A Politically Feasible Social Security Reform with a Two-Tier Structure

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Abstract

This paper investigates the welfare implications and political feasibility of social security reforms with a two-tier structure. We evaluate social security reforms from two points of view: (i) the ex-ante expected value of future generations, and (ii) whether current generations prefer reform to the status-quo system, which we call political feasibility. To evaluate the reforms, we use a large-scale overlapping generations model with idiosyncratic income risk and a two-tier structure. The first tier guarantees a basic pension and the second tier consists of the earnings-related part. Calibrating the parameters of the model to the Japanese economy, we compute the transition path and the two welfare criteria. We find that, given the two-tier structure, an increase in the basic pension and the abolition of the earnings-related part of the social security system improve the welfare of future generations, and ensures political feasibility when a consumption tax is the source of revenue.


Keywords: Social security reform, Political feasibility, Consumption tax, Capital income tax, Aging.

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1 Introduction

Many developed countries have become aging societies. Faced with aging, governments in such countries have taken social security reforms into account seriously to sustain the system. Many studies propose social security reform options for enhancing the social welfare of future generations. However, it is known that there is a status-quo bias problem in implementing reforms, given the pay-as-you-go system (Conesa and Krueger, 1999). It may not be so difficult to find social security reform options that improve the welfare of future generations, compared to the current system, if current generations willingly pay the reform cost. However, such a plan does not usually receive the political support of current generations, and is thus politically infeasible. Therefore, one difficulty is how to implement social security reforms with the agreement of current generations, who have already made, at least in part, consumption-saving decisions over the lifecycle. In what follows, we address the following question: Are there social security reform plans that improve the welfare of future generations with the political support of current generations?

Given a two-tier structure of the social security system, we show that there exists a social security reform that both satisfies political feasibility, and enhances the welfare of future generations. OECD (2007) separates the role of the social security system into two parts: (a) the insurance (annuity) part and (b) the redistribution part. Concerning the insurance part, it is widely believed that the social security system should be actuarially fair. On the other hand, in the redistribution part, a minimum floor is required for the consumption or redistribution of resources through the social security system. For this reason, in many countries, the social security system has a two-tier structure. The first tier comprises three types of redistribution schemes: basic pension schemes, resource-tested plans, and the minimum pension. How the first tier is constructed differs significantly across the developed countries. For example, in the U.S., the government imposes a resource test for the receipt of a public pension. On the other hand, in Japan all eligible retirees receive the same amount of basic pension. Some countries employ a mixture of the three roles. The second tier comprises two typical forms of social security systems, defined benefit and defined contribution. Although a limit is set on the second tier, it is basically earnings-related. Therefore, considering social security reforms with a two-tier structure is applicable to many countries.

To consider social security reforms, we employ an overlapping generations (OLG) model with idiosyncratic income risk. Our model is based on Conesa and Krueger (1999) and Nishiyama and Smetters (2007), who extend the steady state equilibrium model constructed by Huggett (1996) to the transition dynamics. We extend the model to

\[^1\] Using a partial equilibrium model, Hubbard, Skinner, and Zeldes (1995) investigate the redistributive effects of the social security system, especially the effect of the means test. Using steady state comparison, Huggett and Venture (1999) compare the current U.S. system to a two-tier structure system.
include a two-tier structure. There are infinitely many households who face idiosyncratic income shocks, and the government manages the social security system as a pay-as-you-go system. We calibrate the parameters of the model for the Japanese economy and calculate the steady state and the transitional path. We choose the Japanese economy as a target for the following two reasons: First, the Japanese economy employs a two-tier structure: the basic pension and the earnings-related part. Our results in this paper, however, are broadly applicable to any other countries, since many countries have a two-tier structure, as stated above. Second, Japan is one of the most rapidly aging countries in the world. A population projection indicates that the percentage of retired households will exceed 40% by 2055. Therefore, the Japanese economy provides a good example by which to consider social security reforms.

To consider social security reform, we basically propose three reform options, and a mixture of these options: (1) the abolition of social security reform, (2) introducing a consumption tax as a source of revenue for the basic pension, (3) introducing a capital income tax as a source of revenue for the basic pension. Neither the abolition nor the privatization of the social security system is a new idea. For example, Storesletten et al. (1998) consider alternative social security arrangements, including abolition using an OLG model calibrated to the current U.S. system by steady state comparison. Conesa and Krueger (1999) and Nishiyama and Smetters (2007) extend their research to investigate the transition path of privatization. Conesa and Krueger (1999) conclude that privatization improves the welfare of future generations, although it is not supported by hypothetical voting. Because our model includes a two-tier structure, we need to reconsider whether these results apply in our setup. In addition, we consider the choice of sources of revenue to sustain the social security system. In many countries, pensioners (retirees) do not pay social security contributions. One reason why aging reduces the sustainability of the social security system is that, in many countries, social security contributions are based on a payroll tax. Recently, many researchers focus on tax reform by using the consumption tax for social security reforms (e.g., Kotlikoff et al., 1999). A consumption tax may be a good option for improving the welfare of transition generations because it collects revenue from current rich retirees. We add a third plan, capital income tax, as an option that has the potential to improve the welfare, because it too collects revenue from rich retirees.

Many theoretical studies have been conducted on social security reforms using OLG models. Moreover, because research on social security reform requires numerical val-

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2Japan is a country that has a modest public pension system in OECD countries. Japan has a three-tier structure of the social security system: the first tier is a flat-rate basic pension (Kokumin-Nenkin; KN), the second tier is a mandatory defined benefit, i.e., earnings-related (Kosei-Nenkin-Hoken; KHN), and the third floor is private pensions, such as 401k plans. The contribution is based on a flat payroll tax on the monthly standard earnings for an employee. Half the KN benefits are financed by government revenue. For details and a historical view of the social security system in Japan, see Takayama (2003).
ues such as tax rates, quantitative studies of the social security system have attracted attention since the pioneering research by Auerbach and Kotlikoff (1987). Recent research focuses on social security reform in an economy with heterogeneous agents, due to its redistribution effect. For example, İmrohoroğlu et al. (1995), Storesletten et al. (1999), and Huggett and Ventura (1999) investigate an efficient social security system using steady state comparison. Huang et al. (1997), De Nardi et al. (1999), Conesa and Krueger (1999), and Nishiyama and Smetters (2007) consider the transitional dynamics of social security reforms. Our research is an extension of the result by Conesa and Krueger (1999). They show that, given pay-as-you-go systems, there arises the status-quo bias problem for social security reform. Conesa and Garriga (2003) discuss this point, and show that the elimination of compulsory retirement can offset the status-quo bias problem. Büttler (2000) also investigates the political feasibility of several reform options by calibrating the model to the Swiss economy. Using an extension of the Auerbach and Kotlikoff (1987) model, Okamoto et al. (2009) consider the effect of demographic change on welfare and the growth rate in Japan. Their research is more specific to the Japanese economy, and focuses on the actual public pension system in Japan. Compared to their research, we include idiosyncratic income risks, which imply substantial intragenerational income and wealth inequality. Okamoto et al. (2009) conclude that a progressive consumption tax improves the welfare of the economy.

Our conclusion is as follows. Although abolition, or financing choice of the basic pension alone, improves the welfare of future generations, it is not supported by current generations. We find that (i) abolition of the earnings-related part (second tier) and (ii) introducing a consumption tax to finance the basic pension improves the welfare of future generations. The abolition of the earnings-related part improves welfare because the real return from second tier is dominated by the equilibrium interest rate. Moreover, financing the basic pension by consumption tax improves welfare measured in ex-ante value, due to the insurance effect. In addition to these, (iii) expanding the basic pension to guarantee the current middle and old poor households receives political support. Therefore, a combination of these options provides a politically feasible social security reform that improves future generations’ welfare.

