variations are constant for all active firms. Then the first-order condition for profit maximization (2) is now rewritten as

$$0 = P_D(X) + \lambda_i x_i P_D'(X) - C_i'(x_i),$$

which implies that its quasi-supply price is given by

$$v^i(x_i, X, \lambda_i) = C_i'(x_i) - \lambda_i x_i P_D'(X).$$

When $\lambda_i = 0$, the firm does not perceive its market power (i.e., it behaves as a price taker), and its quasi-supply price is just equal to the marginal cost, the supply price in the usual sense.

A specific example for illustrating the use of conjectural variations is the mixed oligopoly in which there are two types of firms in the market, a public firm U seeking to maximize social welfare and a private firm R trying

[10] This production inefficiency is often used to derive several seemingly-counter-intuitive policy prescriptions such as Lahiri and Ono (1988), who show that subsidizing the firms with small market shares lowers social welfare. The smaller firms are small because their marginal costs are higher than the others and hence an expansion of their outputs aggravates the production inefficiency.

to maximize its profit.[11] Adapt Figure 1 to this mixed duopoly, and draw Figure 2.[12]

Since the public firm chooses its output so as to maximize the welfare, its marginal cost is equated with the market price, which means that its marginal cost curve C_uCu' is its quasi-supply price curve. On the other hand, the private firm has the marginal cost curve C_rC_r' and the quasi-supply price curve C_rv_r, where the latter is greater than the former due to the rent required for an oligopolist. The equilibrium for this mixed duopoly requires the public firm's marginal cost to be equal to the private firm's quasi-supply price at point E', i.e., at the equilibrium price p^e. Since the quasi-supply price of the private firm is greater than its marginal cost (given the total output), the equilibrium requires the public firm's marginal cost to be greater than the private firm's. The resulting production inefficiency is measured by the area ABE'.

Note that this welfare loss due to production inefficiency always exists insofar as the number of private firms is exogenously given.

Proposition 2 *Given the number of active private firms, there arises welfare loss from production inefficiency at the mixed oligopoly equilibrium in the sense that the public firm produces more than is socially desired for minimizing the total production costs.*

This proposition is the key to understand that any policy to increase the private firms' output and to decrease the public firm's may enhance social welfare. For example, partial privatization of the public firm makes its quasi-supply price greater than its marginal cost and induces its output reduction. The resulting decrease in the total output increases the loss from the product-mix inefficiency, but it may be overweighed by the smaller loss from the production inefficiency.

Complete privatization of the public firm instead may preclude the production inefficiency when it has the same cost conditions as the private. However, the resulting loss from the product-mix inefficiency may outweigh such a gain from the recovered production efficiency. That is, the society may be worse off than before the privatization.

[11] Although the problem was first addressed by De Fraja and Delbono (1989, 1990), an elegant discussion is made by Matsumura (1998).

[12] When there are many private firms, say firms $1, \ldots, n_r$, the private firm i's quasi-supply price should be equalized at, say v, which implies $v = C_i'(x_i) - x_i P_D'(X)$ (for all $i \in \mathcal{N}_r$), where $\mathcal{N}_r = \{1, \ldots, n_r\}$ represents the set of active private firms. Solve this for x_i and obtain $x_i = S^i(v, X)$. Then its aggregation over the private firms yields $X_r = \sum_{i \in \mathcal{N}_r} S^i(v, X)$. Solving this for v gives rise to the quasi-supply price curve of the private firms as a whole, i.e., $v_r(X)$. Then we can apply the same analysis in the text.

3 Market Failure in Selecting Viable Technologies

Thus far, we have discussed how to describe the short-run equilibrium given the number of active firms as well as the two types of inefficiency involved. These inefficiencies can be eliminated when production taxes and subsidies are available.

However, there is another policy tool, i.e., direct regulation of entry and exit. It should play an important role if the government cannot employ production taxes and subsidies for some reason or other. In fact, as we review later, the ***excess entry theorem*** applies to the Cournot oligopoly market with free entry and exit. The theorem claims that the number of firms in the free-entry equilibrium tends to be socially excessive.[13] This means that there is another type of inefficiency in oligopoly associated with the number of active firms.

Furthermore, we have already known from the preceding discussion that the short-run Cournot equilibrium allows active firms to have diversely-differed cost conditions. One should wonder whether free entry-exit pressure enables the market to select efficient technologies even when the government can employ neither taxes nor subsidies and cannot intervene in the strategic output decisions by active firms. It may cause still another type of inefficiency associated with technology selection by the market.[14]

Since these two inefficiencies are closely related with each other, we extend our theory underlying the quasi-supply curves for its full exploration. To elucidate them, we confine ourselves to the case in which the cost function is linear and identical across firms:

$$C(x_i) = cx_i + f,$$

where $c(> 0)$ is the constant marginal cost, and f the fixed cost. The pair of the marginal cost c and the fixed cost f fully captures the cost condition or production technology used by an active firm, so that we hereafter call it technology (c, f) and the firm using technology (c, f) "$(c, f)-$firm". We also denote its profit function by[15]

$$\widetilde{\pi}(x_i, X_{-i}, c, f) = P_D(x_i + X_{-i})x_i - cx_i - f.$$

3.1 Short-Run Equilibrium in Cournot Competition

We begin with the quasi-reaction function of an active firm. The first-order condition for profit maximization (2) is now rewritten as follows:

$$0 = \frac{\partial \widetilde{\pi}(x_i, X_{-i}, c, f)}{\partial x_i} = P_D(X) + x_i P'_D(X) - c. \tag{9}$$

[13] See Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) for the theorem.

[14] This is the inefficiency explored by Ohkawa et. al (2005) on which this section relies to a great extent.

[15] Since firms are identical, superscript i is dropped from $\widetilde{\pi}$.

Let $r(X,c)$ represent the quasi-reaction function of the firm with the marginal cost c. Unlike in Subsection 2.1, we can solve the above first-order condition for x_i and obtain the quasi-reaction function as follow:

$$r(X,c) = -\frac{P_D(X) - c}{P_D'(X)}. \tag{10}$$

For this best-response output to be well-defined especially for $X = 0$, we assume[16]

Assumption 4 *The demand function $p = P_D(X)$ satisfies either of the following conditions:*

1. $\lim_{X \to +0} P_D(X) < +\infty$ *and* $|\lim_{X \to +0} P_D'(X)| < +\infty$.
2. *The price elasticity of demand ε is a positive constant greater than unity.*

The quasi-reaction function satisfies

$$r_X(X,c) = -1 - \frac{r(X,c)P_D''(X)}{P_D'(X)}, \tag{11}$$

$$r_c(X,c) = \frac{1}{P_D'(X)} < 0. \tag{12}$$

Let us denote by $n(c,f)$ the number of active $(c,f)-$firms and by $\{n(c,f)\}$ the distribution of active $(c,f)-$firms. Then given this distribution of firms, the short-run equilibrium total output X^e is governed by[17]

$$X^e = \sum n(c,f) r(X^e, c).$$

One should note that, when we disregard the integer problem associated with the number of firms, there are numerous distribution of firms giving rise to the same equilibrium total output X^e.

[16] When the first condition is satisfied, the demand curve has a positive intercept with a finite slope. The typical example is a linear demand function. In the case of the second condition, (10) becomes $r(X,c) = \varepsilon \left(X - cX^{1-\frac{1}{\varepsilon}} \right)$. Either condition ensures $\lim_{X \to +0} r(X,c)$ to be well-defined.

[17] Summing (9) over the active firms at the equilibrium, we obtain $NP_D(X^e) + X^e P_D'(X^e) = \sum n(c,f)c$, where $N = \sum n(c,f)$ denotes the total number of active firms. We can use this condition to find out the equilibrium total output X^e. Note that the left-hand side is equal to $(N-1)P_D(X^e) + MR(X^e)$. Therefore, insofar as the industry marginal revenue $MR(X)$ is strictly decreasing in the total output, the equilibrium is unique if it ever exists.

The associated equilibrium profit is expressed by a function of the total output X and the technology used (c, f) as follows:

$$\pi^e(X^e, c, f) = -\frac{(P_D(X^e) - c)^2}{P'_D(X^e)} - f.$$

Let us express the (equilibrium) gross profit earned with

$$F(c, X) = -\frac{(P_D(X) - c)^2}{P'_D(X)}, \tag{13}$$

so that we can rewrite the above equilibrium profit function as follows:

$$\pi^e(X^e, c, f) = F(c, X^e) - f \tag{14}$$

The gross profit function $F(c, X)$ defined by (13) represents the maximum fixed cost which allows the firm with the marginal cost c to earn non-negative profits given the total output X. Hereafter we call it the ***viability function***. We also let $\mathcal{A}(X) = \{(c, f)|\ f \leq F(c, X)\}$, i.e., the set of technologies with which the firm earns non-negative profits given the total output X.

Any short-run equilibrium is sustainable only when active firms earn non-negative profits. That is, active firms use the technologies in $\mathcal{A}(X^e)$, which can be shown by (c, f) along or below the ***viability frontier*** VV' (associated with $f = F(c, X^e)$) in Figure 3.

Note that all the technologies in the region VOV' are not feasible in equilibrium. In fact, there is a constraint due to the feasible technologies. Such a constraint is given by the curve TT', the ***technology frontier***, which shows the minimum feasible fixed cost for each marginal cost. In fact, therefore, the observed viable firms use the technologies in the region VTB^{**} in Figure 3.

