Accounting for persistence and volatility of good-level real exchange rates: the role of sticky information

Mario J. Crucini†, Mototsugu Shintani‡ and Takayuki Tsuruga§

First Draft: October 2007
This version: January 2009

Abstract

Volatile and persistent real exchange rates are observed not only in aggregate series but also in micro-price data at the retail level. Kehoe and Midrigan (2007) recently showed that, under a standard assumption on nominal price stickiness, empirical frequencies of micro price adjustment cannot replicate the time-series properties of the Law of One Price deviations. We extend their sticky price model by combining good-specific price adjustment with information stickiness in the sense of Mankiw and Reis (2002). Our framework allows for multiple cities within a country. Using a panel of U.S.-Canadian city pairs, we estimate a dynamic price adjustment process for 165 individual goods. Under a reasonable assumption on the money growth process, we show that the model fully explains both persistence and volatility of the good-level real exchange rates.

JEL Classification: E31, F31, D40

Keywords: Good-level real exchange rates, Law of One Price, Sticky information, Dynamic panel

---

*We thank Craig Burnside, Eiji Fujii, Carlos Zarazaga and seminar and conference participants at Duke, Hokkaido, North Carolina State and Osaka University, University of Kansas, the Federal Reserve Bank of Dallas, 2008 Midwest Macro Meetings, 2007 9th TCER Macroeconomic Conference for helpful comments and discussions. Mario J. Crucini and Mototsugu Shintani gratefully acknowledge the financial support of National Science Foundation. Takayuki Tsuruga acknowledges the financial support of Grant-in-aid Scientific Research.

†Department of Economics, Vanderbilt University
‡Department of Economics, Vanderbilt University
§Faculty of Economics, Kansai University
1 Introduction

Aggregate real exchange rates are among the most scrutinized of economic variables because their persistence and volatility are much higher than what economists believe is consistent with a plausible degree of price rigidity. The time-dependent pricing model under local currency pricing offers a convenient theoretical link between price stickiness and the stochastic properties of real exchange rates. Chari, Kehoe, and McGrattan (2002, CKM) show that to generate the observed persistence of CPI-based aggregate real exchange rates, prices need to be exogenously fixed for at least one year. This degree of price-stickiness, however, appears implausible based on recent evidence by Bils and Klenow (2004) who find median duration between price changes of only 4.3 months in U.S. micro-data.

An emerging literature using international micro-data finds the half-life of deviations from the Law of One Price (LOP) for the median good in the neighborhood of 18 months, considerably lower than the consensus 3-5 year half-lives of aggregate real exchange rates (Crucini and Shintani (2008)). This evidence suggests that using prices of individual goods, rather than price indices, is a promising approach for evaluating time-dependent pricing models and understanding short-run international relative price dynamics. An important contribution along this line is Kehoe and Midrigan (2007) who allow different price stickiness across individual goods and show that the persistence in LOP deviations is equal to ‘the Calvo parameter,’ the probability of price non-adjustment at the good level. Their empirical analysis using real exchange rates of 66 individual goods across the U.S. and four European countries shows that the frequency of no price adjustment is higher for goods that exhibit more persistent deviations from the LOP, as suggested by the theoretical model. However, the persistence puzzle is still not resolved in the sense that the observed frequencies of price changes are too high to replicate the persistence of real exchange rates for most goods in the cross-section. In addition, the model does not match the time series variability of LOP deviations observed in the micro-data. These theoretical and empirical results point to the need to break the tight link between the frequency of price adjustment and the persistence of LOP deviations predicted by the standard Calvo-type sticky price model.

Our analysis differs from Kehoe and Midrigan (2007) in several ways. First, we break the tight link between the Calvo parameter and LOP persistence by extending the Kehoe-Midrigan model to allow for persistent money growth and information stickiness. We add to the standard Calvo pricing scheme the assumption that only a fraction of firms update their information set each period.
price dynamics become a convolution of price adjustment timing and information updating. In the macroeconomic literature, Mankiw and Reis (2002) show that a model of information stickiness, or inattentiveness, is capable of explaining the observed slow response of aggregate inflation to monetary shocks much better than sticky prices alone. When the information stickiness augments the Calvo-type sticky price mechanism, less frequent information updating leads to higher price persistence, at a given frequency of price adjustment (Dupor, Kitamura, and Tsuruga (2008, DKT)). With plausible assumptions on international money growth processes, a similar effect takes place to increase both the persistence and volatility of real exchange rates.

Second, our theoretical model allows for the presence of multiple cities in each country and for long-run price deviations between the cross-border city pairs to differ by good and city pair. As such, our model allows us to exploit an international retail price survey at the city level which records local currency prices for highly disaggregated individual goods and services spanning most of the CPI basket. Because this survey is conducted by a single agency, the Economist Intelligence Unit, we expect more comparability of the products among international cities than is true of national CPI surveys, bringing the data more in line with the spirit of the model.

Using this survey we expand the number of products from 66 products used in Kehoe and Midrigan (2007) to 165. We also increase locations from five countries (Austria, Belgium, France, Spain and U.S.) to 52 U.S.-Canadian city pairs. An important limitation of our data is its annual frequency and relative short time-span, from 1990 to 2005. As in the case of Crucini and Shintani (2008), the difficulty of estimating persistence with short time-series is mitigated by utilizing the dynamic panel feature of the data.

Third, we examine the effect of the exclusion of sales on the performance of sticky price models in explaining real exchange rate dynamics. Recently, Nakamura and Steinsson (2008) claim that the evidence of the fast price adjustment reported by Bils and Klenow (2004) may be strongly influenced by the presence of sales, or other temporary price reductions. Nakamura and Steinsson (2008) define the regular price change by excluding sales from the observed price change, and report that the median duration between regular price changes increases to the range of 8 to 11 months. Since prices are stickier based on this alternative definition of price change, it elevates the Calvo model’s ability to account for important features of the data. This improvement is subject to the caveat that our model does not explicitly model sales.\(^1\)

\(^1\)Our working paper includes results using the Bils and Klenow frequencies, which are omitted here due to space considerations.
The main conclusions of Kehoe and Midrigan (2007) are robust to the change in the data. Both persistence and volatility are much higher in the EIU data than the prediction of a standard Calvo-type sticky price model even if we use: (i) more disaggregated retail price data, (ii) panel data consisting of multiple cities in the U.S. and Canada, and (iii) the frequency of price changes adjusted for temporary sales.

In contrast to the standard Calvo model, the extended model with information stickiness fully accounts for both persistence and volatility when we calibrate to the frequency of price adjustment levels reported in Nakamura and Steinsson and an average duration between information updates in the neighborhood of one year.\(^2\) The ability of our model to replicate the observed persistence and volatility contrasts to another possible extension of the Calvo model allowing for pricing complementarities. Kehoe and Midrigan (2007) show that complementary leads to a modest improvement in explaining the persistence and little improvement in explaining the variance.

2 The model

Trade is over a continuum of goods between two countries with multiple cities located in each country. Under monopolistic competition, firms set prices in local currency to satisfy demand for a particular good in a particular city. A representative agent in each country chooses consumption over an infinite horizon subject to a cash-in-advance (CIA) constraint. In what follows, the U.S. and Canada represent the home and foreign country, respectively, and the unit of time is one month. Due to the symmetry of the model, we will present most of the equations only for the United States. When we want to emphasize a difference, across countries, we present the Canadian case as well.

The lowest level of aggregation is the brand, \(z\) of a particular good. U.S. brands of each good are indexed \(z \in [0, 1/2]\) while those in Canada are indexed \(z \in (1/2, 1]\). Integrating over brands, we have the CES index for consumption of good \(j\) in a U.S. city \(l\), given by

\[
c_t(j,l) = \left[ \int_0^1 c_t(j,l,z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}},
\]

where \(c_t(j,l,z)\) is consumption of a brand \(z\) of good \(j\) in U.S. city denoted by \(l\). CES aggregation across U.S. cities \(l \in [0, 1]\), gives national consumption of good \(j\) within the U.S.

\[
c_t(j) = \left[ \int c_t(j,l)^{\frac{\theta-1}{\theta}} dl \right]^{\frac{\theta}{\theta-1}},
\]

\(^2^{When we calibrate to Bils and Klenow, average duration between information updates needed to match LOP properties raised to between 14 and 20 months.}
and further CES aggregation across goods in the U.S. gives aggregate U.S. consumption, \( c_t \),

\[
c_t = \left[ \int c_t(j)^{\theta-1} \frac{\theta}{\theta-1} \, dz \right]^{\frac{\theta}{\theta-1}}.
\]

## 2.1 Households

As in Kehoe and Midrigan (2007), complete markets for state-contingent money claims exist. Agents hold the portfolio which yields the (random) payoff in period \( t+1 \). U.S. households hold \( B_{t+1} \) while Canadians hold \( B^*_t \) (both denominated in the U.S. dollars) with the associated nominal stochastic discount factor \( \Upsilon_{t,t+1} \).\(^3\) Also, \( \Upsilon_{t,t+h} \) is the nominal stochastic discount factor by which all firms, regardless of their country of origin, discount profits earned in period \( t+h \) back to the present period \( t \).

Households in each country maximize the discounted sum of \( U(c_t,n_t) = \ln c_t - \chi n_t \ (\chi > 0) \) subject to an intertemporal budget constraint and a CIA constraint. The maximization problem for U.S. households is

\[
\begin{align*}
\max & \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t,n_t), \\
\text{s.t.} & \quad M_t + \mathbb{E}_t(\Upsilon_{t+1} B_{t+1}) = R_{t-1} W_{t-1} n_{t-1} + B_t + (M_{t-1} - P_{t-1} c_{t-1}) + T_t + \Pi_t, \\
& \quad M_t \geq P_t c_t,
\end{align*}
\]

where \( \beta \) is the discount factor of the household satisfying \( 0 < \beta < 1 \) and \( \mathbb{E}_t(\cdot) \) denotes the expectation operator conditional on the information available in period \( t \).

