Immigration Conflicts*
(Preliminary)

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Abstract

Almost all existing literature assumes immigrants immediately assimilate in the host country. In contrast, the present paper considers the case in which immigrants do not immediately assimilate, and analyze immigration conflicts in an overlapping generations dynamic system. We examine three types of conflicts that arise when immigrants come in: skill conflicts that affect the capital rental and also cause the wage gap to change between skilled and unskilled workers; intergenerational conflicts that lead to different impacts on the young and old generations; and distributional conflicts that affect each generation’s life time utility unequally. The degree of substitution between natives and immigrants in high-skilled production plays a key role.

1 Introduction

The current wave of globalization is characterized by freer mobility of not only goods and capital across countries, but also freer mobility of human resources. With the launch of the European Union, NAFTA, APEC, etc., international labor migration is bound to rise. Migrant Watch International based in Switzerland estimates that 130 million people in the world live outside their countries of birth. Immigration is a routine issue in presidential campaigns in the US, Europe and Australia. Even in Japan, a country that has traditionally been closed to foreign migrants, labor shortage especially in agriculture and heavy manual work is forcing the government to reconsider immigration among other alternatives. Already small numbers of seasonal foreign

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workers are being introduced. An example is that Chinese workers were hired for construction in Nagano, Japan, before the Nagano Winter Olympics in 1998. The Japanese government is also drafting a plan to introduce certified foreign nurses. Sooner or later more lenient immigration policies may have to be adopted.

Nevertheless, there is evidence that some unskilled workers, local communities and law enforcement officers are more negative on this issue. They are afraid that immigrants may compete away their jobs, free ride on public services such as schooling, library, park, medicare and other public services, cause cultural conflicts and increase crime, etc. Thus they claim that the net welfare of natives may fall. It is also interesting to note that due to various legal barriers set up by sovereign states, labor rather than capital is perhaps the least mobile factor of production in the real world, while the latter would be least mobile in standard trade models. As a consequence of all these, despite of being an integral part of globalization, immigration is perceived negatively in public opinion. Often when related issues appear in the media, they are about illegal immigration, or some other negative images such as taking jobs away and depressing wages, etc.

A particularly related issue is assimilation. While most analysis in the literature has treated immigrants and natives identically, recent empirical studies have found that immigrants do not exert enough effort to absorb the mainstream culture and language, even though the economic returns (increased productivity and enhanced earnings) to assimilation are high. Both Lazear (1999) and Bauer et al. (2000) find assimilation incentives to be low if it is costly for immigrants to learn the receiving country’s culture and language, or if immigrants’ culture is strongly represented in the host country (such as by a large immigrant community). Djajic (2003) argues that immigrants assimilate at different rates which may also differ from those of their children. These affect the rate of human capital accumulation of immigrant children. He shows that rapid assimilation in certain dimensions serves to increase the rate of human capital accumulation of the second generation, while in other dimensions it may have the opposite effect. Stark and Fan (2006) and Fan and Stark (2007) explain that there could be two main reasons for non-assimilation. One is that immigrants want to keep close ties with families, friends and cultural roots at home, the other is that intensive assimilation may result in migrants’ comparing themselves more with the richer natives and less with fellow migrants, which may reduce the assimilation incentives. They even find that the richer the natives, the weaker the effort to assimilate, other things held the same. More recently, Harrie et al. (2008) argue that immigrants’ utility will be lowered if they assimilate because they have to lose important parts of their own culture.

The present paper attempts to examine the above issues with an overlapping generations (OLG) model. To keep the model simple and tractable, we consider an economy without trade but allows a once-and-for-all permanent immigration, the extent of which is determined by the host country government. To the firms, the number of immigrants they hire is exogenous. As will become clear soon, even this simple structure turns out to be very complicated. Thus, we abstract from modeling illegal or seasonal migration.

Workers are divided into skilled and unskilled, who are combined with capital to produce a single output, with skilled workers being more efficient. Households consume
the output and a public good, the latter of which is provided by the government and is in turn financed through income taxes. To incorporate non-assimilation, we assume that while non-skilled immigrants and natives are perfectly substitutable, skilled immigrants and natives are not. In other words, the possession of skill makes a worker less substitutable. As such, skilled immigrants can be either substitutes or complements to skilled natives. We examine the different impacts of allowing skilled and unskilled immigration.

In particular, we find that immigration at time $t$ brings three types of conflicts:\footnote{One might naturally think that immigration could reduce native employment in certain sectors. This certainly causes conflicts. In the present paper, our concern is that even with full employment of both natives and immigrants, various conflicts still arise.}

i). “Skill conflicts” that cause the wage gap to change between skilled and unskilled workers; ii). “Intergenerational conflicts” that lead to different impacts on the young and old generations at time $t$, stemming from changes in the capital stock, the interest rate and the public good; iii). “Distributional conflicts” that affect each generation’s lifetime utility unequally. The details of each conflict depends crucially on the degree of substitution between natives and immigrants in high-skilled production.

For instance, the arrival of immigrants increases total scale of production, and thus the productivity of native skilled workers rises, this attracts more capital to work with them and increases the skilled wage. It in turn reduces the amount of capital left for unskilled workers, depressing the unskilled wage. At a result, a “skill conflict” arises.

Intergenerational conflicts arise when different generations may face different levels of capital stock, interest rates and public goods. For instance, with regard to public goods, natives not only suffer from an immediate decrease of per capita public-goods consumption after immigrants come in which then leads natives to oppose immigration, but also face the various effects on their wage rates and capital returns, which work in opposite directions on households of different age groups. In addition, young agents earn wage income, while old agents earn income from savings, which could lead to conflicts between workers and ‘capital owners’.

There is voluminous literature on how immigration affects host-country welfare. With regard to models that analyze dynamics effects immigration, Galor and Stark (1991) study the pattern of migration in an OLG model with two countries that are different in technology. Ben-Gad (2008) shows that An influx of high-skilled immigrants lowers the wages of skilled workers, raises the wages of unskilled workers, and because of the relative complementarity between capital and skilled labor, substantially raises the rate of return to native-owned capital. By contrast, an influx of unskilled immigrants produces an opposite effect on wages, and has only a negligible effect on the return to capital. Because of capital skill-complementarity, an increase in the number of skilled immigrants generates an immigration surplus—the overall welfare benefit accruing to the native population—that is approximately ten times larger than the immigration surplus generated by an identical increase in the number of unskilled immigrants. Stark and Wang (2002) argue that a strictly positive probability of migration to a richer country, by raising both the level of human capital formed by optimizing individuals in the home country and the average level of human capital of non-migrants in the
country, can enhance welfare and nudge the economy toward the social optimum. With
regard to static models, Ethier (1985) models the relationship between international
trade and labor mobility, with particular attention paid to the role of international
labor migration in helping to insulate native populations from the rigors of industrial
fluctuations and in helping to preserve import-competing industries that would have
succumbed without the availability of cheap imported labor. However, in all these
models, immigrants and natives are "undifferentiated". In other words, immigrants
assimilate immediately when arriving in the host country.

Empirical studies such as Smith and Edmonston (1997) and Borjas (1999) find weak
positive complementarity effects of immigration and estimate the net benefit that accrues
to the United States population from the present stock of immigrants to be approximately $10 billion. They also conclude that these immigrants are responsible for the
redistribution of approximately 2% of output from native workers to native owners of
capital. Ottaviano and Peri (2005) even find that there are strong complementarities
between comparably skilled immigrants and natives such that overall immigration generates a large positive effect on the average wages of US-born workers. On the other
hand, Davis and Weinstein (2002) show negative terms of trade effects of immigra-
tion based on technology superiority. Felebermayr and Kohler (2007) provide a more
general model that can generate the above-mentioned results as special cases. They
decompose the native welfare effect into a standard complementarity effect, augmented
by a Stolper-Samuelson effect, and a terms-of-trade effect. They calibrate this model
to a generic OECD economy and provide simulation results.

The rest of the paper is organized as follows. Section 2 constructs the basic model,
section 3 derives the dynamic system and steady state of the model, section 4 examines
how each variable is affected pre and post immigration, section 5 analyzes welfare con-
licts, section 6 looks into skill premium, section 7 considers intergenerational conflicts.
Policy issues are discussed in section 8, and finally section 9 includes some concluding
remarks.

2 The Basic Model

Consider an economy where people work in the first period of their lives and retire
in the second period. We distinguish between two types of labor in production: high
skilled and low skilled, which are exogenously given. Skills are born with and cannot
be changed in this model. The native population size is constant at $N$. Let $x$ be the
fraction of the skilled natives; that is, a proportion of the natives, $xN$, is skilled, and
the other proportion, $(1-x)N$, is low skilled.

Before time $t$, only natives live in the host country. At time $t$, the host country
receives a once-and-for-all immigration. The total number of immigrants is $I$, $I^H$ of
which is high skilled and $I^L = I - I^H$ is low skilled. Immigration makes the total
population in the home country $N + I$ at $t$.

