How are catastrophic shocks shared between countries in
the presence of solvency constraints?*

Makoto SAITO† Shiba SUZUKI‡ Tomoaki YAMADA§

February 2008

Abstract

In this paper, we explore how an extremely large and persistent catastrophic
shock is shared between two countries, when markets are complete, but each country
is subject to endogenous solvency constraints. In contrast to a case of incomplete
markets with exogenous borrowing constraints, such a shock is shared effectively
between the two countries with heavy reliance on ex post financing. In ex ante
arrangements, the limited solvency of a nondamaged country tightly constrained
insurance transfer to a damaged country. In the aftermath, however, a damaged
country can clear solvency constraints by intensively building short positions in
Lucas trees and long positions in state contingent claims, thereby financing most of
uncovered losses.

JEL classification: F34, G12, G15.

Keywords: Solvency constraints, Enforcement constraints, Dynamic optimal con-
tract, Catastrophic shocks.

---

*The authors would like to acknowledge Toni Braun, Tomoyuki Nakajima, Hisashi Nakamura, Akihisa
Shibata, and seminar participants at Washington, Hitotsubashi, Osaka, and Tokyo universities, Bank
of Japan for helpful and encouraging comments. The authors are thankful for a grant-in-aid from the
Ministry of Education and Science, Japan, and the Nomura Foundation.

†Correspondence to Makoto Saito, Faculty of Economics, Hitotsubashi University, 2-1, Naka, Kuni-
tachi, Tokyo, 186-8601, Japan, E-mail: makoto@econ.hit-u.ac.jp, phone: +81-42-580-8807, fax: +81-
42-580-8882.

‡Graduate School of Economics, Hitotsubashi University.
§Faculty of Economics, Rissho University.
1 Introduction

In this paper, we explore how an extremely large and persistent catastrophic shock is shared between two countries under the assumption that markets are complete, but each country is subject to endogenous solvency constraints. Catastrophic shocks we consider here constitute of exogenous country-specific events, and have serious repercussions on national outputs; examples include natural disasters and disastrous epidemics.

We introduce solvency constraints into a complete markets setup following the theoretical framework proposed by Lustig (2007). On the one hand, a reason for adopting a complete markets setup is that catastrophic shocks are publicly observable, and markets are therefore likely to exist for such states. On the other hand, a reason for assuming solvency constraints is that it is not easy to force agents to fully recognize outstanding liabilities in a cross-border financial arrangement. In Lustig’s (2007) framework, Lucas trees and all contingent claims are traded among agents, however agents’ solvency is limited in that human capital cannot be used as collateral. An essential aspect of this characterization of solvency constraints is that the current size of short positions in a financial instrument is endogenously determined by the future value of collateral assets (long positions in other financial instruments).

The existing literature intensively explored to which extent the transaction of non-contingent bonds among agents can compensate for the absence of insurance markets in the presence of exogenous borrowing constraints. In this case, the upper limit is exogenously imposed on the current outstanding debts or short positions. The most important result from this setup is that because borrowing constraints keep agents from fully financing uncovered losses by non-contingent bonds, either large or persistent shocks are not shared effectively among agents.

---

1 Lustig and Nieuwerburgh (2005) adopted the same theoretical framework.
2 Using the theoretical framework constructed by Bewley (1986), many papers including Telmer (1993), Lucas (1994), and Aiyagari (1994) have intensively investigated how transitory idiosyncratic shocks are insured against among agents in a setup where insurance markets do not exist, and borrowing constraints are introduced exogenously.
3 Without any borrowing constraint, transitory shocks are insured against almost perfectly through self-insurance, while a severe borrowing constraint reduces substantially the self-insurance ability of agents. On the other hand, persistent shocks are hard to self-insure against even without any borrowing constraint. As shown by Constantinides and Duffie (1996) and others, in an extreme case where idiosyncratic shocks are permanent, self-insurance does not work at all regardless of whether borrowing constraints are present. A major reason for the difficulty with self-insurance against persistent or per-
The major results from our setup differ substantially from those from the above combination of incomplete markets and borrowing constraints in the following respects. First, the constrained efficient allocation is fairly close to the corresponding Pareto optimal allocation. That is, most of large and persistent catastrophic shocks are shared effectively between the two countries.

Second, however, solvency constraints affect profoundly how the two countries trade financial instruments to share catastrophic shocks. As for the *ex ante* financial arrangement or the insurance contract made prior to a catastrophic event, the insurance payment to a damaged country is severely constrained by the limited solvency of a nondamaged country as an insurer, given the expectation that Lucas trees will be heavily discounted in the aftermath.

Third, in the aftermath, on the other hand, a damaged country can clear solvency constraints by intensively building short positions in Lucas trees and long positions in contingent claims, thereby financing most of the uncovered losses. That is, when liquidity is scarce in either country, voluntary and bilateral cross-border transactions themselves will create international collateral for countries in need of liquidity. In the above *ex post* financing, the large long position in Lucas trees accumulated as an opposing position by a nondamaged country can serve as precautionary instruments for a nondamaged country itself, as well as collateral for insurance payments to a damaged country in a repeated catastrophic event. Thus, this financial scheme is powerful enough to cover not only losses in the current income of a damaged country, but also a substantial decrease in lifetime income in the case where catastrophic shocks are persistent.

The literature on international finance intensively investigates the possible effects of solvency constraints because enforcement problems are quite serious in international financial arrangements. They initially addressed whether sovereign debts are sustainable in the presence of solvency constraints, and later investigated how enforcement

---

4It does not necessarily imply that solvency constraints never matter in either resource allocations or asset pricing. In Lustig (2007), solvency constraints have large impacts on equity premiums under the assumption that the volatility of idiosyncratic shocks is counter-cyclical.

5Eaton and Gersovitz (1981) analyzed how sovereign debts are sustained by a reputation mechanism, while Bulow and Rogoff (1989) presented a case in which a reputation mechanism does not necessarily make sovereign bonds sustainable when a borrowing country is allowed to make savings. More recently, Hellwig and Lorenzoni (2005) demonstrated that sovereign bonds are still sustainable given a general equilibrium effect of limited commitment on interest rates. Eaton and Fernandez (1995) surveyed the
constraints influence borrowing ability as well as risk-sharing capacity.\(^6\)

One closely related paper is Kehoe and Perri (2002). They applied the framework proposed by Alvarez and Jermann (2000) to a two-country model. The way solvency constraints are endogenized differs between Alvarez and Jermann (2000) and Lustig (2007). In the former model, once a country is in default, it is excluded forever from international financial trading, while in the latter, even if a country defaults, it can remain in international financial markets after its collateral is confiscated completely. However, both models are similar to each other in that those financial contracts are made such that effective default (insolvency) never occurs in equilibrium. In the latter case, one country lends resources to the other country only up to the value of financial assets as collateral; consequently, a borrowing country is indifferent between default and repayment even when solvency constraints are binding.

In this paper, we employ the framework of Lustig (2007) because it is possible to trace the portfolio positions of Lucas trees and contingent claims within a two-country setup. As in Lustig (2007), thanks to a complete markets setup we solve the two-country model with solvency constraints by translating the market problem into the planner’s problem with time-varying Negishi weights. Unlike in the continuum agent setup of Lustig (2007), our two-agent setup potentially allows us to easily recover market outcomes including portfolio decisions from the equilibrium computed from the planner’s problem.\(^7\) In so doing, we can investigate which financial transactions between the two countries would make the sharing of catastrophic shocks quite effective even in the presence of solvency constraints.

This paper is organized as follows. Section 2 constructs a two-country model employing the framework proposed by Lustig (2007), while Section 3 presents the calibration results. Section 4 offers concluding comments.

---

\(^6\) Using a small country setup, Atkeson and Ríos-Rull (1996) constructed a model with exogenous constraints on borrowing from foreign countries, while Caballero and Krishnamurthy (2001) explored the possible effects of collateral constraints on international borrowing.

