受験番号	番

２０２０年度　一橋大学大学院経済学研究科博士後期課程編入学試験問題

マ ク ロ 経 済 学

実 施 日　２０１９年8月２９日（木）
試験時間　９：３０～１２：３０

注意事項

１．「解答はじめ」の指示があるまでは問題冊子を開いてはいけない。

２．問題冊子は１冊（本文３ページ）、解答用紙は２枚、下書き用紙は１枚である。試験開始後直ちに確認し、枚数が異なる場合は挙手すること。

３．**すべての解答用紙・下書き用紙、問題冊子の表紙に受験番号を記入せよ（解答用紙の2枚目以降にも忘れずに記入すること）。氏名を記入してはならない。**なお、用紙は一切持ち帰ってはいけない。

４．科目名を、解答用紙の科目欄に明記せよ。

５．解答は横書きとする。解答用紙は裏面も使用できる。

６．解答に際しては、原則として問題ごとに解答用紙を分けること。

７．解答用紙の追加配付を希望する受験者には、追加配付を認める。また、解答用紙を汚損した場合、全面的に書き直しを要する場合などは、解答用紙の交換を認める。解答用紙の追加、交換を求める際には、試験中、静かに挙手すること。

８．辞書その他の持ち込みは許可しない。

以上

※その他の試験科目についての注意事項は
問題冊子本文ページ数以外同内容のため割愛

Macroeconomics
2019

There are two problems. Answer all problems in English or Japanese.

Problem 1

Consider the following problem

$$\max_{\{(c_t,k_{t+1})\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t \log c_t$$

subject to

$$c_t + k_{t+1} - (1-\delta)\,k_t = Ak_t^{\alpha} \quad \text{for } t = 0, \cdots, T,$$

$$k_0 > 0 \text{ is given,}$$

and

$$k_{T+1} \geq 0.$$

We assume that $A > 0$, $0 < \alpha < 1$, $0 < \delta < 1$, and $0 < \beta < 1$. Notice that this household knows that the world will end at time $T > 0$. Also notice that choosing

$$k_{t+1} - (1-\delta)\,k_t < 0 \quad \text{for } t = 0, \cdots, T$$

is allowed.

(1) Derive the Euler equation

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} \left(1 - \delta + A\alpha k_{t+1}^{\alpha-1}\right) \quad \text{for } t = 0, \cdots, T-1$$

as optimality conditions.

(2) Show that $k_{t+1} = k_t$ holds for $t \in \{0, \cdots, T-1\}$ if and only if c_t and k_t satisfy

$$c_t = \ell_k(k_t) \equiv Ak_t^{\alpha} - \delta k_t.$$

(3) Show that $c_{t+1} = c_t$ holds for $t \in \{0, \cdots, T-1\}$ if and only if c_t and k_t satisfy

$$c_t = \ell_c(k_t) \equiv Ak_t^{\alpha} + (1-\delta)k_t - k^{ss},$$

where

$$k^{ss} \equiv \left(\frac{A\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{1}{1-\alpha}}.$$

(4) Draw $\ell_k(k_t)$ and $\ell_c(k_t)$ in (k_t, c_t)-space, measuring k_t on the horizontal axis and c_t on the vertical axis. If you did things correctly, the (k_t, c_t)-space should be partitioned into four different regions. In each of the four regions, draw arrows in the directions in which c_t and k_t will evolve.

(5) Suppose that $T = 1$. Derive the optimal $\left\{\left(c_t^*, k_{t+1}^*\right)\right\}_{t=0}^{1}$ that solves the household problem.

(6) Plot in the the phase diagram (k_0, c_0^*) and (k_1^*, c_1^*) where $\{c_0^*, c_1^*\}$ and k_1^* are those you derived in the previous problem.

(7) Now consider the otherwise same problem, including the amount of initial capital k_0, except for T, which is now set at $T = 2$. Derive the optimal $\left\{\left(c_t^*, k_{t+1}^*\right)\right\}_{t=0}^{2}$ that solves the household problem.

