

受験番号	番
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平成27年度 一橋大学大学院経済学研究科博士後期課程編入学試験問題

## マ ク ロ 経 済 学

実 施 日 平成26年9月11日(木)

試験時間 9:30～12:30

### 注意事項

1. 「解答はじめ」の指示があるまでは問題冊子を開いてはいけない。
2. 問題冊子は1冊(本文5ページ)、解答用紙は4枚、下書き用紙は1枚ある。試験開始後直ちに確認し、枚数が異なる場合は挙手のこと。
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4. 科目名を、解答用紙の科目欄に明記せよ。
5. 解答は横書きとする。解答用紙は裏面も使用できる。
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8. 辞書その他の持ち込みは許可しない。

以上

# Macroeconomics

There are four problems.

Answer all of them either in English or in Japanese.

## Problem 1

Consider a variant of the expanding variety model of growth. Time is continuous. The economy is closed and there is no government. There is no physical capital. Homogeneous final goods are produced from a continuum of varieties of differentiated intermediate goods (inputs) and labor. The number of varieties of inputs at time  $t$  is denoted as  $N_t$ . The market for final goods and the labor market are perfectly competitive. The production function for the representative final goods producer is

$$Y_t = \tilde{X}_t^\alpha \cdot L^{1-\alpha}, \text{ where } \tilde{X}_t \equiv \left[ \int_0^{N_t} x_{i,t}^{(\varepsilon-1)/\varepsilon} di \right]^{\varepsilon/(\varepsilon-1)}, \quad \varepsilon > 1, \quad 1 > \frac{\varepsilon}{\varepsilon-1} \cdot \alpha > 0.$$

In the above,  $L$  is labor whose total supply is fixed, and  $x_{i,t}$  is the amount of input of type  $i$  at time  $t$ . The markets for inputs are monopolistically competitive: there are  $N_t$  input producing firms, and each has the monopoly right to produce a single variety of inputs. They can produce 1 unit of input from  $\psi$  units of final goods ( $\psi > 0$ ). We shall take final goods as numeraire (i.e., its price is equal to 1). Price of input of type  $i$  at time  $t$  will be denoted  $p_{i,t}$ .

New firms can enter into the input market by paying an upfront cost of invention: by investing 1 unit of final goods into the R&D activity,  $\eta$  varieties of inputs are invented. Once a firm invents a new variety, it obtains a perpetual monopoly right to produce this type of inputs. We shall denote the present value of those monopoly profits by  $V_t$ .

Assuming the CRRA (Constant Relative Risk Aversion, or CIES) utility, the representative household's Euler equation for consumption of the final goods,  $C_t$ , is:

$$\dot{C}_t / C_t = (1/\theta)(r_t - \rho), \quad \theta > 0, \quad \rho > 0.$$

- (1.1) Explain mathematically that the aggregate productivity is increasing in  $N_t$ .
- (1.2) Derive the final goods producer's demand for  $x_{i,t}$  as a function of  $p_{i,t}$ ,  $\tilde{X}_t$ , and  $L$ .
- (1.3) We can show that all input producers charge the same price which is constant over time:  $p_{i,t} = p$  for all  $i$  and  $t$ . Using this and the result from (1.2), show that profit per input producer, which will be denoted  $\pi_t$ , is decreasing in  $N_t$ .
- (1.4) We shall limit our attention to the steady state in which the interest rate,  $r_t$ , is constant over time:  $r_t = r^*$  for all  $t$ . We can show that  $r^* V_t - \dot{V}_t = \pi_t$  (you do not have to prove this). Give an economic interpretation to this relationship.
- (1.5) Explain why  $\eta V_t \leq 1$  has to hold.
- (1.6) Does this economy exhibit endogenous growth?