This paper is structured as follows. Section 2 provides the details of our model. Section 3 calibrates the parameters of the model for the Japanese economy. Section 4 discusses policy experiment plans, and compares the steady states of the model. Section 5 considers the transitional dynamics, and the political feasibility of several reforms. Finally, Section 6 concludes the paper.
2 Overlapping Generations Model

2.1 Demographic Structure

We consider an overlapping generations model with a continuum of households. In the model, time is discrete. Households have a lifespan of, at most, $J$ periods, but they face mortality risks. The number of households aged $j$ in period $t$ is denoted by $\mu_{j,t}$. A fraction of households $(1 - \phi_{j,t})$ exits the economy owing to death, and $\mu_{j+1,t+1} = \phi_{j,t} \mu_{j,t}$ is the population of households aged $j+1$ at period $t+1$. Let $\mu_t = (\mu_{0,t}, \ldots, \mu_{J,t})'$ denotes the population distribution in period $t$. The population growth rate of age 0 from $t$ to $t + 1$ is represented by $\psi_t$.

2.2 Households

A household born in period $t$ supplies labor inelastically until age $j_r$, and faces idiosyncratic risk with respect to its individual labor endowment. Since households of age $j \in \{0, \ldots, j_r - 1\}$ are workers, they supply the labor endowment. Thereafter, i.e., $j \in \{j_r, \ldots, J\}$, households retire and receive social security benefit from the government.

Age-Efficiency Profile and Idiosyncratic Income Risk Households differ in their labor productivity due to the deterministic age-efficiency profile. Labor efficiency grows when households are young and peaks in middle age, around 50: in other words, the efficiency of a household becomes an inverse U-shape over its working life. We denote the deterministic age-efficiency as $\{\kappa_j\}_{j=0}^{j_r-1}$.

In addition to the deterministic age-efficiency profile, all workers face idiosyncratic risk on labor endowment $z_j$. We assume that the logarithm of labor income of household $i$ at age $j$, $y^i_j$, is specified as follows:

$$\ln y^i_j = \ln \kappa_j + z^i_j, \quad z^i_j = \rho z^i_{j-1} + \eta^i_j, \quad \eta^i_j \sim N(0, \sigma_{\eta,j}^2),$$

where the labor income shock is represented by $\eta^i_j$, and the persistence of the shock is characterized by $\rho$. We assume that the income shock is persistent, and it is independent across households. We assume that the age-efficiency profile $\{\kappa_j\}$ and the stochastic process parameters $(\rho, \sigma_{\eta,j}^2)$ are independent of time $t$.

\footnote{For details, see Appendix A.}

\footnote{We omit the labor supply decision for computational reasons. İmrohoroglu and Kitao (2009) find that the labor supply elasticity does not affect the welfare implications of social security reforms, although it may affect the profile of hours worked.}
Social Security System  In the benchmark case, the government provides social security benefits to retired households through a flat payroll tax on workers. The flat payroll tax rate is denoted as $\tau_{ss}$. In addition, for numerical experiment purposes, we define the consumption tax rate and the capital income tax rate as $\tau_{con}$ and $\tau_{cap}$, respectively. After retirement, a household receives a social security benefit based on a two-tier structure. The first tier provides a constant basic pension $\varphi_{1st}^i$, which is independent of households’ past earning or asset holding level. As a second tier, households receive an earnings-related benefit, which depends on the past labor income of a household. Because of the existence of idiosyncratic income risk, households have different past earnings at $j$. We record the history of the earnings of a household $i$ by the following equation:

$$b_j^i = \begin{cases} 
(j-1)b_{j-1}^i + y_j^i & \text{if } b_j^i \in [b_{min}^i, b_{max}^i] \\
b_{min}^i & \text{if } b_j^i \leq b_{min}^i \\
b_{max}^i & \text{if } b_j^i \geq b_{max}^i 
\end{cases}$$

We call $b_j^i$ monthly standard earnings (MSE). We assume that MSE has an upper and lower bound, i.e., ceilings on pensionable earnings. Thus, the earnings-related part is denoted as $\varphi_{2nd}^j$, where $\varphi_{2nd}^j$ is the accrual rate of the second tier. Both $\varphi_{1st}^i$ and $\varphi_{2nd}^j$ are potentially time-dependent, because these are policy variables. Since the social security benefit is adjusted by the macroeconomic wage level, the total benefit that household $i$ can receive is $w_t(\varphi_{1st}^i + \varphi_{2nd}^j b_j^i)$. We use the macroeconomic wage level as revaluation.

Note that the social security system in our model has redistribution effects because we assume that the social security benefit consists of the constant basic pension and the earnings-related pension with floors $(b_{min}^i, b_{max}^i)$. Although the social security system differs significantly across countries, it generally contains large redistribution mechanisms. The earnings-related part of the social security system has a relatively small impact on the redistribution. On the other hand, the redistribution part has a strong redistributive mechanism.  

A household has some asset holdings $a_{j,t} \geq 0$ at age $j$ and in period $t$. The Bellman equation for a household is defined as follows:

$$V_{j,t}(a, b, z) = \max_{c \geq 0, a \geq 0} \left\{ \frac{c_{j,t}^{1-\gamma}}{1-\gamma} + \phi_{j,t} \beta \sum_{z'} \pi(z'|z) V_{j+1,t+1}(a', b', z') \right\}$$

subject to

$$\begin{align*}
(1 + \tau_{con}^i) c_{j,t} + a_{j+1,t+1} &\leq (1 - (1 - \tau_{cap}^t)) r_t (a_{j,t} + \xi_t) + (1 - \tau_{ss}^i) w_t y_{j,t}, & : \text{Worker} \\
(1 + \tau_{con}^i) c_{j,t} + a_{j+1,t+1} &\leq (1 + (1 - \tau_{cap}^t)) r_t (a_{j,t} + \xi_t) + w_t (\varphi_{1st}^i + \varphi_{2nd}^j b_j^i), & : \text{Retiree}
\end{align*}$$

Imrohoroglu et al. (1995) and Conesa and Krueger (1999) consider an efficient social security system with a constant social security benefit. Storesletten et al. (1999) and Huggett and Ventura (1999) investigate the redistributive effects of the social security system in the U.S. using steady state comparison.
where \( \beta \) is a discount factor, \( r_t \) is the net interest rate at \( t \), \( \xi_t \) is a lump-sum transfer of the accidental bequest explained below, and the primes denote the next period’s variables. We assume that households face a liquidity constraint, i.e., \( a_{j+1,t+1} \geq 0 \).

### 2.3 Production Technology

The aggregate production technology follows a Cobb-Douglas production function,

\[
Y_t = AK_t^\alpha L_t^{1-\alpha},
\]

where \( A \) denotes total factor productivity (TFP), \( K_t \) is aggregate capital, and \( L_t \) is aggregate labor, measured in efficiency units.

The asset holdings and labor income of each household differ even within the same cohort and age group, due to idiosyncratic income risks. We denote a probability measure of households aged \( j \) with asset \( a \), the MSE \( b \), and endowment \( z \) as \( \Phi_{j,t}(a,b,z) \).\(^6\) By construction, \( \int d\Phi_{t,j}(a,b,z) = 1 \) for each \( j \) and \( t \). Aggregate capital and labor are determined by the sums of each generation’s assets and labor:

\[
K_t = \sum_{j=0}^{J} \mu_{j,t} \int a_{j,t} d\Phi_{j,t}(a,b,z), \tag{3}
\]

\[
L_t = \sum_{j=0}^{j_{t-1}} \mu_{j,t} \int y_{j,t} d\Phi_{j,t}(a,b,z). \tag{4}
\]

### 2.4 The Government

We assume that the government collects tax to finance social security benefits, and redistributes it to retired households based on the two-tier structure. We do not consider other government expenditures.\(^7\) The social security system is governed by the Pay-As-You-Go system. Accordingly, the government collects payments from workers and retirees through taxes \( (\tau_{ss}^t, \tau_{con}^t, \tau_{cap}^t) \), and grants social security benefits. In the benchmark case, we assume that the benefit is financed by a payroll tax, as in the Japanese economy, and that the equilibrium payroll tax rate \( \tau_{ss}^t \) is endogenously determined. In the numerical experiments, we consider consumption and capital income tax as the tax bases. We denote aggregate social security payments by the payroll tax as \( T_{lss}^t \), by the consumption tax as \( T_{lcon}^t \), by the capital income tax as \( T_{lcap}^t \), and aggregate social

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\(^6\)For details, see Appendix B.