\^It is natural to consider continuous flow of immigrants. However, in the present model, we show
that a once-and-for-all inflow of immigration is enough to generate several kinds of conflicts.
2.1 Consumers

Let \(i\) represent each four types of agents \(\{NH, IH, NL, IL\}\), respectively for high skilled natives and immigrants, and low skilled natives and immigrants. Let \(p\) represent the number of four types of agents \(\{N^H, I^H, N^L, I^L\}\). Then, \(C^{y,i}_t\) is agent \(i\)'s consumption in young age at time \(t\), and \(C^{o,i}_{t+1}\) is the counterpart when he becomes old; \(G_t\) is a public good at time \(t\), supported by tax at the previous period, \(t-1\). Parks, roads, bridges, hospitals, schools and libraries are examples of the public good. We consider a one-final-good economy. Each type of agent maximizes the utility

\[
\max U = \log C^{y,i}_t + \log G_t + \beta \left( \log C^{o,i}_{t+1} + \log G_{t+1} \right),
\]

(1)

under the following budget constraints:

\[
C^{y}_t = (1 - \tau)w^t_i p_t - S_t,
\]

(2)

\[
C^{o}_{t+1} = R_{t+1} S_t, \quad \text{where} \quad R_{t+1} = 1 + \rho_{t+1},
\]

(3)

where \(\beta\) is the time preference, \(\tau\) is the income tax rate, \(S\) is savings, \(w\) is the wage rate, \(\rho\) is the rate of return from savings, \(R\) is the gross rate of return. All tax revenue, \(T\), is used to produce the public good for the next period,

\[
G_t = \tau(w_{t-1}^t p_{t-1}), \quad T_t = G_{t+1}.
\]

For simplicity, we assume zero population growth. Utility maximization results in the following first order conditions:

\[
\frac{\partial \mathcal{L}}{\partial C^{y,i}_t} = \frac{1}{C^{y,i}_t} - \lambda = 0,
\]

\[
\frac{\partial \mathcal{L}}{\partial C^{o,i}_{t+1}} = \frac{\beta}{C^{o,i}_{t+1}} - \lambda \frac{1}{R_{t+1}} = 0,
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = (1 - \tau)w^i_t h^t_i - C^{y}_t - \frac{C^{o}_{t+1}}{R_{t+1}} = 0.
\]

These conditions give consumptions in the young and old ages and the savings as:

\[
C^{y,i}_t = \frac{1}{1 + \beta}(1 - \tau)w^t_i p_t,
\]

(4)

\[
C^{o,i}_{t+1} = \frac{\beta}{1 + \beta}(1 - \tau)w^t_i p_t R_{t+1},
\]

(5)

\[
S_t = \frac{\beta}{1 + \beta}(1 - \tau)w^t_i p_t.
\]

(6)
2.2 Production

To model the difference between natives and immigrants, we assume that they are combined slightly differently with capital to produce the final good, by the following CES type production function at time $t$,

$$Y_t = (K^H_t)^\alpha \{\gamma^H (N^H_t)^\sigma + \gamma^H (I^H_t)^\sigma\}^{\frac{1-\alpha}{\sigma}} + (K^L_t)^\alpha \{\gamma^L (N^L_t)^\theta + \gamma^L (I^L_t)^\theta\}^{\frac{1-\alpha}{\theta}}, \quad (7)$$

where $K^H_t (K^L_t)$ denotes physical capital used high (low) skilled workers, and $\sigma$ and $\theta$ are parameters. $\gamma^H$ and $\gamma^L$ are also parameters and they regulate the shares of high-skilled natives and immigrants in their respective contributions to output. Production requires both capital and labor. But for labor, either natives or immigrants or both can be used, and both of them can be either high or low skilled. We assume $1 > \gamma^H > \gamma^L$; that is, high skilled agents are relatively more productive than low skilled ones.

Finally, we wish to use the parameters $\sigma$ and $\theta$ to capture possible impacts of the distinct culture and customs of immigrants, which may improve production or cause frictions that delay production. Formally, immigrants and natives can be either substitutes or complements in production, depending on the values of $\sigma$ and $\theta$, which represent the degree of substitutability between them, for high and low skilled agents respectively. It can be understood as follows. Skilled workers possess specific skills that enable them to be less substitutable. As a worker becomes less skilled, he becomes more substitutable, and in the extreme, unskilled natives and immigrants are perfect substitutes.\(^3\)

Let the final good be the numeraire. Then the profit of the representative firm can be written as

$$\pi_t = (K^H_t)^\alpha \{\gamma^H (N^H_t)^\sigma + \gamma^H (I^H_t)^\sigma\}^{\frac{1-\alpha}{\sigma}} + (K^L_t)^\alpha \{\gamma^L (N^L_t)^\theta + \gamma^L (I^L_t)^\theta\}^{\frac{1-\alpha}{\theta}} - (K^H_t + K^L_t) R_t - w^H_t (N^H_t + I^H_t) - w^L_t (N^L_t + I^L_t).$$

Profit maximization yields the return to capital, which must be equalized working with either high or low skilled labor, and it then determines capital allocation between the two types of labor:

$$R_t = \alpha (K^H_t)^{\alpha-1} \{\gamma^H (N^H_t)^\sigma + \gamma^H (I^H_t)^\sigma\}^{\frac{1-\alpha}{\sigma}}; \quad (8)$$

$$R_t = \alpha (K^L_t)^{\alpha-1} \{\gamma^L (N^L_t)^\theta + \gamma^L (I^L_t)^\theta\}^{\frac{1-\alpha}{\theta}}. \quad (9)$$

Firms producing the final good choose the quantity of each type of labor input for a given wage type.

$$w^H_t = (K^H_t)^\alpha (1-\alpha) \{\gamma^H (N^H_t)^\sigma + b^H (I^H_t)^\sigma\}^{\frac{1-\alpha}{\sigma}-1} \gamma^H (N^H_t)^{\sigma-1}, \quad (10)$$

$$w^L_t = (K^L_t)^\alpha (1-\alpha) \{\gamma^L (N^L_t)^\theta + b^L (I^L_t)^\theta\}^{\frac{1-\alpha}{\theta}-1} \gamma^L (N^L_t)^{\theta-1}. \quad (11)$$

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\(^3\)This assumption in fact follows a long research tradition on immigration. For instance, in Borjas (1995) immigrants and natives with equal skills are perfect substitutes. Other studies assume a certain degree of imperfect substitutability, see for example, Ethier (1985), Angrist and Kugler (2003) and Ottaviano and Peri (2005).
We assume that the number of immigrants is determined by the host-country government, which is exogenous to domestic firms. Immigrants receive the same wage rates as natives, given each worker’s type. This can be justified on the grounds that in the absence of illegal immigration, it is unlawful to pay legal immigrants a discriminatory wage that is not skill based.

2.3 Market Equilibrium

By Walras’ law, there are two market equilibrium conditions we need to take care of. One is in the goods market, where final output is consumed by the young and old generations, re-invested as capital and collected as government tax revenue:

\[ Y_t = C_t^{y.t} + C_t^{o.t} + K_{t+1} + T_t. \] (12)

The other is in the capital market, where total capital in each period is combined respectively with high and low skilled workers to produce the final output:

\[ K_t = K_H^t + K_L^t. \] (13)

3 Dynamic System

Now we consider the dynamic impacts of immigration on this economy. After entering in period \( t \), immigrants start to contribute to capital accumulation and government tax. Rewriting (12) using the budget constraints gives the capital movement equation,

\[ K_{t+1} = S_t. \] (14)

Substituting (6) into (14) yields,

\[ K_{t+1} = \beta \left( 1 - \tau \right) \left( w_t^{N.H} N^H + w_t^{N.L} N^L + w_t^{I.H} I + w_t^{I.L} I^L \right). \] (15)

Equating (8) and (9) yields the capital allocation equation,

\[ K_L^t = \Lambda K_H^t, \] (16)

where \( \Lambda = \frac{\left( \frac{\gamma^L}{\gamma^H} \right)^\frac{1}{\gamma^L} \left( \frac{N^L}{N^H} \left( \frac{H^L}{N^L} \right)^{\frac{\sigma}{\gamma^L}} \right)^{\frac{1}{\gamma^L}}}{\left( \frac{\gamma^H}{\gamma^L} \right)^\frac{1}{\gamma^H} \left( \frac{N^H}{N^L} \left( \frac{H^L}{N^H} \right)^{\frac{\sigma}{\gamma^H}} \right)^{\frac{1}{\gamma^H}}}. \) Substituting it into (13) further gives,

\[ K_H^t = \frac{K_t}{1 + \Lambda}, \quad K_L^t = \frac{\Lambda K_t}{1 + \Lambda}. \] (17)