3.2 Free Entry Equilibrium with a Single Technology

The ***excess entry theorem*** shows that free entry in Cournot oligopoly entails a socially excessive number of active firms if and only if the outputs are mutually strategic substitutes. However, one should note that this result is demonstrated only for the case in which firms have the same cost conditions. The question here is what other inefficiency should arise in addition to this inefficiency due to excess entry when there are numerous types of cost conditions available. The above argument implies that it is the one associated with the market failure in selecting the firms with socially desired technologies. We now demonstrate this result.

We first focus our attention on the free-entry equilibrium. Then any active firm should earn zero profits, which implies that the market price should be equal to its average costs. Thus, unlike in the short-run equilibrium one cannot use the average costs for selecting the firms which are socially desired to increase in the number.

Then choose any active single technology (c, f), and consider the free-entry equilibrium with only this $(c, f)-$firms. Then the associate equilibrium number of firms $n^e(c, f)$ is determined by

$$
\begin{aligned}
f &= F(X^e, c) \\
n^e(c, f) &= \frac{X^e}{r(X^e, c)}.
\end{aligned}
$$

The excess entry theorem implies that the second-best number of firms $n^{SB}(c, f)$ should be smaller than $n^e(c, f)$ when the outputs are strategic substitutes. Let us characterize this $n^{SB}(c, f)$.

Proposition 3 *Suppose that all active firms have the same technology type (c, f). Then the number of firms in the free-entry equilibrium is greater than the second-best number of firms, i.e., $n^e(c, f) > n^{SB}(c, f)$ if and only if the outputs of active firms are mutually strategic substitutes.*

Although the original proof of this excess entry theorem employs a rigorous evaluation of the welfare changes associated with entry regulation, it is actually possible to give a simpler alternative proof by considering the industry total costs subject to the strategic output decision by active firms as follows.

Since all active firms use the same technology (c, f), given their strategic output decision described by the quasi-reaction function $r(X, c)$, the industry can produce the total output with the number of firms given by

$$n^r(X, c) = \frac{X}{r(X, c)}, \tag{15}$$

which we call the ***required number of active firms***. Then the associated total costs, i.e., the ***Industry Total Cost*** (ITC) with technology (c, f), is measured by

$$ITC(X, c, f) = cX + \frac{X}{r(X, c)} f.$$

Then we can further define the ***Industry Average Cost*** (IAC) and the ***Industry Marginal Cost*** (IMC) associated with the ITC. Noting $n^r(X, c) = X/r(X, c)$, we obtain

$$IAC(X, c, f) = \frac{ITC(X, c, f)}{X} = c + \frac{f}{r(X, c)}, \tag{16}$$

$$IMC(X, c, f) = \frac{\partial ITC(X, c, f)}{\partial X} = c + \frac{f}{r(X, c, f)} \left(1 - n^r(X, c) r_X(X, c)\right). \tag{17}$$

In view of Assumption 3, the second term on the right-hand side of (17) is strictly positive, so that the IMC is also strictly positive for all outputs given a strictly positive marginal cost c where the segment $\overline{Oc}$ in Figure 4 is equal to $IAC(0, c, f) = c + f/r(0, c)$. The two cost curves given technology (c, f) are shown in Figure 4.

There are two remarks in order here. First, as there holds $IAC(0, c, f) = IMC(0, c, f)$, the two cost curves have the same vertical intercept equal to the marginal cost c. Second, since there holds $ITC(X, c, f) = X \cdot IAC(X, c, f)$ by definition, there follows

$$IMC(X, c, f) = IAC(X, c, f) + X\frac{\partial IAC(X, c, f)}{\partial X}. \tag{18}$$

Then in view of (16) the industry average cost is strictly increasing in the total output if and only if the outputs of active firms are strategic substitutes, i.e., $r_X(X, c) < 0$. Since the average value is increasing along with the output if and only if the average value is smaller than the marginal one, there follows

Lemma 3 *The industry average cost curve and the industry marginal cost curves satisfy the following relations.*

1. $IAC(0, c, f) = IMC(0, c, f)$
2. *The following conditions are equivalent.*
   (a) $IAC(X, c, f) < IMC(X, c, f)$ *for all* $X > 0$.
   (b) $IAC_X(X, c, f) > 0$ *for all* $X > 0$.
   (c) $r_X(X, c) < 0$ *for all* $X > 0$.

The industry cost curves in Figure 4 are drawn for the case of strategic substitutes. One should note that these two industry cost curves are drawn by assuming that the government cannot intervene in any active firm's strategic output decisions but it can only regulate the number of active firms by entry regulation. Free entry and exit makes the price equal to the industry average cost, so that the intersection of the demand curve DD' with the IAC, i.e., point E, gives rise to the free-entry equilibrium with the price p^e and the total output X^e.

However, even if the government cannot directly affect the firms' output decisions by taxes and subsidies, it can change the total output by its direct control on entry and exit. The second-best total output thus achieved should equate the industry marginal cost with the market price, which is shown by the intersection of the demand curve with the IMC, i.e., point B with the price p^{SB} and the total output X^{SB}. Since the IMC is greater than the IAC, this second-best total output X^{SB} is strictly smaller than that in the free-entry equilibrium X^e.

How can we demonstrate that the government can enhance social welfare by entry regulation? For this purpose, one should note that the required number of active firms associated with X^{SB}, i.e., $n^r(X^{SB}, c) = X^{SB}/r(X^{SB}, c)$, is actually the second-best number of firms, $n^{SB}(c, f)$. The assumption of strategic substitutes implies that the individual output of active firms is greater at the second-best equilibrium, which further implies $\left(n^{SB}(c, f) =\right) n^r \left(X^{SB}, c\right) < n^e(c, f)$. The welfare gain from such an entry regulation is measured by the area BEE' in Figure 4.

3.3 Second-Best Choice of Technologies

What if there are enormous types of technologies available in the industry? As was already discussed, active firms may use many possible types of technologies subject to the social technology constraint expressed by the technology frontier shown in Figure 3. Even under the pressure of free entry and exit, several types of technologies can be viable as shown in Figure 5. As each firm should earn zero profits in the free-entry equilibrium using an available technology, the viable technologies expressed by the pairs of the marginal cost and the fixed costs should be found where the technology frontier TT' touches the viability frontier VV' drawn for the total output in the free-entry equilibrium X^e. In Figure 5, they are given by the points along the segment $\overline{F_1F_2}$, and point F_3.[18]

More generally, let us denote by $\mathcal{A}^e$ the set of technology (c, f) with which active firms earn zero profits in the free-entry equilibrium,[19] by $n^e(c, f)$ the number of $(c, f)-$firms in the free-entry equilibrium, and by $\{n^e(c, f)\}_{(c,f)\in\mathcal{A}^e}$ its distribution. Then any distribution of the number of firms $\{n^e(c, f)\}_{(c,f)\in\mathcal{A}^e}$ is compatible with the total output in the free-entry equilibrium X^e whenever the distribution satisfies

$$X^e = \sum_{(c,f)\in\mathcal{A}^e} n^e(c, f) r\left(X^e, c\right).$$

However, from the social point of view, it is better to make the firms with the lowest marginal cost dominate the market. Let us demonstrate this by comparing the second-best optimum given each of the two viable technologies (c_L, f_L) and (c_H, f_H) where $c_L < c_H$ and $f_L > f_H$. Hereafter, we call the technology (c_L, f_L) the lower marginal-cost technology and (c_H, f_H) the higher marginal-cost technology. The following lemma can be proved.[20]

---

[18]As discussed in Appendix A, an increase in total output shifts the viability frontier downward. Thus, the total output in the free-entry equilibrium is obtained by sliding the viability frontier along a change in total output and making it touch the technology frontier.

[19]By definition, any $(c, f) \in \mathcal{A}^e$ should satisfy $f = F(c, X^e)$.

[20]The proof is given in Appendix B.

Lemma 4 *$dIMC(X^e, c, F(c, X^e))/dc < 0$ if either of the following conditions holds.*

1. *$P_D''(X) \leq 0$ holds for all $X \geq 0$.*
2. *The demand function is iso-elastic with price elasticity ε, and the number of firms with technology (c, f) in the free-entry equilibrium satisfies $n^r(X^e, c) > 2\left(\frac{1}{\varepsilon} + 1\right)$.*

This lemma implies that the industry marginal cost at total output in the free-entry equilibrium X^e is greater for the lower marginal-cost technology than for the higher one as shown in Figure 6, given the conditions in Lemma 4.

In view of Lemma 3, the assumption of strategic substitution implies that each of the IAC curves, IAC_i for the technology $(c_i, f_i)(i = L, H)$, starts at c_i and it is strictly upward sloping. The two technologies are viable at the free-entry equilibrium, so that the two IAC curves should cross the demand curve at the same price p^e, the price in the free-entry equilibrium, which is shown by point E.