\(^7\)It is possible that the large government debt in Japan may hinder social security reforms. We omit this effect for computational reasons.
security benefit as $B^1_{t}$ and $B^2_{t}$ for each tier.

$T_t^{SS} = \sum_{j=0}^{j_r-1} \mu_{j,t} \int \tau_t^{ss} w_t y_{j,t} \Phi_j(a, b, z) = \tau_t^{ss} w_t L_t,$

$T_t^{CON} = \sum_{j=0}^{j} \sum_{j=0}^{j_r} \mu_{j,t} \int \tau_t^{con} c_{j,t}(a, b, z) \Phi_j(a, b, z) = \tau_t^{con} C_t,$

$T_t^{CAP} = \sum_{j=0}^{j} \sum_{j=0}^{j_r} \mu_{j,t} \int \tau_t^{cap} r_t \Phi_j(a, b, z) = \tau_t^{cap} r_t K_t,$

$B^1_{t} = \sum_{j=0}^{j} \mu_{j,t} w_t \phi^1_{t}, \quad B^2_{t} = \sum_{j=0}^{j_r} \mu_{j,t} \int w_t \phi^2_{t} \Phi_j(a, b, z).$

Note that the benefit of the second tier depends on the distribution of the MSE. In the benchmark, the government must satisfy the following budget constraints:

$B^1_{t} + B^2_{t} = T_t^{SS}. \quad (5)$

The accidental bequest due to survival risk is collected by the government through tax, and is redistributed in a lump-sum manner. The lump-sum transfer of accidental bequests is determined by the following equation:

$\xi_t = \sum_{j=1}^{J} \mu_{j-1,t-1} (1 - \phi_{j-1,t-1}) a_{j,t}.$

### 2.5 Definition of a Competitive Equilibrium

Our concern is with both the steady state and the transitional dynamics of the economy. Therefore, we need two definitions of equilibrium.

**Definition 1 (Recursive Competitive Equilibrium)** Given the government’s policy $\{\varphi^1_t, \varphi^2_t\}$ and the population dynamics, the Recursive Competitive Equilibrium is a set of value functions $\{V_{j,t}\}$, policy functions $\{g_{j,t}\}$, aggregate capital $\{K_t\}$, aggregate labor $\{L_t\}$, accidental bequest $\{\xi_t\}$, factor prices $\{r_t, w_t\}$, and social security taxes $\{\tau_t^{ss}, \tau_t^{con}, \tau_t^{cap}\}$ that satisfy the following conditions:

(i) A Household’s Optimality: Given the factor prices $\{r_t, w_t\}$ and the social security taxes $\{\tau_t^{ss}, \tau_t^{con}, \tau_t^{cap}\}$, the value function $\{V_{j,t}\}$ solves equation (2), and $\{g_{j,t}\}$ is the associated policy function. The value and policy functions are measurable.

(ii) A Firm’s Optimality: Factor prices are competitively determined as follows:

$r_t = \alpha A_t (K_t/L_t)^{\alpha-1} - \delta, \quad w_t = (1 - \alpha) A_t (K_t/L_t)^{\alpha},$

where $\delta$ is the depreciation rate.
(iii) Market Clearing: The market clearing conditions of equations (3) and (4) are satisfied. From Walras’ law, the goods market also clears.
(iv) the Government’s Budget: The government’s budget (5) clears.
(v) Transition Law of Motion: $\Phi_{t+1} = T(\Phi_t)$.

Definition 2 (Steady Recursive Competitive Equilibrium) The Steady Recursive Competitive Equilibrium is a recursive competitive equilibrium with a stationarity of distribution $\Phi_{j,t+1} = \Phi_{j,t}$ ($\forall t$) for each age $j$.

Given the definition, we compute the transition path following the computational method proposed by Conesa and Krueger (1999), who compute two steady states and their transition paths. Thus, to compute the transition path in our model, we need to calibrate the initial and final steady states.

3 Calibration

3.1 Preference and Technology

We set the initial steady state of the model in the year 2009 in Japan and the final state in the year 2200. A social security reform starts in 2010, if it occurs. We assume that households do not anticipate the reform.

Households enter the economy at age 20, work until 65, and live at the most to 100. As defined in equation (2), we assume that the utility function is of the CRRA type. The parameter for the relative risk aversion is set as $\gamma = 2$. This value is standard in the macroeconomics literature. Abe and Yamada (2009) estimate the preference parameters in Japan by structural estimation using the National Survey of Family Income and Expenditure conducted by the Japanese government, and determine that the risk aversion parameter ranges from 1 to 5. In the model, the discount factor $\beta$ is set at 0.97 to match the equilibrium interest rate of 4%, which is the average return on capital in the 1990s in Japan, as estimated by Hayashi and Prescott (2002).

Finally, we choose the parameters for the production function: capital share and depreciation rate. These parameters are taken from Hayashi and Prescott (2002). The capital share parameter $\alpha$ is fixed at 0.362. The depreciation rate $\delta$ is specified at 0.083. These values are the average of the 1990s in Japan. The TFP level $A$ is determined to normalize the initial period’s equilibrium wage to be one. All the calibrated parameters are summarized in Table 1.

3.2 Labor Income

Idiosyncratic Income Risk  We set the standard deviation $\sigma_{\eta}$ and the persistence parameter $p$ of idiosyncratic income risk to match the actual cross-sectional variance of income in Japan (Figure 1). Ohtake and Saito (1998) show that the logarithm
of the variance of income in Japan increases across age groups. They also show that the shape of the age-variance profile is convex over age groups. To account for the convexity of the variance profile, Abe and Yamada (2009) specify the stochastic labor income process and estimate the parameters. The Data in Figure 1 plot the estimated data used in Abe and Yamada (2009). To incorporate the nonlinearity of the income variances, we use an age-dependent income shock.

We assume that the idiosyncratic labor endowment process follows equation (1). Abe and Yamada (2009) report on the possibility of \( \rho \geq 1 \), because of the convexity of the variance profile. However, incorporating \( \rho \geq 1 \) makes the numerical computation far more difficult, and also difficult to interpret. Thus, we choose the persistence parameter to be close to one, i.e., \( \rho = 0.95 \). In addition, to replicate the convex profile, we assume that the standard deviation of the persistence shock \( \sigma_{\eta_j} \) increases by age, as follows:

\[
\sigma_{\eta_{j+1}} = (1 + \lambda)\sigma_{\eta_j}, \quad \text{if } j \geq \text{Turning Age},
\]

\[
\sigma_{\eta_{j+1}} = \sigma_{\eta_j}, \quad \text{else.}
\]

Assuming the initial value of the labor endowment \( z_0 \) to be zero, we set the initial standard deviation \( \sigma_{\eta_0} = 0.11 \), the growth rate of the standard deviation \( \lambda \) to be 0.2, and the turning age is set at 47. After the specification, we approximate the persistent shock process as a seven-state Markov chain using Tauchen’s (1986) method. As shown in the Simulation in Figure 1, our calibration replicates Japanese earning inequality well.