(8) Plot in the the phase diagram (k_0, c_0^*), (k_1^*, c_1^*), and (k_2^*, c_2^*) where $\{c_0^*, c_1^*, c_2^*\}$ and $\{k_1^*, k_2^*\}$ are those you derived in the previous problem. Explain how the trajectory changes from the case with $T = 1$, i.e., question (6).

Problem 2

Let the set of natural numbers including zero be $\mathbf{N} = \{0, 1, 2, ...\}$ and the set of integers be $\mathbf{Z} = \{..., -2, -1, 0, 1, 2, ...\}$.
Time is discrete and each period is labeled by $t \in \mathbf{N}$. Consider an endowment economy with one non-storable good in each period. Suppose that there are three infinitely lived consumers $(i = 1, 2, 3)$. Let c_t^i be consumer i's consumption at period t. Assume that all the consumers have the same utility function

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma},$$

where $\beta \in (0, 1)$ and $\sigma > 0$. In each period t, consumer i receives the endowment in the following way:

$$\omega_t^i = \begin{cases} 1 & \text{if } t = 3j + i \text{ for some } j \in \mathbf{Z}, \\ 1 & \text{if } t = 3j + i + 1 \text{ for some } j \in \mathbf{Z}, \\ 4 & \text{if } t = 3j + i + 2 \text{ for some } j \in \mathbf{Z}. \end{cases}$$

(1) Define the Arrow-Debreu equilibrium in this economy.

(2) Derive the first order conditions of consumer i's problem in (1).

(3) Characterize the Arrow-Debreu equilibrium in (1).

(4) Let α_i be the Pareto weight for consumer i. Write down the social planner's problem in this economy.

(5) Solve the social planner's problem in (4).

(6) Find the Pareto weight such that the allocations obtained in (3) and (5) are identical.

(7) Define the sequential market equilibrium in this economy.

(8) Show that the allocations in (1) and (7) are equivalent.

Microeconomics

August 2019

There are three problems. Answer all problems either in Japanese or in English. Your style of academic writing will be evaluated as well as mathematics.

Problem 1

Consider an economy with two consumers (A, B), two firms (I,II), and three commodities $(1, 2, 3)$. Consumer A has an initial endowment $e^A = (1, 0, 0)$ and a utility function

$$U^A(x_1, x_2, x_3) = \log x_1 + 2 \log x_2 + 2 \log x_3,$$

where log is a natural logarithm. Consumer B has an initial endowment $e^B = (1, 0, 0)$ and a utility function

$$U^B(x_1, x_2, x_3) = 2 \log x_1 + 2 \log x_2 + \log x_3.$$

Firms I, II are represented by the following production sets, respectively:

$$Y^{\mathrm{I}} = \{(-t, 2t, t) \in \mathbb{R}^3 \,|\, t \geq 0\},$$

$$Y^{\mathrm{II}} = \{(-t, t, 3t) \in \mathbb{R}^3 \,|\, t \geq 0\}.$$

We consider a competitive equilibrium of this economy. The price of commodity 1 is normalized to be one. A price vector is denoted by $p = (1, p_2, p_3)$.

(1) Assuming that firms earn zero profit, derive consumers' demand functions.

(2) Since production sets are represented by cones, firms indeed earn zero profit in equilibrium. Derive an equilibrium price candidate.

(3) Derive a competitive equilibrium allocation.

Problem 2

Imagine an agent with wealth w, who faces a probability $\pi \in (0, 1)$ of incurring a loss L. That is, her wealth becomes $w - L$ with probability π and w with probability $1 - \pi$. The agent can insure against this loss by contracting an insurance policy that will pay out in the event of loss. One unit of insurance costs q and gives a payment of 1 if the loss occurs. Thus, if the agent buys z units of insurance, her wealth becomes $w - qz - L + z$ with probability π, and $w - qz$ with probability $1 - \pi$. Assume that this agent's preferences are represented by an expected utility function with a differentiable, strictly increasing, and strictly concave VNM function u. Assume that the agent chooses z by maximizing her expected utility

$$U(z) = \pi u(w - qz - L + z) + (1 - \pi)u(w - qz).$$

(1) Suppose that the insurance market is competitive, which implies that an insurance company earns zero profit. Show that $q = \pi$ under this assumption.