That is, capital allocation between high-skilled and low-skilled workers depends on labor productivity, more specifically, on \( \Lambda \). An increase in labor productivity (of either type of workers) attracts more capital to work with it, leaving less capital for the other type of workers. This mechanism works through demand linkages in both the capital and labor markets, causing reallocation of capital between workers of different skills.
And substituting (10),(11) and (17) into (15) we have the law of motion of capital as

\[
K_{t+1} = \frac{\beta}{1+\beta} (1-\tau)(1-\alpha) \left( \frac{K_t}{(1+\Lambda)} \right)^\alpha \\
\times \left[ (\gamma^H)^{1-\alpha} (N_t^H)^{-\alpha} E^{1-\alpha-\sigma} (N_t^H + I_t^H) + \Lambda^\alpha (\gamma^L)^{1-\alpha} (N_t^L)^{-\alpha} (E')^{1-\alpha-\sigma} (N_t^L + I_t^L) \right],
\]

(18)

where \( E \equiv \{ 1 + (I_t^H/N_t^H)^{\sigma} \}^{\frac{1}{\beta}} \), \( E' \equiv \{ 1 + (I_t^L/N_t^L)^{\theta} \}^{\frac{1}{\beta}} \) and \( \Lambda \) can then be rewritten as \( \Lambda = \left( \frac{\gamma^L}{\gamma^H} \right)^{\frac{\theta}{\sigma}} \frac{N_t^L}{E} \). Similarly, we can derive the law of motion of public goods as

\[
G_{t+1} = \tau (1-\alpha) \left( \frac{K_t}{(1+\Lambda)} \right)^\alpha \\
\times \left[ (\gamma^H)^{1-\alpha} (N_t^H)^{-\alpha} E^{1-\alpha-\sigma} (N_t^H + I_t^H) + \Lambda^\alpha (\gamma^L)^{1-\alpha} (N_t^L)^{-\alpha} (E')^{1-\alpha-\sigma} (N_t^L + I_t^L) \right]
\]

(19)

Equations (18) and (19) describe the dynamic system of this economy.

### 3.1 Steady State

In order to isolate the impacts of immigration, we assume that the economy is initially in a steady state.

**Definition 1** The original steady state is defined as:

The amount of aggregate capital in period \( t \) is equal to that in period \( t-1 \), i.e., \( K_t = K_{t-1} = K^* \).

After accepting immigrants, it may not stay in the original steady state. Let us see the details of both \( K_t \) and \( K_{t-1} \). Examining the behavior at period \( t-1 \) gives the law of motion of capital at \( t \) as (see the appendix),

\[
K_t = S_{t-1} N_{t-1} \\
= \frac{\beta}{1+\beta} (1-\tau)(1-\alpha) K_{t-1}^\alpha \left\{ (\gamma^H)^{\frac{1}{\sigma}} (N_{t-1}^H)^{1-\alpha} + (\gamma^L)^{\frac{1}{\theta}} (N_{t-1}^L)^{1-\alpha} \right\}^{1-\alpha}.
\]

(20)

The value of capital at the steady state is then

\[
K^* = \left\{ \frac{\beta}{1+\beta} (1-\tau)(1-\alpha) \left( (\gamma^H)^{\frac{1}{\sigma}} (N_{-1}^H) + (\gamma^L)^{\frac{1}{\theta}} (N_{-1}^L) \right) \right\}^{1-\alpha}.
\]

(21)
Since the capital stock is determined by the disposable income, it becomes higher under a lower tax rate and with a higher ratio of skilled workers, which in turn leads to higher output. Substituting (21) into (18), we can further obtain $K_{t+1}$,

$$
K_{t+1} = \left( \frac{K^*}{1 + \Lambda} \right)^{\alpha} \frac{\beta(1 - \tau)(1 - \alpha)}{1 + \beta} \\
\times \left[ (\gamma^H)^{\frac{1-\alpha}{\sigma}} (N^H_t)^{-\alpha} E^{1-\alpha-\sigma} (N^H_t + I^H_t) + \Lambda^\alpha (\gamma^L)^{\frac{1-\alpha}{\sigma}} (N^L_t)^{-\alpha} (E')^{1-\alpha-\sigma} (N^L_t + I^L_t) \right].
$$

Note that (21) and (22) may be different from each other, since immigration increases labor input, even though the economy does not have growth. In other words, after immigration, the economy may move to a new level of capital, or stay at the same one; and exactly which one arises is determined by the productivity of high-skilled labor. When the economy moves to a new steady state where $K_{t+n} = K_{t+n+1} = \hat{K}$, after some periods, we must have,

$$
\hat{K} = \left[ \frac{\beta(1 - \tau)(1 - \alpha)}{1 + \beta} \right]^{\frac{1}{\alpha}} \\
\times \left[ (\gamma^H)^{\frac{1-\alpha}{\sigma}} (N^H_t)^{-\alpha} E^{1-\alpha-\sigma} (N^H_t + I^H_t) + \Lambda^\alpha (\gamma^L)^{\frac{1-\alpha}{\sigma}} (N^L_t)^{-\alpha} (E')^{1-\alpha-\sigma} (N^L_t + I^L_t) \right]^{\frac{1}{\alpha}}.
$$

(23)

The difference between (21) and (22) (or (23) ) is one of the sources that causes various conflicts between natives and immigrants in the present paper, to which we now move on.

4 Immigration Conflicts

We consider conflicts that arise between generations $t$ and $t+1$. Since agents born at $t$ are affected by economic conditions in periods $t-1$, $t$ and $t+1 only, we assume that they do not care about other periods. Moreover, we focus on the case of $\sigma < \theta$; that is, since high-skilled workers possess skills that low-skilled workers do not have, their substitutability must be lower.

4.1 Marginal Productivity

For later use, we need to consider how the marginal productivities of high and low-skilled native workers change with immigration. Note that it is undefined at $\sigma = 0$ and $\theta = 0$. The production function before immigration is obtained by setting $I^H = I^L = 0$ in (7), and in the steady state, we also have wages as;
\[ w_{t-1}^H = (K^*)^\alpha \left( \frac{1}{1 + \Lambda_0} \right)^\alpha \left( 1 - \alpha \right) \left( \gamma^H \right)^{-\frac{1-\alpha}{\sigma}} \left( N_{t-1}^H \right)^{-\alpha}, \quad (24) \]
\[ w_{t-1}^L = (K^*)^\alpha \left( \frac{\Lambda_0}{1 + \Lambda_0} \right)^\alpha \left( 1 - \alpha \right) \left( \gamma^L \right)^{-\frac{1-\alpha}{\sigma}} \left( N_{t-1}^L \right)^{-\alpha}, \quad (25) \]

where \( \Lambda_0 = \frac{(\gamma^L)^{\frac{1}{2}} N_{t-1}^L}{(\gamma^H)^{\frac{1}{2}} N_{t-1}^H} \). The expression \( \frac{1}{1 + \Lambda_0} \left( \frac{\Lambda_0}{1 + \Lambda_0} \right) \) indicates the efficiency of capital use by high (low) skilled workers.

To make the discussion smooth, let us define a few terms that will become handy:
\[ \left( \gamma^H \right)^{\frac{1}{2}}, \left( \gamma^L \right)^{\frac{1}{2}} \equiv \text{the skill difference effect of respectively high and low skilled workers;} \]
\[ \frac{1}{1 + \Lambda_0}, \frac{\Lambda_0}{1 + \Lambda_0} \equiv \text{the capital allocation effect of respectively high and skilled workers before immigration.} \]

As their names show, they tell us as \( \sigma \) and \( \theta \) change, how the effect coming from the difference parameter \( \gamma^H \) and \( \gamma^L \) between skilled and unskilled workers, and how capital allocation between high-skilled and low-skilled workers, change respectively.

After immigration takes place, we obtain,
\[ w^H = (K^*)^\alpha \left( \frac{1}{1 + \Lambda} \right)^\alpha \left( 1 - \alpha \right) \left( \gamma^H \right)^{-\frac{1-\alpha}{\sigma}} \left( N_t^H \right)^{-\alpha} E^{1-\alpha-\sigma} \quad (26) \]
\[ w^L = (K^*)^\alpha \left( \frac{\Lambda}{1 + \Lambda} \right)^\alpha \left( 1 - \alpha \right) \left( \gamma^L \right)^{-\frac{1-\alpha}{\sigma}} \left( N_t^L \right)^{-\alpha} \left( E' \right)^{1-\alpha-\theta} \quad (27) \]

Here \( E \) and \( E' \) (see definition after equation (18)) can be called the immigration effects of respectively allowing in high and low skilled immigrants. Note that without immigration, \( I^H = I^L = 0 \) and \( E = 1 = E' \), then (26) boils down to (24).