**Age-Efficiency Profile** The efficiency of workers at each age \( \{\kappa_j\} \) determines the average earning profile. We conduct the calculation following the method proposed by Hansen (1993). Following Braun et al. (2007), we use the Basic Survey in Wage Structure by the Ministry of Health, Labour, and Welfare in Japan to calculate the age-efficiency profile. We compute the efficiency wage of male workers, including part-time workers. We omit female workers because the earnings-related part is based on each individual’s earnings. Since we do not distinguish between single and married households explicitly, to make the calculation of the social security benefit easier, we consider a married couple with a single earner as a representative household. Table 2 reports the age-efficiency for workers in five-year age groups.

### 3.3 Social Security System with a Two-tier Structure

Social security benefit consists of two tiers. In the first tier, the basic pension is paid to all households in the same amount. An individual receives 66,000 yen a month per capita, and a married couple can receives twice as much, i.e., 132,000 yen a month. In addition, as a second tier, households are granted an earnings-related public pension based on the MSE. According to OECD (2007), the basic pension (first) is 16% of average earnings of workers, i.e., \( \varphi_1^{\text{1st}} = 0.16 \). The second tier is a defined benefit with an accrual rate of
0.55% per year. Thus, we set $\varphi_{t}^{2nd} = 0.22$ (accrual rate of 0.55% per year × 40 year). The benefit is adjusted by wage and price level (valorization). There is a ceilings, and the pension eligibility age in Japan is 65.8

We assume that there are upper and lower bounds on the MSE, i.e., ceilings on pensionable earnings. The lower and upper bounds are calculated as follows: For each month, the government records a household’s monthly earnings. There are upper and lower bounds on the MSE, and the earnings of households that earn above and below the bound are replaced by the upper and lower bound, respectively. The upper bound per month is 600,000 yen and the lower bound is 98,000 yen. Bonus earnings, which are typically paid twice per year, are included in the account, to a maximum of 1,500,000 yen. Therefore, the upper bound of recorded earnings per year is 600,000 yen × 12 months + 1,500,000 yen. The lower bound is 98,000 yen × 12 months. Because the average yearly earnings are about 4,500,000 yen, we set the upper and lower bounds as [0.25, 2.0]. Therefore, the MSE records, at most, twice average earnings. OECD (2007) also reports that $b^{\max} = 2.0$.

3.4 Demographic Structure

We set the demographic parameters to replicate the actual and projected population dynamics. The National Institute of Population and Social Security Research (NIPSSR) provides population projections for each five years, and recent projections are available for the period 2005-2055.9 The NIPSSR provides three alternate variants of fertility and mortality rates, and thus we have nine projections. We set the survival probability $\{\varphi_{j, t}\}_{t=2005}^{2055}$ from the medium variant of the value estimated by the NIPSSR. The fertility rate $\psi_{t}$ is also taken from the medium variant of the projections. Because population growth in our model is represented by the growth rate of newborns, we use the ratio of the projected population of newborns between periods $t$ and $t+1$.

As we need to compute two steady states and the transition paths, we set the initial steady state population distribution in the year 2009 (Figure 2). After the population changes from 2009 to 2055, the population growth rate is assumed to converge to zero over the 10 years between 2055 and 2064. Although the population growth rate of newborns converges immediately, it takes approximately 100 years to reach a new stationary population distribution.

One problem that arises here is how to choose an initial population distribution in the initial steady state. The actual population distribution in 2009 does not seem to be steady state because of the existence of the baby boomer generation.10 However,

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8See Table 1.2 in OECD (2007).
9For details, see Kaneko et al. (2008).
10The population distribution in 2005 is obtained from Population Census by the Ministry of Internal Affairs and Communications in Japan, and the distribution in 2009 is calculated by the mortality and
to compute the initial steady state, a population distribution is required. Therefore, we assume that households in the model believe that the actual population in 2009 is stationary.

4 Efficient Social Security System with a Two-tier Structure: A Steady State Comparison

Before investigating the complicated Japanese economy with aging, we consider a simple demographic structure that focuses on the pure redistributive effect of social security reforms. The population distribution is constant over time, and the population distribution is smooth plotted as Non-Aging, in Figure 2.

4.1 Numerical Experiment Plans and Welfare Evaluation

**Numerical Experiment Plans** In this section, as a first step, we consider an efficient source of revenue choice from a steady state comparison. As another source of revenue for the social security benefit, we consider a consumption and capital income tax. The consumption tax and the capital income tax are used as the source that finances the basic pension, i.e., first tier. Moreover, we calculate the welfare gains, as defined below, of several pairs of \((\varphi^{1st}, \varphi^{2nd})\) to establish the efficient two-tier structure.

**Ex-Ante Value** To evaluate the welfare of households from alternative social security reforms, following Conesa et al. (2008), we employ the ex-ante expected value as follows:

\[
EV_t = \sum \pi(z_0) V_{0,t}(0, b, z_0).
\]

This welfare criterion implies that we use a measure of the expected value of households that enter the economy in period \(t\) at age 20. In other words, it is the lifetime discounted value of each cohort before entering the economy. By assumption, households have no wealth at age 20. In addition, we calculate the following consumption equivalent variation (CEQ) measure

\[
CEQ_t = \left( \frac{EV_t^{Reform}}{EV_t^{Benchmark}} \right)^{1-\gamma} - 1,
\]

to compare the consumption equivalent variation of cohorts between the benchmark \(EV_t^{Benchmark}\) and a social security reform \(EV_t^{Reform}\).

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4.2 Steady State Comparison

4.2.1 Source of Revenue

Table 3 summarizes the macroeconomic statistics in the benchmark case and two social security reform plans, when the population growth rate is zero. When the social security system is financed by a payroll tax, the equilibrium tax rate is 14.5%, and the equilibrium interest rate is 4.1%, which is our calibration target. In the left columns of Table 3 (With Idiosyncratic Risk), compared to the case of a payroll tax, introducing the consumption tax implies capital deepening and, as a result, the interest rate declines by 0.4%. As households need to pay the social security contribution even when retired, old households have more wealth for retirement. On the other hand, a capital income tax provides a disincentive to accumulate wealth, and the interest rate becomes higher. The introduction of a consumption tax for the basic pension requires that tax rate to be set at 5.1%. In the case of the capital income tax, the equilibrium tax rate is about 30%.

In both cases, changing the tax base for the basic pension from earnings (payroll tax) to consumption or capital income improves welfare, as measured by equation (6). Introducing a consumption tax increases the CEQ by 2.16%, as shown in Table 3, which is higher than that of the capital income tax, which is 1.29%. When the revenue to finance the basic pension comes from the payroll tax, households face a more severe liquidity constraint, and the saving decision is distorted. In addition, because households face idiosyncratic income risks, based on the ex-ante welfare criterion, the basic pension has an insurance effect on lifetime income. The insurance effect is also larger in the case of consumption and capital income taxes than with the payroll tax, because the former taxes are also collected from retirees, who are much more unequal than young households.

Does the source of revenue choice depend on the idiosyncratic income risk? To discuss this point, we remove the idiosyncratic income risk, leaving the other parameters fixed. The right columns in Table 3 summarize the results. When households face no income risks, they accumulate less wealth, and the equilibrium interest rate becomes 0.8% higher than in the case with idiosyncratic risk. This is due to the precautionary saving motive. In an economy with no idiosyncratic risks, the welfare gain of introducing a consumption tax is a little higher (2.31%) than in the benchmark case. In other words, when the idiosyncratic income risk is omitted, we overestimate the effect of introducing the consumption tax. When the idiosyncratic income risks are removed, the average consumption profile becomes flatter, and the liquidity constraint becomes less severe. In contrast, the average asset profile becomes steeper, because precautionary savings do not exist. Thus, a capital income tax is likely to be viewed more favorably when households have strong asset accumulation motives.