The figure also shows the IMC curves, IMC_i for technology (c_i, f_i). The discussion in the previous subsection implies that the second-best optimal total output given either technology is given by the intersection of the associated IMC with the demand curve, i.e., point B_i for technology (c_i, f_i). Social welfare at the second-best optimum is then measured by the area $D\bar{c}_iB_i$. Their comparison tells us that the welfare under the lower marginal-cost technology is greater than under the higher marginal-cost one as much as the area $A\bar{c}_L\bar{c}_H$ - the area AB_LB_H, the sign of which seems ambiguous at the first glance.

However, since the two technologies are equally viable at the free-entry equilibrium, the industry total cost to produce total output X^e should be the same, which implies that the area below the two IMC curves should also be the same. Therefore, the area $A\bar{c}_L\bar{c}_H$ is the same in size as the area AE_LE_H. But the area AE_LE_H is larger than the area AB_LB_H, so that social welfare at the second best optimum should be greater under the lower marginal-cost technology.[21]

Proposition 4 *Given any distinct viable technologies at the free-entry equilibrium, social welfare maximized by entry regulation is greater under the lower marginal-cost technology when either of the conditions in Lemma 4 holds.*

[21] Ohkawa et. al (2005) employ a different approach to establish that either (i) a concave demand function or (ii) strategic complementarity implies the assertion in Proposition 4. The present discussion is one of the possible alternative ways to characterize technology which should be chosen from the viewpoint of social welfare.

In view of Figure 6, when the government can observe the viable technologies and strategically select socially better viable technologies, it should choose only the firms with technology expressed by point F_3 in Figure 5 and regulate the entry of those firms. The market fails in general to choose those socially desirable technologies.

3.4 Much Better Technologies?

Even when the government can affect the active firm's strategic output decision only through the entry regulation, there may be much better feasible technologies. To demonstrate this possibility, let us consider the iso-welfare curve given the second-best entry regulations discussed in the previous section.

For this purpose, we first define the maximized social welfare given the technology (c, f) under the entry regulation, which is expressed by

$$W^{SB}(c, f) = \max_{\{X\}} \{U(X) - ITC(X, c, f)\}.$$

It is straightforward to find $W_c^{SB}(c, f) < 0$ and $W_f^{SB}(c, f) < 0$, so that the implicit function theorem allows us to define the iso-welfare curve associated with the welfare level W as follows:

$$f = \omega(c, W) = \{f | \, W^{SB}(c, f) = W\}.$$

Given the conditions in Lemma 4, we know that a decrease in the marginal cost along the viability frontier enhances social welfare. It implies that each iso-welfare curve, such as W_3^{**} and W_B^{**}, cuts the viability frontier from above as shown in Figure 7. The socially best technology given the strategic output decision by active firms is the one where the iso-welfare curve W_B^{**} touches the technology frontier TT', i.e., point B^{**}. In general, this socially best point B^{**} does not lie along the viability frontier but above it, which implies that the society is better off with inviable technologies under free entry and exit in oligopoly.

Proposition 5 *Given the strategic output decision by active firms, the socially best technology is not viable at the free-entry equilibrium in Cournot oligopoly.*

This result recalls us the well-known "infant industry protection" argument discussed by Negishi (1962). When the technology exhibits increasing returns to scale, the firms with the lower marginal cost but with the greater fixed cost hesitates to enter the market, for its entry leads to a great increase in the total output and a large drop in the market price causing a loss to the firm. From the social point of view, however, the resulting increase in the consumer surplus outweighs the firm's loss and hence its entry should be made. What is not explicitly referred to in Negishi (1962) is that such a second-best infant industry protection may require the social choice of the second-best technology as expressed in Proposition 5.

4 Oligopoly in an Open Economy

In this section, we consider international oligopoly. We specifically analyze trade policies applied to foreign exporters which face no domestic competitors. The foreign exporters engage in Cournot competition in the domestic market. In this situation, the domestic government acts as a monopsonist and hence the marginal purchase cost plays a crucial role to determine the optimal policies. Typically, the domestic government imposes tariffs to shift rent from the foreign firms to itself. A seminal work is done by Brander and Spencer (1984) in which a foreign monopolist serves the domestic market. In our analysis, we consider foreign oligopolists. We also examine a case in which the domestic country imports from multiple countries.

4.1 Strategic Tariffs against Foreign Oligopolists

When the domestic market is served by only foreign exporters and the government imposes a specific tariff t on the imports X, domestic welfare is given by

$$\widetilde{W}(X,t) = U(X) - P_D(X)X + tX,$$

which can be rewritten as

$$W = U(X) - \{P_D(X) - t\} X, \tag{19}$$

where $P_D(X) - t$ represents the price paid to the foreign firms, i.e., the international price the importing country faces.

The profit earned by the foreign oligopolist is given by (1) in Section 2. Firm i's quasi-supply price, i.e., the net-of-tax price required to produce the output, is given by

$$v^i(x_i, X) = C_i'(x_i) - x_i P_D'(X),$$

as before from the first-order condition for profit maximization. When the importing country imposes a uniform tariff t over the foreign firms, these quasi-supply prices should be equal, and we can obtain the industry quasi-supply price function $v_S(X)$ as before. This is the price which the importing country must pay for its imports, so that its welfare (19) is now a function only of the total output (or import) as shown below.

$$W(X) = U(X) - v_S(X)X.$$

Then the change in the welfare associated with the total output is expressed by the standard way of decomposing the gains from trade, i.e.,

$$W'(X) = \{P_D(X) - v_S(X)\} - X \cdot v_S'(X),$$

where the first term shows the trade volume effect arising from the differences between the domestic and foreign prices, and the second term the terms of trade effect. Given total output in the free-trade equilibrium X_F, there holds $P_D(X_F) = v_S(X_F)$. Thus, the government should restrict the imports if and only if $v_S'(X_F) > 0$, i.e., the more imports raise the international price. Note that the standard theory of optimal tariffs or import regulation applies here even though the market is imperfectly competitive.

We should also notice that the general rule of welfare maximization requires the marginal benefit of imports, which is equal to the demand price in the importing country $P_D(X)$, to be equal to the marginal purchase cost of imports $MPC(X)$, which is derived from the total purchase cost given by $TPC(X) = v_S(X)X$, i.e., $MPC(X) = v_S(X) + Xv_S'(X)$, which is greater than the quasi-supply price if and only if the quasi-supply price is strictly increasing in the output, i.e., $v_S'(X) > 0$. The optimal total output X^* then should satisfy

$$0 = W'(X^*) = P_D(X^*) - MPC(X^*),$$

which implies the optimal specific tariff rate t^* is given by

$$t^* = P_D(X^*) - v_S(X^*) = X_D^* v_S'(X^*).$$

Thus, we establish the following proposition.

Proposition 6 *When the importing country has no domestic production, the optimal import tariff is strictly positive if and only if the (equalized) quasi-supply price of the exporting countries is strictly increasing in the total output.*

In Figure 8, the free trade equilibrium is given by point F where the demand curve and the industry quasi-supply curve intersects. The optimal imports are determined by point D^* where the MPC curve and the demand curve intersects. To realize this amount of imports, an import subsidy $\overline{D^*S^*}$ must be provided. Brander and Spencer (1984) discuss the possibility of import subsidies as the optimal trade policy for the importing country when the domestic market is served by a foreign monopolist. They conclude that the optimal import tariff can be positive or negative depending on the curvature of the slope of the demand curve.[22] In our analysis, given the downward-sloping quasi-supply curve, it is clear in Figure 8 that the optimal trade policy is import subsidization.

[22] The elasticity of the slope of inverse demand function plays a crucial role. For details, see Brander and Spencer (1984) and Ishikawa (2000).

4.2 Tariff Discrimination

A large importing country can exercise the monopsony power in the international market. Although the GATT/WTO prohibits, the optimal policy for the monopsonist is purchase-price discrimination across the exporting countries.[23] This requires the marginal purchase cost from each exporting country to be equal.[24]

We assume for simplicity that the domestic country imports from two countries, L and H, where the firms have the identical technology insofar as they locate in the same country and that all outputs in both countries are exported to the domestic country. In view of (1) or (2), the first-order condition for profit maximization by the representative firm in country $i (\in \{L, H\})$ is rewritten as follows:

$$0 = P_D(X) + \frac{X_i}{n_i} P'_D(X) - C'_i \left(\frac{X_i}{n_i} \right) - t_i, \tag{20}$$

where $X_i = n_i x_i$ represents the total output by country i and t_i the specific tariff imposed on imports from country i. Then the quasi-supply price of exporting country i is given by

$$v^i(X_i, X, n_i) = P_D(X) - t_i = C'_i \left(\frac{X_i}{n_i} \right) - \frac{X_i}{n_i} P'_D(X), \tag{21}$$

where one should notice that the quasi-supply price depends now also on the number of active firms n_i in country i. It shows the price which the domestic country must pay when importing X_i from country i.

Given the total output, the purchase cost from country i is then given by

$$TPC^i(X_i, X, n_i) = v^i(X_i, X, n_i) X_i, \tag{22}$$

and the total purchase cost depends on the import vector (X_L, X_H), and it is given by

$$TPC(X_L, X_H) = \sum_{i \in \{L,H\}, j \neq i} TPC^i(X_i, X_i + X_j, n_i). \tag{23}$$

Regardless of how much is imported, it is the best for the importing country to minimize the total purchase cost given any total import, which requires the marginal purchase costs to be equalized between the exporting countries. Given the total import volume, the marginal purchase cost from country i defined by $MPC^i(X_i, X) \left(= \partial TPC^i(X_i, X)/\partial X_i\right)$ is expressed as

[23] The following discussion is based on Kiyono (2009).