## Problem 2

Consider an overlapping generations model. Time is discrete. The economy is closed and there is no government. Each household lives for two periods. Denoting the number of households of generation  $t$  (those who are born in period  $t$ ) as  $L_t$ , we have  $L_{t+1}=(1+n)L_t$  ( $n>0$ ) for all  $t$ . Each household of generation  $t$  supplies 1 unit of labor inelastically in period  $t$  and earns wage  $w_t$ , consumes  $c_{1,t}$  and the rest of its wage income becomes its saving,  $s_t$ . This saving can be held either in the form of bonds (the interest rate is denoted  $r_{t+1}$ ) or capital. In period  $t+1$ , it is unable to work, and thus its consumption,  $c_{2,t+1}$ , is equal to the saving plus returns on it. The household utility is:

$$U_t = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}), \quad 1 > \beta > 0.$$

On the production side, there is a continuum of identical firms whose total number is normalized to equal 1. The production function for firm  $i$  is:

$$Y_{i,t} = A \cdot K_t^\gamma \cdot K_{i,t}^\alpha \cdot L_{i,t}^{1-\alpha}, \text{ where } \gamma > 0, \quad 1 > \alpha > 0, \quad 1 > \gamma + \alpha.$$

In the above,  $Y_{i,t}$ ,  $K_{i,t}$  and  $L_{i,t}$  are output, capital stock and labor for this firm. On the other hand,  $K_t$  is the aggregate capital stock at the beginning of period  $t$ , and it represents spillover effects from the economy's stock of knowledge (that comes from the economy's past investment experiences); each firm takes this variable as given. Assume that capital stock depreciates by 100% each period. Consider the competitive equilibrium of this economy.

- (2.1) Explain why  $K_{t+1} = s_t L_t$  holds.
- (2.2) Derive the expressions for the "Private Marginal Product of Capital" (PMPK) and the "Social Marginal Product of Capital" (SMPK).
- (2.3) Derive the law of motion  $k_{t+1} = G(k_t)$  ( $k_t = K_t/L_t$ ).
- (2.4) Derive the steady state  $k_t$ .
- (2.5) Is the steady state resource allocation efficient? Explain why or why not.

### Problem 3

Consider a static consumption allocation problem of a household at a particular period  $t$ . This household consumes two kinds of goods,  $c_t(1)$  and  $c_t(2)$ , to raise the following Cobb-Douglas type static utility  $C_t$ , which is given as a bundle of the two consumption goods:

$$C_t = c_t(1)^\gamma c_t(2)^{1-\gamma}, \quad 0 < \gamma < 1.$$

Suppose that the first consumption good is the numeraire. The relative price of the second good to the first, then, is denoted by  $p_t$ .

(3.1) Write down the static cost minimization problem of the household. Derive (i) the demand functions for the first and second goods given  $p_t$  and  $C_t$ , and (ii) the unit cost of consumption bundle  $C_t$ , which is denoted by  $P_t$ . In particular, verify that unit cost  $P_t$  is proportional to  $p_t^{1-\gamma}$ :  $P_t = Ap_t^{1-\gamma}$ , where  $A$  is a constant.

Now suppose that the household lives in two periods. In each period, the household raises one-period utility by consuming bundle  $C_t$ . The lifetime utility function of the household is

$$U = \frac{C_1^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} + \beta \frac{C_2^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \quad 0 < \sigma, \quad 0 < \beta < 1,$$

where  $\beta$  is the subjective discount factor and  $\sigma$  is the elasticity of the intertemporal substitution.

In each period, the household receives endowment **only** of the first good,  $y_t$ . **There is no endowment of the second good.** However, given the relative price  $p_t$  the household can buy the second good as much as it wants from the corresponding good market. The household also can buy or sell the first good as much as it wants in the corresponding good market. There is a financial market in which the household can buy or one-period bonds in term of the first good,  $B$ , for consumption smoothing across the two periods. The bond has the constant gross rate of return  $R$ . Finally assume that the household has no initial bond holding at the beginning of the first period.

(3.2) Write down the period-by-period budget constraints of the household.

(3.3) Derive the Euler equation as a first order condition for the household's lifetime utility maximization problem. Provide the economic intuition behind the Euler equation.

(3.4) Derive the first period's optimal consumption expenditure of the household,  $P_1 C_1$ .

(3.5) Characterize an effect of a change in the first period's relative price  $p_1$  on the first period's optimal consumption expenditure. Explain how and why the effect depends on the size of the elasticity of intertemporal substitution  $\sigma$ .

(3.6) Verify that when  $\sigma$  approaches one, the one-period utility function  $\frac{C_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}$  converges to the natural log function  $\ln C_t$ . Is there any effect of a change in the first period's relative price  $p_1$  on the first period's optimal consumption expenditure in this limiting case? Discuss why or why not?