To summarize, a consumption tax, or a capital income tax for financing the basic
pension, improves the welfare of the economy from a steady state comparison. Although future households improve their ex-ante welfare by such reforms, the current generation, who have already worked, accumulated wealth, and paid tax, may face a welfare loss from such reforms, because these households need to pay an additional contribution through the consumption tax or the capital income tax. In particular, even though very young households may agree to the reforms, old households may disagree, given the entirely additional nature of these burdens on them. Thus, implementing such reforms becomes a political issue.

4.2.2 Optimal Basic Pension and Accrual Rate

Before trying to establish a politically feasible social security reform, we discuss efficient social security reform with a two-tier structure. Because the basic pension and the earnings-related part have different macroeconomic and welfare implications, we need to investigate the optimal benefit plans separately. Table 4 calculates the CEQ for the first tier (second tier), \( \varphi_{1st} (\varphi_{2nd}) \), given the second tier (first tier) structure as fixed. Thus, the positive CEQ in Table 4 implies that households improve welfare compared to the benchmark case.

As in the previous literature, such as İmrohoroğlu et al. (1995), the optimal replacement rate is zero in the basic pension. Although we find that the optimal benefit plan is the abolition of the social security system, the effects of the first and second tiers differ significantly. When we reduce the benefit coefficients \( \varphi_{1st} \) and \( \varphi_{2nd} \), the earnings-related part improves welfare much more than the counter part of the basic pension. The basic pension has large redistribution effects, and thus offers insurance on the lifetime income risks in addition to insurance on long-living. On the contrary, whether the earnings-related part improves welfare depends on the real return on contributions. Because the accrual rate from our calibration is 0.55% annually, private saving dominates the earnings-related part. Thus, the abolition of the second-tier improves the welfare in equilibrium.

To focus on the insurance effect of the basic pension, we compute the CEQ without idiosyncratic income risk. In Table 4, we find that the CEQ value of zero basic pension in an economy without idiosyncratic risk is larger than that in the case with idiosyncratic risk. This means that the basic pension actually has an insurance role for income risks in our model. In addition, an interesting point is that the expansion of the basic pension causes rising wealth inequality if idiosyncratic income risks exist. The basic pension reduces wealth inequality when households face no idiosyncratic risks, and the extended earnings-related part also implies declining wealth inequality. Due to the redistribution effect, the introduction of the basic pension implies more wealth accumulation for rich households, and poor households need less wealth in an economy with idiosyncratic income risks.
5 Transitional Dynamics and Political Feasibility

5.1 Numerical Experiment Plans

Using a steady state comparison, we showed that the most efficient social security reform is the abolition of the system: the second tier in particular reduces households’ welfare. In addition, changing the tax base of the basic pension, from a payroll tax to a consumption or capital income tax, may improve welfare in an ex-ante sense. As a second step, we compute the equilibrium transition paths of the following scenarios:

1. Abolition of both the basic pension and the earnings-related part (Plan 1)

2. Source of revenue choice

   (a) The basic pension is financed by a consumption tax (Plan 2)
   \[ B_{t}^{1st} = T_{t}^{CON}, \ B_{t}^{2nd} = T_{t}^{SS}. \]

   (b) The basic pension is financed by a capital income tax (Plan 3)
   \[ B_{t}^{1st} = T_{t}^{CAP}, \ B_{t}^{2nd} = T_{t}^{SS}. \]

3. Source of revenue choice and the abolition of the earnings-related part

   (a) The basic pension is financed by a consumption tax and the abolition of the earnings-related part (Plan 4)
   
   (b) The basic pension is financed by a capital income tax and the abolition of the earnings-related part (Plan 5)

4. Source of revenue choice, abolition of the earnings-related part, and expanded basic pension

   (a) The basic pension is financed by a consumption tax, and the accrual rate \( \varphi^{2nd} \) is replaced by the basic pension \( \varphi^{1st} \) (Plan 6)
   
   (b) The basic pension is financed by a capital income tax, and the accrual rate \( \varphi^{2nd} \) is replaced by the basic pension \( \varphi^{1st} \) (Plan 7)
   
   (c) The basic pension is financed by a payroll tax, and the accrual rate \( \varphi^{2nd} \) is replaced by the basic pension \( \varphi^{1st} \) (Plan 8)

We summarize the differences in Table 5. As the efficient social security reform is abolition, we consider it in the transition path in Plan (1).\(^{11}\) In this plan, both the basic pension and the earnings-related part are abolished. We assume that these

\(^{11}\)Storesletten et al. (1999) discuss the abolition of the social security system from steady state analysis.
plans are executed either suddenly in 2010, or gradually for 50 years (Plan 1'). Plans 2 and 3 consider the source of revenue choice for the basic pension. Based on Section 4, these plans will improve the welfare of future generations. The point we focus on is how much such reforms will reduce the current generations’ welfare. These reforms too are implemented in 2010. As the earnings-related part reduces households’ welfare, we consider the abolition of the second tier with the source of revenue choice in Plans (4) and (5).

In Plans (6) to (8), we will discuss how social security reforms can improve welfare and be politically feasible. As we will show later, Plans (1) to (5) improve the welfare of young and future generations. However, as expected, the current middle and old generations do not support such reforms. Thus, to obtain the agreement of the middle and old generations to these reforms, we propose to expand the basic pension $\varphi^{1st}$ and to abolish the second tier, i.e., $\varphi^{2nd} = 0$. We consider three cases: the total coefficient $\varphi_t = \varphi^{1st} + \varphi^{2nd}$ is fixed at 0.38, or increased to 0.45 and 0.5. Note that, if the total coefficient $\varphi_t$ is fixed at the same level, the average pension benefit that households receive is unchanged. Because of the idiosyncratic income risk, poor households who have relatively low MSE may support such reforms. Therefore, based on the two-tier structure, it may be possible to reform the social security system in a way that improves the welfare of both future generations and the current older generations.

5.2 Political Feasibility Criterion

In addition to the ex-ante welfare criterion in equation (6), we introduce the concept of the political feasibility of the existing generations, as in Conesa and Krueger (1999).\footnote{Conesa and Krueger (1999) call the criterion “voting”. This may be inaccurate, because, in general, voting is dynamic, and computationally very difficult in these environments. We call this criterion simply the “agreement rate” or political feasibility. Recently, Corbae et al. (2009) constructed a politico-economic model with idiosyncratic risks.} Suppose that a household weakly prefers to reform the social security system, i.e., $V_{j,2009}^{\text{Reform}}(a,b,z) \geq V_{j,2009}^{\text{Bench}}(a,b,z)$. Then the household with state $(a,b,z)$ of age $j$ votes in agreement with the reform. Then, the total agreement rate of the hypothetical voting by age $j$ is determined as follows:

$$AR_j = \int I(a,b,z,j)d\Phi_{j,2009}(a,b,z),$$

where $I(a,b,z,j)$ is an indicator function defined as follows:

$$I(a,b,z,j) = \begin{cases} 
1, & \text{if } V_{j,2009}^{\text{Reform}}(a,b,z) \geq V_{j,2009}^{\text{Bench}}(a,b,z), \\
0, & \text{else.}
\end{cases}$$

We implicitly assume that the reform occurs once and that the government can commit to it.
5.3 Equilibrium Path and Ex-Ante Welfare

In general, transitional dynamics describes very complicated paths because demographics do not change monotonically. To focus on the social security reforms, we again consider the case of non-aging population distribution: the population distribution is constant over time.