[24] The same results are obtained earlier by Hwang and Mai (1991) and Kiyono (1993).

follows:[25]

$$MPC^i(X_i, X) = v^i(X_i, X, n_i) + X_i \frac{\partial v^i(X_i, X, n_i)}{\partial X_i}.$$

Moreover, the equalized marginal purchase costs should be equated with the social marginal benefit of imports, which is equal to the domestic price in the importing country, i.e., $p = v^i(X_i, X, n_i) + X_i(\partial v^i(X_i, X, n_i)/\partial X_i)$. Let t_i^D represent the discriminatory specific import tariff on imports from country i. Since $t_i^D = p - v^i(X_i, X, n_i)$, the above equation is rewritten as follows:

$$t_i^D = X_i \frac{\partial v^i(X_i, X, n_i)}{\partial X_i}.$$

The property of this discriminatory import tariff policy becomes much clearer when we express it in the form of ad valorem tariffs, i.e.,

$$\tau_i^D = \frac{\partial \ln v^i(X_i, X, n_i)}{\partial \ln X_i}.$$

The right-hand side is equal to the inverse of the price elasticity of the constrained quasi-supply by exporting country i. It is just the same as the optimal tariff formula for perfectly competitive exporters. In this sense, the above formula is a generalized optimal tariff formula.

Let us further elucidate the properties of the optimal discriminatory specific tariff policy. For this purpose, we focus our attention on the case of constant marginal production costs. Let c_i denote the constant marginal production cost of exporting country i. Then, in view of (21), (24) can be rewritten as follows:

$$MPC^i(X_i, X, n_i) = 2v^i(X_i, X, n_i) - c_i. \tag{24}$$

Using $p = MPC^i(X_i, X, n_i) = v^i(X_i, X, n_i) + t_i^D$, we can rewrite the above equation as follows:

$$\begin{aligned} 2\left(p - t_H^D\right) - c_H &= 2\left(p - t_L^D\right) - c_L \\ 2\left(t_L^D - t_H^D\right) &= c_H - c_L \end{aligned}$$

Thus, we obtain the following proposition.

Proposition 7 *Suppose that tariff discrimination is possible. If the marginal production costs in exporting country i, c_i, are constant, then $2\left(t_L^D - t_H^D\right) = c_H - c_L$ holds and hence the exporting country with the lower marginal costs is subject to the higher discriminatory tariff.*

[25] Rigorously speaking, the following is what one may call the "constrained marginal purchase cost", for it is defined given total import volume. See Kiyono (2009).

4.3 Choice of an FTA Partner

The most favored nation (MFN) clause in the GATT/WTO regulations prohibits any member country from imposing discriminatory tariffs on the imports from abroad. However, tariff discrimination is exceptionally allowed under GATT Article 24. As long as the conditions in the article are met, an FTA can be established. In this subsection, we explore the choice of an FTA partner on welfare of the importing country.

As in the last subsection, the domestic country imports from countries L and H. For a welfare comparison between the FTA formation with country H and that with country L, we specifically consider the situation under which the domestic country, initially having an FTA with country L, switches the partner to country H, keeping the same total import volume, X_T.[26]

As is well known, the equilibrium total output in the homogeneous Cournot oligopoly with constant marginal costs is solely determined by the sum of the tariff-inclusive marginal cost, i.e., $\sum_k n_k(c_k + t_k)$. Thus, the following lemma is immediate.

Lemma 5 *Given the numbers of active firms in exporting countries, $n = (n_L, n_H)$, the equilibrium total imports, X_T, are kept constant if $\sum_k n_k t_k$ is constant.*

We focus our attention on the case in which the initial FTA with L imposes a strictly positive external tariff, $t_H^L > 0$. Then the import substitution from the old partner L to the new partner H requires adjustments in the tariff policies from $t^L = (0, t_H^L)$ to $t^H = (t_L^H, 0)$. In view of Lemma 5, the tariff policies associated with this import substitution must satisfy

$$n_H t_H^L = n_L t_L^H.$$

As the total output is unchanged at X_T, the MPC from each country can be replaced with what we may call the *constrained MPC*, which shows the MPC from each exporting country when the total import volume is kept constant.

From (21) and (24), each country's constrained MPC curve is thus linear and strictly upward sloping as illustrated by Figure 9.[27] The line segment $O_L O_H$ is equal to the total imports given by X_T associated with the domestic price P_D in the importing country. The import from country L is measured rightward from point O_L, while that from country H is measured leftward from point O_H. The upward-sloping curve $c_i v_i$ $(i = H, L)$ shows the export price of exporting country i.

[26] For simplicity, we assume that the domestic country imports from both countries before and after switching the partner. For the case without this assumption, see Kiyono (2009).

[27] Note that in (21), $C_i' = c_i$ and $P_D'(X)$ is evaluated at $X = X_T$.

The equilibrium of FTA with country L is shown by point L, where $P_D = v_L$ holds. Of the total import, $O_L L'$ comes from country L, and $O_H L'$ from country H. The tariff imposed on country H, t_H^L, is measured by the difference between its export price and the domestic price, i.e., $t_H^L = LL''$.

Now consider the switch of the FTA partner to country H given the total amount of imports. This requires the export price of country H to be equal to the domestic price, which is given by point H. The imports from country H increase to $O_H H'$, and those from country L decrease to $O_L H'$ facing the tariff of $t_L^H = HH''$.

The change in the total purchase costs of imports are measured by the areas $AL_L L_H$ (showing the decreased costs) and $AH_L H_H$ (showing the increased costs). In view of the figure, we can easily verify that the importing country is better off if and only if the sum of country H's MPC minus that of country L's MPC at two FTA equilibria is strictly positive.[28]

Therefore, we obtain the following proposition.

Proposition 8 *Suppose that the importing country initially forms an FTA with country L. Then the FTA partner switch from country L to country H, while keeping the total import constant, makes the importing country better off if* $c_H > c_L + (n_H/n_L - 1)t_H^L$.

Two remarks are in order here. First, if $n_H = n_L$, then country H is a better partner as long as the total imports are kept constant. Second, if the uniform tariff is set set by the importing country in Figure 9, the equilibrium is determined by point U where $v_H = v_L$ holds. The tariff is UU'' and the total purchase costs of imports are less with the uniform tariff than with the FTA with country L by $U_L U_H L_H L_L$.

5 Conclusion

We explored the production efficiency and product-mix efficiency when firms engage in Cournot competition. To this end, we introduced tools called quasi-reaction functions and quasi-supply curves. These tools allow us to conduct analyses easily by using the standard partial-equilibrium diagram, i.e., the quantity-price plane.

Figure 10 shows how quasi-reaction functions, quasi-supply curves and other concepts developed in our analysis are related. We first considered the short-run case in which the number of firms is fixed. We easily identified welfare losses with the standard Cournot oligopoly on the quantity-price plane. We also depicted the production inefficiency in the case of mixed oligopoly. These inefficiencies can be eliminated when production taxes and

[28]Total purchase costs of imports are minimized when the domestic country imports $O_L A'$ from country L and $O_H A'$ from country H. At point A, the $MPCs$ are equalized between countries L and H.

subsidies are available. In the long-run equilibrium (i.e., the equilibrium with free entry and exit), using the quasi-reaction function, we reexamined the excess entry theorem and illustrated the situation of excess entry in the standard partial-equilibrium diagram. We also considered the issue of technology choices. We showed that entry regulations are useful in those situations. We then reexamined tariffs as a rent-shifting device in open economy settings. Specifically, we investigated the case where there is no domestic supply and foreign firms have different cost structures. Again quasi-supply curves are useful for the analysis.

Although the results themselves are not necessary novel, our analysis makes it possible to easily illustrate inefficiencies in each case and help gain insights intuitively. Reexamination of other public policies using quasi-reaction functions is left for future research.

Appendix

A Entry Dynamics and Stability

Consider the market served by the firms with the same technology (c, f). As shown in (14), the equilibrium profit of the individual firm given the equilibrium total output X is expressed by

$$\pi^e(X, c, f) = F(c, X) - f,$$

where use was made of the definition of the viability frontier (13), i.e.,

$$F(c, X) = -\frac{\{P_D(X) - c\}^2}{P_D'(X)}.$$

Total output X is the short-run equilibrium one if and only if the number of active firms n satisfies

$$X = nr(X, c).$$

In view of Assumption 3, such a total output is uniquely determined given the number of active firms n and their marginal cost c. We express this relation with $\widetilde{X}(n, c)$. Applying the implicit function theorem, one can obtain

$$\widetilde{X}_n(n, c) = \frac{r\left(\widetilde{X}(n, c)\right)}{1 - nr_X\left(\widetilde{X}(n, c), c\right)} > 0.$$

Therefore, the more active firms lead to a greater total output, if and only if the short-run equilibrium is (at least locally) stable as stated in Assumption 3.

Let us assume that a change in the number of firms is subject to the following adjustment rule:

$$\dot{n} = F\left(c, \widetilde{X}(n, c)\right) - f, \tag{25}$$

i.e., the number of active firms increases if and only if an individual firm earns strictly positive profit. The free-entry equilibrium is defined as the number of firms under which this entry dynamics ceases. We denote the number by $n^e(c, f)$.