(2) Assume that the insurance market is competitive. Show that the agent is fully insured against the loss; that is, $z = L$.

(3) Assume that the insurance market is not competitive and an insurance company charges a unit cost of insurance $q > \pi$. Show that the agent is not fully insured against the loss; that is, $z < L$.

(4) Assume $q > \pi$. Moreover, assume that the agent's Arrow-Pratt measure $r^A(x) = -\frac{u''(x)}{u'(x)}$ is strictly decreasing in x. Show that $\frac{\partial^2 U}{\partial w \partial z} < 0$. (Hint: Substitute the first-order condition into $\frac{\partial^2 U}{\partial w \partial z}$.)

(5) Under the same assumption as in (4), show that the agent's optimal choice z will decrease with wealth w.

Problem 3

There are two identical firms, $i = 1, 2$, that produce a homogeneous commodity with the same constant marginal costs $c \in (\frac{1}{6}, \frac{1}{3})$. For the demand side, we assume an inverse demand function $P(X) = 1 - X$, where X is total demand and $P(X)$ is a market clearing price at X. Firm i is owned by owner i. The owner i hires a manager i of the firm and chooses an incentive plan for manager i. Consider the following two-stage duopoly game. At stage 1, owners $i = 1, 2$ simultaneously choose an incentive plan (α_i, f_i), where $\alpha_i \in [0, 1]$ is a weight between profit and revenue and $f_i \in \mathbb{R}$ is a monetary transfer from owner i to manager i (which could be negative). At stage 2, if manager i accepts the incentive plan (α_i, f_i), managers $i = 1, 2$ play a Cournot (quantity) competition; that is, they simultaneously choose an output level x_i. Given (α_i, f_i), manager i's payoff function is given by

$$I_i(x_i, x_{-i}, \alpha_i, f_i) = \alpha_i \pi_i + (1 - \alpha_i) P(x_i + x_{-i}) x_i + f_i,$$

where $\pi_i = (P(x_i + x_{-i}) - c) x_i$ is the profit of firm i. If $\alpha_i = 1$, the manager maximizes profit π_i, while if $\alpha_i = 0$, the manager maximizes revenue $P(x_i + x_{-i}) x_i$. Note that manager i's payoff function is rewritten as

$$I_i(x_i, x_{-i}, \alpha_i, f_i) = P(x_i + x_{-i}) x_i - \alpha_i c x_i + f_i.$$

Moreover, we assume that the managers' reservation utility is zero; that is , if $I_i(x_i, x_{-i}, \alpha_i, f_i) \geq 0$, the manager will accept this incentive plan.

(1) Given (α_i, f_i), $i = 1, 2$, derive a Nash equilibrium at the second stage.

(2) Owner i's payoff function is given by the profit minus payment to the manager; that is, $\pi_i - I_i$. At an optimum of owner's choice, show that I_i can be set to zero by choosing f_i appropriately.

(3) Given the Nash equilibrium at the second stage, derive owner i's reduced form of payoff function, which depends on α_i and α_{-i}.

(4) Derive a Nash equilibrium at the first stage.

Statistics · Econometrics

Answer both Problems 1 and 2 either in Japanese or in English.

Problem 1. Answer EITHER problem 1-1 or problem 1-2.

1-1. (Probability and Statistics) Answer EITHER problem (a) or problem (b).

(a) Let X be a random variable according to an unknown distribution $P_\theta, \theta \in \Theta$ where Θ is a parameter space. Answer questions i and ii.

i. Prove that $\delta(X)$ is uniformly minimum variance unbiased (UMVU) for estimating $g(\theta) = E_\theta[\delta(X)]$ if and only if $\forall\theta, Cov[\delta(X), U(X)] = 0$ for all $U(X)$: unbiased estimator of zero, i.e. $U(X)$ is a statistic satisfying $E_\theta[U(X)] = 0, \forall\theta \in \Theta$.

ii. Suppose that $\delta_1(X)$ and $\delta_2(X)$ are UMVU estimators of $g_1(\theta) = E_\theta[\delta_1(X)]$ and $g_2(\theta) = E_\theta[\delta_2(X)]$, respectively. Prove that $\delta_1(X) + \delta_2(X)$ is the UMVU estimator of $g_1(\theta) + g_2(\theta)$.