\(^7\) Using an extremely advanced technique, Kubler and Schmedders (2003) successfully solve portfolio problems in the presence of collateral constraints in incomplete markets.
2 Model

In this section, we apply the exchange economy model with solvency constraints constructed by Lustig (2007) to a two-country setup in order to analyze how persistent catastrophic shocks are shared between two countries. In this setup, the labor endowment of each country is influenced by country-specific catastrophic shocks, while dividends on Lucas trees are proportional to the world endowment. In terms of market structures, markets are complete with respect to country-specific catastrophic shocks. However, each country is subject to solvency constraints in the sense that net financial positions cannot be negative in every possible future state. That is, either Lucas trees or contingent claims can be used as collateral in cross-border financial transactions; however, nonfinancial assets (human capital) cannot be.

2.1 Setup

**Labor endowment and Lucas trees** A world economy consists of infinite-horizon exchange economies of country \( i \) (\( i = 1 \) or \( 2 \)) in a discrete time setup. It is assumed that the labor endowment is homogenous within each country, but heterogeneous between the two countries. Each country receives labor endowments subject to country-specific catastrophic shocks. There is a fixed supply of Lucas trees whose dividends are subject to world common shocks. The quantity of Lucas trees is standardized to one.

A set of states of country-specific labor endowment is defined as \( y \in Y = \{ y_1, \ldots, y_m \} \), while a set of states of dividends on Lucas trees is denoted as \( z \in Z = \{ z_1, \ldots, z_n \} \). A combination of country-specific and world common states is expressed by \( s_t = (y_t, z_t) \), where \( s_t \) is in \( S = Y \times Z \). In addition, \( s^t = (y^t, z^t) \) denotes a history from time 0 up to time \( t \), while \( s^\tau \succeq s^t \) represents a continuation history from \( s^t \).

Furthermore, we assume that the dividend on Lucas trees \( d(z) \) is proportional to the total labor endowment \( (e^1(y) + e^2(y)) \). Therefore, a combination of country-specific shocks constitutes aggregate states.\(^8\) Both labor endowment of each country and dividends on Lucas trees at time \( t \) are expressed in terms of \( z_t \). Hereafter, \( e^i(y_t) \) denotes the labor endowment of country \( i \), and \( d(z_t) \) denotes the dividend on Lucas trees. The transition probability of the above state variables \( \pi(y', z'|y, z) \) evolves according to the following Markov process: \( \pi(z'|z) = \sum_{y' \in Y} \pi(y', z'|y, z) \), \( \forall z \in Z \), \( \forall y \in Y \).

\(^8\)In Lustig (2007), because idiosyncratic shocks are introduced into labor endowment, by the law of large numbers, the total endowment is influenced only by aggregate shocks.
Given the above processes of labor endowment and dividends, optimal policy functions of consumption and portfolios depend on state $z_t$. As described in Appendix A, all endogenous variables except for asset volume are standardized by the total endowment in a numerical procedure.

**Preferences and resource constraints** Country $i$ maximizes expected lifetime utility with respect to consumption at state $s^t$ ($c^i(s^t)$) as follows:

$$U(\{c^i(s^t)\}_{t=0}^{\infty}(s_0) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t|s_0) u \left[ c^i(s^t) \right], \ i \in \{1,2\},$$

where a preference is characterized as utility with constant relative risk aversion, or

$$u \left[ c^i(s^t) \right] = \frac{c^i(s^t)^{1-\gamma}}{1-\gamma}.$$ 

$\beta$ represents the rate of time preference, while $\gamma$ denotes the degree of relative risk aversion.

The resource constraint of the world economy is given by:

$$e(z_t) = e^1(y_t) + e^2(y_t) + d(z_t).$$

Hereafter, $\alpha$ denotes the ratio of dividends to total labor endowment; that is, $\alpha = \frac{d(z_t)}{e(z_t)}$.

**Complete markets** In this economy with complete markets, both the shares of Lucas trees and one period contingent claims are traded between the two countries. $\theta^i(s^t)$ denotes the share of Lucas trees held by country $i$ at time $t$, while $a^i(s^t, s')$ represents the holding of claims on state $s'$ in the next period given the current state $s^t$. $p(z^t)$ is the price of Lucas trees, and $q(s^t, s')$ is the price of a contingent claim.

The market clearing conditions hold as follows:

$$\theta^1(s^t) + \theta^2(s^t) = 1, \quad (1)$$

$$a^1(s^t, s') + a^2(s^t, s') = 0, \ for \ all \ s' \in S. \quad (2)$$

The budget constraint of country $i$ at time $t$ is expressed by:

$$c^i(s^t) + p(z^t)\theta^i(s^t) + \sum_{s' \in S} q(s^t, s')a^i(s^t, s') \leq w^i(s^t), \quad (3)$$

where $w^i(s^t)$ denotes the initial wealth at time $t$, which is described below.
\textbf{Enforcement constraints} In this economy, the outstanding liability in short positions is enforceable up to the value of financial assets as collateral. That is, net positions of financial assets cannot be negative in every possible one-period ahead state. That is:

\[ [p(z^{t+1}) + d(z_{t+1})] \theta^t(s^t) \geq -a^t(s^t, s'), \forall s' \in S. \tag{4} \]

We can call the above enforcement constraint a \textit{collateral constraint}. This formulation of enforcement constraints implies that when equation (4) is binding, a debtor country is indifferent between default with confiscation and repayment at maturity.

If a collateral constraint or equation (4) is binding for country \( i \), then the possessed financial assets are confiscated, and the next period’s wealth \( w^i(s^{t+1}) \) is equal to:

\[ w^i(s^{t+1}) = e^i(y_{t+1}), \]

otherwise, it is equal to:

\[ w^i(s^{t+1}) = e^i(y_{t+1}) + [p(z^{t+1}) + d(z_{t+1})] \theta^i(s^t) + a^i(s^t, s_{t+1}). \]

\subsection{2.1.1 Competitive equilibrium with collateral constraints}

In sequential trading, each country maximizes expected lifetime utility subject to budget constraint (3) and collateral constraint (4):

\[
\begin{align*}
\max_{\{e^i\}, \{\theta^t\}, \{a^t\}} & \sum_{t=0}^{\infty} \sum_{s^t \in S} \beta^t \pi(s^t | s_0) u \left[ e^i(s^t) \right], \\
\text{s.t.} & \quad e^i(s^t) + p(z^t) \theta^t(s^t) + \sum_{s' \in S} q(s^t, s') a^t(s^t, s') \leq w^i(s^t), \\
& \quad [p(z^{t+1}) + d(z_{t+1})] \theta^t(s^t) \geq -a^t(s^t, s'), \forall s' \in S. \tag{7} 
\end{align*}
\]

A collateral constrained competitive equilibrium is defined as follows. Given the initial wealth \( \{w_0^i, w_0^j\} \), the trading strategy \( \{a^i(s^t, s'_t)\}, \{c^t(s^t)\}, \{\theta^t(s^t)\} \), the pricing function \( \{q(s^t, s')\} \) and \( \{p(z^t)\} \), each country maximizes (5) subject to equations (6) and (7), and market clearing conditions (1) and (2) are satisfied.

In the above competitive equilibrium, once a collateral constraint is binding, then intertemporal efficiency conditions

\[ q(s^t, s_{t+1}) u^t \left[ e^i(s^t) \right] - \beta \pi(s_{t+1} | s_t) u^t \left[ c^i(s^{t+1}) \right] > 0, \forall s^{t+1} \succeq s^t \]

and

\[ p(z^t) u^t \left[ c^i(s^t) \right] - \beta \sum_{s^{t+1} \succeq s^t} [p(z^{t+1}) + d(z_{t+1})] u^t \left[ c^i(s^{t+1}) \right] > 0 \]

hold with strict weak inequality. These inequalities are often called \textit{Euler inequalities}. 