This free-entry equilibrium is stable when the gross profit is strictly decreasing in the number of firms. Since the total output is increasing in the number of firms, however, the stability condition is in fact that the gross profit is strictly decreasing in the total output. To derive this condition, it is more convenient to use $F(c, X) = -\{r(X, c)\}^2 P'_D(X)$ instead of the original expression above, the one given by (13). In fact, its partial differentiation with respect to the total output X gives rise to the following stability condition:

$$F_X(c, X) = r(X, c)P'_D(X) \{1 - r_X(X, c)\} < 0.$$

Therefore, we have established

Lemma 6 *Suppose that all active firms use the same technology* (c, f)*. Then the free-entry equilibrium resulting from the entry dynamics governed by (25) is stable if* $F_X(c, X) < 0$*, i.e.,* $r_X(X, c) < 1$*, holds for all* $X \geq 0$

B Changes in Technologies and IMC

We prove Lemma 4 in this appendix. From (15) and (17), we obtain

$$IMC(X, c, f) = c + \frac{f}{r(X, c)} \left\{1 - \frac{X}{r(X, c)} r_X(X, c)\right\}.$$

Then a change in the marginal cost c gives rise to

$$\begin{aligned}
&IMC_c(X, c, f) \\
=&1 - \frac{f}{\{r(X, c)\}^2} r_c(X, c) + \frac{X r_X(X, c)}{\{r(X, c)\}^3} r_c(X, c) f - \frac{X f}{\{r(X, c)\}^2} r_{Xc}(X, c) \\
=&1 - \frac{f}{\{r(X, c)\}^2 P'_D(X)} + \frac{X f}{\{r(X, c)\}^3 P'_D(X)} r_X(X, c) + \frac{X f}{\{r(X, c)\}^2} \frac{P''_D(X)}{\{P'_D(X)\}^2} \\
&\left(\because r_c(X, c) = \frac{1}{P'_D(X)} \text{ and } r_{Xc}(X, c) = -\frac{P''_D(X)}{\{P'_D(X)\}^2}\right) \\
=&1 - \frac{f}{\{r(X, c)\}^2 P'_D(X)} \{1 - n^r(X, c) r_X(X, c)\} + \frac{n^r(X, c) f}{\{r(X, c)\}^2 P'_D(X)} \{-1 - r_X(X, c)\} \\
&\left(\because n^r(X, c) = \frac{X}{r(X, c)} \text{ and } r_X(X, c) = -1 - \frac{r(X, c) P''_D(X)}{P'_D(X)}\right) \\
&= 1 - \frac{f}{\{r(X, c)\}^2 P'_D(X)} \{n^r(X, c) + 1\}.
\end{aligned}$$

Thus, form (10) and (13), there holds

$$IMC_c(X, c, F(c, X^e)) = 1 + \frac{\{r(X^e, c)\}^2 P_D'(X^e)}{\{r(X, c)\}^2 P_D'(X)} \{n^r(X, c) + 1\}. \tag{26}$$

Similarly, a change in the fixed cost f gives rise to

$$IMC_f(X, c, f) = \frac{1}{r(X, c)} \{1 - n^r(X, c) r_X(X, c)\}. \tag{27}$$

Then in view of $F_c(c, X^e) = -2r(X^e, c)$, (26) and (27) jointly imply

$$\begin{aligned}
\frac{dIMC_c(X^e, c, F(c, X^e))}{dc} &= n^r(X^e, c) + 2 - 2\{1 - n^r(X^e, c) r_X(X^e, c)\} \\
&= n^r(X^e, c)(1 + 2r_X(X^e, c)) \\
&= n^r(X^e, c)\left(-1 - 2\frac{r(X^e, c)P_D''(X^e)}{P_D'(X^e)}\right),
\end{aligned}$$

which is strictly negative when $P_D''(X) \leq 0$ by virtue of (11), i.e., $r_X(X, c) = 1 - r(X, c)P_D''(X)/P_D'(X)$.

Similarly, in view of Assumption 2, we obtain

$$\begin{aligned}
\frac{dIMC_c(0, c, F(c, X^e))}{dc} &= 1 + \frac{\{r(X^e, c)\}^2}{\{r(0, c)\}^2} \frac{P_D'(X^e)}{P_D'(0)} - 2\frac{r(X^e, c)}{r(0, c)} \quad (\because n^r(0, c) = 0) \\
&= \frac{P_D'(X^e)}{P_D'(0)} \left\{ \left(\frac{r(X^e, c)}{r(0, c)} \right)^2 - 2\frac{P_D'(0)}{P_D'(X^e)} \frac{r(X^e, c)}{r(0, c)} + \frac{P_D'(0)}{P_D'(X^e)} \right\} \\
&= \frac{P_D'(X^e)}{P_D'(0)} \left[\left\{ \frac{r(X^e, c)}{r(0, c)} - \frac{P_D'(0)}{P_D'(X^e)} \right\}^2 + \frac{P_D'(0)\{P_D'(X^e) - P_D'(0)\}}{\{P_D'(X^e)\}^2} \right].
\end{aligned}$$

When $P_D''(X) = 0$, there hold $P_D'(0) = P_D'(X^e)$ and $r_X(X, c) < 0$, the latter of which implies $r(X^e, c) < r(0, c)$. Therefore, the right-hand side of the above equation is strictly positive. The same argument applies to the case for $P_D'(0) > P_D'(X^e)$ in general where $P_D'(X) < 0$ ensures $r(X^e, c) < r(0, c)$.

Now consider the iso-elastic demand function given by $P_D(X) = X^{-\frac{1}{\varepsilon}}$. Then there holds[29]

$$\begin{aligned}
-1 - 2\frac{r(X^e, c)P_D''(X^e)}{P_D'(X^e)} &= -1 + \frac{2}{n^r(X^e, c)} \left(\frac{1}{\varepsilon} + 1\right) \\
&\propto 2\left(\frac{1}{\varepsilon} + 1\right) - n^r(X^e, c).
\end{aligned}$$

Therefore, we establish that even when the demand function is iso-elastic, $n^r(X^e, c) > 2\left(\frac{1}{\varepsilon} + 1\right)$ assures $dIMC_c(X^e, c, F(c, X^e))/dc < 0$.

[29]It can easily be verified that $1/\varepsilon + 1 = -XP_D''(X)/P_D'(X)$ holds.

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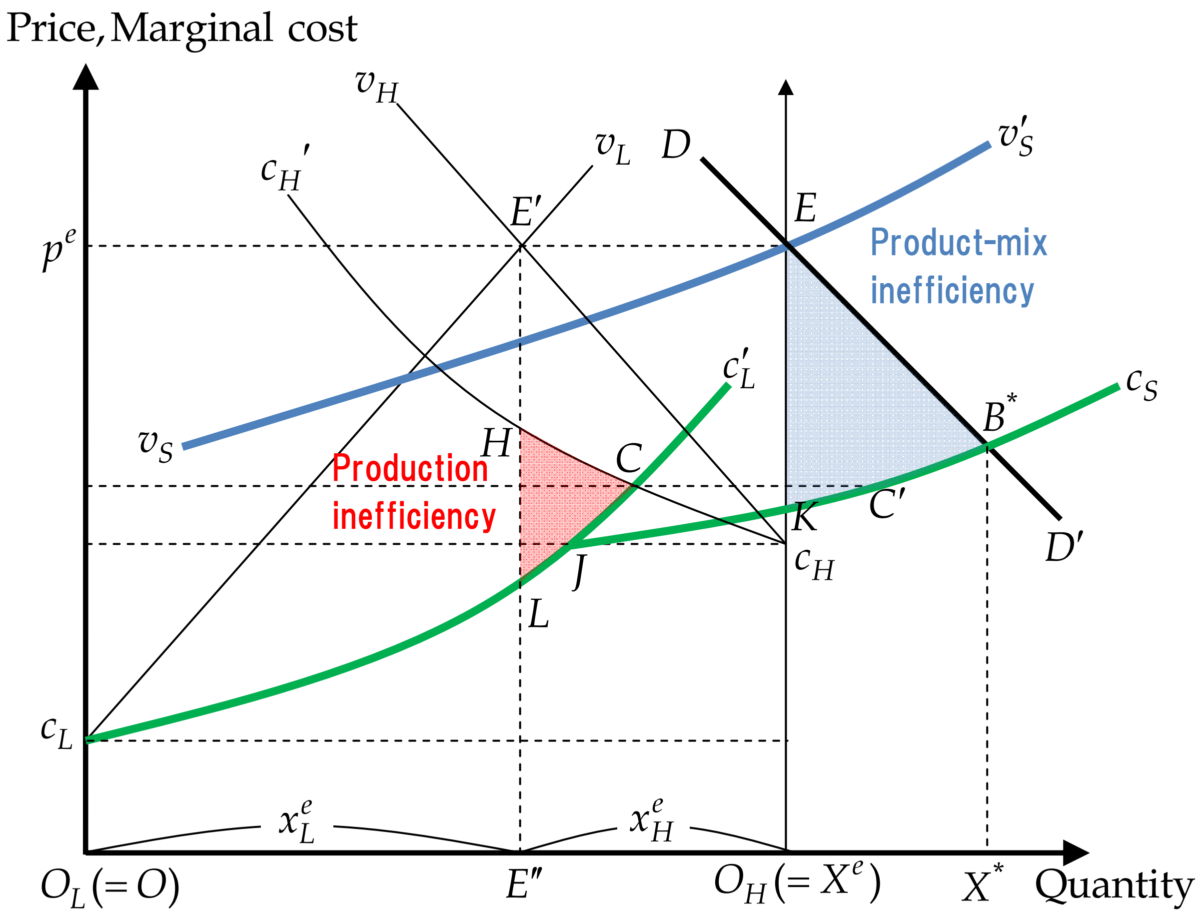