Figure 3 shows the transitional dynamics of the equilibrium interest rate. The economy is in a steady state in the year 2009, and a social security reform occurs in the year 2010. In the equilibrium path, the interest rate declines when the government abolishes the social security system (Plan 1), which is not surprising because households need to accumulate more wealth for their retirement. The interest rate declines when the basic pension part is financed by a consumption tax (Plan 2). In contrast, introducing a capital income tax increases the interest rate, because the capital income tax offers a disincentive to accumulate assets (Plan 3). When the abolition of the second tier is combined with the financing choice, the interest rate falls, even in the case of a capital income tax (Plan 5). Although the social security reform occurs in 2010, the convergence to a new steady state takes more than 30 years. The long period to converge implies that the amount of current generations who receive a benefit from the social security reform is small, even if households will obtain welfare gains in the future. Thus, as Conesa and Garriga (2003) discuss, it is difficult to implement such a reform politically when convergence is very slow.

In Figure 4, we plot the ex-ante value of each cohort based on equation (7). In the long run, the CEQ of the social security reforms converges to a new steady state value. However, the reforms have different effects on the current and near-future generations. If the lines in Figure 4 are below 0%, such generations do not support the reforms. Not surprisingly, gradual abolition (Plan 1′) damages the welfare of the current generations significantly, due to the low benefits from slow convergence, and the so-called double burdens (Panel a in Table 4). Thus the CEQ value is below the benchmark for all generations who enter the economy before 2009. In contrast, a sudden cut in the social security benefit (Plan 1) improves the welfare of the current young generations who were born before 1989. Although the sudden cut reduces the welfare of the older generations by more than 5%, as measured by the CEQ, such a policy improves the welfare of the young cohorts, as they bear little cost, and the optimal replacement rate is zero.

Introducing a consumption tax (Plan 2) and a capital income tax (Plan 3) also improves the welfare of the current young and middle generations (Panel b in Figure 4). Unlike the case of the capital income tax, the consumption tax shares the burden across all generations equally, because of the flat consumption profile. Introducing a capital income tax improves the welfare of the young generations sharply, because they hold few assets. The current young generations start to accumulate wealth, and the equilibrium interest rate becomes higher than in the new steady state. Thus, the current young
generations obtain large welfare gains compared to future generations. Interestingly, when abolishing the earnings-related part, both a consumption tax and a capital income tax have similar welfare effects (Plans 4 and 5).

Old households prefer to stay with the status-quo social security system for the five reforms. This result is the same as in Conesa and Krueger (1999). The question we address in this paper is whether there are social security reform plans that improve the welfare of future generations with political support. In other words, we need to find a social security reform that at least some middle and old households prefer.

5.4 Political Feasibility of Reform Plans

Political feasibility by age is plotted in Figure 5. Consistent with Figure 4, middle and old households do not agree with any of the social security reforms in Plans 1–5. In Plan 1, only very young households, such as the 20s, agree to the abolition of the social security system, which is again consistent with Conesa and Krueger’s (1999) result. Table 6 shows the agreement rate over the total population. For example, in the case of Plan 1, only 18% of the total population agrees with such a social security reform. Thus, it is politically infeasible, even if it would improve future generations’ welfare substantially as in Panel a in Figure 4. Introducing the consumption tax and capital income tax is more favorable to the 30s than abolition (Plans 2 and 3). However, such reforms are also politically infeasible, as shown in Table 6. Only 30% agree with these plans. Because there is heterogeneity in a cohort, opinion is divided even within the same age group. For example, the asset rich do not prefer a capital income tax although the asset poor of the same age do.

Simple social security reforms, such as changing the tax base or abolition, are not supported by current generations. This is the status-quo bias problem. However, we show that a tax base choice that expands the basic pension could resolve this problem. The current middle and old generations do not agree to reforms that reduce the social security benefit of both tiers, even if future generations would obtain large welfare gains. On the contrary, the current older generation, especially poor households, prefer the plan in which the basic pension is increased, although the welfare of the young and future generations is reduced, since the optimal basic pension is zero. The simple abolition of the earnings-related pension is preferred only by the current young and future generations. However, if we expand the basic pension by abolishing the earnings-related pension, middle and old households with relatively low MSE may improve their welfare, because the shift from the earnings-related to the basic pension may raise their total benefit. Whether or not the mixture of both options improves welfare is a quantitative issue.

\[^{13}\text{Total agreement is calculated by the agreement rate by age and the population distribution.}\]
Panels d and e in Figure 5 execute the mixed plans. In Panel d in Figure 5, we abolish the earnings-related pension by increasing $\varphi_{1st}$ from 0.16 to 0.38, 0.45 and 0.50. As expected, a part of old households agrees with the reforms even if we abolish the earnings-related element. If $\varphi_{1st}$ is set at 0.45 (0.5)—that is, all retired households receive half of the average macroeconomic wage level—the agreement rate becomes 57% (65%), as shown in Table 6. Thus Plans 6$'$ and 6$''$ are politically feasible. Moreover, as shown in Panel d in Figure 4, the welfare of future generations is also improved by such reforms. Therefore, Plans 6$'$ and 6$''$ are welfare improving plans with political feasibility. On the contrary, the capital income tax (Plan 7) also improves the welfare of old generations by increasing $\varphi_{1st}$. However, by increasing $\varphi_{1st}$, the ex-ante welfare of future generations declines, as shown in Panel e in Figure 4. This is due to an excessively high capital income tax rate. Therefore Plan 7 is politically feasible, but it does not improve the welfare of future generations. Lastly, we abolish the earnings-related pension, and expand the basic pension by a payroll tax (Plan 8). When the macroeconomic replacement rate is fixed at the same level, more than half the population prefers this reform. Because of the idiosyncratic income risk, about half of middle and old households prefer it to the basic pension. However, the expansion of the basic pension damages the welfare of future generations, although the agreement rate is more than 50%.

5.5 Japanese Economy

In the transition path, many factors reallocate resources, including aging, an increasing tax burden, and changing factor prices. In particular, based on the population projection discussed in Section 3, the proportion of retirees in the total population exceeds 40% in Japan. Thus, rapid aging with an increasing tax burden may change our results.

Figure 6 plots the general equilibrium paths of the interest rate in alternative plans. As Figure 2 shows, the population distribution in Japan is not very smooth due to the baby boomers and their children. Therefore, the general equilibrium interest path fluctuates erratically, compared with Figure 3. Contrary to Figure 3, the interest rate declines in many plans. Significant capital deepening is predicted, and aggregate capital increases for 30 years. Because of aging, the equilibrium payroll tax rate also increases sharply, and without no reforms it becomes more than 30%. Comparing Figures 7 and 5, the CEQ level by undertaking a social security reform becomes larger than in the case without aging.

Political feasibility, from Panel a-c in Figure 8, is similar to that in Figure 5. However, the result changes in Panels d and e in Figure 8. In Panel d in Figure 5, Plan 6$''$ is supported by almost all young households. However, young households do not agree to such plans in Figure 8, although middle and old households support the plan, as before. This is due to the excessively large distortion effect of the basic pension. The high tax rate on the high basic pension distorts the lifetime allocation of consumption
and saving. In the Japanese economy, to finance the basic pension by the consumption tax, the equilibrium tax rate must be more than 25%. This is a huge burden that pays too high a benefit, and reduces young households’ welfare.\textsuperscript{14} The total agreement rate with Plan 6-6 is less than 40% (Table 6). Thus, these plans may, unfortunately, be politically infeasible if rapid aging cannot be moderated. However, the social security reform plans discussed above would be feasible in any of the other countries.

6 Concluding Remarks

In this paper, we investigate the welfare implications and political feasibility of social security reforms, using an overlapping generations model with idiosyncratic income risks. We examined such alternative reform options as (1) the abolition of the social security system, (2) the introduction of a consumption tax to finance the basic pension, (3) the introduction of a capital income tax, and a mixture of these options based on a two-tier structure. We show that introducing a consumption tax and a capital income tax as the tax base of the basic pension improves welfare from a steady state comparison. Moreover, the abolition of the social security system creates large welfare gains for future generations. To establish the political feasibility of implementing these plans, we evaluate alternative reform options based on two criteria: (i) ex-ante welfare and (ii) the political support of current generations. We find that, given a two-tier structure, abolishing the earnings-related part and expanding the basic pension improve the welfare of future generations, and is also politically feasible when the basic pension is financed by a consumption tax. Therefore, this plan offers a solution to the “status-quo bias” problem.