The left hand side of the intertemporal budget constraint (2) represents the nominal value of total wealth of the household brought into the beginning of period \( t+1 \). It consists of cash holding \( M_t \) and bond holdings \( B_{t+1} \). As shown on the right-hand-side of (2), the household receives nominal labor income \( W_{t-1} n_{t-1} \) in period \( t-1 \) which earns gross nominal interest \( R_{t-1} \) per unit of labor income until period \( t \) in terms of U.S. currency.\(^4\) Households carry nominal bonds in amount \( B_t \) and cash holding remaining after consumption expenditures \( (M_{t-1} - P_{t-1} c_{t-1}) \) into period \( t \); \( P_t \) is the aggregate price index defined below. Finally, \( T_t \) and \( \Pi_t \) are nominal lump sum transfers from the U.S. government and nominal profits of firms operating in the U.S., respectively.\(^5\)

\(^3\) As Kehoe and Midrigan (2007) argue, it does not matter if foreign (Canadian) consumers hold complete and state-contingent bonds denominated in the foreign currency (Canadian dollars). It would be simply a redundant assumption under state-contingent bond markets.

\(^4\) We assume that the government pays interest rate \( R_t (= 1/\mathbb{E}_t \Upsilon_{t+1}) \) on labor income in period \( t \). This assumption allows households’ intratemporal optimality condition to be undistorted.

\(^5\) We assume that government’s lump sum transfers and firms’ profits in a country go to households in that country.
Equation (3) is the CIA constraint. The aggregate price $P_t$ is given by $P_t = \int P_t(j)^{1-\theta} dj \frac{1}{1-\theta}$, where $P_t(j)$ is the aggregate price index for good $j$; it is a CES aggregate over city-specific prices for that good: $P_t(j) = \int P_t(j, l)^{1-\theta} dl \frac{1}{1-\theta}$. The price index for good $j$ in a particular city $l$ used in this aggregation is given by

$$P_t(j, l) = \left[ \int P_t(j, l, z)^{1-\theta} dz \right] \frac{1}{1-\theta}.$$

Households in Canada solve the analogous optimization problem except we must convert their U.S. dollar bond holdings into Canadian dollars at the spot nominal exchange rate, $S_t$. Thus the Canadian-dollar intertemporal budget constraint is

$$M_t^* + \frac{E_t(\Upsilon_{t,t+1} B_{t+1}^*)}{S_t} = S_{t-1} R_{t-1} W_{t-1}^* + \frac{B_{t}^*}{S_t} (M_{t-1}^* - P_{t-1}^* c_{t-1}) + T_t^* + \Pi_t^*.$$  (4)

The key first-order conditions of households are as follows:

$$\frac{W_t}{P_t} = \chi c_t,$$  (5)

$$\frac{M_t}{P_t} = P_t c_t,$$  (6)

$$\Upsilon_{t,t+1} = \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{P_t}{P_{t+1}} \right] = \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{S_t P_t^*}{S_{t+1} P_{t+1}^*} \right].$$  (7)

Equation (5) represents intratemporal substitution between labor and consumption while (6) indicates that the CIA constraint always binds. The intertemporal conditions for optimal consumption choices across adjacent months in the U.S. and Canada are given by (7). The counterparts of (5) and (6) apply to Canada as well.

Combining the intratemporal conditions with the CIA constraints, for the U.S., we have

$$W_t = \chi M_t.$$

The nominal wage rate in the U.S. is proportional to the stock of money held by households in the U.S., an analogous condition holds for Canada.

The aggregate real exchange rate is determined by combining the home and foreign intertemporal conditions:

$$q_t = \frac{S_t P_t^*}{P_t} = \kappa \frac{c_t}{c_t^*},$$  (8)

where $\kappa = q_0 c_0^* / c_0$.

The nominal exchange rate is determined by combining the home and foreign CIA constraints with (8):

$$S_t = \kappa M_t.$$  (9)
2.2 Firms

The output of brand \( z \) of good \( j \) in the U.S. is equal to the number of hours allocated to that activity:

\[
y_t(j, z) = n_t(j, z).
\]

Goods are perishable, so the consumption of each good across all cities equals output of that good in the current period:

\[
\int c_t(j, l, z) dl + \int [1 + \tau(j, l^*)] c_t^*(j, l^*, z) dl^* = y_t(j, z).
\] (10)

We allow for long-run deviations from the LOP across borders through \( \tau(j, l^*) \), an iceberg transportation cost in exporting good \( j \) from the U.S. to a Canadian city indexed by \( l^* \). A firm must ship \( 1 + \tau(j, l^*) \) units of good \( j \) to city \( l^* \) for one unit of that good to arrive at the destination.

An analogous market clearing condition holds for each of the Canadian goods:

\[
\int [1 + \tau(j, l)] c_t(j, l, z) dl + \int c_t^*(j, l^*, z) dl^* = y_t^*(j, z).
\] (11)

2.3 Price adjustment and information updating

This section begins by reviewing Calvo pricing used by Kehoe and Midrigan (2007) and then presents our extension to allow for information updating as in Mankiw and Reis (2002).

2.3.1 Calvo pricing

We model the nominal price rigidities as in Calvo (1983) and Yun (1996): each month a fraction of firms \( 1 - \lambda_j \) are randomly drawn and allowed to reset their prices. As suggested by the subscript, the frequency of price changes varies by good \( j \), but not by country.

All U.S. firms \( z \in [0, 1/2] \) that sell their good \( j \) in U.S. city \( l \) choose the same optimal price when they adjust prices in period \( t \). The price \( P_{H,t}(j, l) \) solves the following maximization problem:

\[
\max_{P_{H,t}(j, l)} \mathbb{E}_t \sum_{h=0}^{\infty} \lambda_j^h \tau_{t+h} [P_{H,t}(j, l) - W_{t+h}] \left( \frac{P_{H,t}(j, l)}{P_{t+h}} \right)^{-\theta} c_{t+h},
\]

for all cities \( l \in [0, 1] \). Here we have used the fact that since the elasticity of substitution is the same across all sub-aggregators, we can express demand for good \( j \) in city \( l \) as

\[
c_t(j, l) = \left( \frac{P_t(j, l)}{P_t} \right)^{-\theta} c_t.
\]
The first-order condition for $P_{H,t}(j,l)$ satisfies
\[
\mathbb{E}_t \sum_{h=0}^{\infty} \lambda_j^h \Upsilon_{t,t+h}\left(\frac{P_{H,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h} = \frac{\theta}{\theta - 1} \mathbb{E}_t \sum_{h=0}^{\infty} \lambda_j^h \Upsilon_{t,t+h}\left(\frac{P_{H,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h}. \tag{12}
\]
Similarly, the first-order condition for Canadian firms’ price in U.S. city $l$ in U.S. dollars, conditional on adjustment, $P_{F,t}(j,l)$, satisfies
\[
\mathbb{E}_t \sum_{h=0}^{\infty} \lambda_j^h \Upsilon_{t,t+h}\left(\frac{P_{F,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h} = \frac{\theta}{\theta - 1} \mathbb{E}_t \sum_{h=0}^{\infty} \lambda_j^h \Upsilon_{t,t+h}\left(\frac{1 + \tau(j,l)S_{t+h}W_{t+h}^s}{P_{F,t}(j,l)}\right)\left(\frac{P_{F,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h}. \tag{13}
\]

2.3.2 Calvo pricing with infrequent information updating

We now add information stickiness to the model following Mankiw and Reis (2002). Consider firms facing two nominal rigidities. First, each firm has a constant probability of price resetting $1 - \lambda_j$ as before. Second, with probability of $1 - \omega$, a firm receives an information update in the current month. The fraction of firms that fail to get updates, $\omega$, use the information available from their most recent update. DKT develop this combined stickiness structure to explain persistent inflation dynamics as we specified above. In DKT, infrequent price changes arise due to the Calvo assumption of price changes. However, when firms compute their optimal reset prices, a fraction of firms use the newest information set and the remaining firms use the stale information set to determine prices. Following DKT, we employ this structure and refer to it as “dual stickiness” pricing.

All U.S. firms that sell their good $j$ in city $l$ choose different prices according to the vintage of their information set. When firms are allowed to adjust prices, those with the same vintage of information choose the same price. Let $P_{H,t}^k(j,l)$ be the optimal price reset by U.S. firms conditional on information of vintage $k$, its age in months. The price $P_{H,t}^k(j,l)$ for these firms solves
\[
\max_{P_{H,t}^k(j,l)} \mathbb{E}_{t-k} \sum_{h=0}^{\infty} \lambda_j^h \Upsilon_{t,t+h}\left[P_{H,t}^k(j,l) - W_t\right]\left(\frac{P_{H,t}^k(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h},
\]
for $k = 0, 1, 2, \cdots$ and for all cities $l \in [0, 1]$. Note the only difference between this problem and the standard Calvo problem is that the expectation is taken with respect to information of vintage $k$ and reset prices are indexed both by the time period they are reset and the vintage of the information used at the point they are reset, $P_{H,t}^k(j,l)$. 
The first-order condition for \( P_{H,t}^{k,j,l} \) is
\[
E_{t-k} \sum_{h=0}^{\infty} \lambda_j^h \Upsilon_{t,t+h} \left( \frac{P_{H,t}^{k,j,l}}{P_{t+h}} \right)^{-\theta} c_{t+h}
\]
\[
= \frac{\theta}{\theta - 1} E_{t-k} \sum_{h=0}^{\infty} \lambda_j^h \Upsilon_{t,t+h} \left( \frac{W_{t+h}}{P_{H,t}^{k,j,l}} \right) \left( \frac{P_{H,t}^{k,j,l}}{P_{t+h}} \right)^{-\theta} c_{t+h},
\]
for \( k = 0, 1, 2, \ldots \). Canadian firms change prices to satisfy an analogous first-order condition to (14):
\[
E_{t-k} \sum_{h=0}^{\infty} \lambda_j^h \Upsilon_{t,t+h} \left( \frac{P_{F,t}^{k,j,l}}{P_{t+h}} \right)^{-\theta} c_{t+h}
\]
\[
= \frac{\theta}{\theta - 1} E_{t-k} \sum_{h=0}^{\infty} \lambda_j^h \Upsilon_{t,t+h} \left( \frac{1 + \tau(j,l) S_{t+h} W_{t+h}^*}{P_{F,t}^{k,j,l}} \right) \left( \frac{P_{F,t}^{k,j,l}}{P_{t+h}} \right)^{-\theta} c_{t+h},
\]
for \( k = 0, 1, 2, \ldots \).