Figure 1　Welfare Losses in Cournot Oligopoly

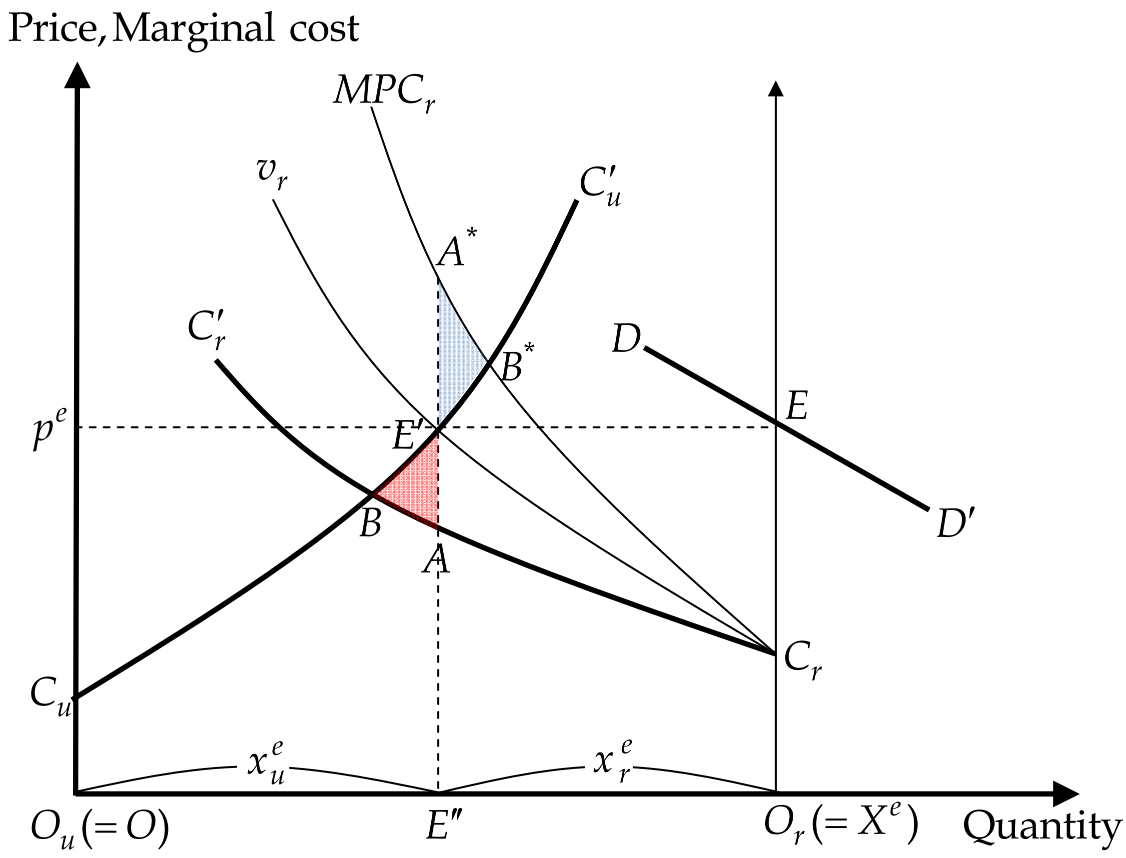

Figure 2　Production Inefficiency at the Mixed Duopoly Equilibrium

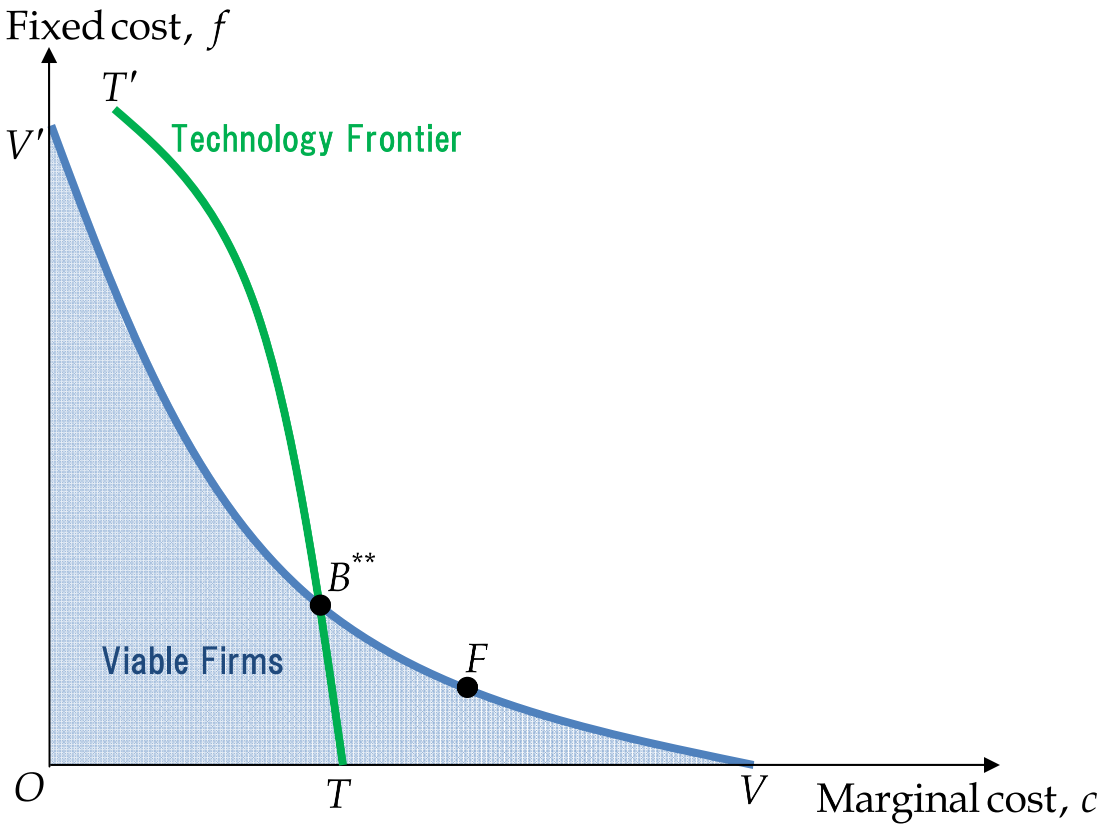

Figure 3 Fixed and Marginal Costs of Viable Firms

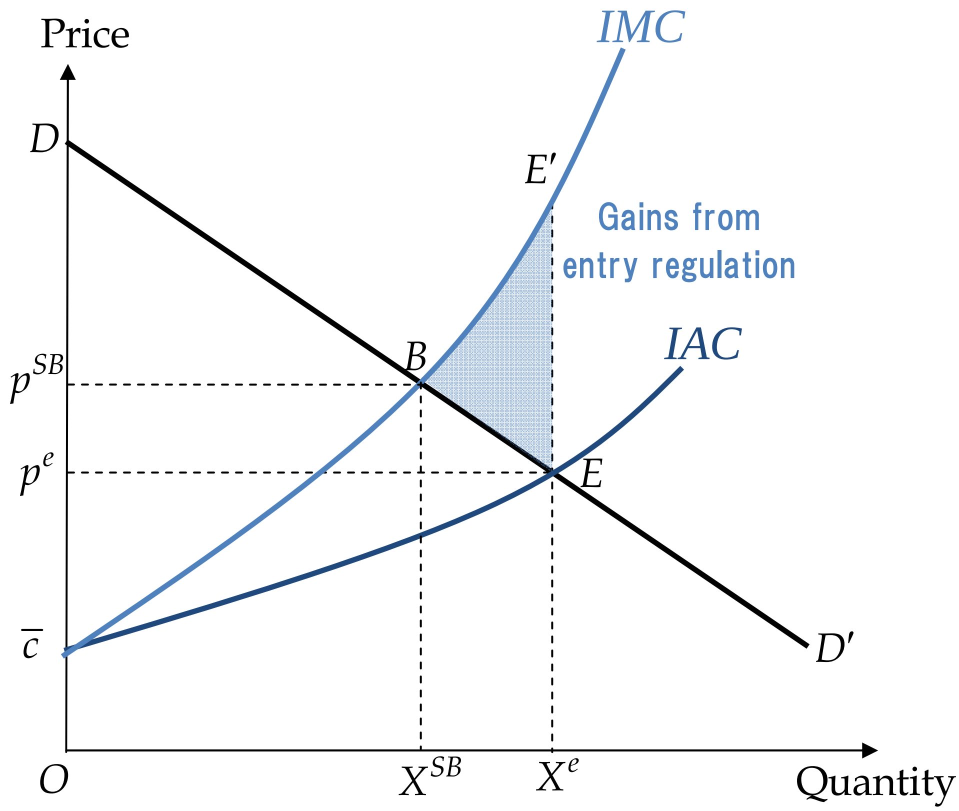

Figure 4 Industry Average and Marginal Cost Curves, Free-Entry Equilibrium and the Second-Best Total Output