6
2.2 Construction of the time 0 problem using Negishi weights

2.2.1 Time 0 cost minimization problem

It is impossible to directly solve the above sequential trading problem because of the presence of collateral constraints. Following Lustig (2007), we first translate the sequential trading problem into the time 0 problem. More precisely, we convert the sequential trading utility maximization problem to its dual problem or cost minimization at time 0 together with a promise keeping constraint. In the time 0 problem, a value function itself summarizes a history of binding collateral constraints, and serves as a state variable. As discussed later, the above time 0 cost minimization setup allows us to link the time 0 market problem with the planner’s problem at time 0 in the context of complete markets. However, one important deviation from the standard planner’s problem is that the Negishi weights (Negishi, 1960) are no longer fixed, but time varying as a consequence of binding collateral constraints.

We may rewrite collateral constraints for a country whose Lucas trees are entirely confiscated following default at state $s^t$:

$$
\sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) c^i(s^j) = \sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) e^i(y_j),
$$

where $Q(s_0, s^t) = q(s_0, s^1) \cdot q(s^1, s^2) \cdots q(s^{t-1}, s^t)$, which corresponds to a stochastic discount factor between state $s^t$ and state $s_0$. If collateral constraints are not binding, then:

$$
\sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) c^i(s^j) > \sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) e^i(y_j).
$$

Hereafter, the above constraint is called a solvency constraint.

Employing the above solvency constraint, we can translate the sequential trading problem into the time 0 problem:

$$
\max_{\{c^i\}} u \left[ c^0_0(s_0) \right] + \sum_{t=1}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t | s_0) u \left[ c^i(s^t) \right],
$$

s.t. $\sum_{t \geq 0} \sum_{s^t \in S^t} Q(s_0, s^t) \left[ c^i(s^t) - e^i(y_t) \right] \leq w^i_0,$

$$
\sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) c^i(s^j) \geq \sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) e^i(y_j), \forall s^t \in S^t, ~ t \geq 0.
$$

When a country is in default, the last constraint (solvency constraint) is binding.

---

9 See Proposition 3.1 in Lustig (2007).
The dual problem to the above time 0 problem, that is, the cost minimization problem to attain lifetime utility \( v_i^0 \) at time 0, is characterized as follows:

\[
C_i (s_0) = \inf_{\{c_i\}} \left\{ \sum_{t=1}^{\infty} \sum_{s't \in S^t} Q(s_0, s') c_i(s'), \right. \\
\left. \text{s.t. } \sum_{t \geq 0} \sum_{s' \in S^t} \beta^t \pi(s' | s_0) u \left[ c_i(s') \right] = v_i^0, \right. \\
\sum_{j \geq t} \sum_{s' \geq s^t} Q(s', s^t) c_i(s^t) \geq \sum_{j \geq t} \sum_{s' \geq s^t} Q(s', s^t) e_i(y_j), \ \forall s^t \in S^t, \ t \geq 0.
\]

The second equation is called a *promise keeping constraint* in the sense that the optimal solution allows a consumer to attain at least lifetime utility \( v_i^0 \).

### 2.2.2 Time-varying Negishii weights

An important feature of the above cost minimization problem is that the multiplier associated with the solvency constraint \( (\tau_i(s^t)) \) may be either zero or positive depending on whether a default occurs, while the multiplier associated with the promise-keeping condition \( (\mu_i^0) \) is fixed over time.

To record a history of binding solvency constraints (a history of \( \tau_i(s^t) \)), we define the cumulative multiplier \( \chi_i(s^t) \) as

\[
\chi_i(s^t) = \chi_i(s^{t-1}) - \tau_i(s^t) \Pi_{s^t}[\{e^t\}],
\]

where

\[
\Pi_{s^t}[\{e^t\}] = \sum_{j \geq t} \sum_{s' \geq s^t} Q(s', s^t) e_i(y_j), \ \forall s^t \in S^t, \ t \geq 0.
\]

If Lucas trees are equally endowed at time 0, then \( \chi_1^0 = \chi_2^0 = 1 \).

We here define $\zeta^i(s^t)$ as $\frac{\mu^i_0}{\chi^i(s^t)}$. It is easy to demonstrate that from the first order condition of the above RSPP, the ratio of marginal utility between the two countries is equal to the ratio of $\zeta^i(s^t)$ between the two.

$$\left[\frac{c^1(s^t)}{c^2(s^t)}\right]^{-\gamma} = \frac{\zeta^2(s^t)}{\zeta^1(s^t)}$$

The above equation implies that $\zeta^i(s^t)$ is equivalent to Negishi weights (an individual weight on lifetime utility) in the following time 0 planner’s objective:

$$\max \left\{ c^1, c^2 \right\} \left[ \zeta^1_0 u(c^1_0(s_0)) + \zeta^2_0 u(c^2_0(s_0)) + \sum_{t=1}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t | s_0) \left[ \zeta^1(s^t)u\left[c^1(s^t)\right] + \zeta^2(s^t)u\left[c^2(s^t)\right] \right] \right].$$

Given that $h(s^t) \equiv \zeta^1(s^t)^{\frac{1}{\gamma}} + \zeta^2(s^t)^{\frac{1}{\gamma}}$, the consumption of country $i$ is derived as $c^i(s^t) = \frac{\zeta^i(s^t)^{\frac{1}{\gamma}}}{h(s^t)} e(z_t)$. In addition, $\omega^i(s^t) \equiv \frac{\zeta^i(s^t)^{\frac{1}{\gamma}}}{h(s^t)}$ corresponds to the consumption share of each country.

Without any solvency constraint, the Negishi weight is constant over time. Accordingly, the cross-country consumption share does not change dynamically at all. With solvency constraints, however, the consumption share between the two countries fluctuates over time. The Negishi weight $\zeta^i(s^t)$ is constant unless country $i$ is in default at time $t$, but otherwise, it is revised upward as a result of positive $\tau^i(s^t)$ (the multiplier associated with solvency constraints). Therefore, the consumption share of country $i$ at time $t$ increases when country $i$ is in default at time $t$. More precisely, one country is not able to transfer goods from time $t + 1$ to time $t$ as a result of a solvency constraint at a certain state at time $t + 1$. Consequently, the time $t + 1$ consumption share of that country becomes large relative to the time $t$ share. In other words, this country yields higher consumption growth toward a state in which a solvency constraint is binding.

With due consideration for the above features of the Negishi weight, the sum of (nonlinearly transformed) Negishi weights $h(s^t)$ may reveal the extent to which solvency constraints are binding. Given the growth of $h(s^t)$, or

$$g(s^{t+1}) \equiv \frac{h(s^{t+1})}{h(s^t)},$$

a higher $g(s^{t+1})$ implies that either of the two countries face severe solvency constraints between time $t$ and time $t + 1$. By construction, $g(s^{t+1})$ is always one or higher. Lustig (2007) called $g(s^{t+1})$ liquidity shocks.
As demonstrated by Lustig (2007),\textsuperscript{12} thanks to a complete markets setup, a stochastic discount factor between state $s$ and state $s'$ can be defined as a function of the aggregate endowment and the above liquidity shock, or

$$\pi(s'|s) \left(\frac{e(z')}{e(z)}\right)^{-\gamma} g(s'|s)^\gamma.$$  

Without any solvency constraint ($g(s'|s) = 1$), a stochastic discount factor reduces to a standard one or $\pi(s'|s) \left(\frac{e(z')}{e(z)}\right)^{-\gamma}$.

The above aspect of the time 0 problem simplifies substantially the numerical computation procedure because it is possible to compute a stochastic discount factor between state $s'$ and state $s_0$ ($Q(s_0, s')$) given optimal policy functions. Following Lustig (2007), Appendix A briefly reviews the numerical method to solve the underlying model.