Accordingly, we can call \( E^{1-\alpha-\sigma} \) and \( \left( E' \right)^{1-\alpha-\theta} \) their respective productivity effects. \( E^{1-\alpha-\sigma} \) increases (decreases) the productivity and therefore wages of high-skilled workers when \( \sigma < (>) 1 - \alpha \), since it enters (26) positively (negatively). Similarly, the low skilled wage increases (decreases) if \( \theta < (>) 1 - \alpha \). These effects are supported by empirical studies (e.g. Lazear, etc.), which show that skill complementarity increases productivity and wages.

With immigration, the capital allocation effects change from \( \frac{1}{1 + \Lambda_0} \) and \( \frac{\Lambda_0}{1 + \Lambda_0} \) to respectively \( \frac{1}{1 + \Lambda} \) and \( \frac{\Lambda}{1 + \Lambda} \). Whether a particular change is positive or not depends on the labor productivity after immigration. If the productivity of the high skilled increases more than that of the low skilled, then the former can attract more capital and raise the skilled wage; otherwise the opposite arises. Finally, \( (N_t^H)^{-\alpha} \) and \( (N_t^L)^{-\alpha} \) stem from the scale of immigrants, negatively, and they thus lower the wages since more immigration increases labor supply.
4.2 The Skill Conflicts

The previous subsection shows that the marginal productivity of labor depends on $\sigma$ and $\theta$. Moreover, due to the productivity effects $E_1^{1-\sigma}$ and $(E')^{1-\sigma}$, immigration may bring opposite changes to the marginal productivities and wage rates of high and low skilled workers, which in turn causes conflicts to arise between the two types of workers.

Let us first examine the marginal productivity conditions before and after immigration.

Lemma 2 Given $\sigma < \theta$ by assumption, suppose $\frac{N_t^H}{N_t^F} = \frac{N_t^L}{N_t^F} < 1$, i.e., the ratio of high-skilled immigrants to high-skilled natives is equal to that of the low-skilled, then we have $E - E' > 0$, i.e., the immigration effect of high skilled workers dominates that of low skilled workers.

Proof. Note that $\frac{1}{\sigma} > \frac{1}{\theta}$. From the definition of $E$ and $E'$, we see that the sign of $E - E'$ depends on their component difference $(I_t^H/N_t^H)^\sigma - (I_t^L/N_t^L)^\theta > 0$. Hence $E - E' > 0$. $\blacksquare$

4.2.1 The high-skilled wage

Now we analyze how immigration affects the high skilled wage. Taking the ratio of the high-skilled wage between periods $t-1$ and $t$ gives

$$\frac{w_t^H}{w_{t-1}^H} = \left(\frac{(\Delta_H + \Delta_L) E}{\Delta H E + \Delta L E'} \right)^\alpha (E)^{1-\sigma},$$

(28)

where $\Delta_H \equiv (\gamma^H)^{\frac{1}{\sigma}} (N_{t-1}^H)$ and $\Delta_L \equiv (\gamma^L)^{\frac{1}{\theta}} (N_{t-1}^L)$. Equation (28) says that the wage ratio of high-skilled workers is determined by the productivity effect $(E)^{1-\sigma}$ and the ratio of capital allocation for high-skilled workers before and after immigration. If $\sigma < (>)1 - \alpha$, the productivity effect increases (decreases) the high-skilled wage.

Regarding the expression in large parenthesis, subtracting the denominator from the numerator yields,

$$f^H(E, E') \equiv (\gamma^L)^{\frac{1}{\theta}} (N_{t-1}^L) (E - E'),$$

(29)

which is positive fromLemma 1. This means that an increase in high-skilled immigration reallocates more capital to high-skilled workers, raising their productivity and wage rate.
4.2.2 The low-skilled wage

Calculations give

$$\frac{w_t^L}{w_{t-1}^L} = \left( \frac{(\Delta H + \Delta L) E'}{\Delta H E + \Delta L E'} \right)^{\alpha} (E')^{1-\alpha-\theta}, \quad (30)$$

where \((E')^{1-\alpha-\theta}\) is the productivity effect, and the rest of the equation on the RHS is the capital allocation effect. When \(\theta < (<) 1 - \alpha\), the productivity effect raises (reduces) the low skilled wage.

On the capital allocation effect, subtracting the denominator from the numerator and using Lemma 1 yield,

$$f^L(E, E') \equiv \left( \gamma^H \right)^{\frac{1}{\alpha}} \left( N_{t-1}^H \right) (E' - E) < 0. \quad (31)$$

That is, an increase in low-skilled immigration reallocates less capital to low-skilled workers, since their productivity is lower than that of the high skilled. This in turn reduces the low skilled wage.

4.2.3 Skill Conflicts on Wages

Combining both the productivity effect and the capital allocation effect, we can determine if wages increase after immigration. Recall that the capital allocation effect raises (reduces) the high (low) skilled wage. However, whether the productivity effect is positive or not depends on the value of \(\sigma\) or \(\theta\). If \(\sigma\) (\(\theta\)) is given in Region A in Figure 1, the productivity effect raises the high (low) skilled wage. However, in Reasion B, this effect reduces the high (low) skilled wage. Thus, we consider the following three cases\(^\text{5}\).

Case I : both \(\sigma\) and \(\theta\) are given in Region A;
Case II : both \(\sigma\) and \(\theta\) are given in Region B;
Case III : \(\sigma\) is given in Region A but \(\theta\) is given in Region B.

First, consider Case I. Each productivity effect increases its respective wage, and the capital allocation effect raises the high skilled wage but reduces the low skilled wage. It follows that the former wage increases after immigration. However, on the low skilled wage, if it increases, (30) must be greater than 1. Then, we obtain the following condition;

$$\Delta^L E' \left[ (E')^{\frac{1-\alpha-\theta}{\alpha}} - 1 \right] + \Delta^H \left[ E' (E')^{\frac{1-\alpha-\theta}{\alpha}} - E \right] > 0. \quad (32)$$

The first term in (32) is positive since \(\theta < 1 - \alpha\). We focus on the second term. If
\n\(E' (E')^{\frac{1-\alpha-\theta}{\alpha}} - E > 0\), i.e., \((E')^{\frac{1-\theta}{\alpha}} - E > 0\), then the low skilled wage increases after immigration. Some intuition follows. Since \(\theta < 1 - \alpha\), we have \(\frac{1-\theta}{\alpha} > 1\). And thus \((E')^{\frac{1-\theta}{\alpha}} > E\) implies that the productivity effect of low skilled workers dominates the

\(^5\text{Since we assume } \sigma < \theta, \text{ we do not consider the case where } \sigma \text{ is in Region B and } \theta \text{ in Region A.}\)
immigration effect of high skilled workers. If this force is strong enough, low skilled workers attract more capital and their wage increases after immigration.

In Case II, each productivity effect negatively works on its respective wage. The low skilled wage decreases since the capital allocation effect works in the same direction. However, the capital allocation effect and the productivity effect work in opposite directions on the high skilled wage. Let us examine which effect dominates. For the high skilled wage to decrease, we must have,

\[
\Delta^H E \left( E^{\frac{1-\alpha - \sigma}{\alpha}} - 1 \right) + \Delta^L \left( EE^{\frac{1-\alpha - \sigma}{\alpha}} - E' \right) < 0. \tag{33}
\]

The first term of (33) is negative because of \(1 - \alpha < \sigma\). When \(E^{\frac{1-\alpha}{\alpha}} < E'\), the second term of (33) becomes negative. This means the productivity effect of low skilled workers is high enough such that these workers attract more capital. As a consequence the high skilled wage decreases, since the negative productivity effect dominates the positive capital allocation effect.

In case III, both the capital allocation and productivity effects work positively on the high skilled productivity, but negatively on the low skilled productivity. Hence the high skilled wage increases and the low skilled wage decreases.

### 4.2.4 Summary on Wages

Table 1.1 summarizes the impact of immigration on both wages, where sign “+” (“−”) means the wage is higher (lower) with immigration than without. From the Table, whether wages increase or not depends on the productivity effect and the capital allocation effect, which cause changes in the marginal productivity of native skilled workers. The capital allocation effect is positive for the high skilled but negative for the low skilled, because the immigration effect is larger for the former than for the latter, \(E > E'\), according to Lemma 1. Therefore, high-skilled workers attract more capital, making their marginal productivity increase even more. On the other hand, the productivity effect depends on \(\sigma\) and \(\theta\). These contrasting effects stem from workers’ skill differences, and cause conflicts between skilled and unskilled workers.

<table>
<thead>
<tr>
<th>Table 1.1 The impact of immigration on wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>on (w^H)</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>(\sigma &lt; 1 - \alpha)</td>
</tr>
<tr>
<td>(\sigma &gt; 1 - \alpha)</td>
</tr>
<tr>
<td>on (w^L)</td>
</tr>
<tr>
<td>−</td>
</tr>
</tbody>
</table>

Table 1.2 combines both the productivity effect and the capital allocation effect, and summarizes the main comparative statics results on wages. In case I, both \(\sigma\) and \(\theta\) are smaller than \(1 - \alpha\), and both types of immigrants tend to be complementary to
natives. In this case, both wages increase with immigration. However, case II is the exact opposite, where \( \sigma, \theta > 1 - \alpha \) and both wages decrease. The above two cases do not create conflicts between high and low skilled workers.