We believe that this result actually offers a feasible plan for aging countries that have a social security system with a two-tier structure. However, the social security reform we proposed does not imply an efficient reallocation. Note that in our model the optimal basic pension is zero. Thus the politically feasible plan tends to move away from the optimum. Recently, Conesa and Garriga (2008) consider the optimal fiscal policy design for social security reforms. Conesa et al. (2008) find that the optimal capital income tax is positive, and that the progressivity of the labor income tax should be low. Although we do not consider the fiscal policy mix, the progressivity of alternative tax options may enhance our results. In this paper, the political feasibility mechanism is very simple. Thus, more sophisticated modeling of the politico-economic decisions regarding alternative reforms, such as Corbae et al. (2009), may prove to be a promising approach. This point will be considered in our future research.

\textsuperscript{14}We confirm that young generations do not agree to Plans 6 and 7 even in the case of the constant population distribution if the $\varphi^{\text{1st}}$ becomes more than 0.6.
References


A  Demographics

We assume that households begin economic activity at 20, retire before 65, and lives at the most to 100. Because households are in their childhood before age 20, they do not engage in consumption, but they are included in the population dynamics for computing the future fertility rate. The population dynamics in our economy are expressed in the following matrix form:

\[
\mu_{t+1} = \begin{bmatrix}
1 + \psi_t & 0 & 0 & \cdots & 0 \\
\phi_{t,0} & 0 & 0 & \cdots & 0 \\
0 & \phi_{t,1} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \phi_{t,100} & 0 \\
\end{bmatrix} \mu_t,
\]

where \(\psi_t\) is the population growth rate of age 0 from \(t\) to \(t+1\).

Since the most recent available data on population distribution is for 2005, in our simulation we begin computing the transition equation (9) with population distribution in 2005. In the model, we compute the steady state using the population distribution in 2009 and 2200.

B  Details of the Model and Numerical Procedure

B.1  The Model

First Order Conditions  From the Bellman equation (2) and the envelope theorem, the intertemporal first-order condition is as follows:

\[
c_{j,t}^{-\gamma} \geq \frac{(1 + \tau_{t+1}^{\text{cap}})}{(1 + \tau_{t+1}^{\text{con}})} (1 + (1 - \tau_{t+1}^{\text{cap}})\tau_{t+1}) \beta E_t c_{j+1,t+1}^{-\gamma},
\]

with inequality if the liquidity constraint binds.

Law of Motion  Define the probability space as \(((A \times B \times Z), B((A \times B \times Z)), \Phi_j)\) where \(B((A \times B \times Z))\) is a Borel \(\sigma\)-field, and \(\Phi_{t,j}(X)\) is a probability measure over \(X \in B((A \times B \times Z))\). From the policy function, the transition probability of labor skill \(\pi(z'|z) = \Pr(z'|z)\), and the transition function of public pension account, the transition probability \(Q_t(\cdot, \cdot)\) over household’s states \((a, b, z)\), and the distribution function \(\Phi_{t,j}(a, b, z)\) are computable. The probability measure, which is defined over household’s state, represents the fraction of households with state \(X\). Because we assume that households of age 0 have zero assets, \(\Phi_0\) is equal to one on \(a_{0,t} = 0\). The transition function

\[^{15}\text{See Ríos-Rull (2001).}\]
\( Q_j : (A \times B \times Z) \times B((A \times B \times Z)) \rightarrow [0, 1] \) is defined as

\[
Q_j ((A \times B \times Z), X) = \sum_{z' \in Z} \begin{cases} 
\pi(z'|z) & \text{if } g_{j,t}(a,b,z) \in X, \text{ and } h_j(b,z) \in X, \\
0 & \text{else,}
\end{cases}
\]

where \( h_j(b,z) \) is the transition of the public pension account.

Given initial distribution \( \Phi_{t,0} \), the distribution function \( \{\Phi_{t,j}\}_{j=1}^{J} \) for each \( j \) is mapped by the following equation.

\[
\Phi_{t+1,j+1}(X) = \int Q_j ((A \times B \times Z), X) \, d\Phi_{j,t}, \quad (\forall X \in B(A \times B \times Z)), \ j = 0, \ldots, J - 1.
\]

### B.2 Computational Algorithm

**Computation of the Steady State** The computation of the steady state is the same as in Huggett (1996). There are three markets in the model: goods, labor, and capital. However aggregate labor is determined by the idiosyncratic risks and the average productivity profile, \( L^{ss} = \sum_{j=0}^{J-1} \mu_{j,t} \kappa_{j} \sum_{i=1}^{n} \pi_{j}(i) \exp z_{j,t}(i) \), and the labor market clears by assumption.\(^{16}\) The factor prices \( (r, w) \) are determined from the capital–labor ratio \( K/L \). By the Walras law, we concentrate on \( K/L \) and government budget clearing of \((\tau_{ss}, \tau_{con}, \tau_{cap})\).\(^{17}\)

1. Given a labor income process, we compute the distribution of the pension account at 65 to compute the required total benefit of the earnings-related part. Because we assume that the labor income process is exogenous, this procedure is implementable without difficulty.

2. Compute an aggregate labor supply \( L^{ss} \). Given an initial guess of \( K^0 \), compute a pair of \( (r^0, w^0) \). We also need an initial guess of \( C^0 \) for consumption tax.

3. Given \( (r^0, w^0, K^0, C^0) \) and an exogenous policy set \((\tau_{con}, \tau_{cap}, \varphi^{1st}, \varphi^{2nd})\), compute the payroll tax rate \( \tau_{ss}^0 \) from the government budget condition.

4. Given \( (r^0, w^0, \tau_{ss}^0; \tau_{con}, \tau_{cap}) \), compute the policy function using the Endogenous Gridpoint Method (EGM) by Carroll (2006), and obtain the distribution function \( \Phi^0 \) for each age. Computation of the transition function requires policy functions \( a' = g(a,b,z) \) and \( b' = h(b,z) \).

5. Integrating the distribution function \( \Phi^0 \), we obtain the aggregate capital \( K^1 \).

\(^{16}\)Note that, because we assume that \( z_0 \) starts at \( z_0 = 1 \), we approximate the initial transition endowment state to be near one. We start the expected value of the endowment at \( j = 0 \) to be one. The probability distribution for each state \( \pi_j(i) \) must be computed separately.

\(^{17}\)We take 50 grids on asset \( a \) for computing the policy function, and to compute the distribution we take 1000 grids.
6. If new $K^1$ and old $K^0$ are sufficiently close to each other, then stop: we have equilibrium prices for the given $\tau_{ss,0}$.

7. From a new equilibrium condition $(r^1, w^1, K^1, C^1)$, re-compute a new payroll tax $\tau_{ss,1}$. Repeat steps 3 – 5. If the iteration error of $\tau_{ss}$ is sufficiently small, stop. We have an equilibrium.

**Transition Dynamics** After the computation of the steady state in 2009 and 2200, we compute the transitional path between the steady states. The basic idea here is the same as in Conesa and Krueger (1999) and in Nishiyama and Smetters (2005).

1. Set an exogenous path of basic pension and accrual rate ($\varphi^{1st}, \varphi^{2nd}$). Guess an equilibrium sequence of $\{r_t, w_t, \tau_{ss}^t, \tau_{con}^t, \tau_{cap}^t, \xi_t\}_{t=2009}^{2200}$, which is needed to solve a household’s problem.

2. Because we have the policy function of the final steady state in 2200, we compute a sequence of policy functions using the EGM by backward induction.

