Intertemporal efficiency and cross sectional distribution within dynamic contracts

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Motivation

1. Empirical questions: Which kind of restrictions are imposed on intertemporal efficiency and cross sectional distribution in the presence of dynamic contract constraints?
   (a) interaction between dynamic contract constraints and growth/cycles
   (b) asymmetric treatment in the previous approach
      i. complete markets: structural restrictions
      ii. incomplete markets: rather reduced form restrictions
2. Policy questions: Which kind of policy issues would emerge with dynamic contract constraints?
   (a) Vickrey- Mirrlees approach
   (b) dynamic general equilibrium version of \textit{distribution versus efficiency}
3. To which macroeconomic field are dynamic contract constraints applicable?
Characteristics of contract approach in macroeconomics

1. Very simple in essential parts (want to show today)

2. Hard parts: dynamics and general equilibrium (want to give a sort of flavor today)

3. Cumbersome parts: numerical computation (not mention today)
A plan of talk

1. Implications from complete markets

2. Hidden information
   (a) a three period setup
   (b) alternative opportunities (hidden saving)
   (c) enforcement constraints
   (d) infinite horizon setup (promise keeping constraints)
3. Hidden actions
   (a) a two period setup
   (b) an inverse version of Euler equation
   (c) alternative opportunities (hidden saving)

4. Policy issues
   (a) money and credit
   (b) tax and distribution policy
5. Setup:

(a) abstracting aggregate shocks in most cases.

(b) principal-agent model: a principal commits, but agents may not.

(c) market model: decentralization by tax instruments.
Starting from complete markets

1. many consumers identical at time 0 and standardized as population size equal to 1:

2. stochastic endowment process of consumer $i$:
   
   $y^i_t = s_h Y_t$ with probability 0.5 and $y^i_t = s_l Y_t$ with probability 0.5, where $s_h > s_l$. 
3. individual maximization

$$\max \sum_{t=1}^{\infty} \sum_{\omega_t \in \Omega_t} \left[ \frac{1}{(1 + \rho)^t} \pi(\omega_t) u(c^i(\omega_t)) \right]$$

s. t.

$$\sum_{t=1}^{\infty} \sum_{\omega_t \in \Omega_t} \left[ p(\omega_t)(y^i(\omega_t) - c^i(\omega_t)) \right] = 0$$
4. consequence (1): complete insurance (cross sectional efficiency among *ex ante* identical consumers)

\[ c_t^i = E[Y_t] \]

5. consequence (2): Euler equation with respect to one period non-contingent bonds (intertemporal efficiency)

\[ P(\omega_t) = \frac{1}{1 + \rho} E\left[ \frac{u'(c_t^{i+1})}{u'(c_t^i(\omega_t))} \mid \omega_t \right] \]

at any \( \omega_t \).
Difficulty with complete markets

1. Any state is **publicly observable**.

2. Any contract is **enforceable**. In other words, both parties on contract commit to an original contract at any state of any time.
Macroeconomic Issues

1. Asymmetric information as well as enforcement problems may potentially impose completely different restrictions on intertemporal efficiency and cross sectional distribution!

2. They may also open up new macroeconomic policy issues!
Hidden information and efficiency conditions

A simple example based on Townsend [26]

1. three period exchange economy: zero income at time 0, \( y_1 = y_h \) with probability \( \frac{1}{2} \), \( y_1 = y_l \) with probability \( \frac{1}{2} \), \( y_2 = \frac{y_h + y_l}{2} \).

2. maximization problem:

\[
\max \frac{1}{2} \left[ u(c_l) + u(c_{2l}) \right] + \frac{1}{2} \left[ u(c_h) + u(c_{2h}) \right]
\]
Complete markets:

1. planner’s problem:

\[ \frac{1}{2} u(y_l + \psi) + \frac{1}{2} u(y_h - \psi) + u(c_2) \]

2. FOC:

\[ u'(y_l + \psi) = u'(y_h - \psi) \]

3. insurance transfer:

\[ \psi = \frac{y_h - y_l}{2} \]