### 3 Calibration Exercises

#### 3.1 Setup

This section explores numerically how country-specific catastrophic shocks are shared between the two countries, and to what extent solvency constraints matter in sharing those shocks \textit{ex ante} and \textit{ex post}.

The existing empirical literature documents the size and persistence of catastrophic shocks on national outputs. Using US data for the period between 1869 and 1985, Cecchetti, Lam, and Mark (1990) identify catastrophic shocks on GDP. In their estimation, total annual output declines by 15.1\% in the catastrophic regime, while it grows by 2.5\% in the normal regime. The normal regime moves to the catastrophic regime with probability of 1.8\% per year. Once the economy enters the catastrophic state, the state repeats itself with probability 51.0\%. Their estimations of probability and loss magnitudes of catastrophic states is quite comparable with those estimated by Barro (2006). Barro (2006) argued that the annual probability of catastrophic states is around 1.7\%, and that the loss amounts to 15\% through 64\% of total output through intensively collecting data of developed and developing countries. These papers find that such catastrophic shocks permanently reduce the level of national output.

\textsuperscript{12}In Lustig (2007), when the aggregate endowment declines (that is, $\left(\frac{e(z')}{e(z)}\right)^{-\gamma}$ is larger), more consumers face solvency constraints as a result of more volatile idiosyncratic shocks (that is, $g(s'|s)^\gamma$ is larger). Accordingly, stochastic discount factors tend to heavily discount payoffs in a future recession state; this is a source of a larger risk premium in his model.
In the setup described in Section 2, we introduce persistent transitory catastrophic shocks with the limited number of states $z$. We set the degree of persistence of catastrophic states at extremely high levels partly to capture the fact that some types of catastrophic shocks have permanent effects on national output as discussed above, and partly to consider the fact that catastrophically damaged countries often experienced eventual recoveries, as discussed in Gourio (2007). Note that the direct introduction of permanent country-specific shocks would make the number of states $z$ extremely large in the current setup, and consequently make the numerical analysis rather difficult.\textsuperscript{13}

Following the estimation by Cecchetti et al. (1990) and Barro (2006), we assume that the labor endowment of a country declines by 20\% or 40\% with probability 1.8\% per year. Without the realization of catastrophic shocks, the labor endowment remains at a given level. When we consider the case of extremely persistent catastrophic shocks, we assume that a catastrophic state repeats itself with probability 80\%, once a catastrophic state hits one country. That is, a catastrophic state continues for five years ($\frac{1}{1-0.8}$) on average. A catastrophic shock is assumed to be country-specific and uncorrelated between the two countries.

We shed light on cases where solvency constraints are severely binding by making the ratio of dividends to the world labor endowment ($\alpha$) rather low. At time 0, both labor endowment and Lucas trees are equally distributed between the two countries. The rate of time preference is 5\% ($\beta = 0.95$), and the degree of relative risk aversion is five ($\gamma = 5$).

### 3.2 Welfare evaluation

We evaluate the welfare cost of solvency constraints in terms of deviations from certainty equivalence of consumption shares. From simulation results, we first obtain the series of the consumption share $\omega^i$ on $(z, z^k)$, and generate its distribution. We then calculate the share of $\omega_t$, and the expected utility based on relative consumption ($\Omega = E \left[ \sum_{t=0}^{\infty} \beta^t \frac{\omega_t^{1-\gamma}}{1-\gamma} \right]$). The certainty equivalence of consumption shares ($\bar{\omega}$) is defined as:

$$\Omega = \sum_{t=0}^{\infty} \beta^t \frac{\bar{\omega}^{1-\gamma}}{1-\gamma}.$$
The welfare cost of solvency constraints is evaluated by $0.5 - \bar{\omega}$. Without any solvency constraint, the consumption share between the two countries is constant with a fifty-fifty share over time. Therefore, the welfare cost reduces to zero. With solvency constraints, the consumption share becomes volatile. Accordingly, welfare costs becomes positive in this measure.

In addition to the unconditional measure, we compute the welfare cost conditional on a state in which only one country is in a catastrophic state.

### 3.3 Derivation of portfolio positions

While a portfolio problem is implicit in solving a planner’s problem, thanks to a two-country setup, it is possible to recover the portfolio positions of country 1 and country 2 as follows. From the budget constraint (3), we obtain the following system of equations to determine portfolio rules together with the market clearing conditions (1) and (2):

$$c^1(s^t) + p(z^t)\theta^1(s^t) + \sum_{s' \in S} q(s^t, s') a^1(s^t, s') = e^1(y_t) + [p(z_t) + d(z_t)] \theta^1(s^{t-1}) + a^1(s^{t-1}, s_t),$$

and

$$c^2(s^t) + p(z^t)\theta^2(s^t) + \sum_{s' \in S} q(s^t, s') a^2(s^t, s') = e^2(y_t) + [p(z_t) + d(z_t)] \theta^2(s^{t-1}) + a^2(s^{t-1}, s_t).$$

Note that both consumption and asset prices are standardized by the total endowment. We hereafter ignore an extremely rare state in which both countries receive catastrophic shocks simultaneously.

It is possible to identify from simulation results which state and which country faces a solvency constraint. These identified facts simplify the above system of equations. When a solvency constraint is binding on country 1 in state $s'$ at time $t$, country 1 repays up to:

$$-a^1(s^{t-1}, s') = [p(z^t) + d(z_t)] \theta^1(s^{t-1}).$$

When a solvency constraint is binding on country 2 in state $s'$ at time $t$, country 1 is repaid by:

$$a^1(s^{t-1}, s') = [p(z^t) + d(z_t)] (1 - \theta^1(s^{t-1})).$$

After simplifying the system, we approximate portfolio rules by $\theta^i(s^t) = \nu^i_0 + \nu^i c^i(s^t)$ and $a^i(s^t, s') = \alpha^i_0 + \alpha^i c^i(s^t)$. Given the simulated series of asset prices and consumption shares, we identify the values of $\nu^i_0$, $\nu^i$, $\alpha^i_0$, and $\alpha^i$ that minimize the sum of squared
residuals of the above system for a certain range of $c'(s')$. In so doing, we classify current states (time $t$ states) into three possible states, including (1) neither country 1 nor country 2 receives adverse shocks, (2) only country 1 receives shocks, and (3) only country 2 receives shocks.

We may have a special and convenient case in which as of time $t - 1$, either country would be subject to solvency constraints in any possible state of a one-period ahead period (time $t$). In this case, binding solvency constraints can identify portfolio positions precisely, and we can obtain exact positions without using any approximation.

3.4 Calibration results

3.4.1 Purely transitory case

We first consider a case with purely transitory and relatively mild catastrophic shocks to understand how risk-sharing works between two countries in the presence of solvency constraints. In particular, a country-specific catastrophic shock reduces labor endowment by 20% with probability 1.8% per year, but without any persistence. When $\alpha$ is quite small, a damaged country suffers from $1 - \alpha$ or about 5.6% losses in endowment shares relative to a fifty-fifty share. Table 1 reports the average equity prices and equity premiums for various dividend-to-endowment ratios ($\alpha$); all pricing variables represent the ratio relative to the world endowment. Table 2 reports the unconditional and conditional welfare costs for each value of $\alpha$.