In contrast, in case III, the high skilled wage increases, but the low skilled wage decreases. Hence a conflict arise, and we name it the skill conflict. Note that if the productivity effect dominates, then a conflict will not occur.

Table 1.2 Immigration conflicts

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w^H )</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( w^L )</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>conflict or not</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

Note that in Case II, since \( 1 - \theta > \alpha \) and \( E' < E \) from Lemma 1, we must have \( (E')^{\frac{1-\theta}{\alpha}} > E \). Similarly we have \( E^{\frac{1-\theta}{\alpha}} < E' \) in Case III.

4.3 Effects on goods and interest income

We have considered how immigration impacts wages. The wage changes may affect the household’s behavior in consumption and savings patterns, eventually leading to changes in the capital stock, public goods, and interest rate. We examine them in detail now.

4.3.1 Capital goods and the interest rate

Comparing (21) and (22) yields

\[
\frac{K_{t+1}}{K^*} = \frac{w^H_t}{w^H_{t-1}} \left( \frac{\gamma^H}{N^H_t} \right) + \frac{w^L_t}{w^L_{t-1}} \left( \frac{\gamma^L}{N^L_t} \right) \left( \frac{N^L_t + I^L_t}{N^L_t} \right) + \frac{\Delta^H}{\Delta^H + \Delta^L} + \frac{\Delta^L}{\Delta^H + \Delta^L} . \tag{34}
\]

\( K^* \) is from natives only, but \( K_{t+1} \) is generated by the disposable income of the four types of agents. The ratio of capital stock before and after immigration depends on not only the ratio of high and low skilled workers, but also the scale of immigration. Here we assume the scale effect to be dominated by the wage effect (otherwise immigration always increases the capital stock). As shown in the previous section, in Case I (respectively Case II), both the high and low skilled wages increase (decrease), and the capital stock at \( t + 1 \) is higher (lower) than at \( t \).

In contrast in Case III, \( \frac{w^H_t}{w^H_{t-1}} \) increases but \( \frac{w^L_t}{w^L_{t-1}} \) decreases, and hence we need to consider which term dominates. Rewriting (34) gives

\[
\frac{K_{t+1}}{K^*} = \left( 1 + \frac{I^H_t}{N^H_t} \right) \left[ \frac{w^H_t}{w^H_{t-1}} \frac{\Delta^H}{\Delta^H + \Delta^L} + \frac{w^L_t}{w^L_{t-1}} \frac{\Delta^L}{\Delta^H + \Delta^L} \right] . \tag{35}
\]
where we use the assumption $\frac{H}{N_h} = \frac{l_t}{N_t}$ from Lemma 1. If the scale effect $\frac{H}{N_h}$ is large enough, (35) is always greater than 1, irrespective of $\frac{w_H}{w_{l-1}}$ and $\frac{w_L}{w_{l-1}}$. Therefore, we ignore the scale effect for the moment. The terms $\frac{\Delta^H}{(\Delta^H + \Delta^L)}$ and $\frac{\Delta^L}{(\Delta^H + \Delta^L)}$ indicate how the ratio of high and low skilled wages respectively contribute to the ratio of capital stock, and they sum up to 1, such that $\frac{\Delta^H}{(\Delta^H + \Delta^L)} + \frac{\Delta^L}{(\Delta^H + \Delta^L)} = 1$. We name it the contribution ratio. Thus, to confirm whether the capital stock increases or not with immigration, i.e., (35) > (<)1, we need the second bracket in (35) to be greater (smaller) than 1. The condition is given by

$$\left( E^{1-\sigma} - \left[ \frac{E'}{E} \right]^\alpha \left( E' \right)^{1-\sigma} \right) > \left( \frac{\Delta^H + \Delta^L}{\Delta^H} \right) \left[ \frac{\Delta^H E + \Delta^L E'}{E (\Delta^H + \Delta^L)} \right]^\alpha - \left[ \frac{E'}{E} \right]^\alpha \left( E' \right)^{1-\sigma}. \tag{36}$$

The LHS is the difference of the productivity effects between the low and high skilled. Now, since in case III, $1 - \alpha > \sigma$ and $1 - \alpha < \theta$, the value of the LHS can be bigger than 1. On the other hand, the RHS of (36) shows the difference between the inverse of the capital allocation effect of high skilled workers and the productivity effect of low skilled workers. This difference is small because the former is less than 1. Moreover, if the contribution ratio of the high skilled to the capital stock is not too far from that of the low skilled, then we can also ignore $\frac{\Delta^H + \Delta^L}{\Delta^H}$. Accordingly, condition (35) > 1 is easily satisfied, and we conclude that $\frac{K_t}{K^{*}} > 1$ in case III.

Summarizing these effects, we can obtain the sign of (34) for all cases as in table 2.

<table>
<thead>
<tr>
<th>Capital stock effect</th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

If $K_{t+1} \neq K^{*}$, then the economy jumps to a new trajectory, from which it moves to another steady state, away from the initial one where $K_{t-1} = K_t = K^{*}$. As a consequence, the next generation of households would have a different level of capital stock. It is especially important to note that in case II, the total capital stock falls after immigration, causing what we can call *intergenerational conflicts*.

Further, the ratio of interest rates can be derived as

$$\frac{R_{t+1}}{R_t} = \left( \frac{K_{t+1}}{K^{*}} \right)^{\alpha-1}. \tag{37}$$

Thus, changes in the interest rate depend on changes of the capital stock, in the opposite direction. This again could lead to *intergenerational conflicts*.
4.3.2 Public goods

Note that the public goods in each period is provided by the tax revenue of the previous period;

\[ G_t = \tau \left( w_{t-1}^H N_t^H + w_{t-1}^L N_t^L \right), \]
\[ G_{t+1} = \tau \left( w_t^H N_t^H + w_t^L N_t^L + w_{t-1}^I, H I_t^H + w_{t-1}^I, L I_t^L \right). \]

Then the difference between pre and post immigration is,

\[ \frac{G_{t+1}}{G_t} = \frac{K_{t+1}}{K^*}. \] (38)

Similar to the capital stock, the provision of public goods increases (decreases) after immigrants come in, again leading to intergenerational conflicts. Also note that even if skilled immigrants and skilled natives were completely the same (\( \sigma = 1 \)), since immigration reduces per-capita \( G \), conflicts still arise.

5 Welfare

In this section, we examine how immigration affects the host country’s per capita welfare of generation \( t \). Using (1), (4), (5) and (19), we obtain the welfare difference with and without immigration as;

\[ U_t^{\text{with,NI}} - U_t^{\text{without,NI}} = (1 + \beta) \left( \log \left( w_t^{\text{with,NI}} N_t^I \right) - \log \left( w_t^{\text{without,NI}} N_t^I \right) \right) \]
\[ + (1 + \beta) (\log N - \log (N + I)) \]
\[ + \beta \log R_{t+1}^{\text{with}} - \beta \log R_{t+1}^{\text{without}} \]
\[ + \beta \log \left( w_t^{\text{with,NI}} N_t^H + w_t^{\text{with,NI}} N_t^L + w_{t-1}^{I, H I_t^H} + w_{t-1}^{I, L I_t^L} \right) \]
\[ - \beta \log \left( w_t^{\text{without,NI}} N_t^H + w_t^{\text{without,NI}} N_t^L \right). \]

Substitution and rearrangement of equations lead to,

\[ U_t^{\text{with,Native}} - U_t^{\text{without,Native}} = (1 + \beta) \log \frac{w_t^{\text{with,NI}} N_t^I}{w_t^{\text{without,NI}} N_t^I} + \beta \log \frac{R_{t+1}^{\text{with}}}{R_{t+1}^{\text{without}}} + \beta \log \frac{G_{t+1}^{\text{with}}}{G_{t+1}^{\text{without}}} + (1 + \beta) \log \frac{N}{N + I}. \] (39)

The immigration-induced change in the per-capita welfare can be decomposed into four effects: the wage effect, the interest effect, the public-goods effect, and the scale effect, which can be explained as follows. We know by assumption that utility is obtained
from consumption at young and old ages, and from the public goods. Consumption at young age comes from wage income, and that at old age depends on savings and interest income. Moreover, the public goods at \( t + 1 \) is produced by tax revenue at \( t \), which also comes from wage income. Therefore, the welfare difference with and without immigration depends on the difference of wages, interest rates, public goods and the scale effect of immigration—the ratio of immigrants to total population. We assume that the scale effect is dominated by other effects for the rest of the analysis, i.e., the ratio of immigrants to the total population is small.