(b) For each n, $X_{n,i}, i = 1, 2, \cdots, n$ are $i.i.d.$ random variables according to a discrete distribution with a probability function given by

$$P\{X_{n,i} = k\} = \left(1 - \frac{\beta}{n}\right)\left(\frac{\beta}{n}\right)^k, \quad k = 0, 1, 2, \cdots,$$

where $\beta > 0$ is a constant. Let $S_n = \sum_{i=1}^n X_{n,i}$. Prove that as $n \to \infty$, S_n converges weakly to the non-degenerate limit and identify the limiting distribution.

1-2. (Econometrics) Answer EITHER problem (a) or problem (b).

(a) Consider the following regression model:

$$y = X\beta + \varepsilon,$$

where y is an n by 1 dependent variable, X is an n by k regressor, ε is an n by 1 disturbance, and β is a k by 1 unknown parameter. We would like to estimate β under restrictions given by $H_0 : R\beta = r$, where R is q by k with $rank(R) = q$, r is q by 1, and R and r are known. We thus consider the following Lagrangian:

$$\mathcal{L} = \frac{1}{n}(y - X\beta)'(y - X\beta) + 2(R\beta - r)'\lambda. \qquad (1)$$

We also assume that the following relations hold:

$$\frac{1}{n}X'X \to_p \Sigma, \quad \frac{1}{\sqrt{n}}X'\varepsilon \to_d N(0, V),$$

where Σ and V are positive definite. Answer questions i-iv.

i. Derive the first order conditions for the estimation using (1).

ii. Let $\tilde{\beta}$ and $\tilde{\lambda}$ be the solutions of the first order conditions derived in question i. Prove that they are expressed as

$$\begin{aligned}\tilde{\beta} &= \hat{\beta} - \left(\frac{1}{n}X'X\right)^{-1} R'\tilde{\lambda}, \\ \tilde{\lambda} &= \left[R\left(\frac{1}{n}X'X\right)^{-1} R'\right]^{-1} (R\hat{\beta} - r),\end{aligned}$$

where $\hat{\beta}$ is the OLS estimator of β without restrictions.

iii. Derive the limiting distribution of $\sqrt{n}\tilde{\lambda}$ under H_0.

iv. Suppose that $\hat{V}$ is the consistent estimator of V. The validity of restrictions H_0 can be investigated by testing whether $\lambda = 0$ or not. Explain how to test $\lambda = 0$ by using $\tilde{\lambda}$.

(b) Consider the following simple dynamic panel data model with unobserved individual effects:

$$y_{i,t} = \alpha y_{i,t-1} + \eta_i + u_{i,t}, \quad \text{for } i = 1, \cdots, N \text{ and } t = 1, \cdots, T,$$

where $|\alpha| < 1$ and $y_{i,0} = \eta_i/(1-\alpha)$. Let the individual effects η_i follow $i.i.d.(0, \sigma_\eta^2)$ and the idiosyncratic errors $u_{i,t}$ follow $i.i.d.(0, \sigma_u^2)$, where $\{\eta_i\}$ and $\{u_{i,t}\}$ are independent. The goal is to consistently estimate α under N and $T \to \infty$. Answer questions i-v.

i. Show that the pooled OLS estimator $\hat{\alpha}_{OLS}$, i.e. regressing $y_{i,t}$ on $y_{i,t-1}$, is inconsistent.

ii. Show that the within-group fixed effect estimator $\hat{\alpha}_{WG}$, i.e. regressing $(y_{i,t} - \bar{y}_i)$ on $(y_{i,t-1} - \bar{y}_i)$ where $\bar{y}_i = T^{-1}\sum_{t=1}^{T} y_{i,t}$, is inconsistent.

iii. Show that the OLS in first-differences fixed effect estimator $\hat{\alpha}_{FD}$, i.e. regressing $(y_{i,t} - y_{i,t-1})$ on $(y_{i,t-1} - y_{i,t-2})$, is inconsistent.

iv. Propose an instrumental variable (IV) estimator to consistently estimate α. For simplicity, you may use only one instrumental variable.

v. Derive the asymptotic distribution of the IV estimator proposed in question iv.