The most important observation about these tables is that such catastrophic shocks are almost perfectly shared between the two countries even if $\alpha$ is extremely low. For example, when $\alpha$ is 0.1%, the average equity premium (0.994%) is much closer to the perfect insurance premium that emerges when solvency constraints are absent (0.937%) than to the closed economy premium that emerges when cross-border risk-sharing is absent (4.981%). As shown in Table 2, the unconditional welfare cost is almost zero. Even the conditional welfare cost amounts to only 0.191%, much lower than 5.6% damages in labor endowment shares. Figure 1 plots the consumption share between a nondamaged country (country 1) and a damaged country (country 2) in the case where a catastrophic state takes place only in time 0. According to this figure, the consumption share of the damaged country declines only by about 0.2% when a catastrophic shock is realized in time 0, and the damaged country suffers from only 0.1% permanent losses unless another catastrophic shock hits this country.
However, almost perfect insurance outcomes do not necessarily imply that solvency constraints are never binding. In recovering perfect insurance outcomes in the aftermath, about 5.6% of the world endowment should be transferred from the nondamaged country to the damaged country. When a catastrophe hits country 2 for the first time, the damaged country finances consumption partly by gross returns from previously invested Lucas trees \((p + \alpha)\theta^2\) where \(\theta^2 = 0.5\), and partly by insurance payments from the nondamaged country. However, the nondamaged country can make insurance payments only up to its solvency or gross returns from previously invested Lucas trees as collateral \((p + \alpha)\theta^1\) where \(\theta^1 = 0.5\). The two financial sources in \((p + \alpha)(0.5 + 0.5)\) are equal to the total value of Lucas trees (relative to the total endowment). Therefore, if the total equity value is short of the 5.6% endowment losses in the aftermath, then solvency constraints are binding in insurance arrangements made prior to catastrophe. As shown in Table 1, unless \(\alpha\) is larger than 0.5%, the equity is valued at an average of less than 5.6% in the aftermath. When \(\alpha\) is 0.1%, the aftermath equity value is only 1.4%, and the damaged country cannot finance sufficiently endowment losses through \textit{ex ante} arrangements.

The above reasoning indicates that the damaged country covers endowment losses not only by insurance receipts from the nondamaged country, but also by \textit{ex post} financing from the nondamaged country. Table 3 reports financial transactions between the two countries when a catastrophe hits country 2 in time 0 without any further catastrophe in either country in the case of \(\alpha = 0.1\). In the aftermath (time 0), given 5.6% endowment losses, the damaged country receives only 0.7% insurance payments from the nondamaged country. However, the damaged country finances 3.9% by building intensively long positions in contingent claims (44.0%) and short positions in Lucas trees (−47.9%). Together with gross returns from previously invested Lucas trees (0.7%), the damaged country can mitigate endowment losses substantially from 5.6% to 0.2%. As the opposite side, the nondamaged country builds large long positions in Lucas trees (49.3%) and short positions in contingent claims (−44.0%). Note that the nondamaged country (the damaged country) is the insurer (the insured) in contingent claim markets in time 0.

Why do such cross-border financial transactions with heavily leveraged positions help to achieve almost perfect insurance outcomes? The large long position in Lucas trees built by the nondamaged country plays an essential role in the above risk-sharing scheme, and serves two purposes. First, Lucas trees invested by the nondamaged country
in time 0 can be used as precautionary instruments for the nondamaged country in a case where they experience a catastrophe shock in time 1. Second, the long position in Lucas trees in time 0 improves the solvency of the nondamaged country in making insurance payments to the damaged country when the damaged country faces a catastrophic event again in time 1.

In time 1, a similar transaction between the two countries facilitates almost perfect risk-sharing. From time 2 onwards, the damaged country in turn holds long positions in Lucas trees for the above two purposes; in this case, the previously damaged country plays a role as the insurer in contingent claim markets. As a consequence of the above financial transactions involved with heavily leveraged positions, the damaged country is subject to only 0.1% permanent losses.

3.4.2 Extremely persistent case

We next move to a case with extremely large and persistent catastrophic shocks. That is, a country-specific catastrophic state reduces labor endowment by 40% with probability 1.8% per year, while it repeats itself with probability 80%. When $\alpha$ is rather small, a damaged country suffers from 12.5% endowment losses relative to fifty-fifty shares. Table 4 reports the average equity pricing for various values of $\alpha$, while Table 5 reports the unconditional and conditional welfare costs. Again, all pricing variables represent the ratio relative to the world endowment.

A major difficulty with insuring persistent shocks is that a damaged country is forced to finance not only a decrease in current labor endowment (12.5%), but also a substantial decline in lifetime income. Therefore, if the solvency of a nondamaged country is limited severely, such a large insurance transfer is not feasible at all. For example, when $\alpha = 0.2\%$, the average equity premium is 7.657% per year. If the risk premium on labor endowment is equal to the equity premium, the expected loss in lifetime income approximately equals $\sum_{t=0}^{\infty} [0.8 (1 + 0.07657))^t \times 12.5\%] = 90.1\%$. As shown in Table 4, however, the aftermath equity valuation (8.904%) is significantly less than the expected loss in lifetime income; an insurance transfer from a nondamaged country to a damaged country is not sufficient to cover permanent income losses.\footnote{The equity valuation is larger in the case with persistent shocks than in the case with purely transitory shocks as a result of stronger precautionary demand for Lucas trees.}

Nevertheless, almost perfect risk-sharing emerges in this case, even when $\alpha$ is extremely small. For example, with $\alpha = 0.2\%$, the average equity premium (7.657%) is
much closer to the perfect insurance premium (7.397%) than to the closed economy premium (25.076%). As shown in Table 5, the unconditional welfare cost is only 0.551%, while even the conditional welfare cost is less than 4% regardless of substantial declines in lifetime income (about 90% as discussed above). That is, most of the persistent catastrophic shocks are shared between the countries regardless of severe solvency constraints. When $\alpha = 7.4\%$, the aftermath equity valuation (82.366%) is large enough to cover the required insurance transfer between the two countries, and the average equity premium is accordingly exactly equal to the perfect insurance premium.

The \textit{ex post} financing mechanism is the same as in the previous case with purely transitory shocks. Table 6 reports financial transactions between the two countries when a catastrophe hits country 2 in time 0 without any further catastrophe in either country in the case of $\alpha = 0.2\%$. Figure 2 plots the time series of the consumption share in the corresponding case. In the aftermath (time 0), given 12.5% current endowment losses, the damaged country receives only 4.6% insurance payments from the nondamaged country. However, the damaged country finances 1.6% by building intensively long positions in contingent claims (85.7%) and short positions in Lucas trees (−87.3%). Together with gross returns from previously invested Lucas trees (4.6%), the damaged country mitigates current endowment losses substantially from 12.5% to 1.9%.

At the same time, the nondamaged country builds large net financial positions (10.5%) by having long positions in Lucas trees (96.2%) and short positions in contingent claims (−85.7%). The long position in Lucas trees accumulated by the nondamaged country meets the insurance needs of both countries partly by improving the insurance solvency provided by the nondamaged country to the damaged country, and partly by building precautionary instruments for the nondamaged country. In time 1, a similar transaction between the two countries facilitates almost perfect risk-sharing. From time 2 onwards, the damaged country in turn holds long positions in Lucas trees. Consequently, the damaged country is subject to only 1.4% permanent losses.

To demonstrate that the above risk-sharing mechanism is fairly powerful, we present two more cases with extremely large and persistent shocks; one where a catastrophic state continues for the expected length of time, or it hits country 2 five times in a consecutive manner (Table 7 and Figure 3), and the other where a catastrophic shock in turn hits country 1 in time 1 (Table 8 and Figure 4). In the former case, the maximum loss of the consumption share of the damaged country is at most 3.2% in time 5, while the permanent loss falls to 1.5%. As Table 7 illustrates, the large long (short) position in
Lucas trees held by the nondamaged (damaged) country facilitates a large-scale transfer between the two countries. In the latter case, on the other hand, thanks to own Lucas trees as precautionary instruments, the newly damaged country (country 1) suffers from only 1.1% permanent loss in its consumption share. In this case, the previously damaged country (country 2) in turn offers the insurance protection by holding large long positions in Lucas trees in time 1.

4 Conclusion

In this paper, we have examined the \textit{ex ante} and \textit{ex post} sharing of country-specific persistent catastrophic shocks between countries. Applying the theoretical framework proposed by Lustig (2007), we construct a two-country model with solvency constraints. In this constrained exchange economy, the exogenous endowment process consists of labor income subject to country-specific persistent catastrophic shocks, and dividends from Lucas trees subject to common world shocks. Only financial assets, including Lucas trees and contingent claims, can serve as collateral for cross-border financial transactions of catastrophe insurance (\textit{ex ante} arrangements) and \textit{ex post} financing.