The ratio of interest rates depends negatively on the ratio of capital stock from (37). And the ratio of public goods is (38), which works in the same direction as the capital stock effect. The following subsections provide more details.

### 5.1 Low-skilled natives

First, we analyze the immigration-induced change in the welfare of low-skilled natives. Substituting (30) (37) and (38) into (39), we have

\[
U^{\text{with,NL}} - U^{\text{without,NL}} = (1 + \beta) \log \frac{w_t^L}{w_{t-1}^L} + \beta \log \left( \frac{K_{t+1}}{K^*} \right)^{\alpha-1} + \beta \log \left( \frac{K_{t+1}}{K^*} \right) + (1 + \beta) \log \frac{N}{N+I}. \tag{40}
\]

As previously shown, the capital stock effect works in the same direction with the public goods effect, but opposite to the interest rate effect. To figure out whether the welfare of low skilled workers increases or not, let us consider which effect dominates in (40). Firstly, we examine the interest rate effect and the public goods effect, because both of them depend on the capital goods effect. Here, we obtain

\[
\beta \log \left( \frac{K_{t+1}}{K^*} \right)^{\alpha-1} + \beta \log \left( \frac{K_{t+1}}{K^*} \right) = \alpha \beta \log \left( \frac{K_{t+1}}{K^*} \right). \tag{41}
\]

With the help of Table 2, we find that the capital goods effect positively (negatively) works on the economy in Case I and Case III (Case II). (41) shows that the public goods effect dominates the interest effect. If the capital goods increases with immigration, the marginal productivity of capital will decrease. A decrease in the interest rate reduces consumption utility in old age. However, the public goods effect works in the opposite direction, and it dominates the interest rate effect.

Next, we look into the wage effect. We obtain the welfare difference with and without immigration as,

\[
U^{\text{with,NL}} - U^{\text{without,NL}} = (1 + \beta) \log \frac{w_t^L}{w_{t-1}^L} + \alpha \beta \log \left( \frac{K_{t+1}}{K^*} \right). \tag{42}
\]

\(^6\)In this subsection, we consider how immigration effects affect the economy in the host country when the host country accepts immigrants at \( t \). Therefore, now, we have to compare the interest rate with the interest rate with and without immigrants at \( t \). However, the interest rate without immigrants is the same as the interest rate at \( t - 1 \). Therefore, we can consider the ratio of interest rates as (??).
Substituting (35) into (42), we can rewrite (43) as,

\[
U_{\text{with}, NL} - U_{\text{without}, NL} = (1 + \beta) \log \frac{w_t^L}{w_{t-1}^L} + \alpha \beta \log \left(1 + \frac{I_t^H}{N_t^H} \right) \left[ \frac{w_t^H}{w_{t-1}^H} - z + \frac{w_t^L}{w_{t-1}^L} (1 - z) \right]
\]

(43)

where \( z = \frac{\Delta^H}{(\Delta^H + \Delta^L)} \) and \( \frac{\Delta^H}{(\Delta^H + \Delta^L)} + \frac{\Delta^L}{(\Delta^H + \Delta^L)} = 1 \). In Case I (Case II), from Table 1.2, since both wages increase (decrease), (43) increases (decreases) as well. On the other hand, in Case III, the high skilled wage increases, but the low skilled wage decreases. We thus need to determine whether the wage effect dominates the interest and public goods effects. Here, we focus on \( \frac{w_t^H}{w_{t-1}^H} z + \frac{w_t^L}{w_{t-1}^L} (1 - z) \), and rewrite it as,

\[
\frac{w_t^H}{w_{t-1}^H} z + \frac{w_t^L}{w_{t-1}^L} (1 - z) = z \left( \frac{w_t^H}{w_{t-1}^H} - \frac{w_t^L}{w_{t-1}^L} \right) + \frac{w_t^L}{w_{t-1}^L} \]

(44)

(44) is a linear function of \( z \). If \( z = 0 \), then (44) becomes \( \frac{w_t^L}{w_{t-1}^L} \). Therefore, (43) can be rewritten as

\[
U_{\text{with}, NL} - U_{\text{without}, NL} = (1 + \beta) \log \frac{w_t^L}{w_{t-1}^L} + \alpha \beta \log \frac{w_t^L}{w_{t-1}^L}
\]

(45)

In this case, since \( \frac{w_t^L}{w_{t-1}^L} < 1 \), the low skilled welfare will decrease.

If \( z = 1 \), (44) becomes \( \frac{w_t^H}{w_{t-1}^H} \), and (43) is rewritten as

\[
U_{\text{with}, NL} - U_{\text{without}, NL} = (1 + \beta) \log \frac{w_t^L}{w_{t-1}^L} + \alpha \beta \log \frac{w_t^H}{w_{t-1}^H}
\]

(46)

Note that \( \Delta^L = 0 \) when \( z = 1 \). Substituting (28) and (??) into (46), and using the condition, \( \Delta^L = 0 \), and \( E \equiv \left\{ 1 + \left( \frac{I_t^H}{N_t^H} \right)^\theta \right\}^{\frac{1}{\theta}} \), \( E' \equiv \left\{ 1 + \left( \frac{I_t^L}{N_t^L} \right)^\theta \right\}^{\frac{1}{\theta}} \) gives

\[
U_{\text{with}, NL} - U_{\text{without}, NL} = (1 + \beta) \log \frac{w_t^L}{w_{t-1}^L} + \alpha \beta \log \frac{w_t^H}{w_{t-1}^H} = (1 + \beta)(1 - \theta) \log (E') - (\alpha + \alpha^2 \beta + \alpha \beta \sigma) \log E
\]

\[
= \frac{(1 + \beta)(1 - \theta)}{\theta} \log \left(1 + \left( \frac{I_t^L}{N_t^L} \right)^\theta \right) - \frac{(\alpha + \alpha^2 \beta + \alpha \beta \sigma)}{\sigma} \log \left(1 + \left( \frac{I_t^H}{N_t^H} \right)^\theta \right)
\]

(47)

Lemma 1 gives

\[
\left(1 + \left( \frac{I_t^L}{N_t^L} \right)^\theta \right) < \left(1 + \left( \frac{I_t^H}{N_t^H} \right)^\theta \right),
\]

18
Therefore, if the coefficient of the first (second) term dominate that of the second (first) term in (47), then the low skilled welfare increases (decrease) with immigration. In Case III, $1 - \theta < \alpha$. Suppose $(1 - \theta) = \alpha$, substracting the coe®cient of the first term from the second term gives

$$\frac{(1 + \beta)(1 - \theta)}{\theta} - \frac{(\alpha + \alpha^2\beta + \alpha\beta\sigma)}{\sigma}$$

$$= \frac{\alpha(\sigma - \theta)(1 + \alpha\beta)}{\theta\sigma} < 0$$

(48)

By assumption, (48) is negative. Thererfore, the second term dominates the first term. This shows that the low skilled welfare decreases. Moreover, since $0 < 1 - \theta < \alpha$, the minimum value of $1 - \theta$ is slightly larger than 0. If $1 - \theta = 0$, then we immediately know that the second term dominates the first term.

From the above discussion, we can conclude that the low skilled welfare always decreases irrespective of the value of $z$. Thus, we can summarize these results in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>wage effect</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>interest effect</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>public goods effect</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>scale effect</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>total</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3 shows that the wage effect dominates the other effect in welfare. As shown in (40), the coe®cient of the wage effect, $1 + \beta$, is the largest in three effects, where "1" shows the contribution to young consumption of the wage effect and "$\beta$" is to old. That is, the contribution of the wage effect to welfare is the largest. Immigration change in the production function in the host country, and the wage is directly affected by the productivity effect. Thus, change in wages brings change in consmption and savings. This is an indirectly effect. Therefore, the wage effect which is the direct effect of immigration dominates the other effect.

5.2 High-skilled natives

The utility of a high skilled native worker changes as follows,

$$U^{with, Native} - U^{without, Native}$$

$$= (1 + \beta) \log \frac{w^H_t}{w^H_{t-1}} + \beta \log \left( \frac{K_{t+1}}{K^*} \right)^{a-1} + \beta \log \left( \frac{K_{t+1}}{K^*} \right) + (1 + \beta) \log \frac{N}{N + I}$$

(49)

Again suppose that the three effects are large enough to dominate the scale effect. Then, the welfare of skilled natives with immigration changes as in table 4.
<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>wage effect</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>interest effect</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>public goods effect</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>scale effect</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>total</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
</tbody>
</table>

5.3 Summary on Welfare

From Tables 3 and 4, the welfare of both high and low skilled workers increases in Case I but decreases in Case II. In these two cases, skill conflicts between high and low skilled workers do not occur. In other words, if high and low skilled immigrants bring complementary technology with high and low skilled natives, then welfare increases for both high and low skilled natives, as in Case I. On the other hand, if the host country accepts immigrants with high $\sigma$ and $\theta$, then the welfare of natives (irrespective of skill) will decrease, as in Case II.