2.4 Equilibrium

The monetary authority in each country sets the growth rate of the money stock such that it follows an AR(1):
\[
\ln \mu_t = \rho \ln \mu_{t-1} + \varepsilon_t,
\]
where \( \varepsilon_t \) and \( \varepsilon_t^* \) are mean-zero i.i.d shocks and \( \mu_t = M_t/M_{t-1} \) and \( \mu_t^* = M_t^*/M_{t-1}^* \). The steady state (log) money growth rates are set to zero and the common persistence parameter satisfies \( \rho \in [0, 1) \).

Total transfers from the government to individuals in each country equal domestic money injections minus the lump sum tax from the government paying interest. For the U.S., we have \( T_t = M_t - M_{t-1} - (R_{t-1} - 1)W_{t-1} n_{t-1} \). The total transfers in Canada are of the same form up to currency conversions: \( T_t^* = M_t^* - M_{t-1}^* - (S_{t-1} R_{t-1}/S_t - 1)W_{t-1}^* n_{t-1} \).

The profits of U.S. firms accrue exclusively to U.S. households. In other words, \( \Pi_t = \int_j \int_{z=0}^{\frac{1}{2}} \Pi_t(j,z)dzdj \), where \( \Pi_t(j,z) \) is the profit of a U.S. firm. Similarly, the profits of Canadian firms accrue exclusively to Canadian households: \( \Pi_t^* = \int_j \int_{z=\frac{1}{2}}^{1} \Pi_t^*(j,z)dzdj \), where \( \Pi_t^*(j,z) \) is the profit of a Canadian firm.

Recall, market clearing conditions for good markets were given by (10) and (11). The labor
market clearing conditions are
\[ n_t = \int_j \int_{z=0}^{\frac{1}{2}} n_t(j, z) d z d j, \]
\[ n^*_t = \int_j \int_{z=\frac{1}{2}}^{1} n^*_t(j, z) d z d j. \]

Last, but not least, the contingent bond markets clear at each date and state: \( B_t + B^*_t = 0 \) for all \( t \).

An equilibrium of the model is a collection of allocations and prices:

- \( \{c_t(j, l, z)\}_{j, l, z}, M_t, B_{t+1}, n_t \) for U.S. households;
- \( \{c^*_t(j, l^*, z)\}_{j, l, z}, M^*_t, B^*_{t+1}, n^*_t \) for Canadian households;
- \( \{P_t(j, l, z), P^*_t(j, l^*, z), n_t(j, z), y_t(j, z)\}_{j, l, l^*, z \in [0, 1/2]} \) for U.S. firms;
- \( \{P_t(j, l, z), P^*_t(j, l^*, z), n^*_t(j, z), y^*_t(j, z)\}_{j, l, l^*, z \in (1/2, 1]} \) for Canadian firms;
- Nominal wages and bond prices satisfy the following conditions:
  1. Households’ allocations solve their maximization problems;
  2. Prices and allocations of firms solve their maximization problems;
  3. All markets clear;
  4. The money supply process and transfers satisfy the specifications above.

### 3 Theoretical implications for LOP dynamics

We now contrast the implications of the two variant models for the persistence and volatility of deviations from the LOP.

#### 3.1 Calvo pricing

The model allows money to be a non-stationary random variable. As a result, we normalize all nominal variables by the nominal money stock. For example, the normalized U.S. dollar price becomes \( p_{H,t}(j, l) = P_{H,t}(j, l)/M_t \). We work with log-linear deviations of these transformed variables from their steady-state levels and these deviations are denoted with ‘*’s over them. The reset price
based on this normalization, in log-deviations from their steady-state level, for U.S. firms selling
good \( j \) in city \( l \), is (using (12)):

\[
\hat{p}_{H,t}(j,l) = (1 - \lambda_j \beta) \sum_{h=0}^{\infty} (\lambda_j \beta)^h \mathbb{E}_t (\hat{\mu}_{t+1,t+h}) = \left[ \frac{\lambda_j \beta \rho}{1 - \lambda_j \beta \rho} \right] \hat{\mu}_t,
\]

where

\[
\hat{\mu}_{t+1,t+h} = \begin{cases} 
0 & \text{for } h = 0 \\
\sum_{d=1}^{h} \hat{\mu}_{t+d} & \text{for } h = 1, 2, \ldots
\end{cases}
\]

Here we have used the proportionality of money and nominal wages, \( W_t = \chi M_t \), to replace endogenous wages with exogenous money. We see that firms adjusting their prices in period \( t \) adjust them in proportion to the present discounted value of future marginal cost changes during periods of price non-adjustment.

Analogously, we can derive the log-deviation of reset price relative to the U.S. nominal money supply for Canadian firms from (13):

\[
\hat{p}_{F,t}(j,l) = (1 - \lambda_j \beta) \sum_{h=0}^{\infty} (\lambda_j \beta)^h \mathbb{E}_t (\hat{\mu}_{t+1,t+h}) = \left[ \frac{\lambda_j \beta \rho}{1 - \lambda_j \beta \rho} \right] \hat{\mu}_t,
\]

where we have used the fact that under the monetary approach, the change in the nominal exchange rate is proportional to the change in domestic relative to foreign money supplies. As it turns out \( \hat{p}_{F,t}(j,l) = \hat{p}_{H,t}(j,l) \) so the short-run dynamics of the optimal prices are the same for home and foreign firms selling the same good at the same location in spite of the transportation costs which drive a wedge between the prices in the long-run.

Thus, the log-deviation of price index for \( p_t(j,l) = P_t(j,l)/M_t \) under Calvo pricing becomes

\[
\hat{p}_t(j,l) = \lambda_j \hat{p}_{t-1}(j,l) - \lambda_j \hat{\mu}_t + (1 - \lambda_j) \left[ \frac{\lambda_j \beta \rho}{1 - \lambda_j \beta \rho} \right] \hat{\mu}_t.
\] (18)

The analogous expression for the Canadian price index for good \( j \) and Canadian city \( l^* \) is

\[
\hat{p}_t^*(j,l^*) = \lambda_j \hat{p}_{t-1}^*(j,l^*) - \lambda_j \hat{\mu}_t^* + (1 - \lambda_j) \left[ \frac{\lambda_j \beta \rho}{1 - \lambda_j \beta \rho} \right] \hat{\mu}_t^*,
\] (19)

and the log bilateral real exchange rate for good \( j \) across cities \( l \) and \( l^* \) is \( \hat{q}_t(j,l,l^*) = \ln q_t(j,l,l^*) - \ln q(j,l,l^*) \), where \( q_t(j,l,l^*) \) is given by

\[
q_t(j,l,l^*) = \frac{S_t P_{t*}^*(j,l^*)}{P_t(j,l)},
\]

and \( q(j,l,l^*) \) is its steady state value.

The next proposition characterizes the short-run, good-level, real exchange rate dynamics under Calvo pricing with a slight generalization of Kehoe and Midrigan (2007).
Proposition 1. Under the preference assumption \(U(c, n) = \ln c - \chi n\), the CIA constraints, the assumption of money growth (16) and (17) and good-specific Calvo pricing, the good-level real exchange rate between any cities \(l\) and \(l^*\) follows an AR(2) process of the form:

\[
\hat{q}_t(j, l, l^*) = (\lambda_j + \rho)\hat{q}_{t-1}(j, l, l^*) - \lambda_j \rho \hat{q}_{t-2}(j, l, l^*) + \theta_j \eta_t, \tag{20}
\]

where \(\theta_j = \lambda_j - (1 - \lambda_j)\lambda_j \beta \rho / (1 - \lambda_j \beta \rho)\), and \(\eta_t = \varepsilon_t - \varepsilon_t^*\) is i.i.d. \((0, \sigma^2_{\eta})\).

Proof. From (8) and (9), \(\hat{q}_t(j, l, l^*) = \hat{p}^*_t(j, l^*) - \hat{p}_t(j, l)\). Subtracting (18) from (19) yields \(\hat{q}_t(j, l, l^*) = \lambda_j \hat{q}_{t-1}(j, l, l^*) + \theta_j (\hat{\mu}_t - \hat{\mu}_t^*)\). Because \(\hat{\mu}_t - \hat{\mu}_t^*\) follow an AR(1) from (16) and (17), we obtain (20) and proved Proposition 1.

Proposition 1 of Kehoe and Midrigan (2007) is a special case of the one above: when money growth rates follow an i.i.d. process \((\rho = 0)\) equation (20) reduces to an AR(1) model with its coefficient \(\lambda_j\) and \(\theta_j = \lambda_j\) as Kehoe and Midrigan (2007) prove.6

3.2 Calvo pricing with infrequent information updating

Let \(p^k_{H,t}(j, l)\) be the log deviation of \(P^k_{H,t}(j, l)/M_t\) from the steady state. Log-linearizing (14) around the steady state yields

\[
\hat{p}^k_{H,t}(j, l) = (1 - \lambda_j \beta) \sum_{h=0}^{\infty} (\lambda_j \beta)^h E_{t-k} (\hat{\mu}_{t+1,t+h}), \text{ for } k = 0
\]

and

\[
\hat{p}^k_{H,t}(j, l) = (1 - \lambda_j \beta) \sum_{h=0}^{\infty} (\lambda_j \beta)^h E_{t-k} (\hat{\mu}_{t+1,t+h}) + E_{t-k} (\hat{\mu}_t + \hat{\mu}_{t-1} + \cdots + \hat{\mu}_{t-k+1}) - (\hat{\mu}_t + \hat{\mu}_{t-1} + \cdots + \hat{\mu}_{t-k+1}), \text{ for } k = 1, 2, 3, \ldots.
\]

The only differences between this equation and Calvo pricing are that \(\hat{p}^k_{H,t}(j, l)\) and \(E_t\) are replaced with \(\hat{p}^k_{H,t}(j, l)\) and \(E_{t-k}\) and that, under \(k > 0\), forecast errors are accumulated from the period in which firms last update information.7

We must now determine and keep track of prices based on the vintage \(k\) of information being used. As in Calvo pricing, it is possible to show that the (normalized) price index for good \(j\) in

---

6The same proposition was independently derived by Carvalho and Nechio (2008) who focus on the role of sticky prices in producing aggregation bias in the persistence of the aggregate real exchange rate.