Problem 2. Answer EITHER problem 2-1 or problem 2-2.

2-1. (Probability and Statistics) Answer BOTH problem (a) and problem (b).

(a) Let $X_i, i = 1, \cdots, m$ be $i.i.d. N(\mu, \sigma^2)$ and $Y_j, j = 1, \cdots, n$ be $i.i.d. N(\mu, \tau^2)$, where $\{X_i\}$ and $\{Y_j\}$ are independent. Answer questions i-v.

i. Find a set of jointly minimal sufficient statistics for (μ, σ^2, τ^2).

ii. Is the statistics in question i. complete sufficient?

In questions iii-v, assume $\sigma^2 = k\tau^2$, where $k > 0$ is a known constant.

iii. Find a set of jointly complete sufficient statistics for (μ, σ^2).

iv. Find the uniformly minimum variance unbiased (UMVU) estimator of μ.

v. Find the UMVU estimator of σ^2.

(b) Let X and Y be $i.i.d.$ geometric random variables with probability $P\{X = k\} = P\{Y = k\} = pq^k, k = 0, 1, \cdots, p + q = 1, 0 < p < 1$. Answer questions i-iii.

i. Find the probability distribution of $U = \min\{X, Y\}$.

ii. Find the probability distribution of $V = X - Y$.

iii. Prove that U and V are independent.

2-2. (Econometrics) Answer BOTH problem (a) and problem (b).

(a) Consider the following regression model with structural change in variance:

$$y_t = x_t'\beta + \varepsilon_t, \quad \text{for} \quad t = 1, \cdots, T,$$

where x_t is a k by 1 regressor, β is a k by 1 parameter, and $\{\varepsilon_t\}$ is a sequence of independent normal random variables with $E(\varepsilon_t) = 0$ for all t, $E(\varepsilon_t^2) = \sigma_1^2$ for $t = 1, \cdots, [T/2]$, and $E(\varepsilon_t^2) = \sigma_2^2$ for $t = [T/2]+1, \cdots, T$ with $[a]$ denoting the integer part of a. The regressor $\{x_t\}$ is independent of $\{\varepsilon_t\}$ and the following weak law of large numbers and the central limit theorem hold:

$$\frac{1}{T}\sum_{t=1}^{T} x_t x_t' \to_p \Sigma, \qquad \left\{ \frac{1}{\sqrt{T}} \sum_{t=1}^{[T/2]} x_t \varepsilon_t, \frac{1}{\sqrt{T}} \sum_{t=[T/2]+1}^{T} x_t \varepsilon_t \right\} \to_d \{z_1,\ z_2\},$$

where Σ is positive definite, $z_1 \sim N\left(0, \frac{\sigma_1^2}{2}\Sigma\right)$, $z_2 \sim N\left(0, \frac{\sigma_2^2}{2}\Sigma\right)$, and z_1 is independent of z_2. Answer questions i-v.

i. Derive the limiting distribution of the OLS estimator of β, $\hat{\beta}_{LS}$.

ii. Suppose that σ_1^2 and σ_2^2 are known. Write down the GLS estimator of β, $\hat{\beta}_{GLS}$.

iii. Derive the limiting distribution of $\hat{\beta}_{GLS}$.

iv. Show that the asymptotic variance of the GLS estimator is smaller than that of the OLS estimator and that they are the same only in the case where $\sigma_1^2 = \sigma_2^2$.

v. In practice, we do not know the true values of σ_1^2 and σ_2^2. Explain how to construct the feasible GLS estimator.