In Case III, immigration increases the high skilled wage but decreases the low skilled wage, since high skilled natives benefit from increases in more capital allocation and from a positive productivity effect, but low skilled natives suffer from less capital and a negative productivity effect. This is because high skilled immigrants have complementary technology with high skilled natives, but low skilled immigrants substitutional ones with low skilled natives.

5.4 The Skill Premium

How does the skill-premium (i.e., the wage gap between high and low skilled workers) change after immigration takes place? Before immigrants come in, the skill-premium can be written as,

$$\frac{w^H_{t-1}}{w^L_{t-1}} = \frac{(\gamma^H)^{\frac{1}{\sigma}}}{(\gamma^L)^{\frac{1}{\sigma}}}.$$  

(50)

After accepting immigrants, the ratio changes to,

$$\frac{w^H_t}{w^L_t} = \frac{(\gamma^H)^{\frac{1}{\sigma}} E^{1-\sigma}}{(\gamma^L)^{\frac{1}{\sigma}} (E')^{1-\sigma}}.$$  

(51)

Taking their difference yields

$$\frac{w^H_t}{w^L_t} - \frac{w^H_{t-1}}{w^L_{t-1}} = \frac{(\gamma^H)^{\frac{1}{\sigma}}}{(\gamma^L)^{\frac{1}{\sigma}}} \left[ \frac{E^{1-\sigma}}{(E')^{1-\sigma}} - 1 \right].$$  

(52)

Since $E^{1-\sigma}$ is larger than $(E')^{1-\theta}$, by using Lemma 1, we have,

$$\frac{w^H_t}{w^L_t} - \frac{w^H_{t-1}}{w^L_{t-1}} > 0.$$
That is, the wage difference widens with immigration. The economic intuition follows. In Case I and Case II, both high and low skilled wages move in the same direction, but the high skilled workers attract more capital due to their higher productivity. And in Case III, immigration raises the high skilled wage but reduces the low skilled wage as shown in the previous section.

6 Intergenerational Conflicts

Next, we examine how immigration affects different generations living at time $t$, which could possibly lead to intergenerational conflicts. The $t$-period welfare of the old and young generations$^7$ are respectively

$$U^o_t = \log C^o_t + \log G^o_t, \quad U^y_t = \log C^y_t + \log G_t.$$

Since the public goods at $t$ is provided by tax collected at $t-1$, both generations enjoy the same amount of $G_t$. It follows that the welfare difference is determined by the difference in consumption (or income) as below,

$$\frac{U^y_t - U^y_{t-1}}{U^o_t - U^o_{t-1}} = \frac{\log \frac{w^L_{t-1} N^L_t + w^H_{t-1} N^H_t}{w^L_{t-1} N^L_{t-1} + w^H_{t-1} N^H_{t-1}}}{\log R_t / R_{t-1}} = \frac{\log \left[ E^a \Delta H E^{1-a-\sigma} + (E')^a \Delta L (E')^{1-a-\theta} \right]}{\log [\Delta^H E + \Delta^L E']}$$

(53)

This expression depends on the wage ratio and the interest rate ratio. Subtracting the antilogarithm of the denominator from that of the numerator in (53) yields,

$$g(E, E') = \Delta^H E \left( E^{-\sigma} - 1 \right) + \Delta^L E' \left( (E')^{-\theta} - 1 \right) < 0,$$

which says that the welfare increase is smaller for the young generation than for the old generation. Some economic intuition follows. First, the welfare of young natives is affected by the wage effect, which depends on the capital allocation effect and the productivity effect, since the capital stock does not change between periods $t$ and $t-1$. Second, the difference in the interest effects between $t$ and $t-1$ depends on the immigration effect with or without immigration. As our definition shows, the productivity effect, $E^{1-a-\sigma} (E')^{1-a-\theta}$ is smaller than the immigration effect, $E (E')$. This is because the interest rate is directly affected by taking immigration, the marginal rate of labor is indirectly affected through production function. Moreover, capital allocation effect positively work on high skilled wage, but negatively work on low skilled wage. Therefore, this effect ambiguously work on the total income. As a result, the effect which works on old-generation natives is larger than that on the young-generation.

$^7$We consider all young and old irrespective of their skills.
7 Distributional Conflicts

In this section, we look into how immigration affects the life-time utility of each generation. It may seem that immigration at \( t \) may lower the welfare of the natives born at \( t - 1 \) irrespective of their skill levels, because the public goods they contributed must be divided by immigrants also, i.e., by the number \( N + I \) at \( t \). In other words, immigrants can enjoy the benefits at \( t \), although they did not pay tax for the public goods at \( t - 1 \). However, since the steady state and each variable also change, natives born at \( t - 1 \) are not always harmed by immigration, compared with \( t \) or \( t + 1 \) generations. In this section, we investigate this issue in detail.

7.1 Welfare of Low-Skilled Natives

7.1.1 Low-skilled natives born at \( t - 1 \)

First, the welfare difference between generations \( t - 1 \) and \( t \) is;

\[
U_t^{NL} - U_{t-1}^{NL} = (1 + \beta) \log \frac{w_t^L}{w_{t-1}^L} + \beta \log \left( \frac{K_{t+1}}{K^*} \right)^{\alpha - 1} + \beta \log \left( \frac{K_{t+1}}{K^*} \right) + (1 + \beta) \log \frac{N}{N + I}.
\]

This is the same as equation (40), which is presented in Table 3.

7.1.2 Low skilled natives born at \( t + 1 \)

Next, we compare the welfare levels of generations \( t \) and \( t + 1 \). Substituting and using results from previous sections give,

\[
U_{t+1}^{with,NL} - U_t^{with,NL} = \log \left( \frac{R_{t+1}}{R_t} \right) + \beta \log \left( \frac{g_{t+1}}{g_t} \right) + \log \left( \frac{g_{t+2}}{g_{t+1}} \right)
\]

At \( t + 1 \), young immigrants who came to the host country at \( t \) become old. We also assume that their children do not assimilate in the host country. Therefore, the productivity effect and capital allocation effect do not change between \( t \) and \( t + 1 \) periods. Then wages are different between \( t \) and \( t + 1 \) only because the capital stock has changed. Agents who save at \( t \) receive interest income at \( t + 1 \). Further, the difference in interest income between \( t + 1 \) and \( t + 2 \) depends on the difference in the capital stock between the two periods;
\[
\frac{K_{t+2}}{K_{t+1}} = (1 + \Lambda) \left( \frac{K_{t+1}}{K_t} \right)^\alpha
\]

The capital stock ratio between \(t + 2\) and \(t + 1\) depends on itself one period earlier and also on the ratio of capital allocation.

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) wage effect</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>(2) interest effect</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>(3) public goods effect</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>(4) scale effect</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>total</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

Using tables 3 and 5, if the scale effect is dominated by the other three effects, we can obtain the following table that shows which generations can obtain the most benefit from immigration.

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) wage</td>
<td>(P_1)</td>
<td>(P_2)</td>
<td>(P_3)</td>
</tr>
<tr>
<td>(2) interest</td>
<td>(P_2)</td>
<td>(P_1)</td>
<td>(P_2)</td>
</tr>
<tr>
<td>(3) public goods</td>
<td>(P_1)</td>
<td>(P_2)</td>
<td>(P_1)</td>
</tr>
<tr>
<td>Welfare</td>
<td>(P_1)</td>
<td>(P_2)</td>
<td>(P_3)</td>
</tr>
</tbody>
</table>

where Pattern 1(\(P_1\)) means that \(V_{t-1} < V_t < V_{t+1}\), Pattern 2 (\(P_2\)) is \(V_{t+1} < V_t < V_{t-1}\), Pattern 3(\(P_3\)) is \(V_t < V_{t+1}, V_{t-1}\), and Pattern 4 (\(P_4\)) is \(V_t > V_{t+1}, V_{t-1}\), where \(V_t\) stands for wage, interest rate, public goods and welfare at \(t\), respectively.

The welfare difference between \(t\) and \(t-1\) comes from the change in the capital allocation effect and the productivity effect after immigration. In contrast the welfare difference between \(t\) and \(t+1\) comes from change in the capital stock. It follows that in Case I, the welfare after immigration is the highest. Let us consider the economic intuition. Per capita welfare of generation \(t\) is given by

\[
U = \log C_t^{\mu_p} + \log G_t + \beta \left( \log C_{t+1}^{\mu_p} + \log G_{t+1} \right)
= \left( 1 + \beta \right) \log w_t^{p} + \beta \log R_{t+1} + \left( \log w_{t-1}^{p} + \beta \log w_t^{p} \right)
\]

(54)

Welfare comes from consumption and public goods. (54) shows that total consumption depends on wage and the interest rate, because households save a part of wage for consumption in old. If immigrants increase in the native wage, then their welfare will increase. However, as we discussed in the previous sections, since the interest rate is equal to the marginal productivity on capital, increase in capital stock in the host country through taking immigrants, the interest rate will decrease. This leads to
decrease in welfare. Which does the effect dominate, the wage effect or the interest rate effect? Comparing the coefficients of \( w_t \) and \( R_{t+1} \), the coefficient of wage is larger than that of the interest rate. Moreover, as (8) and (9) shown, when we think the contribution of capital to welfare, adding \( \beta \) times \( \alpha - 1 \) as a coefficient. Therefore, it seems reasonable to suppose that the wage effect dominates the interest rate.