3. Given the policy functions, compute the distribution function from 2009 onwards and compute aggregate variables, $\{K_t, r_t, w_t\}_{t=2009}^{2200}$.

4. Check whether each market clearing condition and government budget balances are satisfied. If these are not in equilibrium, up-date the price sequences and repeat steps 2 – 3.

5. If all markets clear in all periods, stop computation.
C Tables and Figures

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Discount factor $\beta$</td>
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<td>Relative risk aversion $\gamma$</td>
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<td>Capital share $\alpha$</td>
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<td>Depreciation rate $\delta$</td>
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<td>Persistence of Shock $\rho$</td>
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<td>Initial standard deviation $\sigma_0$</td>
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<td>Growth rate of shock $\lambda$</td>
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<td>Turning age</td>
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<td>Basic pension $\varphi_{\text{1st}}$</td>
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<tr>
<td>Earnings-related coefficient $\varphi_{\text{2nd}}$</td>
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Table 1: Calibrated Parameters

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<tr>
<th>Age</th>
<th>$\kappa_j$</th>
<th>Age</th>
<th>$\kappa_j$</th>
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<td>20–24</td>
<td>0.576</td>
<td>45–49</td>
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<td>25–29</td>
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<td>50–54</td>
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<tr>
<td>40–44</td>
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Table 2: Age-Efficiency Profile
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<th>With Idiosyncratic Risk</th>
<th>Without Idiosyncratic Risk</th>
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<td>Interest Rate</td>
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<td>Wage</td>
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<td>$\tau^{ss}$</td>
<td>14.5%</td>
<td>8.4%</td>
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<tr>
<td>$\tau^{con}$</td>
<td>0.0%</td>
<td>5.1%</td>
</tr>
<tr>
<td>$\tau^{cap}$</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>$K/Y$</td>
<td>2.93</td>
<td>3.01</td>
</tr>
<tr>
<td>CEQ</td>
<td>0.00%</td>
<td>2.16%</td>
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</table>

Table 3: Optimal Sources of Finance for Basic Pension

<table>
<thead>
<tr>
<th></th>
<th>Benchmark ((\phi^{1st} = 0.16))</th>
<th>First tier: (\phi^{1st})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00</td>
<td>0.05</td>
</tr>
<tr>
<td>With</td>
<td>Interest Rate</td>
<td>4.1%</td>
</tr>
<tr>
<td>Idiosyncratic</td>
<td>Wealth Gini</td>
<td>0.563</td>
</tr>
<tr>
<td>Risk</td>
<td>CEQ</td>
<td>0.0%</td>
</tr>
<tr>
<td>No</td>
<td>Interest Rate</td>
<td>4.9%</td>
</tr>
<tr>
<td>Idiosyncratic</td>
<td>Wealth Gini</td>
<td>0.509</td>
</tr>
<tr>
<td>Risk</td>
<td>CEQ</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Benchmark ((\phi^{2nd} = 0.22))</th>
<th>Second tier: (\phi^{2nd})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00</td>
<td>0.05</td>
</tr>
<tr>
<td>With</td>
<td>Interest Rate</td>
<td>4.1%</td>
</tr>
<tr>
<td>Idiosyncratic</td>
<td>Wealth Gini</td>
<td>0.563</td>
</tr>
<tr>
<td>Risk</td>
<td>CEQ</td>
<td>0.00%</td>
</tr>
<tr>
<td>No</td>
<td>Interest Rate</td>
<td>4.9%</td>
</tr>
<tr>
<td>Idiosyncratic</td>
<td>Wealth Gini</td>
<td>0.509</td>
</tr>
<tr>
<td>Risk</td>
<td>CEQ</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 4: Optimal Benefit Plans When the Benefit is Financed by Payroll Tax
<table>
<thead>
<tr>
<th>Plan</th>
<th>1st-tier</th>
<th>2nd-tier</th>
<th>Transition Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varphi_{1st}^{1})</td>
<td>(\varphi_{2nd}^{2})</td>
<td>Tax Base</td>
<td>Tax Base</td>
</tr>
<tr>
<td>Plan (1)</td>
<td>0.0</td>
<td>-</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (1)'</td>
<td>0.0</td>
<td>Earning</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (2)</td>
<td>0.16</td>
<td>Consumption</td>
<td>0.22</td>
</tr>
<tr>
<td>Plan (3)</td>
<td>0.16</td>
<td>Capital Income</td>
<td>0.22</td>
</tr>
<tr>
<td>Plan (4)</td>
<td>0.16</td>
<td>Consumption</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (4)'</td>
<td>0.16</td>
<td>Consumption</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (5)</td>
<td>0.16</td>
<td>Capital Income</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (5)'</td>
<td>0.16</td>
<td>Capital Income</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (6)</td>
<td>0.38</td>
<td>Consumption</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (6)'</td>
<td>0.45</td>
<td>Consumption</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (6)''</td>
<td>0.50</td>
<td>Consumption</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (7)</td>
<td>0.38</td>
<td>Capital Income</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (7)'</td>
<td>0.45</td>
<td>Capital Income</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (7)''</td>
<td>0.50</td>
<td>Capital Income</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (8)</td>
<td>0.38</td>
<td>Earning</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (8)'</td>
<td>0.45</td>
<td>Earning</td>
<td>0.0</td>
</tr>
<tr>
<td>Plan (8)''</td>
<td>0.50</td>
<td>Earning</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 5: Social Security Reform Plans

<table>
<thead>
<tr>
<th>Plan</th>
<th>Non-Aging</th>
<th>Japanese Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan 1</td>
<td>18%</td>
<td>12%</td>
</tr>
<tr>
<td>Plan 2</td>
<td>32%</td>
<td>24%</td>
</tr>
<tr>
<td>Plan 3</td>
<td>29%</td>
<td>22%</td>
</tr>
<tr>
<td>Plan 4</td>
<td>26%</td>
<td>21%</td>
</tr>
<tr>
<td>Plan 5</td>
<td>26%</td>
<td>21%</td>
</tr>
<tr>
<td>Plan 4'</td>
<td>9%</td>
<td>6%</td>
</tr>
<tr>
<td>Plan 5'</td>
<td>11%</td>
<td>6%</td>
</tr>
<tr>
<td>Plan 6</td>
<td>45%</td>
<td>36%</td>
</tr>
<tr>
<td>Plan 6'</td>
<td>57%</td>
<td>39%</td>
</tr>
<tr>
<td>Plan 6''</td>
<td>65%</td>
<td>31%</td>
</tr>
<tr>
<td>Plan 7</td>
<td>42%</td>
<td>34%</td>
</tr>
<tr>
<td>Plan 7'</td>
<td>50%</td>
<td>34%</td>
</tr>
<tr>
<td>Plan 7''</td>
<td>53%</td>
<td>29%</td>
</tr>
<tr>
<td>Plan 8</td>
<td>67%</td>
<td>66%</td>
</tr>
<tr>
<td>Plan 8'</td>
<td>64%</td>
<td>65%</td>
</tr>
<tr>
<td>Plan 8''</td>
<td>67%</td>
<td>67%</td>
</tr>
</tbody>
</table>

Table 6: Agreement Rate over Total Population
Figure 1: Variance of Log-Income Profile

Figure 2: Population Distribution
Figure 3: Interest Rate Paths When the Population Distribution is Constant.
Figure 4: Ex-Ante Value When the Population Distribution is Constant.
Figure 5: Political Feasibility When the Population Distribution is Constant.
Figure 6: Interest Rate Path When the Demographic Dynamics is Calibrated to Japanese Economy.
Figure 7: Ex-Ante Value When the Demographic Dynamics is Calibrated to Japanese Economy.
Figure 8: Political Feasibility When the Demographic Dynamics is Calibrated to Japanese Economy.