4. full insurance outcomes:

\[ c_h = c_l = c_2 = \frac{y_h + y_l}{2} \]
Self insurance

1. for low income earners: \( u(y_l + \psi) + u(y_2 - R\psi) \)
2. for high income earners: \( u(y_h - \psi) + u(y_2 + R\psi) \)
3. market clearing pricing: \( R = 1 \)
4. self insurance transfer
   \[ \psi = \frac{y_h - y_l}{4} \]
5. partially insured outcomes:
   \[ c_l = c_{2l} = \frac{y_h + 3y_l}{4} \]
   \[ c_h = c_{2h} = \frac{3y_h + y_l}{4} \]
Simple dynamic contract

1. insurance payoffs for high income earners: \(-\psi\) at time 1 and \(+\alpha\psi\) at time 2

2. insurance payoffs for low income earners: \(+\psi\) at time 1 and \(-\alpha\psi\) at time 2

3. IC for high income earners:

\[ u(y_h - \psi) + u(y_2 + \alpha\psi) \geq u(y_h + \psi) + u(y_2 - \alpha\psi) \]

4. IC for low income earners:

\[ u(y_l + \psi) + u(y_2 - \alpha\psi) \geq u(y_l - \psi) + u(y_2 + \alpha\psi) \]
5. under quadratic preference $u(c) = c - bc^2$

\[
\frac{u'(y_h)}{u'(y_2)} \leq \alpha \leq \frac{u'(y_l)}{u'(y_2)}
\]

6. binding IC for high income earners:

\[
0 < \tilde{\alpha} = \frac{u'(y_h)}{u'(y_2)} < 1
\]
7. planner’s problem:

\[
\frac{1}{2} \left[ u(y_h - \psi) + u(y_2 + \hat{\alpha}\psi) \right] + \frac{1}{2} \left[ u(y_l + \psi) + u(y_2 - \hat{\alpha}\psi) \right]
\]

8. dynamic insurance transfer:

\[
\psi = \frac{y_h - y_l}{2(\hat{\alpha}^2 + 1)}
\]

9. comparison among three regimes:

\[
\frac{y_h - y_l}{4} < \frac{y_h - y_l}{2(\hat{\alpha}^2 + 1)} < \frac{y_h - y_l}{2}
\]
Issue 1: Hidden saving with hidden income

Cole and Kocherlakota [6]

dynamic contracts are forced to compete with alternative investment opportunities

\[ \text{lending rates} \leq \alpha - 1 \leq \text{borrowing rates} \]
**Issue 2: Enforcement constraint**

\[ u(y_2 - \alpha \phi) \geq u \]

**Issue 3: Inequality and insurance incentives**

\[ 0 < \hat{\alpha} = \frac{u'(y_h)}{u'(y_2)} < 1 \]

\[ \psi = \frac{y_h - y_l}{2(\hat{\alpha}^2 + 1)} \]
Extension to infinite horizon

Green [12]
Thomas ans Worrall [24]
Atkeson and Lucas [3]

1. history of reported income:

\[ h^t = \{y_1, y_2, y_3, \ldots, y_t \} \]

2. insurance payoff contract:

\[ b_t = f_t(h^t) \]
3. planner’s problem:

$$\max E \left[ \sum_{t=1}^{\infty} \frac{1}{(1 + \rho)^t} u(f_t(h^t) + y_t) \right]$$

with incentive compatibility constraints (truth-telling constraints). But, it is extremely hard to solve.
4. reformulating its dual problem (cost minimization) in a recursive manner: promised utility $v$ as a state variable

\[ Q(v) = \max \left[ \frac{1}{2}(-b_h + \frac{1}{1+\rho}Q(w_h)) \right] + \left[ \frac{1}{2}(-b_l + \frac{1}{1+\rho}Q(w_l)) \right] \]
s. t.  