(b) The data is generated by the following linear model:

$$y_i = x_{1,i}\beta_1 + x_{2,i}\beta_2 + u_i, \quad \text{for } i = 1, \cdots, n,$$

where y_i is a dependent variable, $x_{1,i}$ and $x_{2,i}$ are scalars of nonrandom regressors with unknown coefficients β_1 and β_2. The error term u_i follows $i.i.d.(0, \sigma^2)$. The goal is to estimate β_1 while you may or may not control $x_{2,i}$ in your regression. Let $\hat{\beta}_1$ be an OLS estimator in a regression of y_i on $x_{1,i}$ and $x_{2,i}$ and let $\tilde{\beta}_1$ be an OLS estimator in a regression of y_i on $x_{1,i}$ only. Answer questions i-iii.

i. Derive the bias, the variance, and the mean squared error (MSE) of $\hat{\beta}_1$.

ii. Derive the bias, the variance, and the MSE of $\tilde{\beta}_1$.

iii. Show, step-by-step, that the MSE of $\tilde{\beta}_1$ is smaller than the MSE of $\hat{\beta}_1$ if and only if

$$r_{1,2}^2 > 1 - \frac{\sigma^2}{\beta_2^2(\sum_{i=1}^n x_{2,i}^2)},$$

where

$$r_{1,2}^2 = \frac{(\sum_{i=1}^n x_{1,i}x_{2,i})^2}{(\sum_{i=1}^n x_{1,i}^2)(\sum_{i=1}^n x_{2,i}^2)}.$$

政治経済学

Political Economy

次の第１題から第４題のうち、２題を選択して解答しなさい。
（解答の冒頭に、選択した問題の番号を明記すること。）
Answer only two of the following four problems.
(Write the number of the problem at the beginning of each answer.)

第１題

マルクス（Karl Marx）の商品論・貨幣論に関する以下の問いに答えなさい。

（１）商品の「価値（value）」について説明しなさい。
（２）「１単位の商品Ａは、x 単位の金に値する」という商品の価値表現のしくみについて、説明しなさい。

Problem 1:

Answer the following two questions about Karl Marx's theory of commodities and money.

1. Describe the "value" of commodities.
2. Explain the mechanism of the following expression of commodity value: "one unit of commodity A is worth x units of gold."

第２題

「価値の生産価格への転化」について論じなさい。

Problem 2:

Discuss the "transformation of values into prices of production."

第３題

宮本憲一は『環境経済学』において「近代経済学の環境経済論の限界」を論じている。その内容を、以下の用語を用いて簡潔に説明しなさい。

「市場の失敗」、「社会的費用」、「公共的介入」、「費用便益分析」、「経済的手段」

Problem 3:

In his book entitled *Environmental Economics*, Ken'ichi Miyamoto discussed "the limitations of environmental economics in modern economics." Explain the contents briefly using the following terms: "market failure", "social cost", "public intervention", "cost-benefit analysis", and "economic tools."

第４題

社会科学における「比較」の意義について、他の学問分野とりわけ自然科学との対比の上で論じなさい。

Problem 4:

Discuss how the comparative approach in the social science is different from that of other research fields such as the natural sciences.

経済史
Economic History

次のすべての問題に日本語もしくは英語で解答しなさい。
Answer all questions either in Japanese or English.

第 1 題

経済史研究における理論の役割について、斎藤修『新版　比較史の遠近法』(書籍工房早山、2015 年)にもとづき、以下の 2 点に留意しながら、記述しなさい。

(1) 歴史研究のリアリズムの系譜、たとえば英国の Gregorian realism と Maltusian realism など
(2) 比較史への展望

Question 1

Explain the role of theory or theoretical thinking in the research of economic history from the following viewpoints as defined in *Hikakushi-no-Enkin-ho: Shinpan* (Shoseki-kobo Hayama 2015) by Osamu Saito.

(1) from the history of realism in social science such as Gregorian realism and Malthusian realism in Great Britain
(2) from the perspective of comparative history

第 2 題

C. A. ベイリ(平田雅博ほか訳)『近代世界の誕生―グローバルな連関と比較　1780-1914―』(名古屋大学出版会、2018 年)で定義されている「初期グローバル化」局面から「国際主義」局面への移行が及ぼした社会経済的影響について、具体的な事例を用いて説明しなさい。

Question 2

Through the use of a concrete case, explain the socio-economic influences of the transformation from the phase of “Archaic Globalization” to that of “Internationalism” as defined in *The Birth of the Modern World, 1780-1914: Global Connections and Comparisons* (Blackwell 2004) by Christopher A. Bayly.