### Problem 4

Consider a household who lives in infinite periods. The household can buy and sell stock  $s_t$  under price  $v_t$ . In each period, the stock gives the household stochastic dividend  $\pi_t$  per stock. Moreover, the household buys or sells state-non-contingent bond  $b_t$  with gross interest rate  $R_t$ . The household maximizes the following expected lifetime utility:

$$E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}), \quad 0 < \beta < 1, \quad u' > 0, \quad u'' < 0$$

subject to the budget constraint

$$c_t + v_t s_t + b_t = (v_t + \pi_t) s_{t-1} + R_{t-1} b_{t-1}.$$

Assume that the No Ponzi Game condition is satisfied.

**(4.1)** Construct the Bellman equation for the above problem. Derive the first order conditions, the Benveniste and Scheinkman conditions, and the transversality conditions.

**(4.2)** Drive the Euler equations with respect to the two assets. Provide the stochastic discount factor.

**(4.3)** Let  $R_t^*$  denote the gross rate of return of the stock,  $R_t^* \equiv (v_t + \pi_t)/v_{t-1}$ . Prove that the expected equity premium  $E_t R_{t+1}^*/R_t$  depends negatively on the covariance between the stochastic discount factor and the gross rate of return of the stock. Discuss the economic intuition behind this theoretical result.

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平成27年度 一橋大学大学院経済学研究科博士後期課程編入学試験問題

## ミクロ経済学

実施日 平成26年9月12日(金)

試験時間 9:30～12:30

### 注意事項

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以上

# Microeconomics

September 12, 2014

There are three problems. Answer all problems either in Japanese or in English.

## Problem 1

[A Characterization of Pareto Efficiency] Consider an exchange economy  $\mathcal{E} = (X^i, u^i(\cdot), \omega^i)_{i \in \mathcal{I}}$ , where  $\mathcal{I} = \{1, \dots, I\}$  is the set of consumers; there are  $L$  commodities;  $X^i = \mathbb{R}_+^L$  is  $i$ 's consumption set;  $u^i : X^i \rightarrow \mathbb{R}$  is  $i$ 's utility function; and  $\omega^i \in X^i$  is  $i$ 's initial endowment. Assume that each  $u^i(\cdot)$  is continuous, strongly increasing, and concave. Let

$$V = \left\{ (v_1, \dots, v_I) \in \mathbb{R}^I \mid \begin{array}{l} \text{there exists a feasible allocation } x = (x^1, \dots, x^I) \\ \text{such that } v_i \leq u^i(x^i) \text{ for all } i \in \mathcal{I} \end{array} \right\}$$

be the set of attainable utility levels. Answer the following questions.

- (1) Show that  $V$  is a convex set in  $\mathbb{R}^I$ .
- (2) Show that if  $x = (x^1, \dots, x^I)$  is a Pareto efficient allocation, then,  $(u^1(x^1), \dots, u^I(x^I))$  is “not” in the interior of  $V$ .
- (3) Provide the precise statement of the separating hyperplane theorem.
- (4) Show that if  $x = (x^1, \dots, x^I)$  is a Pareto efficient allocation, then there exists  $\alpha = (\alpha_1, \dots, \alpha_I) \in \mathbb{R}^I \setminus \{\mathbf{0}\}$  such that  $\sum_{i \in \mathcal{I}} \alpha_i u^i(x^i) = \max_{v \in V} \alpha \cdot v$ .
- (5) Let  $v = (v_1, \dots, v_I) \in V$ . Show that for any  $\tilde{v} \in \mathbb{R}^I$ , if  $\tilde{v} \leq v$  (i.e.,  $\tilde{v}_i \leq v_i$  for each  $i \in \mathcal{I}$ ), then  $\tilde{v} \in V$ .
- (6) Let  $x$  be a Pareto efficient allocation and suppose  $\sum_{i \in \mathcal{I}} \alpha_i u^i(x^i) = \max_{v \in V} \sum_{i \in \mathcal{I}} \alpha_i v_i$  for some weights  $(\alpha_i)_{i \in \mathcal{I}} \in \mathbb{R}^I \setminus \{\mathbf{0}\}$ . Show that one can choose these weights  $(\alpha_1, \dots, \alpha_I)$  such that  $\alpha_i \geq 0$  for each  $i \in \mathcal{I}$  and  $\sum_{i \in \mathcal{I}} \alpha_i = 1$  (Hint: Use the property claimed in the previous question).