(promise keeping constraint)

\[
\frac{1}{2} (u(y_h + b_h) + \frac{1}{1 + \rho} w_h) + \frac{1}{2} (u(y_l + b_l) + \frac{1}{1 + \rho} w_l) = v
\]

(IC for high income earners)

\[
u(y_h + b_h) + \frac{1}{1 + \rho} w_h \geq u(y_h + b_l) + \frac{1}{1 + \rho} w_l
\]

(IC for low income earners)

\[
u(y_l + b_l) + \frac{1}{1 + \rho} w_l \geq u(y_l + b_h) + \frac{1}{1 + \rho} w_h
\]
5. solution properties: analogous to Townsend’s case
(binding IC for high income earners)

\[ u(y_h + b_h) + \frac{1}{1 + \rho} w_h = u(y_h + b_l) + \frac{1}{1 + \rho} w_l \]

(up-front and down-front payoff structure)

\[ b_l \geq b_h \text{ and } w_l \leq w_h \]
6. immiseration phenomena

\[ v \] converges to negative infinity.

more future penalties on current low income earners

(current beneficiaries)

contrasting with upward consumption profiles with

strong precautionary savings.

7. more complicated in serially correlated

idiosyncratic shocks, Fernandes and Phelan [8]

8. general equilibrium by component planners,

Atkeson and Lucas [3]
Application of hidden information: Tobin’s effects

Saito and Takeda [23]

1. fiat money, basically free of contract constraints (private information and enforcement).

2. enforcement constraints → reducing insurance effects → creating money demand for a precautionary reason

3. money as an alternative investment opportunity (coexistence of money and credit)
4. welfare effects of inflation

(a) costs: holding costs of money

(b) benefits: lowering returns on money $\rightarrow$ enhancing insurance effects as a result of relaxed incentive constraints

(c) mild inflation would result in welfare improvement ('Tobin’s effects).
Hidden actions and efficiency conditions

A simple example based on Rogerson [22]

1. time 1 income: \( y_1 \)

2. time 2 income: \( y_h \) with probability \( \pi(n) \), and \( y_l \) with probability \( 1 - \pi(n) \)

3. time 1 effort \( n \): \( \pi'(n) > 0 \), \( \pi''(n) < 0 \)

4. utility maximization:

\[
u(c_1) - v(n) + \frac{1}{1 + \rho} [\pi(n)u(c_h) + (1 - \pi(n))u(c_l)]\]
5. incentive constraints: given \{c_1, c_h, c_l\},

\[
\max_{n} u(c_1) - v(n) + \frac{1}{1 + \rho} [\pi(n)u(c_h) + (1 - \pi(n))u(c_l)]
\]

(FOC)

\[
v'(n) = \frac{1}{1 + \rho} \pi'(n) [u(c_h) - u(c_l)]
\]
6. planner's problem:

\[
\max_{c_1, c_h, c_l} u(c_1) - v(n) + \frac{1}{1 + \rho} \left[ \pi(n)u(c_h) + (1 - \pi(n))u(c_l) \right]
\]

s. t.

(resource constraints)

\[
\pi(n)c_h + (1 - \pi(n))c_l = (1 + r)(y_1 - c_1) + \pi(n)y_h + (1 - \pi(n))y_l
\]

(incentive compatibility constraints)

\[
v'(n) = \frac{1}{1 + \rho} \pi'(n) [u(c_h) - u(c_l)]
\]
7. planner’s Lagrangean problem:

\[
L = u(c_1) - v(n) + \frac{1}{1 + \rho} \left[ \pi(n)u(c_h) + (1 - \pi(n))u(c_l) \right] \\
+ \eta \left[ (1 + r)(y_1 - c_1) + \pi(n)y_h + (1 - \pi(n))y_l - \pi(n)c_h - (1 - \pi(n))c_l \right] \\
+ \mu \left[ -v'(n) + \frac{1}{1 + \rho} \pi'(n) \left[ u(c_h) - u(c_l) \right] \right]
\]

8. FOC:

\[
u'(c_{1***}) - \eta(1 + r) = 0,
\]
\[
\frac{\pi(n)}{1 + \rho} u'(c_{h***}) - \eta \pi(n) + \mu \frac{\pi'(n)}{1 + \rho} u'(c_{h***}) = 0,
\]
\[
\frac{1 - \pi(n)}{1 + \rho} u'(c_{l***}) - \eta(1 - \pi(n)) - \mu \frac{\pi'(n)}{1 + \rho} u'(c_{l***}) = 0.
\]
9. an inverse version of Euler equation!!:

\[
\frac{1}{u'(c_{1^{***}})} = \frac{1}{1 + \rho} \left[ \pi(n) \frac{1}{u'(c_{h^{***}})} + (1 - \pi(n)) \frac{1}{u'(c_{l^{***}})} \right]
\]

\[
= \frac{1}{1 + \rho} E \left[ \frac{1}{u'(c_{2^{***}})} \right]
\]

implying

\[
u'(c_{1^{***}}) < \frac{1 + r}{1 + \rho} E u'(c_{2^{***}})
\] (1)

violating traditional intertemporal efficiency conditions.
Hidden saving with hidden actions