Summarizing the above discussion, in Case I, where both wage and capital stock increase, wage at \( t \) is larger than that at \( t - 1 \). Adding the positive capital stock effect on wage at \( t + 1 \), wage at \( t + 1 \) is larger than that at \( t \). In this case, although the interest rate negatively work on welfare, since wage effect dominates the other effect, the welfare in \( t + 1 \) is the highest in Case I.

Similarly to this discussion, we can show that the welfare before immigration is the highest in Case II, where both the wage and the capital stock decrease after immigration. In Case III, while we cannot decide whether welfare is higher at \( t - 1 \) or \( t + 1 \), we find that it is the lowest at \( t \). This is because we cannot decide which is larger, the positive productivity effect or the positive capital stock effect. However, we know that welfare at \( t \) is the smallest.

Thus, we see distributional conflict occurs among different periods by taking immigrants.

7.2 Welfare of High-Skilled Natives

7.2.1 High-skilled natives born at \( t - 1 \)

Using the same procedure as in the previous subsection yields the welfare difference between high skilled workers of generations \( t \) and \( t - 1 \) as follows,

\[
U_{t}^{\text{with,NH}} - U_{t-1}^{\text{NH}} = (1 + \beta) \log \left( \frac{w_t^H}{w_{t-1}^H} \right) + \beta \log \left( \frac{K_{t+1}}{K^*} \right)^{\alpha-1} + \beta \log \left( \frac{K_{t+1}}{K^*} \right) + (1 + \beta) \log \left( \frac{N}{N + I} \right).
\]

This is the same as equation (49), which is presented in Table 4.

7.2.2 High skilled natives born at \( t + 1 \)

The welfare difference between high skilled workers at generations \( t \) and \( t + 1 \) is,

\[
U_{t+1}^{\text{with,NH}} - U_{t}^{\text{with,NH}} = (1 + \beta) \log \left( \frac{K_{t+1}}{K^*} \right) + \beta \log \left( \frac{K_{t+2}}{K_{t+1}} \right)^{\alpha-1} + \beta \log \left( \frac{K_{t+1}}{K^*} \right).
\]

This difference depends on the level of capital stock, because the economy does not take new immigrants, and hence the productivity does not change between \( t + 1 \) and \( t \). We thus obtain Table 5 again. Combining Tables 4 and 5 gives Table 7 like Table 6.
Table 7

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
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<td>$P_2$</td>
<td>$P_1$</td>
</tr>
<tr>
<td>Welfare</td>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$P_1$</td>
</tr>
</tbody>
</table>

The welfare difference between $t$ and $t-1$ comes from the change in the capital allocation effect and the productivity effect after immigration. In contrast, the difference between $t$ and $t+1$ comes from the change in the capital stock. Table 7 shows that in Case I and Case III, the welfare at $t+1$ is the highest among the three periods. In Case I and Case III, wage at $t$ is larger than that at $t-1$, because both the capital allocation effect and the productivity effect positively work on welfare at $t$. Moreover, since capital stock increases at $t+1$, wage at $t+1$ is larger than at $t$. Therefore, Pattern 1 emerges in this economy. As discussed before, since the wage effect dominates other effects, welfare at $t+1$ in both cases will be the highest.

In Case II, the welfare at $t-1$ becomes the highest; that is, welfare decreases after immigration. This is because the wage decreases at $t$ and this leads to capital stock at $t+1$.

8 Result Comparison and Policy

These results make followings clear. In Case I, the welfare of both low and high skilled natives born at $t$ increases with immigration. But in Case II, their welfare decreases. In these two cases, conflicts between high and low skilled natives do not occur because their respective welfare changes in the same direction. However, at period $t$ ($t+1$), welfare which people born at $t$ get is higher than that of $t-1$, and, at $t+1$, the welfare is smaller than that of $t+1$. This leads to conflict between generation $t$ and $t-1$ ($t+1$). The benefit from immigration is not equally divided among generations.

In contrast, in Case III, the welfare of the low skilled native born at $t$ is the lowest among generations $t-1$, $t$ and $t+1$. On the other hand, the welfare of the high skilled native born at $t$ is higher with immigrants. In this Case, if the government tries to take immigrants, high skilled natives are with it, but low skilled are without. That is, conflict between high and low natives will occur again. Moreover, since the fruit from immigrants is not equally distributed among generations, we can also see the distributional conflict.

What do we suggest to immigration policies? We consider them on the view of skill conflict, intergenerational conflict and distributional conflict. Firstly, let us consider the policy which the government takes immigration of Type-Case I. Since immigrants increase wages of both high and low skilled, the government need not care about skill conflict. Moreover, the natives’ welfare with immigrants economy becomes larger than without. Therefore, natives will be with taking immigrants. However, the government
have to care about distribution problems. Skill premium enlarges and the increasing rate between young and old is different.

Next, in Case II, both natives wages decreases, as a result, welfare also reduces. Therefore, natives at $t$ may oppose immigration. Thus, it is reasonable for the government not to take immigrants. If the welfare of both high and low skilled workers moves in the same direction like Case I and Case II, then policy guidance is simple and clear. Problems arise if one of them rises while the other one falls. In this case, fine tuning, such as transfers and other redistribution measures are needed.

Finally, in Case III, high skilled natives will benefit from immigration, but low skille will not. From the point of view on distributional conflict, low skilled natives at generation $t$ is the lowest, but high skilled can increase their welfare with immigrants. In this case, if the government take immigrants, it should give policies for increasing in the generation $t$’s low skilled welfare. For example, the government reallocate more public goods to the low skilld in $t$.

9 Concluding Remarks

There exists volumes literature on the impact of immigration on the host-country economy, and especially in terms of the labor market. However, almost all of them assume that immigrants immediately assimilate in the host country. In contrast, in the present paper, we consider the opposite case in which immigrants do not immediately assimilate. It is believed that immigration conflicts may arise, in terms of tax contribution and the consumption of public goods. We have analyzed such conflicts in an overlapping generations dynamic system, with which possible intergenerational conflicts are also examined. We find that immigration brings contrasting effects on the skilled and unskilled workers, and intergenerations, depending on the degree of substitution between natives and immigrants, conflicts arise among native population.

Appendix

9.1 Derivation of Equilibrium

The goods market condition in $t-1$ period can be written as

$$Y_{t-1} = (K_{t-1}^H)^\alpha \{\gamma^H (N_{t-1}^H)^\sigma \}^{\frac{1-\alpha}{\sigma}} + (K_{t-1}^L)^\alpha \{\gamma^L (N_{t-1}^L)^\theta \}^{\frac{1-\alpha}{\sigma}}.$$

Equalization of capital earnings in period $t-1$ gives

$$\alpha (K_{t-1}^H)^{\alpha-1} \{\gamma^H (N_{t-1}^H)^\sigma \}^{\frac{1-\alpha}{\sigma}} = \alpha (K_{t-1}^L)^{\alpha-1} \{\gamma^L (N_{t-1}^L)^\theta \}^{\frac{1-\alpha}{\theta}},$$

$$K_{t-1}^L = \Lambda_0 K_{t-1}^H.$$
where \( \Lambda_0 = \left( \frac{\gamma^L (N_{t-1}^L)^{1-\alpha}}{\gamma^H (N_{t-1}^H)^{1-\alpha}} \right)^{\frac{1}{\alpha}} \)

Capital in period \( t \) is determined by savings in period \( t - 1 \),

\[
K_t = S_{t-1}N_{t-1} = \frac{\beta}{1+\beta} (1-\tau)w^p_{t-1}X^p_{t-1}.
\]

The same procedure in previous section gives us

\[
K_t = \frac{\beta(1-\tau)(1-\alpha)}{1+\beta} \left( \frac{1}{1+\Lambda_0} \right)^\alpha \left\{ (\gamma^H)^{\frac{1-\alpha}{\alpha}} (N_{t}^H)^{1-\alpha} + (\Lambda_0)^\alpha (1-\alpha) (\gamma^L)^{\frac{1-\alpha}{\alpha}} (N_{t}^L)^{1-\alpha} \right\} (K_{t-1})^\alpha.
\]

References


Figure 1

Region A
\( \sigma < 1 - \alpha \)
\( \theta < 1 - \alpha \)

Region B
\( \sigma > 1 - \alpha \)
\( \theta > 1 - \alpha \)