Our calibration exercises demonstrate that as to the insurance contract made prior to a catastrophic event, the insurance payment to a damaged country is severely constrained by the limited solvency of a nondamaged country as an insurer, given the expectation that Lucas trees will be heavily discounted in the aftermath. In the aftermath, nevertheless, a damaged country can clear solvency constraints by building up assets and liabilities, thereby financing most of its uncovered losses. In addition, this risk-sharing scheme can work even for extremely persistent country-specific shocks. Therefore, the resulting outcome is almost equivalent to the perfect insurance outcome. Surprisingly, almost perfect risk-sharing still emerges even if one country faces several catastrophic shocks in a consecutive manner.

We have the following implications from our investigation. First, the literature on asset pricing, including Rietz (1988), Cecchetti, Lam, and Mark (1993), and Barro (2006), demonstrated that catastrophic risks help to generate large equity premiums in a closed economy setup. However, this statement may have to be revised in an open economy context to the extent that country-specific catastrophic shocks are insured against among countries. As demonstrated in this paper, if catastrophic shocks are shared effectively among countries despite the presence of solvency constraints, catastrophic shocks may
not be fundamentally responsible for large risk premiums in an open economy.

Second, one of the most important findings in this paper is that in the aftermath, voluntary cross-border transactions themselves create international collateral for countries in need of liquidity. Accordingly, what is needed during uneven allocation of liquidity among countries is not unilateral provision of liquidity from nondamaged countries (likely their public financial institutions) to damaged countries, but bilateral large-scale transactions of different financial instruments including derivatives between nondamaged and damaged countries. Such active and voluntary cross-border transactions allow a damaged country to clear solvency constraints and finance most of uncovered losses. That is, cross-border financial transactions accompanied by actively building assets and liabilities in both countries would emerge as a powerful risk-sharing scheme for catastrophic shocks. In this regard, when solvency constraints matter fundamentally, strict regulations on cross-border derivative transactions and severe restrictions on short positions in globally traded financial instruments would seriously limit opportunities to effectively insure against country-specific catastrophic shocks, likely repeated, among countries.

Appendix A: The numerical computation methods

As mentioned in Section 2, it is almost impossible to directly solve the sequential trading problem characterized by equation (5) because of the presence of solvency constraints. Following Lustig (2007), we instead solve the time 0 cost minimization problem dual to the utility maximization problem. We omit the time subscript $t$ because the problem is formulated in a recursive manner.

For expository purposes, we standardize all endogenous variables except for asset volume by the total world endowment.$^{15}$ Accordingly, we transform the stochastic discount factors as follows:

$$\hat{\pi}(s'|s) = \frac{\pi(s'|s) \left( \frac{e(z')}{e(z)} \right)^{1-\gamma}}{\sum_{s' \in S} \pi(s'|s) \left( \frac{e(z')}{e(z)} \right)^{1-\gamma}}, \quad (9)$$

$$\hat{\beta}(s) = \beta \sum_{s' \in S} \pi(s'|s) \left( \frac{e(z')}{e(z)} \right)^{1-\gamma}. \quad (10)$$

$^{15}$Alvarez and Jermann (2000) adopted the same transformation to make endogenous variables stationary in the case where the total endowment is growing.
We can use as a state variable the stationary consumption share \( \omega^i(s') = \frac{\zeta^i(s')}{k(s')} \in [0, 1] \) instead of the Negishi weight \( \zeta^i(s') \). As the Negishi weight is revised upward upon default, the consumption share is revised upward based on a cutoff rule as described below.

There are two steps in finding the equilibrium pricing and allocation. Given the initially guessed liquidity shocks \( g^{\text{guess}}(s'|s) \), the first step consists of solving the cost minimization problem given the sequence of prices, and of deriving optimal policy functions. In the second step, the sequence of consumption and asset pricing is computed from the simulation based on the derived policy functions; it is possible to map from liquidity shocks \( g^{\text{guess}}(s'|s) \) to stochastic discount factors \( \beta\left( \frac{e(z')}{e(z)} \right)^{-\gamma} g(s'|s)\gamma \), and to compute equilibrium asset pricing. We repeat this two-step procedure until the initially guessed liquidity shocks \( g^{\text{guess}}(s'|s) \) coincide with the newly generated liquidity shocks \( g^{\text{new}}(s'|s) \).

In solving the cost minimization problem, the current history is replace by a truncated history \( z^k \). Then the control variable is not current consumption, but a consumption share \( \omega^i \) in a detrended version of the cost function (8), and it is rewritten in a recursive manner:

\[
\hat{C}(\omega^i, s, z^k) = \min_{\omega^i} \left[ \omega^i + \hat{\beta}(s) \sum_{s' \in S} \hat{\pi}(s'|s)g(s'|s)\gamma \hat{C}(\omega^i|s', z^k') \right],
\]  

where \( \hat{\pi}(s'|s) \) and \( \hat{\beta}(s) \) are defined in equations (9) and (10). Note that \( \hat{\pi}(s'|s)g(s'|s)\gamma \) in the cost function corresponds to a stochastic discount factor or a pricing kernel; as a result of detrending, \( \left( \frac{e(z')}{e(z)} \right) \) is always equal to one. Similarly, a detrended version of the present value of the endowment sequence is written as follows:

\[
\hat{C}^e(s, z^k) = \hat{e}'(s) + \hat{\beta}(s) \sum_{s' \in S} \hat{\pi}(s'|s)g(s'|s)\gamma \hat{C}^e(s', z^h'),
\]

where \( \hat{e}'(s) \) is the share of individual labor endowment to the aggregate endowment.

Lustig (2007) found that \( \omega^i \) is bounded from \( \omega^i(s) \) as a result of binding solvency constraints, and constructs the following cutoff rule to revise a state variable \( \omega^i \) upward: that is, if \( \omega^i > \omega^i(s) \), then \( \omega^i = \frac{\omega^i(s)}{g(s'|s)} \), and if \( \omega^i \leq \omega^i(s) \), then \( \omega^i = \frac{\omega^i(s)}{g(s'|s)} \). Because \( \omega^i \) is bounded from below upon default, the lower bound of \( \omega^i \) or \( \omega^i(s) \) is determined by:

\[
\hat{C}(\omega^i(s), s, z^k) = \hat{C}^e(s, z^k).
\]

Given the exogenous endowment process \( (s, z^k) \), the social planner solves the above cost minimization problem together with the cutoff rule by adjusting the current consumption share \( \omega^i \) and the share promised in the next period \( \omega^i' \).
Because equation (11) is a standard dynamic programming problem, we can solve it by guess and verify procedure (a value function iteration). For this purpose, the cost function is approximated by a cubic spline interpolation with 100 grids for state $\omega^i \in [0, 1]$. In addition, it is assumed that $k = 3$ for the history parameter. It is possible to derive a policy function $\omega' = f(\omega, s, z^k)$ from the computed cost function. It is also possible to obtain the share of consumption of the two countries from the sequence of promised consumption shares $\omega^i$.

In simulation, we first generate the sequence of aggregate and idiosyncratic shocks $\{s_t\}_{t=1}^{31,000}$ for 31,000 periods while omitting the initial 1000 periods. Given this generated sequence, we derive the sequence of consumption shares from the computed policy function, and then compute asset pricing and liquidity shocks. As mentioned above, we repeat this procedure until the generated liquidity shocks converge.