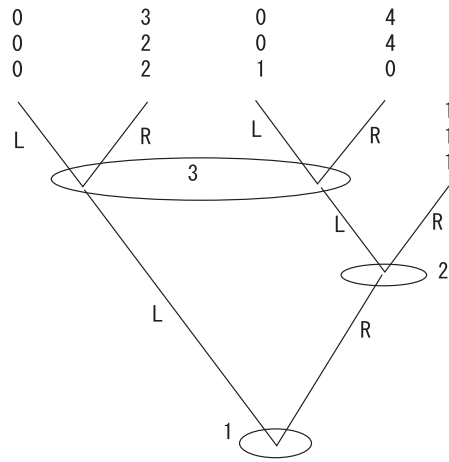


Figure 1: 3-person extensive game for Problem 2

## Problem 2

Answer all the questions below in a clear, logical and readable manner. Your way of academic writing is evaluated as well as mathematics.

- (1) What is a difference between a Nash equilibrium and a subgame perfect equilibrium for a game in extensive form? In your view, which of the two equilibrium concepts is reasonable for a non-cooperative solution of a game in extensive form? Why? Provide your answers by giving a numerical example.
- (2) Consider an  $n$ -person game in strategic form where every player has a finite set of pure strategies. Explain a procedure to prove the existence of a Nash equilibrium in mixed strategies. If you use mathematical theorems, state them precisely. You need not to provide a complete proof.
- (3) Consider a 3-person game in extensive form above. Each player  $i$ ,  $i = 1, 2, 3$ , has only one information set indexed by  $i$  where he has two actions  $R$  and  $L$ . The numbers of each payoff vector indicate payoffs of players 1, 2 and 3 from top to bottom.
  - (i) Find all Nash equilibria in pure strategies. Your answer should explain why your specified strategy profiles are Nash equilibria, and also why other strategy profiles are not.

- (ii) Compute all sequential equilibria in pure strategies. Your answer should explain why your specified strategy profiles are sequential equilibria, and also why other strategy profiles are not. The notion of a consistent belief in the sense of Kreps and Wilson should be used.

### Problem 3

There are two agents and three alternatives. Let  $N = \{1, 2\}$  be the set of agents and  $X = \{x_1, x_2, x_3\}$  the set of alternatives. Each agent  $i \in N$  is endowed with a complete and transitive preference relation  $\succsim_i$ . Let  $\mathcal{R}$  be the set of all complete and transitive preference relations on  $X$ , and  $\mathcal{B}$  the set of all binary relations on  $X$ .

1. Consider the following rule to construct social preferences on alternatives. Agent 1's weak preferences  $\succsim_1$  are always accepted as social weak preferences. That is, for all  $x_i, x_j \in X$ , if  $x_i \succsim_1 x_j$ , then  $x_i$  is ranked socially at least as good as  $x_j$ . However, the alternative  $x_1$  is a special one, and agent 2 can "veto" socially ranking  $x_1$  strictly below another alternative  $x_j$  if he strictly prefers  $x_1$  to  $x_j$ . That is, for all  $x_j \in X$ , if  $x_1 \succ_2 x_j$ , then  $x_1$  is ranked socially at least as good as  $x_j$ .

Let us now formally define the above rule as the social preference function  $F : \mathcal{R}^N \rightarrow \mathcal{B}$ . For all  $\succsim_N = (\succsim_1, \succsim_2) \in \mathcal{R}^N$  and all  $x_i, x_j \in X$ ,  $x_i F(\succsim_N) x_j$  if and only if (i)  $x_i \succsim_1 x_j$  or (ii)  $x_i = x_1$  and  $x_i \succ_2 x_j$ . (Recall that  $x_i F(\succsim_N) x_j$  means that  $x_i$  is socially at least as good as  $x_j$  at  $\succsim_N$ .) Let  $F_P(\succsim_N)$  denote the strict part of  $F(\succsim_N)$ .

Answer whether each of the following statements (1)-(6) is true or false. If your answer is "true," then prove the statement. If your answer is "false," then provide a counter-example.