7Note that \(\hat{p}^0_{H,t}(j, l) = \hat{p}_{H,t}(j, l)\) because of the equivalence between (12) and (14) when \(k = 0\).
location \( l \) can be expressed in terms of fixed, reset prices and money growth rates. However, the reset prices are our weighted average of price resets given different vintage of information:

\[
\hat{p}_t(j,l) = \lambda_j \hat{p}_{t-1}(j,l) - \lambda_j \hat{\mu}_t + (1 - \lambda_j) \hat{x}_t(j,l),
\]

where \( \hat{x}_t(j,l) \) is the (normalized) weighted average for the newly set prices for good \( j \) in city \( l \) of the U.S., based upon different information vintages,

\[
\hat{x}_t(j,l) = (1 - \omega) \sum_{k=0}^{\infty} \omega^k \mathbb{E}_{t-k} \hat{p}_{H,t}(j,l) + (1 - \omega) \sum_{k=1}^{\infty} \omega^k \left[ \mathbb{E}_{t-k} (\hat{\mu}_t + \hat{\mu}_{t-1} + \cdots + \hat{\mu}_{t-k+1}) - (\hat{\mu}_t + \hat{\mu}_{t-1} + \cdots + \hat{\mu}_{t-k+1}) \right],
\]

which is similar in mathematical formulation to the price index in Mankiw and Reis (2002, p.1300).

Canadian versions of these expressions are

\[
\hat{p}^*_t(j,l) = \lambda_j \hat{p}^*_{t-1}(j,l) - \lambda_j \hat{\mu}_t^* + (1 - \lambda_j) \hat{x}^*_t(j,l),
\]

\[
\hat{x}^*_t(j,l) = (1 - \omega) \sum_{k=0}^{\infty} \omega^k \mathbb{E}_{t-k} \hat{p}^*_{H,t}(j,l) + (1 - \omega) \sum_{k=1}^{\infty} \omega^k \left[ \mathbb{E}_{t-k} (\hat{\mu}_t^* + \hat{\mu}_{t-1}^* + \cdots + \hat{\mu}_{t-k+1}^*) - (\hat{\mu}_t^* + \hat{\mu}_{t-1}^* + \cdots + \hat{\mu}_{t-k+1}^*) \right].
\]

The next proposition establishes the rich short-run dynamics of the good-level real exchange rate emerging from the extended model.

**Proposition 2.** Under the preference assumption \( U(c,n) = \ln c - \chi n \), the CIA constraints, the assumption of money growth (16) and (17), along with good-specific Calvo pricing and Mankiw-Reis information updating, the good-level real exchange rate between any cities \( l \) and \( l^* \) follows an ARMA(4,2) process of the form:

\[
\hat{q}_t(j,l,l^*) = \sum_{r=1}^{4} \phi_{j,r} \hat{q}_{t-r}(j,l,l^*) + \sum_{r=0}^{2} \theta_{j,r} \eta_{t-r}
\]

where the coefficient are known functions of \( \beta, \rho, \lambda_j, \) and \( \omega \).

When \( \omega = 0 \) this proposition reduces to Proposition 1.\(^8\) Below, we show that both persistence and volatility of good-level real exchange rates predicted by the dual stickiness pricing can be quite high. Moreover, this is true even if the price adjustment is relatively fast, which is essential in matching the cross-sectional evidence which contains goods with frequent price changes and, yet, high persistence and variability in their LOP deviations.

\(^8\)In particular, we obtain \( \phi_{j,1} = \lambda_j + \rho, \phi_{j,2} = -\lambda_j \rho, \) and \( \phi_{j,3} = \phi_{j,4} = 0 \) for the AR parameters and \( \theta_{j,0} = \theta_j \) and \( \theta_{j,1} = \theta_{j,2} = 0 \) for the MA parameters. The proofs are available in the working paper version.
4 Quantitative theoretical implications for LOP persistence and volatility

The theoretical model places important restrictions on the relationship between the structural parameters of the model, $\rho$, $\lambda_j$ and $\omega$ and the parameters of the univariate ARMA model we estimate in the next section. Here we explore the quantitative implications of the theory by parameterizing the theory to flesh out some intuition for how the structural parameters impact LOP persistence and volatility.

There are only two free parameters in this exercise, $\rho$ and $\omega$, since the $\lambda_j$'s are pinned down by the estimates of the frequency of price adjustments existing in the literature. We consider two extreme values for each of the free parameters.

The equilibrium conditions of our model imply that the persistence of changes in the nominal exchange rate, changes in the nominal money stock and changes in nominal GDP are equal to each other. What the data suggest otherwise reflects a combination of model mis-specification and difficulty in precisely estimating the persistence parameter. Our approach is to present the extremes in this section since this enhances intuition, and to consider points in between these extremes in the estimation section where empirical plausibility is tested.

For the persistence parameter, $\rho$, two key benchmarks are 0 and 0.83. The former is familiar from the famous Meese and Rogoff (1983) puzzle, the inability of the monetary model to beat a random walk model of the nominal exchange rate. With this benchmark, the monetary model produces a random walk nominal exchange rates, but fails to rationalize the literal interpretation of the simple CIA model of money demand. This is also the value used by Kehoe and Midrigan (2007). The latter benchmark is the monthly analog to the CKM calibration for M1 growth.\(^9\) However, using nominal GDP growth, we may also parameterize $\rho = 0.75$, slightly lower than M1 growth persistence.\(^10\)

For the information updating parameter, $\omega$, we have less prior evidence to restrict its range. To frame our discussion of its role in accounting for persistence in real exchange rates of individual

\(^9\)The CKM estimate of the autoregressive coefficient is 0.68 using quarterly U.S. data for M1 growth. We transformed this quarterly persistence of M1 growth into the monthly persistence by solving $\text{Cov}(\hat{M}_t - \hat{M}_{t-3}, \hat{M}_{t-3} - \hat{M}_{t-6})/\text{Var}(\hat{M}_t - \hat{M}_{t-3}) = 0.68$ for $\rho$ and obtained 0.83.

\(^10\)We estimated the autoregressive coefficient for quarterly U.S. nominal GDP growth from 1947:Q3 to 2008:Q2 and obtained a coefficient of 0.55. To obtain the monthly persistence of 0.75, we again transformed the quarterly persistence, using the formula shown in the footnote 9.
goods and services, we pick values that encompass the diverse range of persistence estimates we find in the EIU micro-data. In the empirical section we estimate the information updating parameter by fixing the persistence of the money shocks and minimizing the distance between the persistence implied by the restricted model and the estimated persistence in the micro-data.

4.1 Calvo pricing

4.1.1 Persistence

Turning to implications for persistence of the good-level real exchange rates we employ the sum of autoregressive coefficients (SAR) as the persistence metric. This is often the case in applied work when moving beyond the AR(1) model (e.g., Andrews and Chen (1994) and Clark (2006)) because the SAR has a one-to-one relationship to the cumulative long-run impulse response to a shock. We denote the SAR by $\alpha_j$.

Under Proposition 1, the SAR measure of persistence is $\alpha_j = \lambda_j + \rho(1 - \lambda_j)$; it simplifies to $\alpha_j = \lambda_j$ when $\rho = 0$. Obviously, the SAR is strictly increasing in $\rho$ regardless of the degree of price stickiness under $\lambda_j \in [0, 1)$. The left panel of Figure 1 shows the effect of increasing $\rho$ on the persistence for the two goods: a good with relatively slow price adjustment ($\lambda_j = 0.95$) and a good with relatively fast price adjustment ($\lambda_j = 0.5$).

The right panel of Figure 1 plots the SAR against $\lambda_j$. The figure compares the impact of changing $\rho$ from 0 to 0.83. The impact of introducing persistence in $\hat{\mu}_t$ and $\hat{\mu}_t^*$ on the SAR is clear. When $\rho = 0$, the model predicts that the SAR equals $\lambda_j$, so the two parameters lie on the 45 degree line in the figure. On the other hand, when $\rho > 0$, the model predicts a flatter line. Thus, a high persistence of the money growth rates increases the persistence of LOP deviations, regardless of the frequency of price adjustment, but the quantitative impact is greatest when the frequency of price adjustment is highest.

To see the intuition behind the persistent dynamics it is instructive to express the current LOP deviation as a function of its lagged self and the change in the nominal exchange rate:

$$\hat{q}_t(j,l,l^*) = \lambda_j \hat{q}_{t-1}(j,l,l^*) + \theta_j \Delta \hat{S}_t,$$

where $\Delta \hat{S}_t = \hat{\mu}_t - \hat{\mu}_t^*$ from (9). When $\rho = 0$ as in Kehoe and Midrigan (2007), $\Delta \hat{S}_t$ is an i.i.d shock and the good-level real exchange rate follows an AR(1) with persistence parameter, $\lambda_j$. When international money growth differential is positively autocorrelated ($\rho > 0$) so is the change in the nominal exchange rate, which contributes to increased persistence in the real exchange rate.
4.1.2 Volatility

Throughout, real exchange rate volatility will be measured relative to the standard deviation of the change in the nominal exchange rate: \( \sigma_j = \text{std}(q_t(j,l,l^*)) / \text{std}(\Delta S_t) \). When the nominal exchange rate follows a random walk, \( \rho = 0 \), the model predicts the normalized standard deviation to be \( \sigma_j = \sigma_1(\lambda_j) = \lambda_j / \sqrt{1 - \lambda_j^2} \) and a good with larger \( \lambda_j \) will exhibit more variability. When \( \rho > 0 \), the normalized standard deviation is predicted to be of the form \( \sigma_j = \sigma_2(\lambda_j, \rho, \beta) \) and may be obtained using the variance formula of an AR(2) process along with \( \text{std}(\Delta S_t) = \text{std}(\eta_t) / \sqrt{1 - \rho^2} \).

Importantly, the volatility function depends not just on \( \lambda_j \), but also on \( \rho \) and \( \beta \).

An implication of this is that increased persistence in money growth, while helpful in resolving the persistence puzzle, may actually make the volatility puzzle worse because \( \sigma_j = \sigma_2(\lambda_j, \rho, \beta) \) turns out not to be monotonic in \( \rho \). Even more disturbing is that the shape of the relationship with \( \rho \) depends on the frequency of price adjustment, which we know differs across goods. The practical thrust of this is: changes in money growth persistence will have differential impacts across goods.