Golosov and Tsyvinski [11]

1. individual optimization problem: given \( \{c_1, c_h, c_l\} \),

\[
\begin{align*}
v &= \max_{n, \hat{c}_1, \hat{c}_h, \hat{c}_l} u(\hat{c}_1) - v(n) + \frac{1}{1 + \rho} \left[ \pi(n)u(\hat{c}_h) + (1 - \pi(n))u(\hat{c}_l) \right] \\
\text{s. t.} \\
\hat{c}_h &= (1 + r)(c_1 - \hat{c}_1) + c_h \\
\hat{c}_l &= (1 + r)(c_1 - \hat{c}_1) + c_l
\end{align*}
\]
2. planner’s problem with hidden saving:

\[
\max_{c_1, c_h, c_l} u(c_1) - v(n) + \frac{1}{1 + \rho} \left[ \pi(n) u(c_h) + (1 - \pi(n)) u(c_l) \right]
\]

s. t.

(resource constraints)

\[
\pi(n)c_h + (1 - \pi(n))c_l = (1 + r)(y_1 - c_1) + \pi(n)y_h + (1 - \pi(n))y_l
\]

(hidden saving constraints and IC)

\[
u(c_1) - v(n) + \frac{1}{1 + \rho} \left[ \pi(n) u(c_h) + (1 - \pi(n)) u(c_l) \right] \geq v
\]

3. Euler equation still holds: 

\[
u'(c_1) = \frac{1 + r}{1 + \rho} Eu'(c_2),
\]

deteriorating effort levels by hidden saving.
Application of hidden actions: Capital income taxation

Golosov, Kocherlakoto, and Tsyvinski [9] (history-dependent capital taxation and uniform cross-commodity consumption taxes)

Kocherlakota [14] (history-dependent zero expected wealth taxes)

Albanesi and Sleet [1] (general equilibrium by component planners, capital taxation dependent on capital and labor supply)

Kocherkakota [15] (a survey)
1. to decentralize optimal allocation a la Prescott and Townsend [21]

2. to restore \( \{c_1^{***}, c_h^{***}, c_l^{***}\} \) by capital income taxation.

3. capital income taxation:
\[
1 + \bar{r}(\hat{c}_2) = (1 - \tau(\hat{c}_2))(1 + r)
\]

4. Euler equation in competitive markets:
\[
u'(c_1^{***}) = E \left[ \frac{1 + \bar{r}(\hat{c}_2)}{1 + \rho} u'(c_2^{***}) \right]
\]
5. optimal taxation:

\[ \tau(\hat{c}_2) = 1 - \frac{1}{u'(\hat{c}_2)E\left[\frac{1}{u'(c^{***)}}\right]} \]

6. zero expected taxes: \( E\tau(\hat{c}_2) = 0 \)

7. distributional effects of optimal capital taxes:

\[ \tau(c_h) < 0 < \tau(c_l) \]
Where to go?

1. the performance of credit/insurance markets depends on macroeconomic environment including idiosyncratic income volatility (often business cycle related) and returns on alternative investment opportunities, Krueger and Perri [17] (enforcement constraints).
2. requiring *ex post* inequality in income to have optimal contracts
   → reconsidering redistribution policies.

3. requiring lower growth on the average
   → reconsidering growth policies and capital accumulation, Bohacek [4], Bohacek [5].

5. public insurance programs: asset tests, Golosov and Tsyvinski [10].

6. potential effects of inflation in coexistence of money and credit (optimal contracts)
7. asset pricing implications, difficulty with aggregate shocks, Kocherlakota [13], Kocherlakota and Pistaferri [16] (private information), enforcement problems may be more significant than private information, Lustig [19] (enforcement constraints with complete markets).

8. in sum, may opening up new dimensions in fiscal and monetary policies.

References