- (1)  $F$  is dictatorial.
- (2)  $F$  satisfies Weak Pareto.
- (3)  $F$  satisfies Pairwise Independence.
- (4) For all  $\succsim_N \in \mathcal{R}^N$ ,  $F(\succsim_N)$  is complete.
- (5) For all  $\succsim_N \in \mathcal{R}^N$ ,  $F(\succsim_N)$  is quasi-transitive (that is,  $F_P(\succsim_N)$  is transitive).
- (6) For all  $\succsim_N \in \mathcal{R}^N$ ,  $F(\succsim_N)$  is acyclic (that is,  $F_P(\succsim_N)$  contains no cycle).

2. Based on the social preference function  $F$  defined above, define the social choice function  $f : \mathcal{R}^N \rightarrow X$  as follows. For all  $\succsim_N \in \mathcal{R}^N$ , first let  $G(\succsim_N$

$\}) = \{x_i \in X \mid \forall x_j \in X, x_i F(\succsim_N) x_j\}$ , and then let  $f(\succsim_N)$  be the alternative  $x_i \in G(\succsim_N)$  such that for all  $x_j \in G(\succsim_N)$ ,  $i \leq j$ . That is, the social choice function  $f$  chooses the alternative with the smallest index in the set of the greatest elements of  $F(\succsim_N)$  in  $X$ .

Answer whether each of the following statements (7)-(10) is true or false. If your answer is “true,” then prove the statement. If your answer is “false,” then provide a counter-example.

(7)  $f$  is dictatorial.

(8)  $f$  is weakly Paretian.

(9)  $f$  is strategy-proof (or truthfully implementable in dominant strategies).

(10)  $f$  is monotonic.

受験番号	番
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平成27年度 一橋大学大学院経済学研究科博士後期課程編入学試験問題

## 統計学・計量経済学

実施日 平成26年9月11日(木)

試験時間 14:00～17:00

### 注意事項

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以上

# Statistics · Econometrics

Answer both problems 1 and 2 either in Japanese or English.

Problem 1. Answer EITHER problem 1-1 or problem 1-2.

1-1. (Probability and Statistics) Answer EITHER problem (a) or problem (b).

(a) Let  $X_1, \dots, X_n$  be independent and normally distributed random variables with  $E[X_i] = \xi_i \mu$  and  $Var[X_i] = \sigma^2$ . Here the  $(n+2)$  parameters  $\xi_i \in \{-1, +1\}$ ,  $\mu \geq 0$ , and  $\sigma^2 > 0$  are all unknown. Answer questions i and ii.

i. Find maximum likelihood estimators  $\hat{\xi}_i$ ,  $\hat{\mu}$ , and  $\hat{\sigma}^2$ .

ii. Show that  $\hat{\mu}$  and  $\hat{\sigma}^2$  are not consistent estimators of  $\mu$  and  $\sigma^2$  as  $n \rightarrow \infty$ .

(b) Let  $X_1, X_2, \dots$  be a sequence of i.i.d. random variables with

$$P\{X_i = 1\} = p = 1 - P\{X_i = -1\}, \quad 0 < p < 1.$$

Let  $S_0 = 0$  and  $S_n = \sum_{i=1}^n X_i$ ,  $n \geq 1$ . Prove that  $P\{S_n = 0 \text{ infinitely often}\} = 1$  if and only if  $p = 1/2$ . (You may use Stirling's formula  $n! \sim \sqrt{2\pi n} n^{n+1/2} e^{-n}$  as  $n \rightarrow \infty$ ).

1-2. (Econometrics) Answer EITHER problem (a) or problem (b).

(a) Consider the following dynamic model:

$$y_t = \beta_0 + \beta_1 x_t + \rho y_{t-1} + \varepsilon_t - \theta \varepsilon_{t-1}$$

for  $t = 1, 2, \dots, T$ , where  $|\rho| < 1$ ,  $|\theta| < 1$ ,  $\rho \neq \theta$  and  $\{\varepsilon_t\} \sim i.i.d.(0, \sigma^2)$ . Answer questions i – v.

i. Derive the short-run impact from  $x$  to  $y$ .

ii. Derive the long-run impact from  $x$  to  $y$ .

iii. Let  $\beta_1 = 0$ . Derive  $E[y_t]$ .

iv. Suppose that the lag polynomial  $(1 - \rho L)^{-1}(1 - \theta L)$  is expressed as

$$(1 - \rho L)^{-1}(1 - \theta L) = \sum_{j=0}^{\infty} \psi_j L^j,$$

where  $L$  is the lag operator. Express  $\psi_j$  for  $j = 0, 1, \dots$  using  $\rho$  and  $\theta$ .

v. Let  $\beta_1 = 0$ . Derive the variance and the first order autocovariance of  $y_t$ .