The left panel of Figure 2 plots the normalized standard deviations \( \sigma_j = \sigma_2(\lambda_j, \rho, \beta) \) against \( \rho \).\(^{11}\) For a good with relatively infrequent price changes (\( \lambda_j = 0.95 \)), volatility of the real exchange rate rises over most of the range of money growth persistence, before falling sharply as money growth approaches a random walk. In contrast, for a good with relatively frequent price changes (\( \lambda_j = 0.5 \)), the volatility of the relative price is declining in the money growth rate persistence throughout. The right panel of Figure 2 shows the ambiguous impact of introducing a positive \( \rho \) on the volatility from another dimension. The normalized standard deviation is smaller for \( \rho = 0.83 \) than for \( \rho = 0 \) when price adjustment is fast. When the price adjustment is slow, we have a larger normalized standard deviation for \( \rho = 0.83 \) than for \( \rho = 0 \).

4.2 Calvo pricing with infrequent information updating

4.2.1 Persistence

The SAR in dual stickiness pricing is given by

\[
\alpha_j = \sum_{r=1}^{4} \phi_{j,r} = 1 - (1 - \lambda_j)(1 - \omega)(1 - \omega \rho)(1 - \rho).
\]

Clearly, the slower the speed of information updating adjustment is (\( \omega \rightarrow 1 \)), the larger the SAR becomes.

\(^{11}\)We set the discount factor \( \beta \) to 0.99.
For a general ARMA process without parameter restrictions, it is not conventional to use the SAR as a measure of persistence, because of the presence of MA terms. However, if our model is correctly specified, we can show that both long-run impact of cumulative impulse response of a unit monetary shock on real exchange rates and the SAR are strictly increasing function of $\lambda_j$, $\omega$, and $\rho$. Furthermore, using the SAR is also convenient in computation and for the purpose of making comparison with simpler models introduced in the previous subsection. For these reasons, we continue to focus on the SAR as an approximate measure of persistence under the assumption that (21) is correctly specified.

The dual stickiness pricing works well in generating the persistence of real exchange rates. The left panel of Figure 3 shows the SAR among different $\omega$’s. The persistence is increasing in $\omega$ and is very high regardless of the infrequency of price changes.$^{12}$ The right panel of Figure 3 plots the persistence against $\lambda_j$. This panel compares cases of two extreme values of $\omega$. One is the case in which firms producing good $j$ updates their information every month. (i.e., $\omega = 0$.) The other is the case in which firms, on average, update information every 50 months (i.e., $\omega = 0.98$). For the former case, the obtained SAR corresponds to the upper line in the right panel of Figure 1 since we set $\rho = 0.83$ in the computation. In the latter case, the persistence measure is very close to one whether prices are sticky or flexible.

### 4.2.2 Volatility

Having improved the potential of the model in accounting for persistence of real exchange rates, we ask if it helps along the dimension that was more ambiguous in the baseline model, variability. We calculate the new normalized standard deviation $\sigma_j = \sigma_3(\omega, \lambda_j, \rho, \beta)$, using the fact that the good-level real exchange rates now follow the ARMA(4,2) process according to Proposition 2. The left panel of Figure 4 plots the normalized standard deviations against $\omega$. It shows that the volatility grows exponentially as $\omega$ increases. The right panel of Figure 4 shows the effect of increasing $\lambda_j$ on the normalized standard deviations under the two extreme cases: $\omega = 0$ and 0.98. It shows that real exchange rate volatility becomes substantially greater when the information adjustment is slower. Thus, the introduction of information stickiness enhances the real exchange rate volatility to a large extent.

The question we pose next is what lengths of information delays are needed to match key properties of the micro-data, conditional on the model and the observed frequency of price changes.

$^{12}$Even if $\omega = 0$, $\hat{q}_t(j,l,l^*)$ is already somewhat persistent, because of the AR(1) money growth.
The key properties are the persistence and volatility of good-level real exchange rates.

5 Empirical results

5.1 Data

The retail prices come from the *Worldwide Cost of Living Survey* compiled by the Economist Intelligence Unit (EIU). It is an extensive annual survey of international retail prices that was originally designed to help managers determine compensation levels of their employees residing in different cities. The coverage of goods and services is broad enough to overlap significantly with what appears in a typical urban consumption basket (see Rogers (2007), for more detail on the comparison between EIU data and the CPI data from national statistical agencies). A notable advantage of the EIU data is the fact that all the individual good prices are listed in absolute terms with the survey conducted by a single agency in a consistent manner over time. Because of this convenient panel data format, a number of recent studies on international price dynamics have used this data, including Crucini and Telmer (2007), Crucini and Shintani (2008), Engel and Rogers (2004), Parsley and Wei (2007) and Rogers (2007).

For a limited number of countries, the EIU data contains observations from multiple cities. In our empirical analysis, we focus on U.S.-Canadian city pairs since the assumption of the common probability of price adjustment for each good seems to be a reasonable approximation between the two neighboring countries.\footnote{Amirault, Kwan, and Wilkinson (2006) report frequencies of price adjustment for Canadian firms that match up reasonably well with the U.S. evidence. In other contexts, one may use the average of price change frequencies between the two countries, an approach employed in Kehoe and Midrigan (2007), when data from both countries are available.}

After removing missing observations to construct a balanced panel for the period from 1990 to 2005, three of the 16 U.S. cities available in the survey are dropped, while all four Canadian cities remain. This results in a total of 52 unique city pairs.

For each good $j$, the log of $q_t(j, l, l^*)$ for each year $t (= 1, ..., 16)$ is computed using the price level in a U.S. city $l (= 1, ..., 13)$ expressed in U.S. dollars ($P_t(j, l)$), the price level in a Canadian city $l^* (= 1, ..., 4)$ expressed in Canadian dollars ($P^*_t(j, l^*)$), and the spot U.S.-Canadian dollar exchange rate ($S_t$), all from the EIU data.

Next, for the price stickiness parameter, $\lambda_j$, we utilize the frequency of price changes, $f_j$ and
transform it with $\lambda_j = 1 - f_j$ for good $j$. Since the EIU data is annual, it is not useful for constructing estimates of the frequency of price changes. Here we rely on Nakamura and Steinsson (2008) who revisited Bils and Klenow’s analysis using more detailed and updated BLS data. Using the CPI Research Database created by the BLS, they re-estimated the frequencies of price change after removing temporary price changes associated with sales. They found that the median duration between regular price changes was 8 - 11 months depending on the treatment of substitutions, considerably higher than the 4.3 months for the median good, found by Bils and Klenow (2004). In what follows, we use their data as our benchmark.\textsuperscript{14}

Specifically, we took the monthly average frequency of price changes, $f_j$, and matched them with the 165 goods in the EIU sample. Since we require paired persistence and frequency adjustment parameters to evaluate the model, we use only these 165 matched pairs in our analysis. We assume that the frequency of price changes applies to the entire sample period of 1990-2005 in our EIU data set.\textsuperscript{15} In addition, we assume a common frequency of price changes between the U.S. and Canadian cities, good-by-good.

For the nominal exchange rate changes required for the theoretical volatility calculation, we use monthly changes in the log of the end-of-month U.S.-Canadian dollar spot rates. While both price stickiness parameter (frequency of no price changes) and nominal exchange rates are available in monthly series, real exchange rates are only observed annually. The small number of time series observation at the annual frequency is the major limitation of the EIU data. In the next subsection, we briefly discuss how to reconcile the mixed frequencies of observation in the dynamic panel estimation and describe the procedure to estimate the time series models.

\section*{5.2 Estimation}

Table 1 shows how monthly ARMA processes predicted by the model are transformed into the ones which have non-zero coefficients for multiples of 12 month lags and finite MA terms. The first row of the table shows the easiest transformation. In Calvo pricing with $\rho = 0$, the equation (20)

\textsuperscript{14}The working paper version of this paper compared the two.

\textsuperscript{15}In some countries which experienced a structural shift in inflation, an assumption of constant frequency of price changes over years may not be satisfied. For example, Ahlin and Shintani (2007) use Mexican price data on 44 goods and report that the average monthly frequency of price changes was 28\% in 1994 and as large as 50\% in 1995. We expect that this issue is less serious in our case since both U.S. and Canada had stable inflation during the period under consideration.
directly implies that
\[ \hat{q}_t(j,l,l^*) = \lambda_j \hat{q}_{t-1}(j,l,l^*) + \lambda_j \eta_t. \]

By repeated substitutions, we get
\[ \hat{q}_t(j,l,l^*) = \lambda_j^{12} \hat{q}_{t-12}(j,l,l^*) + \lambda_j \Lambda_j(L) \eta_t, \]
where \( \Lambda_j(L) = \sum_{r=0}^{11} \lambda_j^r L^r \). In this equation, the AR term is the 12th lag (in months) and the order of the MA term is 11. This ARMA(12,11) is equivalent to an AR(1) sampled annually since \( \lambda_j \Lambda_j(L) \eta_t \) and \( \hat{q}_{t-12}(j,l,l^*) \) are not correlated.

Such a transformation is not necessarily possible with a general ARMA process including AR(2) and ARMA(4,2) processes. However, thanks to a special dynamic feature of the theoretical model, it is possible that we can make the AR parameters non-zero only if the lags are multiples of 12 and the MA parameters are finite under our extended models (20) and (21).

Previously, \( l \) and \( l^* \) were used for the U.S. and Canadian cities, respectively. Here, they are replaced by a new single index \( i (= 1, \ldots, 52) \) each representing a city pair spanning a national border. In addition, the sampling frequency for the model was assumed to be monthly. With some abuse of notation, our new time subscript now represents the time in annual frequency. Namely, if the true data process is generated for each month \( t^* = 1, \ldots, T^* \), we now only observe the series annually at the months of \( t = 12 \times t^* = 1, \ldots, T \) with \( T = T^*/12 \). With this newly introduced index, we define \( q^j_{i,t} \) as the log of the real exchange rate for good \( j \) between the city pair indexed by \( i \) at year \( t \):
\[ q^j_{i,t} = \ln q_t(j,l,l^*). \]

Thus, the former log deviation from the steady state \( \hat{q}_t(j,l,l^*) \) can be rewritten as \( q^j_{i,t} - q^j_i \), where \( q^j_i \) is the long-run value which equals
\[ q^j_i = \ln q(j,l,l^*) = \ln \frac{[1 + \kappa^{1-\theta}(1 + \tau(j,l^*))^{1-\theta}]^{1/\theta}}{[1 + \kappa^{1-\theta}(1 + \tau(j,l))^{1-\theta}]^{1/\theta}}, \]
Intuitively, the relative price of a good in the long-run is higher in the destination market with the higher shipping cost from the source. Thus if city \( l^* \) is, say, farther from the source of the good than city \( l \), \( q^j_i \) is positive. These heterogeneous long-run deviations justify the presence of individual effects (the time invariant city pair-specific effect) in the panel estimation.