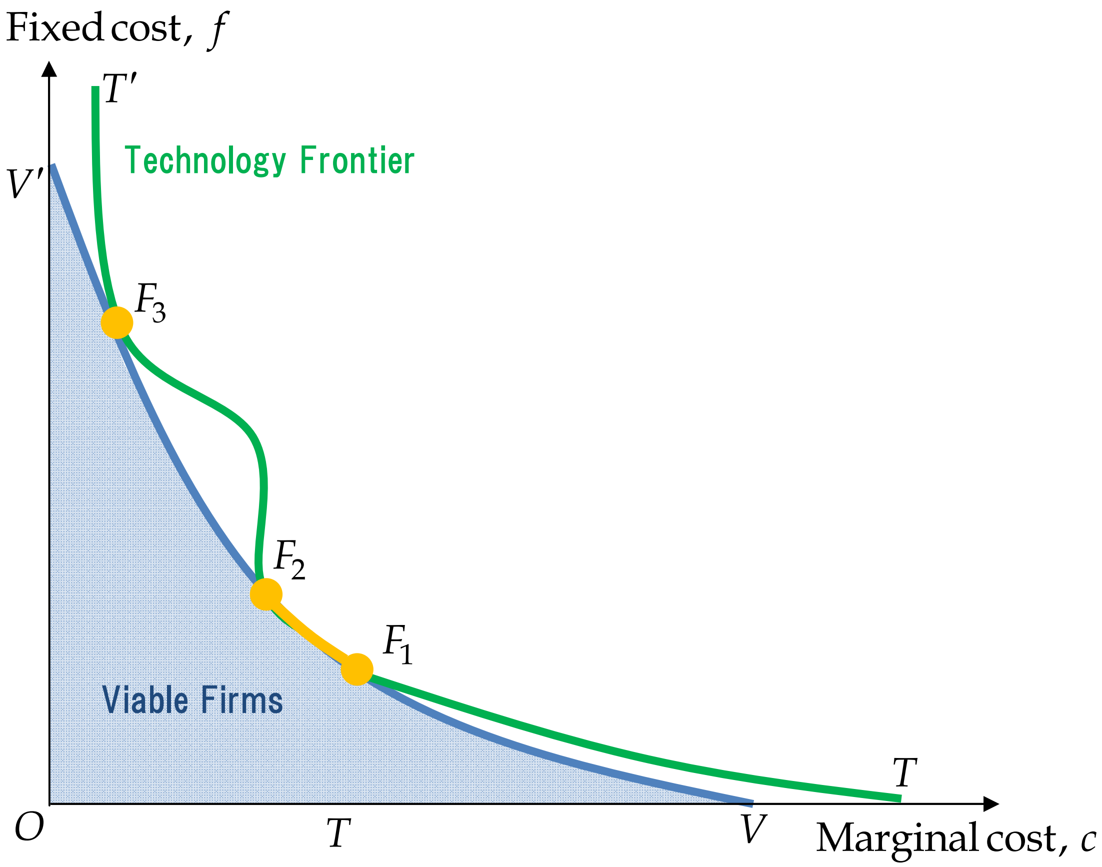

Figure 5 Viable and Second-Best Technologies

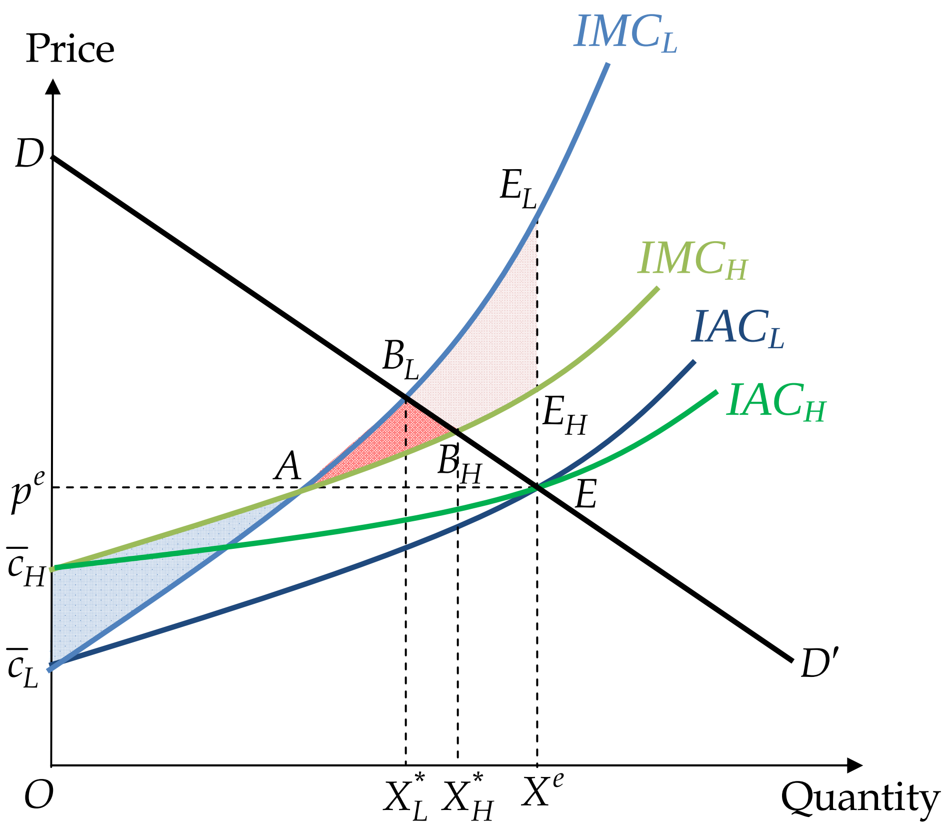

Figure 6　Second-Best Choice of Technologies

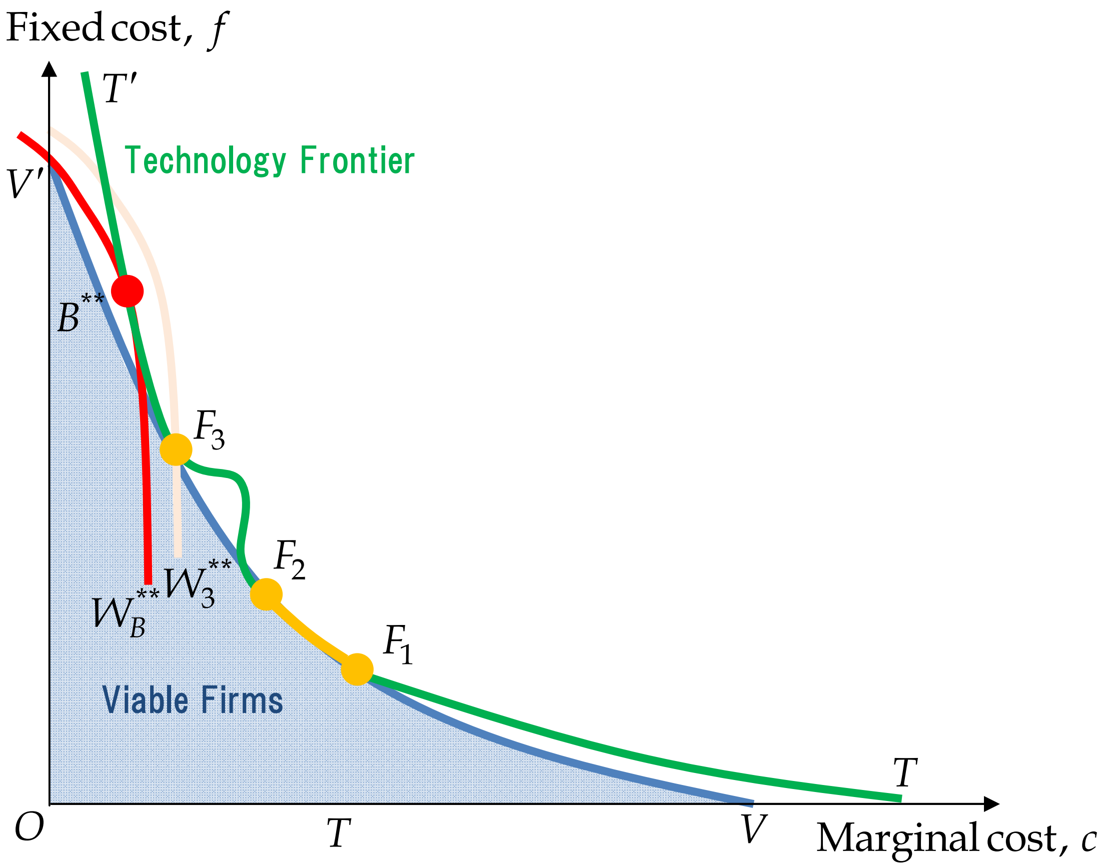

Figure 7　Technology Selection by the Market and the Government