(b) Consider the following linear panel data model with a constant regressor

$$y_{it} = \beta + e_{it}, \quad \text{for } i = 1, 2, \dots, N \text{ and } t = 1, \dots, T,$$

where the errors have an individual component so that

$$e_{it} = \eta_i + u_{it}.$$

We assume that  $\eta_i \sim i.i.d.(0, \sigma_\eta^2)$  and  $u_{it} \sim i.i.d.(0, \sigma_u^2)$  and they are mutually independent. Answer questions i – iv.

- i. Derive the asymptotic variance of the pooled OLS estimator  $\hat{\beta}_{OLS}$  under  $N \rightarrow \infty$  and  $T$  fixed.
- ii. Explain how to construct the consistent variance estimator for  $\hat{\beta}_{OLS}$  step-by-step.

Consider now the following estimator of  $\beta$  that takes an average of cross-sectional OLS estimators over the sample period.

$$\tilde{\beta} = T^{-1} \sum_{t=1}^T \hat{\beta}_t,$$

where

$$\hat{\beta}_t = N^{-1} \sum_{i=1}^N y_{it}.$$

- iii. Derive the asymptotic variance of  $\tilde{\beta}$  under  $N \rightarrow \infty$  and  $T$  fixed. Does it coincide with the asymptotic variance of the pooled OLS estimator?
- iv. Explain if the variance estimator proposed in the literature

$$s_{\tilde{\beta}}^2 = T^{-1} \sum_{t=1}^T \frac{(\hat{\beta}_t - \tilde{\beta})^2}{T-1},$$

can consistently estimate the asymptotic variance of  $\tilde{\beta}$  as  $N \rightarrow \infty$  and  $T$  fixed. If not, write down additional assumptions that guarantee its consistency.

**Problem 2.** Answer EITHER problem 2-1 or problem 2-2.

2-1. (Probability and Statistics) Answer BOTH problem (a) and problem (b).

- (a) Let  $X_1, \dots, X_m, m \geq 4$  be i.i.d.  $N(\mu_1, \sigma_1^2)$  random variables and  $Y_1, \dots, Y_n, n \geq 2$  be i.i.d.  $N(\mu_2, \sigma_2^2)$  random variables independent of  $X_i$ 's. All parameters  $\mu_i, \sigma_i^2, i = 1, 2$  are unknown. Answer questions i and ii.
  - i. Find the uniformly minimum variance unbiased (UMVU) estimator of  $\sigma_1^2 - \sigma_2^2$ .
  - ii. Find the UMVU estimator of  $\sigma_2^2 / \sigma_1^2$ .

(b) In the following questions,  $\{X_n\}$  is a sequence of random variables on some probability space  $(\Omega, \mathcal{F}, P)$  such that  $X_n(\omega) \rightarrow X(\omega), \forall \omega \in \Omega$ . Suppose  $X_n$ 's and  $X$  are  $P$ -integrable. Answer questions i – iii.

i. Suppose  $X_n \geq 0$  and there exists a  $P$ -integrable random variable  $Y$  such that  $X_n(\omega) \leq Y(\omega), \forall \omega$  and  $\forall n$ . Prove that

$$\limsup_{n \rightarrow \infty} E[X_n] \leq E[X]$$

from the Monotone Convergence Theorem.

ii. Suppose  $X_n \geq 0$ . Prove that  $E[|X_n - X|] \rightarrow 0$  if and only if  $E[X_n] \rightarrow E[X]$ .

iii. Assume that  $X_n$  are not necessarily non-negative. Prove that  $E[|X_n - X|] \rightarrow 0$  if  $E[|X_n|] \rightarrow E[|X|]$ .

2-2. (Econometrics) Answer BOTH problem (a) and problem (b).