Based on the annual transformation shown in Table 1, all the dynamics of the real exchange rate for good \( j \) can be written as
\[ q^j_{i,t} = \sum_{r=1}^{m} \Phi_{j,r} q^j_{i,t-r} + \zeta^j_i \kappa^j_i + u^j_i + \nu^j_{i,t}, \]

20
where $\zeta^j_i$ is the time invariant unobserved city pair-specific effect which allows long-run price difference between two cities, $u^j_i$ is the common time effect which represents the exchange rate shocks and $\nu_{i,t}^j$ is a good-specific residual term.

This empirical model nests all the theoretical models under consideration: (i) Calvo pricing with $\rho = 0$ implies $m = 1$; (ii) Calvo pricing with $\rho \neq 0$ implies $m = 2$; and (iii) dual stickiness pricing implies $m = 4$. For the individual specific effect $\zeta^j_i$, we can easily see its relationship to the long-run mean and the persistence from $q_{i,t}^j = \zeta^j_i / (1 - \alpha_j)$ where $\alpha_j = \sum_{r=1}^m \Phi_{j,r}$. For the common time effect $u^j_i$, Calvo pricing with $\rho \neq 0$ predicts a serial correlation of order one, while dual stickiness pricing predicts a serial correlation of order three. However, in a short panel asymptotic with finite $T$, the common time effects can be treated as unknown parameters to be estimated with time dummies.

Since our main interest is to estimate the SAR, $\alpha_j$, it is convenient to rewrite the model into the augmented Dickey-Fuller (ADF) form. Thus, the nested model is given by

$$q_{i,t}^j = \alpha_j q_{i,t-1}^j + \sum_{r=1}^{m-1} \Phi_{j,r} \Delta q_{i,t-r}^j + u_j^T D_t + \zeta^j_i + \nu_{i,t}^j,$$

where $\Delta q_{i,t-r}^j = q_{i,t-r}^j - q_{i,t-r-1}^j$, $\Phi_{j,r} = \sum_{v=r+1}^m \Phi_{j,v}$ for $r = 1, ..., m-1$, $u_j = (u_{m+1}^j, ..., u_T^j)^\top$ is a vector of constants, $D_t$ is a $(T - m) \times 1$ time dummy vector with one in the $t$-th position and zero elsewhere.

To estimate this short dynamic panel model, we employ the generalized method of moments (GMM) estimator in the first differenced form for the purpose of eliminating the individual effect $\zeta^j_i$. We follow Arellano and Bond (1991) in the choice of instruments and initial weighting matrix, the moment condition is given by

$$\mathbb{E} \left[ q_{is}^j \left( \Delta q_{it}^j - \alpha_j \Delta q_{i,t-1}^j - \sum_{r=1}^{m-1} \gamma_{j,r} \Delta^2 q_{i,t-r}^j - \delta_j^T D_t \right) \right] = 0,$$

for $s = 1, ..., t-m-1$ and $t = m+2, ..., T$, where $\Delta^2 q_{i,t-r}^j = \Delta q_{i,t-r}^j - \Delta q_{i,t-r-1}^j$ $\delta_j = (\Delta u_{m+2}^j, ..., \Delta u_T^j)^\top$ is a vector of constants, $D_t$ is a $(T - m - 1) \times 1$ time dummy vector with one in the $t$-th position and zero elsewhere. The total number of parameters to be estimated is $T - 1$ with the number of moment conditions given by $(T - m)(T - m - 1)/2$.\(^{16}\) This GMM estimator for $\alpha_j$ is consistent under large $N$ fixed $T$ asymptotics.

\(^{16}\)For the model to be (over-) identified, at least $T = 4$ is required for $m = 1$, $T = 6$ is required for $m = 2$, and $T = 9$ is required for $m = 4$. Since $T = 16$ is available in our sample, the number of over-identifying restrictions is 51, 76, and 90, respectively, for $m = 1, 2, \text{and} 4$.\)
5.3 Persistence

In this subsection, we evaluate the Kehoe-Midrigan model and its extension in explaining the observed persistence of the real exchange rate for each good. Following the theoretical analysis, our empirical persistence measure is the good-specific SAR, $\alpha_j$.

We first revisit the original Kehoe-Midrigan model with an assumption of an i.i.d. money growth ($\rho = 0$). In this case, the theory predicts an AR(1) model and thus $\alpha_j$ is simply an AR(1) coefficient. A GMM estimation of $\alpha_j$ yields a median of 0.56 with the median standard error equal to 0.030 using annual U.S.-Canadian city pairs data. In terms of monthly frequency, our value corresponds to $0.56^{1/12} = 0.95$, which is slightly less than 0.98, the median value obtained by Kehoe and Midrigan (2007) based on bilateral real exchange rates of 66 goods between the U.S. and European countries.

The first panel in Figure 5 plots the estimated persistence measure $\alpha_j$ against the (annual) infrequency of the price adjustment $\lambda_j^{12} = (1 - f_j)^{12}$ computed based on $f_j$ from Nakamura and Steinson (2008). A cross-sectional regression of $\alpha_j$ on $\lambda_j^{12}$ yielded a significantly positive slope coefficient estimate of 0.25 (with a standard error of 0.06) which is consistent with the theoretical prediction at least in direction: more price stickiness implies higher persistence. However, 84 percent of the goods have persistence levels that lie above the 45 degree line ($\alpha_j = \lambda_j^{12}$) in the scatter plot. If the model performance is evaluated by computing the ratio of the predicted persistence (on the 45 degree line) to the observed persistence for each good, the median good obtains a ratio of 48 percent. This confirms Kehoe and Midrigan’s claim that a simple model of price stickiness alone is quantitatively insufficient to reproduce the observed persistence in good-level real exchange rates.

We next consider the effect of introducing serially correlated money growth ($\rho = 0.83$). On the whole, the persistence estimate $\alpha_j$ remains almost unchanged with a median value of 0.57 based on the AR(2) model with the median standard error rising somewhat to 0.035. The regression slope shown in the second panel of Figure 5 is 0.25 and is again significantly positive. Recall that for a given $\lambda_j$, $\alpha_j$ is a monotonically increasing function of $\rho$ (see the left panel of Figure 1). In annual frequency, the predicted SAR from Table 1 is given by

$$\alpha_j = 1 - (1 - \rho^{12})(1 - \lambda_j^{12}).$$

(22)

The effect of increasing $\rho$ can be seen in the median value of the ratio of predicted and estimated

---

17This value lies between the medians for OECD city pairs (0.65) and LDC city pairs (0.51) obtained by Crucini and Shintani (2008) based on the same data source.
values provided in the first row of Table 2. In terms of the median, the theoretical persistence equals the estimated persistence when $\rho$ is between 0.90 and 0.95. However, this value is higher than $\rho = 0.83$, the value estimated by CKM and $\rho = 0.75$, our estimate from nominal GDP growth and much higher than 0.25, the estimated monthly autocorrelation of changes in the Canadian-U.S. nominal exchange rate.

Indeed, when $\rho = 0.83$, about 66 percent of the persistence can be explained by the model. Even at this high value, the inability of persistent money growth to account for the persistence of real exchange rates can be seen from the scatter plot. Notice that from (22), increasing $\rho$ pivots the theoretical line upward and leftward from the 45 degree line such that it becomes flatter with a higher intercept. At $\rho = 0.83$, expressed on the annual frequency basis, we draw the theoretical prediction in the second panel of Figure 5. Relative to zero persistence, the intercept of this line rises from 0 to $\rho_{12}^{12} = 0.83^{12} = 0.11$, while the slope flattens from 1 to $1 - \rho_{12}^{12} = 0.89$. Yet, about 76 percent of data points remain above the theoretical line. Thus, persistence in money growth helps a bit, but the model with Calvo pricing remains largely unsuccessful in explaining the persistence with a reasonable choice of money growth process.

Turning to the role of information delay in explaining $\alpha_j$, the persistence estimates based on the AR(4) model become somewhat lower with a median value of 0.51 (with a median standard error of 0.035), but still are much higher than the level predicted by the standard Calvo pricing without information delay (which corresponds to the $\omega = 0$ line shown in the lower panel of Figure 5). Recall that from the left panel of Figure 3, for a fixed value of $\lambda_j$ and $\rho = 0.83$, $\alpha_j$ is strictly increasing in $\omega$. This pattern is preserved in the SAR expressed in annual frequency:

$$\alpha_j = 1 - (1 - \rho_{12}^{12})(1 - \lambda_j^{12})(1 - \omega_{12}^{12})(1 - (\omega \rho)_{12}^{12}).$$

(23)

Notice the perfect symmetry of $\rho$ and $\omega$ in the theoretical SAR expression. In effect, the information delay takes some of the burden-off of the persistence of the money growth rate in accounting for persistence in LOP deviations. In the lower panel of Figure 5, we present a very high information delay case, ($\omega = 0.98$ which corresponds to the 50 month average duration of information updates) and the no delay case ($\omega = 0$), which reduces the model back to the Calvo pricing framework, both with persistence of money growth at $\rho = 0.83$. The line for the very high information delay case has an intercept of 0.82 and a slope of $1 - 0.82 = 0.18$. These are computed from (23).

Taking these two polar cases of information delay as upper and lower limits defines the shaded
triangular region in Figure 5. The regression line through the scatter of empirical estimates falls within this triangle over a substantial fraction of its range. The fitted line has an intercept of 0.44 and a slope is 0.42. These changes relative to the other panels are brought about by estimating the AR(4) model instead of the AR(1) or AR(2) model in the other panels; the frequencies of price changes (the x-coordinates) are the same across the panels.

Turning to the ability of the model to match the median level of persistence, Table 2 reports the ratio of predicted persistence to estimated persistence of the median good for various parameterizations of the theoretical and empirical model.