References


Table 1: Unconditional averages of equity prices and dividend ratios (shock size: 20%, persistence: i.i.d., unit: %)

<table>
<thead>
<tr>
<th>dividend ratio (α)</th>
<th>average equity price</th>
<th>average price-dividend ratio</th>
<th>average equity premium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>normal state</td>
<td>shock state</td>
<td>normal state</td>
</tr>
<tr>
<td>0.1</td>
<td>2.024</td>
<td>1.353</td>
<td>2024.3</td>
</tr>
<tr>
<td>0.2</td>
<td>3.931</td>
<td>2.580</td>
<td>1965.7</td>
</tr>
<tr>
<td>0.3</td>
<td>5.879</td>
<td>3.857</td>
<td>1959.8</td>
</tr>
<tr>
<td>0.4</td>
<td>7.819</td>
<td>5.130</td>
<td>1954.8</td>
</tr>
<tr>
<td>0.5</td>
<td>9.680</td>
<td>6.351</td>
<td>1936.1</td>
</tr>
</tbody>
</table>

Note: All pricing variables represent the ratio relative to the entire world endowment.

Table 2: Welfare costs and dividend ratios (shock size: 20%, persistence: i.i.d., unit: %)

<table>
<thead>
<tr>
<th>dividend ratio (α)</th>
<th>welfare cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>unconditional</td>
</tr>
<tr>
<td>0.1</td>
<td>0.002</td>
</tr>
<tr>
<td>0.2</td>
<td>0.004</td>
</tr>
<tr>
<td>0.3</td>
<td>0.003</td>
</tr>
<tr>
<td>0.4</td>
<td>0.002</td>
</tr>
<tr>
<td>0.5</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table 3: Portfolio transaction behavior with one-time shock (shock size: 20%, persistence: i.i.d., $\alpha=0.1\%$)

<table>
<thead>
<tr>
<th>Labor endowment</th>
<th>Realized</th>
<th>Insurance</th>
<th>Consumption</th>
<th>Invested</th>
<th>Invested</th>
<th>Invested</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share</td>
<td>Consumption share</td>
<td>realized tree value</td>
<td>(p + $\alpha$)$\theta$</td>
<td>$a$</td>
<td>$\theta'$</td>
<td>$p\theta'$</td>
</tr>
<tr>
<td>non-damaged country</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time -1</td>
<td>0.500</td>
<td>0.011</td>
<td>0.000</td>
<td>0.500</td>
<td>0.5</td>
<td>0.010</td>
</tr>
<tr>
<td>time 0</td>
<td>0.556</td>
<td>0.007</td>
<td>-0.007</td>
<td>0.502</td>
<td>36.5</td>
<td>0.493</td>
</tr>
<tr>
<td>time 1</td>
<td>0.500</td>
<td>0.777</td>
<td>-0.756</td>
<td>0.502</td>
<td>819.3</td>
<td>16.632</td>
</tr>
<tr>
<td>time 2</td>
<td>0.500</td>
<td>17.369</td>
<td>-17.348</td>
<td>0.502</td>
<td>-101.6</td>
<td>-2.052</td>
</tr>
<tr>
<td>time 3</td>
<td>0.500</td>
<td>-2.154</td>
<td>2.175</td>
<td>0.501</td>
<td>-103.0</td>
<td>-2.081</td>
</tr>
<tr>
<td>damaged country</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time -1</td>
<td>0.500</td>
<td>0.011</td>
<td>0.000</td>
<td>0.500</td>
<td>0.5</td>
<td>0.010</td>
</tr>
<tr>
<td>time 0</td>
<td>0.444</td>
<td>0.007</td>
<td>0.007</td>
<td>0.498</td>
<td>-35.5</td>
<td>-0.479</td>
</tr>
<tr>
<td>time 1</td>
<td>0.500</td>
<td>-0.756</td>
<td>0.756</td>
<td>0.498</td>
<td>-818.3</td>
<td>-16.611</td>
</tr>
<tr>
<td>time 2</td>
<td>0.500</td>
<td>-17.348</td>
<td>17.348</td>
<td>0.499</td>
<td>102.6</td>
<td>2.073</td>
</tr>
<tr>
<td>time 3</td>
<td>0.500</td>
<td>2.175</td>
<td>-2.175</td>
<td>0.499</td>
<td>104.0</td>
<td>2.101</td>
</tr>
</tbody>
</table>

Note: All variables except for the number of invested Lucas’ trees ($\theta$) represent the ratio relative to the entire world endowment.

Table 4: Unconditional averages of equity prices and dividend ratios (shock size: 40%, persistence: 80%, unit: %)

<table>
<thead>
<tr>
<th>Dividend ratio ($\alpha$)</th>
<th>Average equity price</th>
<th>Average price-dividend ratio</th>
<th>Average equity premium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal state</td>
<td>Shock state</td>
<td>Normal state</td>
</tr>
<tr>
<td>0.2</td>
<td>14.962</td>
<td>8.904</td>
<td>7481.0</td>
</tr>
<tr>
<td>1.0</td>
<td>47.147</td>
<td>26.465</td>
<td>4714.7</td>
</tr>
<tr>
<td>2.0</td>
<td>62.651</td>
<td>31.982</td>
<td>3132.5</td>
</tr>
<tr>
<td>3.0</td>
<td>85.117</td>
<td>42.552</td>
<td>2837.2</td>
</tr>
<tr>
<td>4.0</td>
<td>107.270</td>
<td>53.210</td>
<td>2681.8</td>
</tr>
<tr>
<td>5.0</td>
<td>126.450</td>
<td>62.205</td>
<td>2529.1</td>
</tr>
<tr>
<td>6.0</td>
<td>147.410</td>
<td>72.314</td>
<td>2456.9</td>
</tr>
<tr>
<td>7.0</td>
<td>162.130</td>
<td>78.781</td>
<td>2316.2</td>
</tr>
<tr>
<td>7.4</td>
<td>169.760</td>
<td>82.366</td>
<td>2294.1</td>
</tr>
</tbody>
</table>

Note: All pricing variables represent the ratio relative to the entire world endowment.
Table 5: Welfare costs and dividend ratios (shock size: 40%, persistence: 80%, unit: %)

<table>
<thead>
<tr>
<th>dividend ratio (α)</th>
<th>unconditional</th>
<th>conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.551</td>
<td>3.967</td>
</tr>
<tr>
<td>1.0</td>
<td>0.593</td>
<td>3.972</td>
</tr>
<tr>
<td>2.0</td>
<td>0.673</td>
<td>3.815</td>
</tr>
<tr>
<td>3.0</td>
<td>0.565</td>
<td>3.357</td>
</tr>
<tr>
<td>4.0</td>
<td>0.338</td>
<td>2.528</td>
</tr>
<tr>
<td>5.0</td>
<td>0.148</td>
<td>1.613</td>
</tr>
<tr>
<td>6.0</td>
<td>0.079</td>
<td>1.149</td>
</tr>
<tr>
<td>7.0</td>
<td>0.003</td>
<td>0.164</td>
</tr>
<tr>
<td>7.4</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 6: Portfolio transaction behavior with one-time shock (shock size: 40%, persistence: 80%, α=0.2%) 