(a) Consider the following model:

$$y_i = c + \beta x_i + u_i \quad (1)$$

$$x_i = \gamma' z_i + e_i \quad (2)$$

$$u_i = \delta e_i + v_i, \quad (3)$$

for  $i = 1, 2, \dots, n$ , where  $z_i = [1, z_i^*]'$  with  $z_i^*$  being a one dimensional random variable,  $\gamma = [\gamma_1, \gamma_2]'$  with  $\gamma_1 = 0$  and  $\gamma_2 \neq 0$ , and  $[z_i^*, e_i, v_i]'$  is an i.i.d. random vector with mean  $[0, 0, 0]'$  and variance  $diag\{\sigma_z^2, \sigma_e^2, \sigma_v^2\}$ , having the finite fourth moment. The variables  $y_i$ ,  $x_i$  and  $z_i^*$  are observable while  $u_i$ ,  $e_i$  and  $v_i$  are unobservable. Suppose that we are interested in the statistical inference about  $\beta$ . Answer questions i – iv.

i. Explain what the following hypothesis implies:

$$H_0 : \delta = 0 \quad \text{vs.} \quad H_1 : \delta \neq 0. \quad (4)$$

ii. Let  $\hat{\gamma}$  be the OLS estimator of  $\gamma$  in (2). Derive the limiting distribution of  $\sqrt{n}(\hat{\gamma} - \gamma)$ .

iii. Let  $\hat{e}_i$  be the OLS residuals in (2). Derive the probability limits of

$$\frac{1}{n} \sum_{i=1}^n \hat{e}_i, \quad \frac{1}{n} \sum_{i=1}^n x_i \hat{e}_i, \quad \text{and} \quad \frac{1}{n} \sum_{i=1}^n \hat{e}_i^2.$$

iv. Consider the regression of  $y_i$  on a constant,  $x_i$  and  $\hat{e}_i$ , that is,

$$y_i = c + \beta x_i + \delta \hat{e}_i + \text{regression error}.$$

We would like to test (4) using the above regression. Derive the limiting distribution of  $\sqrt{n}(\hat{\delta} - \delta)$  under the null hypothesis.

(b) Consider the linear model with one unobservable regressor  $x_i^*$

$$y_i = \beta x_i^* + u_i, \quad (5)$$

for  $i = 1, 2, \dots, n$ . We are interested in consistently estimating the coefficient  $\beta$ . Suppose that observable random variable  $x_i$  is known to be related to  $x_i^*$  by the equation

$$x_i = x_i^* + e_i, \quad (6)$$

and  $x_i^*$  are determined by

$$x_i^* = \gamma' z_i, \quad (7)$$

where  $z_i$  is a vector of  $k$  non-random exogenous variables and  $\gamma$  is a vector of unknown parameters. In (5) and (6),  $u_i \sim i.i.d.(0, \sigma_u^2)$ ,  $e_i \sim i.i.d.(0, \sigma_e^2)$ , and  $E(u_i e_i) = \rho$ . We also assume

$$p \lim n^{-1} \sum_{i=1}^n z_i z_i' = Q,$$

exists and is positive definite. Answer questions i – iii.

- i. Discuss if the OLS estimator of  $\beta$  from the regression of  $y_i$  on  $x_i$  is consistent. If not, derive its asymptotic bias.
- ii. Explain how your result in (i) changes or does not change, if  $\rho = 0$ .
- iii. Show that an estimator of  $\beta$  obtained from the regression of  $y_i$  on  $\hat{x}_i$  where  $\hat{x}_i$  is a fitted value in the regression of  $x_i$  on  $z_i$  is consistent.



受験番号	番
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平成27年度 一橋大学大学院経済学研究科博士後期課程編入学試験問題

## 政治経済学

実施日 平成26年9月12日(金)

試験時間 9:30～12:30

### 注意事項

1. 「解答はじめ」の指示があるまでは問題冊子を開いてはいけない。
2. 問題冊子は1冊(本文1ページ)、解答用紙は2枚、下書き用紙は1枚である。試験開始後直ちに確認し、枚数が異なる場合は挙手のこと。
3. **すべての解答用紙・下書き用紙、問題冊子の表紙に受験番号を記入せよ(解答用紙の2枚目以降にも忘れずに記入すること)。氏名を記入してはならない。**なお、用紙は一切持ち帰ってはいけない。
4. 科目名を、解答用紙の科目欄に明記せよ。
5. 解答は横書きとする。解答用紙は裏面も使用できる。
6. 解答に際しては、原則として1題ごとに1枚の解答用紙を使用すること。
7. 解答用紙の追加配付を希望する受験生には、追加配付を認める。また、解答用紙を汚損した場合、全面的に書き直しを要する場合、解答用紙の交換を認める。解答用紙の追加、交換を求める際には、試験中、静かに挙手すること。
8. 辞書その他の持ち込みは許可しない。