We see in this Table the symmetry of \(\omega\) and \(\rho\) very clearly. For example, the model predicts the same ratio for parameterization \((\omega, \rho) = (0.90, 0.95)\) as it does for \((\omega, \rho) = (0.95, 0.90)\). Focusing on the persistence consistent with the CKM parameterization, namely \(\rho = 0.83\), an information delay of \(\omega = 0.90\) which corresponds to 10 months of average duration between information updates reconciles the theory with the evidence on median LOP persistence.

Other researchers have alternatively used the persistence of nominal GDP or the persistence of nominal exchange rate changes to parameterize the persistence of the nominal shocks driving the model. We estimate the persistence parameter to be 0.25 and 0.75, using the Canadian-U.S. nominal exchange rate and nominal U.S. GDP, respectively. As is evident from Table 2, information delay would need to increase by only two months, to 12 months to accommodate these alternatives.

The duration between information updates needed to reconcile LOP persistence is broadly consistent with estimates found in existing aggregative studies. Using the aggregate data on inflation over 1960:Q1 - 2007:Q2, DKT find that information delay, on average, is 7.1 months with 95 percent confidence intervals between 5.0 and 16.1 months. Knotek (2006) introduces information stickiness into the fixed menu cost model and finds the average duration between information updates to be 20.4 months over 1983:Q1 - 2005:Q4. Therefore, at least in terms of the median, dual stickiness pricing with a reasonable money growth process is capable of replicating the observed persistence.

5.4 Volatility

The second puzzle brought up in Kehoe and Midrigan (2007) is the observation of too much volatility in good-level real exchange rates which is inexplicable by either a simple sticky price model or a model with pricing complementarities. In this subsection, we evaluate the role of information stickiness in terms of explaining the observed volatility.

We recognize, as other authors have, that the simple monetary model of nominal exchange rates
fails to produce the level of nominal exchange rate variability observed in the data when calibrated to
match the empirics of the monetary aggregates. Accordingly, we focus on the normalized standard
deviation of LOP deviation, as we did in the quantitative theoretical section earlier.

The performance of the model is evaluated by the ratio of the ‘theoretical’ normalized standard
deviation to the ‘observed’ normalized standard deviation. The procedure of computing each stan-
dard deviation is as follows. First, to compute ‘theoretical’ normalized standard deviation, note
that the standard deviation of real exchange rates predicted by the theory has the same implica-
tion to both annually sampled data and monthly sampled data. Therefore, unlike the measure of
persistence that required transformations shown in Table 1, using annual data poses no complica-
tion. For each good, the theoretical normalized standard deviation \( \sigma_j \) can be directly obtained by
substituting \( \lambda_j = 1 - f_j \) into the formula discussed in Section 4.

Second, to compute the ‘observed’ normalized standard deviation, note that using a pooled
sample variance as a volatility measure is not appropriate since it includes the variance component
due to the dispersion of long-run real exchange rate \( q_i^j \) among city pairs in our panel data. In
addition, the theory predicts volatility caused by the nominal exchange rate fluctuation which is
common to all the products, but is not designed to incorporate the idiosyncratic variance component
such as the one due to time-varying city specific shocks. For this reason, we conduct a variance
decomposition based on a standard two-way error components model and focus on the extracted
variance component due to a time specific shock. This decomposition seems to be a reasonable
choice in our study because it is consistent with the idea of using time dummies in the dynamic panel
estimation to incorporate the common time specific shocks in our previous analysis of persistence.
We thus use the observed standard deviation of time specific component normalized by the sample
standard deviation of monthly nominal exchange rate growth.

The first row of Table 3 shows the median of the ratio of the theoretical to observed normalized
standard deviation. The original Kehoe-Midrigan setting with \( \rho = 0 \) can explain only 23 percent of
the variation in the data. Thus, the evidence of excess volatility discovered by Kehoe and Midrigan
(2007) is also confirmed in the EIU panel data of the U.S.-Canadian city pairs. Can we explain this
observed volatility with an introduction of serially correlated money growth? Unfortunately, unlike
the persistence, the predicted volatility is not a monotonically increasing function of \( \rho \). Examples
presented in the left panel of Figure 2 show that the volatility decreases monotonically for goods
with small \( \lambda_j = 1 - f_j \) and increases only in some range of \( \rho \) for goods with a larger \( \lambda_j \). As a
result of the combination of the two effects for many goods, none of the median ratios presented in
the first row of Table 3 is above one, though the maximum value does reach 42 percent at shock persistence in the range of 0.75 to 0.83.

In contrast to the effect of \( \rho \), the left panel of Figure 4 shows that the volatility increases monotonically with \( \omega \) in dual stickiness pricing for any given values of \( \lambda_j \) and \( \rho \). Table 3 also presents the ratio of standard deviations based on dual stickiness pricing with various \( \omega \). With an introduction of the information delay under \( \rho = 0.83 \), the volatility can now be fully explained for the median good when the average duration between information updates is 12 months. When we take \( \rho = 0.75 \), the dual stickiness pricing can account for the volatility with 15 months of information delay. In this sense, the information delay plays an essential role in explaining volatility even though the model produces too little variability of real exchange rates at low values of the shock persistence.

We believe it is more reasonable to expect the model to match the median and cross-sectional patterns of persistence than volatility. As Crucini and Telmer (2007) have shown, deviations from the LOP seem to be largely driven by shocks specific to the good rather than a common shock such as the nominal exchange rate. In fact, they show that in this data, the variance of changes in real exchange rates that are common across goods is only about 7 percent. This means that the bulk of the time series variation we see in the micro-data is due to good-specific shocks, which are abstract from here, and we should aim lower in a model with only an aggregate shock. With this in mind, we delve into the role of heterogeneous information updating using estimated persistence and not volatility.

5.5 Heterogeneous information updating

Up to this point, good-specific parameter heterogeneity has been restricted to the price adjustment parameter, \( \lambda_j \) using the Nakamura and Steinsson estimates while the information updating parameter has been the same for all goods. This asymmetry is more a reflection of the current state of measurement, than reality. The invention of scanner bar codes has made the direct cost of price changes practically zero in recent decades, weakening the literal interpretation of menu cost models as a rationalization for Calvo pricing.

The same technology has facilitated the collection of detailed consumer demand information at most large-scale retail firms and likely complicated the decision theory needed to determine optimal markups. Thus the time dependent information updating process may capture, in a reduced-form sense, asymmetries across goods in this underlying decision problem.
All of our theoretical propositions and dynamic equations are valid when information updating is good-specific, with $\omega$ replaced by $\omega_j$ in the relevant places. In this section, we use the theory to infer good-specific information updates by minimizing the difference between estimated persistence at the level of individual goods and what the theory implies given the frequency of price changes for each good.

We also ask whether the cross-sectional patterns of these estimates are correlated with the distribution share. The distribution share is the difference between what consumers pay in retail markets and what manufactures receive, divided by what consumers pay. By construction it includes transportation costs, wholesale costs, retail costs and markups in the movement of goods from the factory door to the retail floor. We focus on this characteristic because models in which retail prices are jointly determined by the prices of traded and non-traded inputs are gaining popularity in accounting for both the times series and cross-sectional behavior of LOP deviations.

We estimated a good-specific information updating parameter, $\omega_j$ for $j = 1, 2, ..., 165$ by solving the problem:

$$\min_{\omega_j \in [0,1]} [\hat{\alpha}_j - \alpha(\omega_j | \lambda_j, \rho)]^2,$$

where $\hat{\alpha}_j$ denotes the SAR estimate of the AR(4) model and $\alpha(\omega_j | \lambda_j, \rho)$ is the theoretical SAR given by (23) evaluated at $\rho = 0.83$ and $\lambda_j = 1 - f_j$ from the frequency of price changes calculated by Nakamura and Steinsson (2008).

We first ask whether our results are, on the whole, consistent with evidence from micro studies on prices. No micro studies provide directly comparable distribution of information delay among goods, but survey results on price reviews done by firms may serve our purpose. Fabiani, Druant, Hernando, Kwapił, Lau, Loupias, Martins, Matha, Sabbatini, Stahl, and Stokman (2005) argue that the frequency of price reviews rather than price changes “could be related to the arrival of information.” According to Fabiani et. al. (2005), when additional information on the state of the economy infrequently arrives, it is sensible for firms to review prices infrequently. In this sense, we can exploit survey results for price reviews.

We compare the cross-sectional distribution of information delay parameters to Blinder, Canetti, Lebow, and Rudd (1998) survey of U.S. firms about price setting behavior in the beginning of 1990s. They ask firms what the customary interval (e.g., daily, weekly, monthly, quarterly, and yearly) was between price reviews for their most important product. Table 4 compares our distributions of duration of information updates with Blinder et. al. (1998) survey results. Overall, our distributions of duration between information updates seem to match the distribution of price reviews.
Finally, we examine the relationship between information updating infrequency and the distribution share. To accomplish this we first take averages of the $\omega_j$ across $j$ falling into each sector, $s$, for which we have a distribution share parameter. A regression of the infrequency of information updating on the distribution share yields the following:

$$\omega_s = 0.878 + 0.0711 \gamma_s + e_s,$$

where $\gamma_s$ is the distribution share for sector $s$. The 164 goods in our micro-panel fall into 23 unique sectors, with distribution shares ranging from 0.17 to 0.94. An example of an EIU item from the first category is a low-priced automobile and an example from the latter category is an annual premium for automobile insurance.

The coefficient on the distribution share suggests that firms selling retail goods involving larger shares of non-traded inputs update their information sets less often. One interpretation of this correlation is the view that information flows are more frequent for traded inputs where globally centralized spot markets exist than for the inputs used in the distribution sector, which often entail confidential wage and rental contract information.

6 Conclusion

Using highly disaggregated price data from U.S. and Canadian cities, we have confirmed Kehoe and Midrigan’s main finding that the standard Calvo-type sticky price model fails to explain the persistence and volatility of good-level real exchange rates. We found that this puzzling but stimulating result remains robust to the use of the frequency adjusted for temporary sales and consideration of novel panel data collected by the EIU. The robustness of their finding suggests that the baseline model is deficient.