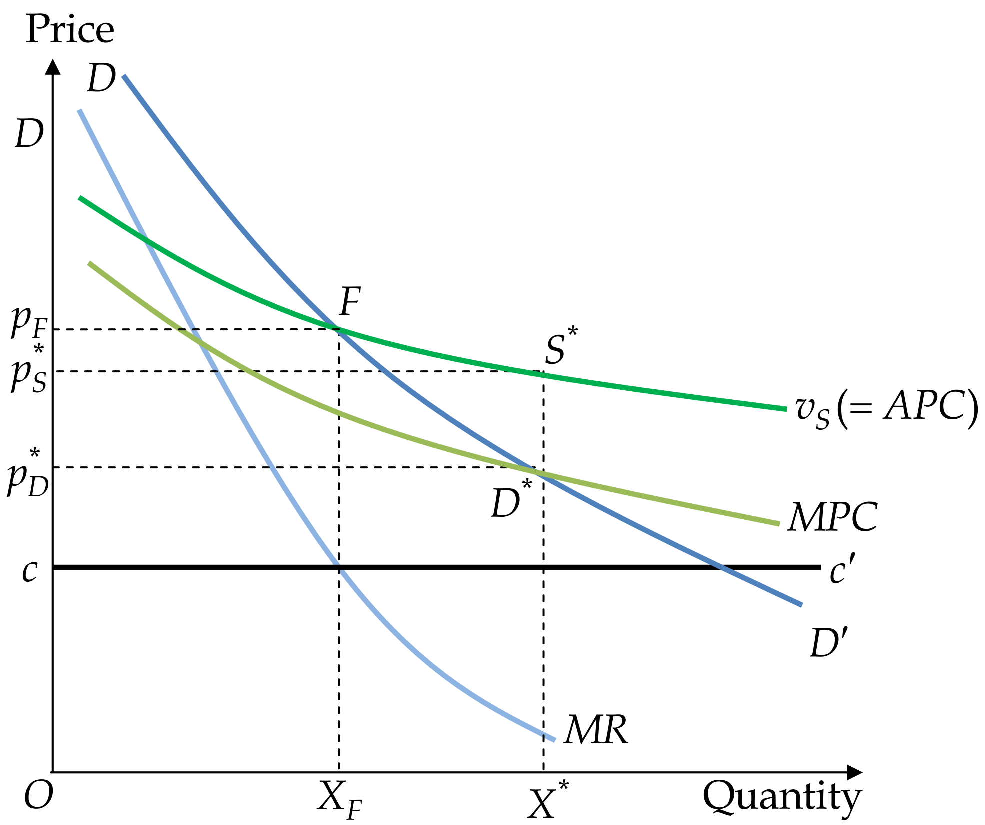

Figure 8 Downward-Sloping Quasi-Supply Curve and Import Subsidies As the Best Trade Policy

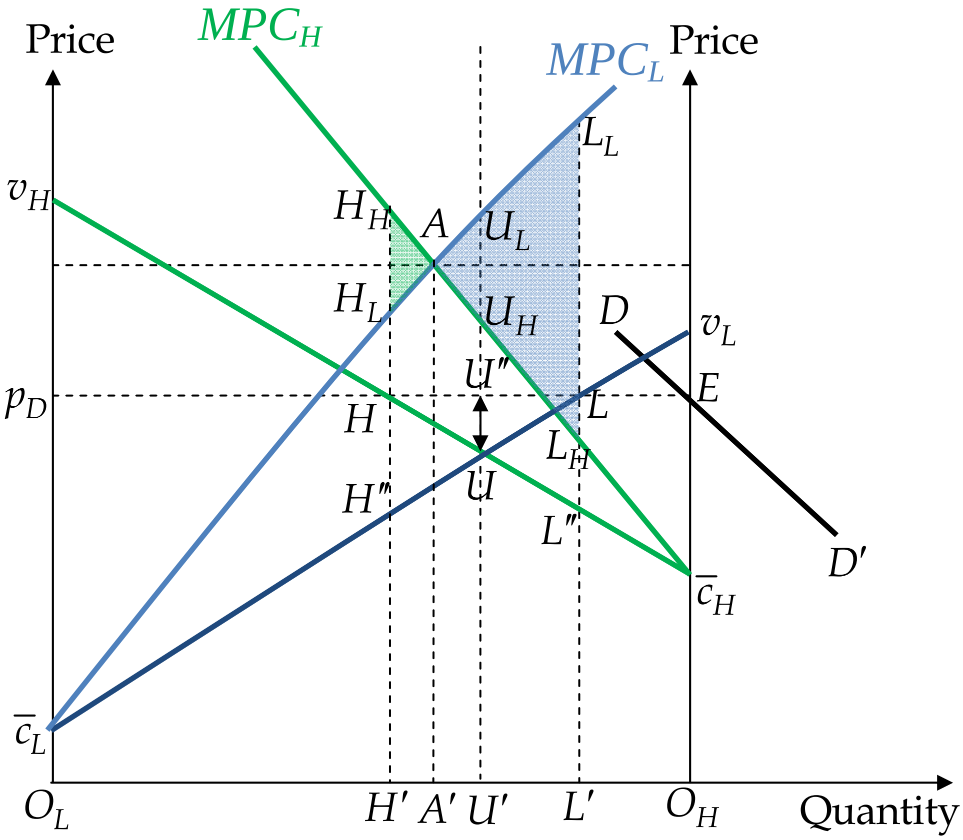

Figure 9 Choice of Exporting Countries as FTA Partners

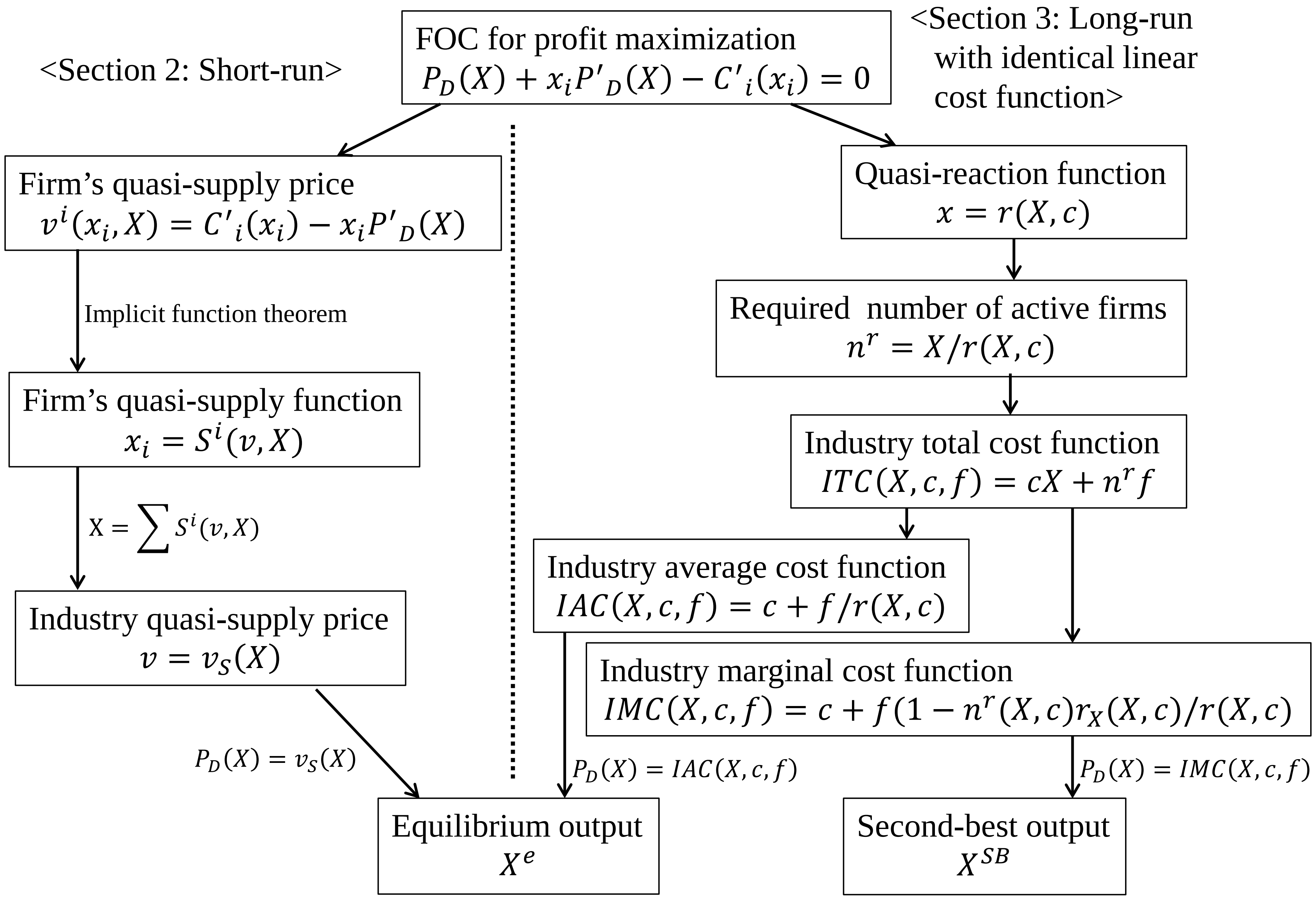

Figure 10 Quasi-reaction and the related concepts