以上

# 政治経済学

次の問(1)から問(6)のうち、2問を選択して解答しなさい。  
(解答の冒頭に、選択した問題の番号を明記すること。)

## 問(1)

労働価値説の意義について論じなさい。

## 問(2)

マルクス (K. Marx) の再生産表式における「資本循環と資本循環のからみ合い」および「資本循環と所得流通のからみ合い」について論じなさい。

## 問(3)

労働市場の規制緩和が労働者に及ぼす影響について論じなさい。

## 問(4)

現代経済における実物部門と金融部門の関連について論じなさい。

## 問(5)

経済学の分野では、社会的に望ましくない事象が発生する原因を、「市場の失敗 (market failure)」、「政府の失敗」 (government failure)、さらに近年では「制度の失敗」 (institutional failure) として説明されることが多い。

以下の設問に答えなさい。

- (1) 「市場の失敗」とはどのようなことか。具体的事例を挙げて、簡潔に説明しなさい。
- (2) 「政府の失敗」とはどのようなことか。具体的事例を挙げて、簡潔に説明しなさい。
- (3) 「制度の失敗」とはどのようなことか。具体的事例を挙げて、簡潔に説明しなさい。

## 問(6)

社会科学における「比較」の意義について論じなさい。

受験番号	番
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平成27年度 一橋大学大学院経済学研究科博士後期課程編入学試験問題

## 経 済 史

実 施 日 平成26年9月11日(木)

試験時間 14:00～17:00

### 注意事項

1. 「解答はじめ」の指示があるまでは問題冊子を開いてはいけない。
2. 問題冊子は1冊(本文1ページ)、解答用紙は2枚、下書き用紙は1枚である。試験開始後直ちに確認し、枚数が異なる場合は挙手のこと。
3. **すべての解答用紙・下書き用紙、問題冊子の表紙に受験番号を記入せよ(解答用紙の2枚目以降にも忘れずに記入すること)。氏名を記入してはならない。**なお、用紙は一切持ち帰ってはいけない。
4. 科目名を、解答用紙の科目欄に明記せよ。
5. 解答は横書きとする。解答用紙は裏面も使用できる。
6. 解答に際しては、原則として1題ごとに1枚の解答用紙を使用すること
7. 解答用紙の追加配付を希望する受験生には、追加配付を認める。また、解答用紙を汚損した場合、全面的に書き直しを要する場合、解答用紙の交換を認める。解答用紙の追加、交換を求める際には、試験中、静かに挙手すること。
8. 辞書その他の持ち込みは許可しない。

以上

# 経済史

以下の第1題～第3題のうち2題を選択して、それぞれ別紙を用いて解答しなさい。（解答用紙の冒頭に、選択した問題番号を明記すること）

## 第1題

以下の2つの問題について解答しなさい。

- (1) アブナー・グライフ『比較歴史制度分析』NTT出版(2009) (A. Greif, (2006) *Institutions and the Path to the Modern Economy: Lessons from Medieval Trade*. Cambridge University Press.)の分析視角に従って、制度とは何かを、簡潔に説明しなさい。
- (2) グライフ（前掲書）の11章「理論-歴史対話型の文脈に依存した分析」では、比較歴史制度分析における実証方法の有り方を説明している。それはどのようなものなのか、グライフ（前掲書）の第2章「自立的秩序による契約履行制度：マグリブ貿易商の結託」の事例を利用して簡潔に説明しなさい。

## 第2題

斎藤修『比較経済発展論』（岩波書店、2008年）で示されている「スミスの発展」の内容を説明し、それに即して、近世のヨーロッパと日本における経済発展のあり方を比較史的に論じなさい。

## 第3題

I. ウォーラースティンが提唱した「近代世界システム」論に即して、20世紀初頭までの「中核諸国」の展開過程を概観するとともに、「長い19世紀」における「中道自由主義」の歴史的意義を考察しなさい。