We offer a possible solution to this puzzle by extending the Kehoe-Midrigan model such that only a fraction of firms have the up-to-date information when resetting prices. Due to the infrequent arrival of information, real exchange rates become more persistent and track the volatile nominal exchange rate even if price adjustment is relatively fast. Our model can explain estimated persistence and a sizable fraction of volatility with a plausible common duration of information updating.

Our results are subject to the caveat that we require a departure from the existing rational
expectations approach which presumes continuous information updating and the ability of agents to process the data instantaneously in the sense of knowing the complete stochastic general equilibrium model. Our sense is that decision theory in which some information is released at discrete intervals (e.g., government provision of aggregate data) and managers or analysts vary in their attentiveness is what this time dependent model attempts to emulate. Just as the plausibility of the Calvo framework initially required some notion of how sticky prices were, our analysis begs the question regarding information updating and processing. We see no reason that this process would be less costly or time consuming than menu costs of price adjustment. We hope to provide more concrete evidence on this dimension in future work.\footnote{Examples of models that consider modifications of decision theory along these lines, include Sims (2003), Woodford (2008) and Gorodnichenko (2008).}

We have limited our attention to the implications of our model under many simplifying assumptions. Therefore, there are numerous promising avenues for future research. For example, what would happen to the prediction of our model if pricing complementarities are included? What would be the impact on good-level real exchange rate dynamics if the non-traded inputs in producing a good are included in the model?\footnote{See Crucini, Telmer, and Zachariadis (2005) for this line of research.} We believe that answering these questions would help us further understand the dynamics of price adjustment within and across countries.

References


Figure 1: Persistence without information delay: function of money growth parameter (\(\rho\)) and Calvo parameter (\(\lambda_j\))

![Persistence of AR(2) process](image)

Figure 2: Volatility without information delay: function of money growth parameter (\(\rho\)) and Calvo parameter (\(\lambda_j\))

![Volatility of AR(2) process](image)

**NOTES:** The discount factor \(\beta\) is 0.99.
Figure 3: Persistence with information delay: function of information stickiness parameter ($\omega$) and Calvo parameter ($\lambda_j$)

Figure 4: Volatility with information delay: function of information stickiness parameter ($\omega$) and Calvo parameter ($\lambda_j$)

NOTES: The discount factor $\beta$ is 0.99.
Figure 5: Real exchange rate persistence and price stickiness: Nakamura and Steinsson (2008)
Table 1: Summary of transformations from monthly to annual specification

<table>
<thead>
<tr>
<th></th>
<th>Monthly specification</th>
<th>Annual specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calvo ($\rho=0$)</td>
<td>$\hat{q}<em>t(j,l,l^*) = \lambda_j \hat{q}</em>{t-1}(j,l,l^*) - \lambda_j \eta_t$</td>
<td>$\hat{q}<em>t(j,l,l^*) = \lambda_j^{12} \hat{q}</em>{t-12}(j,l,l^*) - \lambda_j \Lambda_j(L) \eta_t$</td>
</tr>
<tr>
<td>Calvo ($\rho &gt; 0$)</td>
<td>$\hat{q}<em>t(j,l,l^*) = (\lambda_j + \rho) \hat{q}</em>{t-1}(j,l,l^*)$</td>
<td>$\hat{q}<em>t(j,l,l^*) = (\lambda_j^{12} + \rho^{12}) \hat{q}</em>{t-12}(j,l,l^*)$</td>
</tr>
<tr>
<td></td>
<td>$-\lambda_j \rho \hat{q}_{t-2}(j,l,l^*) - \theta_j \eta_t$</td>
<td>$-\lambda_j^{12} \rho^{12} \hat{q}_{t-24}(j,l,l^*) - \lambda_j \Lambda_j(L) R(L) \eta_t$</td>
</tr>
<tr>
<td>Dual stickiness</td>
<td>$\hat{q}<em>t(j,l,l^*) = \sum</em>{r=1}^{4} \phi_{j,r} \hat{q}<em>{t-r}(j,l,l^*) + \sum</em>{r=0}^{2} \theta_{j,r} \eta_{t-r}$</td>
<td>$\hat{q}<em>t(j,l,l^*) = \sum</em>{r=1}^{4} \Phi_{j,r} \hat{q}_{t-12r}(j,l,l^*) + \Theta_j(L) \eta_t$</td>
</tr>
</tbody>
</table>

NOTES: The left panel shows the original monthly ARMA processes which are in the main text. The right panel shows corresponding conversions such that autoregressive coefficients are non-zero only if the lags are multiples of 12 and that moving average terms are finite. These conversions allow us to estimate the original monthly ARMA process with annually sampled data. The autoregressive parameters $\Phi_{j,r}$ and moving average polynomials, $\Lambda_j(L)$, $R(L)$ and $\Theta_j(L)$ are given in Appendix of our working paper.

Table 2: Proportions of explained persistence of good-level real exchange rates: Nakamura and Steinsson (2008)

<table>
<thead>
<tr>
<th>Information delay</th>
<th>$\rho$</th>
<th>0</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>0.83</th>
<th>0.90</th>
<th>0.95</th>
<th>0.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>No delay ($\omega = 0$)</td>
<td></td>
<td>0.48</td>
<td>0.51</td>
<td>0.51</td>
<td>0.57</td>
<td>0.66</td>
<td>0.92</td>
<td>1.23</td>
<td>1.52</td>
</tr>
<tr>
<td>10 months ($\omega = 0.90$)</td>
<td></td>
<td>0.92</td>
<td>0.91</td>
<td>0.91</td>
<td>0.93</td>
<td>1.02</td>
<td>1.20</td>
<td>1.48</td>
<td>1.68</td>
</tr>
<tr>
<td>12 months ($\omega = 0.92$)</td>
<td></td>
<td>0.98</td>
<td>1.00</td>
<td>1.00</td>
<td>1.02</td>
<td>1.11</td>
<td>1.29</td>
<td>1.52</td>
<td>1.70</td>
</tr>
<tr>
<td>15 months ($\omega = 0.93$)</td>
<td></td>
<td>1.09</td>
<td>1.10</td>
<td>1.10</td>
<td>1.14</td>
<td>1.20</td>
<td>1.35</td>
<td>1.59</td>
<td>1.72</td>
</tr>
<tr>
<td>17 months ($\omega = 0.94$)</td>
<td></td>
<td>1.13</td>
<td>1.16</td>
<td>1.16</td>
<td>1.18</td>
<td>1.26</td>
<td>1.42</td>
<td>1.62</td>
<td>1.73</td>
</tr>
<tr>
<td>20 months ($\omega = 0.95$)</td>
<td></td>
<td>1.23</td>
<td>1.21</td>
<td>1.21</td>
<td>1.26</td>
<td>1.33</td>
<td>1.48</td>
<td>1.66</td>
<td>1.74</td>
</tr>
</tbody>
</table>

NOTES: Numbers are median ratios of the theoretical persistence, predicted by Nakamura and Steinsson (2008), to observed persistence measured by the SAR estimated from real exchange rate data. Theoretical persistence for the first row is the SAR for various $\rho$ when Calvo pricing is used. Theoretical persistence from the second to the bottom row is the SAR for various $\rho$ with information delay from 10 to 20 months when dual stickiness pricing is used. Median SAR estimates for AR(1), AR(2) and AR(4) models are 0.563, 0.568, and 0.508, respectively.
Table 3: Proportions of explained volatility of good-level real exchange rates: Nakamura and Steinsson (2008)

<table>
<thead>
<tr>
<th>Information delay</th>
<th>( \rho )</th>
<th>0</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>0.83</th>
<th>0.90</th>
<th>0.95</th>
<th>0.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>No delay (( \omega = 0 ))</td>
<td>( 0.23 )</td>
<td>0.29</td>
<td>0.35</td>
<td>0.42</td>
<td>0.42</td>
<td>0.40</td>
<td>0.31</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>10 months (( \omega = 0.90 ))</td>
<td>0.41</td>
<td>0.51</td>
<td>0.64</td>
<td>0.84</td>
<td>0.88</td>
<td>0.91</td>
<td>0.88</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>12 months (( \omega = 0.92 ))</td>
<td>0.43</td>
<td>0.54</td>
<td>0.69</td>
<td>0.90</td>
<td>1.01</td>
<td>1.03</td>
<td>0.98</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>15 months (( \omega = 0.93 ))</td>
<td>0.46</td>
<td>0.58</td>
<td>0.75</td>
<td>1.02</td>
<td>1.12</td>
<td>1.24</td>
<td>1.19</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>17 months (( \omega = 0.94 ))</td>
<td>0.48</td>
<td>0.61</td>
<td>0.78</td>
<td>1.07</td>
<td>1.21</td>
<td>1.33</td>
<td>1.33</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>20 months (( \omega = 0.95 ))</td>
<td>0.51</td>
<td>0.64</td>
<td>0.83</td>
<td>1.15</td>
<td>1.31</td>
<td>1.46</td>
<td>1.51</td>
<td>1.29</td>
<td></td>
</tr>
</tbody>
</table>

NOTES: Numbers are median ratios of the theoretical volatility, predicted by Nakamura and Steinsson (2008), to observed volatility measured by normalized standard deviation of real exchange rate data. Theoretical volatility for the first row is the normalized standard deviation for various \( \rho \) when Calvo pricing is used. Theoretical volatility from the second to bottom row is the normalized standard deviation for various \( \rho \) with common information delay from 10 to 20 months when dual stickiness pricing is used. The normalized sample standard deviation of real exchange rate is the extracted standard deviation component due to time specific shocks in the two-way error component model.

Table 4: Intervals between information update

<table>
<thead>
<tr>
<th>one month or less</th>
<th>1.01-5.99 months</th>
<th>6-11.99 months</th>
<th>12 months or above</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blinder et. al.'s survey</td>
<td>25.6</td>
<td>13.2</td>
<td>16.5</td>
</tr>
<tr>
<td>Nakamura and Steinsson</td>
<td>33.3</td>
<td>12.7</td>
<td>18.2</td>
</tr>
</tbody>
</table>

NOTES: The numbers in the first row represent the distribution, in percentages, of the frequency of price reviews reported in Blinder, Canetti, Lebow, and Rudd (1998, Table 4.7 in p. 90). The second row shows the distribution of information delay implied by the observed persistence of real exchange rates based on Nakamura and Steinsson’s (2008) data on the frequency of regular price changes is used.