<table>
<thead>
<tr>
<th>labor endowment</th>
<th>realized</th>
<th>insurance</th>
<th>consumption</th>
<th>invested</th>
<th>invested</th>
<th>invested</th>
</tr>
</thead>
<tbody>
<tr>
<td>share</td>
<td>tree value</td>
<td>receipt</td>
<td>share</td>
<td>trees</td>
<td>θ'</td>
<td>pθ'</td>
</tr>
<tr>
<td>(p + α)θ</td>
<td>a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>non-damaged country</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time -1</td>
<td>0.500</td>
<td>0.076</td>
<td>0.000</td>
<td>0.500</td>
<td>0.5</td>
<td>0.075</td>
</tr>
<tr>
<td>time 0</td>
<td>0.625</td>
<td>0.046</td>
<td>-0.046</td>
<td>0.519</td>
<td>10.8</td>
<td>0.962</td>
</tr>
<tr>
<td>time 1</td>
<td>0.500</td>
<td>1.733</td>
<td>-1.572</td>
<td>0.516</td>
<td>10.9</td>
<td>1.727</td>
</tr>
<tr>
<td>time 2</td>
<td>0.500</td>
<td>1.653</td>
<td>-1.501</td>
<td>0.514</td>
<td>-22.8</td>
<td>-3.411</td>
</tr>
<tr>
<td>damaged country</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time -1</td>
<td>0.500</td>
<td>0.076</td>
<td>0.000</td>
<td>0.500</td>
<td>0.5</td>
<td>0.075</td>
</tr>
<tr>
<td>time 0</td>
<td>0.375</td>
<td>0.046</td>
<td>-0.046</td>
<td>0.481</td>
<td>-9.8</td>
<td>-0.873</td>
</tr>
<tr>
<td>time 1</td>
<td>0.500</td>
<td>-1.572</td>
<td>1.572</td>
<td>0.484</td>
<td>-9.9</td>
<td>-1.569</td>
</tr>
<tr>
<td>time 2</td>
<td>0.500</td>
<td>-1.501</td>
<td>1.501</td>
<td>0.486</td>
<td>23.8</td>
<td>3.561</td>
</tr>
</tbody>
</table>

Note: All variables except for the number of invested Lucas' trees (θ) represent the ratio relative to the entire world endowment.
Table 7: Portfolio transaction behavior with five consecutive shocks (shock size: 40%,
persistence: 80%, $\alpha=0.2\%$)

<table>
<thead>
<tr>
<th></th>
<th>labor endowment share</th>
<th>realized tree value $(p + \alpha)\theta$</th>
<th>insurance receipt $a$</th>
<th>consumption share</th>
<th>invested trees $\theta'$</th>
<th>invested contingent claims $\sum q\theta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-damaged country</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time -1</td>
<td>0.500</td>
<td>0.076</td>
<td>0.000</td>
<td>0.500</td>
<td>0.5</td>
<td>0.075</td>
</tr>
<tr>
<td>time 0</td>
<td>0.625</td>
<td>0.046</td>
<td>-0.046</td>
<td>0.519</td>
<td>10.8</td>
<td>0.962</td>
</tr>
<tr>
<td>time 1</td>
<td>0.625</td>
<td>0.916</td>
<td>-0.916</td>
<td>0.526</td>
<td>33.2</td>
<td>2.748</td>
</tr>
<tr>
<td>time 2</td>
<td>0.625</td>
<td>2.815</td>
<td>-2.815</td>
<td>0.529</td>
<td>32.0</td>
<td>2.649</td>
</tr>
<tr>
<td>time 3</td>
<td>0.625</td>
<td>2.713</td>
<td>-2.713</td>
<td>0.531</td>
<td>31.3</td>
<td>2.591</td>
</tr>
<tr>
<td>time 4</td>
<td>0.625</td>
<td>2.654</td>
<td>-2.654</td>
<td>0.532</td>
<td>30.9</td>
<td>2.558</td>
</tr>
<tr>
<td>time 5</td>
<td>0.500</td>
<td>4.958</td>
<td>-4.797</td>
<td>0.532</td>
<td>8.2</td>
<td>1.299</td>
</tr>
<tr>
<td>time 6</td>
<td>0.500</td>
<td>1.243</td>
<td>-1.092</td>
<td>0.522</td>
<td>-19.8</td>
<td>-2.963</td>
</tr>
<tr>
<td>time 7</td>
<td>0.500</td>
<td>-3.002</td>
<td>3.154</td>
<td>0.517</td>
<td>-21.7</td>
<td>-3.247</td>
</tr>
<tr>
<td>time 8</td>
<td>0.500</td>
<td>-3.290</td>
<td>3.442</td>
<td>0.515</td>
<td>-22.5</td>
<td>-3.366</td>
</tr>
</tbody>
</table>

|                  |                       |                                           |                       |                    |                          |                                  |
| non-damaged country |                       |                                           |                       |                    |                          |                                  |
| time -1          | 0.500                 | 0.076                                    | 0.000                 | 0.500              | 0.5                      | 0.075                           | 0.000                            |
| time 0           | 0.375                 | 0.046                                    | 0.046                 | 0.481              | -9.8                     | -0.873                          | 0.857                            |
| time 1           | 0.375                 | -0.831                                   | 0.916                 | 0.474              | -32.2                    | -2.666                          | 2.650                            |
| time 2           | 0.375                 | -2.730                                   | 2.615                 | 0.471              | -31.0                    | -2.566                          | 2.554                            |
| time 3           | 0.375                 | -2.628                                   | 2.713                 | 0.469              | -30.3                    | -2.508                          | 2.498                            |
| time 4           | 0.375                 | -2.569                                   | 2.654                 | 0.468              | -29.9                    | -2.475                          | 2.466                            |
| time 5           | 0.500                 | -4.797                                   | 4.797                 | 0.468              | -7.2                     | -1.141                          | 1.172                            |
| time 6           | 0.500                 | -1.092                                   | 1.092                 | 0.478              | 20.8                     | 3.112                           | -3.091                           |
| time 7           | 0.500                 | 3.154                                    | -3.154                | 0.483              | 22.7                     | 3.396                           | -3.380                           |
| time 8           | 0.500                 | 3.442                                    | -3.442                | 0.485              | 23.5                     | 3.516                           | -3.502                           |

Note: All variables except for the number of invested Lucas’ trees ($\theta$) represent the ratio relative to the entire world endowment.
Table 8: Portfolio transaction behavior with alternate shocks (shock size: 40%, persistence: 80%, $\alpha=0.2\%$)

<table>
<thead>
<tr>
<th>Time</th>
<th>Labor Endowment Share $(p + \alpha)\theta$</th>
<th>Realized Tree Value $a$</th>
<th>Insurance Receipt $\theta'$</th>
<th>Consumption Share $p\theta'$</th>
<th>Invested Trees $\sum q\theta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>initially non-damaged country</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time -1</td>
<td>0.500</td>
<td>0.076</td>
<td>0.000</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>time 0</td>
<td>0.625</td>
<td>0.046</td>
<td>-0.046</td>
<td>0.519</td>
<td>-10.8</td>
</tr>
<tr>
<td>time 1</td>
<td>0.375</td>
<td>0.916</td>
<td>-0.831</td>
<td>0.489</td>
<td>-54.9</td>
</tr>
<tr>
<td>time 2</td>
<td>0.500</td>
<td>-8.809</td>
<td>8.809</td>
<td>0.489</td>
<td>-10.7</td>
</tr>
<tr>
<td></td>
<td>initially damaged country</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time -1</td>
<td>0.500</td>
<td>0.076</td>
<td>0.000</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>time 0</td>
<td>0.375</td>
<td>0.046</td>
<td>-0.831</td>
<td>0.481</td>
<td>-9.8</td>
</tr>
<tr>
<td>time 1</td>
<td>0.625</td>
<td>-0.831</td>
<td>0.831</td>
<td>0.511</td>
<td>55.9</td>
</tr>
<tr>
<td>time 2</td>
<td>0.500</td>
<td>8.969</td>
<td>-8.809</td>
<td>0.511</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Note: All variables except for the number of invested Lucas’ trees ($\theta$) represent the ratio relative to the entire world endowment.
Figure 1: Consumption shares with one-time shock (shock size: 20%, persistence: i.i.d., \( \alpha = 0.1\% \))

Figure 2: Consumption shares with one-time shock (shock size: 40%, persistence: 80%, \( \alpha = 0.2\% \))
Figure 3: Consumption shares with five consecutive shocks (shock size: 40%, persistence: 80%, \( \alpha = 0.2\% \))

![Figure 3: Consumption shares with five consecutive shocks](image)

Figure 4: Consumption shares with alternate shocks (shock size: 40%, persistence: 80%, \( \alpha = 0.2\% \))

![Figure 4: Consumption shares with alternate shocks](image)