受験番号	番

平成２９年度　一橋大学大学院経済学研究科博士後期課程編入学試験問題

マ ク ロ 経 済 学

実 施 日　平成２８年９月８日（木）
試験時間　９：３０～１２：３０

注意事項

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２．問題冊子は１冊（本文６ページ）、解答用紙は４枚、下書き用紙は１枚である。試験開始後直ちに確認し、枚数が異なる場合は挙手すること。

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以上

Macroeconomics

There are four problems.

Answer all of them either in English or in Japanese.

Problem 1

Consider the standard overlapping generations model with capital income tax. Time is discrete. The economy is closed. Each household lives for two periods. Denoting the number of households of generation *t* (those who are born in period *t*) as L_t, we have $L_{t+1} = (1+n)L_t$ ($n > 0$) for all *t*. Each household of generation *t* supplies 1 unit of labor inelastically in period *t* and earns wage w_t, consumes $c_{1,t}$ and the rest of its wage income becomes its saving, s_t. This saving can be held either in the form of bonds (the interest rate is denoted r_{t+1}) or capital stock. There is a tax on income from the latter. In period *t*+1, it is unable to work, and thus there is no labor income. Its consumption, $c_{2,t+1}$, is equal to the saving plus its returns (net of taxes), plus lump sum transfers distributed by the government. The household utility is:

$$U_t = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}), \quad 1 > \beta > 0.$$

On the production side, the representative firm's production function is:

$$Y_t = A \cdot K_t^{\alpha} \cdot L_t^{1-\alpha}, \text{ where } 1 > \alpha > 0.$$

In the above, Y_t, K_t and L_t are output, capital stock and labor. Assume that capital stock depreciates by 100% each period.

The government imposes capital income tax on the old and distributes the revenues to the old as lump-sum transfers. Denoting the capital income tax rate (assumed to be constant over time) by τ, lump-sum transfers (note this is aggregate, not per capita) by T_t and the rental rate of capital by R_t, the government budget constraint is given by

$$\tau \cdot R_t \cdot K_t = T_t \quad .$$

(1.1) Why do we say $1 + r_{t+1} = (1-\tau)R_{t+1}$ holds?

(1.2) Write down the budget constraints for a generation *t* household, for each period, separately. Then combine the two to derive its lifetime budget constraint.

(1.3) Solve the household's optimization problem and derive an expression for s_t.

(1.4) Why do we say that $K_{t+1} = s_t L_t$?

(1.5) Derive the equilibrium law of motion $k_{t+1} = G(k_t)$ (where $k_t \equiv K_t / L_t$, and the initial condition is given by $k_0 = \overline{k_0} > 0$). Is the steady state unique? Is it stable?

(1.6) State the definition of "Golden Rule". Can the government achieve the Golden Rule by setting τ appropriately? If yes, is such a policy desirable? If no, explain the intuition why the Golden Rule cannot be achieved in this model.

Problem 2

Consider a closed-economy, continuous time growth model with two sectors, the consumer goods producing sector ("C sector") and the investment goods producing sector ("I sector"). There is no government. In each of the two sectors, firms are identical, and the number of firms in each sector is normalized to be equal to 1. We denote outputs of those sectors at time t by Y_{Ct} and Y_{It}, respectively. Production functions are given by:

$$Y_{Ct} = K_{Ct} \text{ and } Y_{It} = A_t \cdot K_{It}^{\alpha} \cdot G^{1-\alpha}, \text{ where } 1 > \alpha > 0. \qquad (2\text{-}1)$$

In the above, K_{Ct} and K_{It} represent capital stock used in each of the sectors, and G is land and it is in fixed supply. For simplicity, we assume G=1. In equilibrium,

$$K_{Ct} + K_{It} = K_t, \qquad (2\text{-}2)$$

where K_t is total capital stock at time t. The productivity in the I sector is determined by:

$$A_t = B \cdot \bar{K}_t^{1-\alpha}, \text{ where } B > 0. \qquad (2\text{-}3)$$

In equation (2-3), the term $\bar{K}_t$ denotes aggregate capital stock, and it represents externality from the aggregate stock of knowledge which is accumulated along with capital stock. Each firm (or household) takes this variable as given. We shall normalize the price of a C good to be equal to 1, and denote the price of an I good by P_{It}.

The representative household is characterized by the logarithmic utility function:

$$U = \int_0^{\infty} e^{-\rho t} \cdot \ln(C_t)\, dt, \text{ where } \rho > 0. \qquad (2\text{-}4)$$

It owns and rents capital and earns the rental rate of capital which is equal to R_t per unit. It also has access to bonds with the interest rate r_t. (It also rents land to the I sector firms and earns land rents.) In equilibrium, we must have:

$$C_t = Y_{Ct}, \text{ and } \dot{K}_t = Y_{It} - \delta K_t, \text{ where } \delta > 0. \qquad (2\text{-}5)$$

Note that, in equilibrium, $K_t = \bar{K}_t$ must hold. In what follows, we shall focus on the steady state in which growth rates of all the variables are constant over time (but not necessarily the same across variables). We denote the growth rate of variable X_t by γ_X.

(2.1) Why do we say that, in the steady state, we must have $K_{It} = z \cdot K_t$, where z is an unknown constant (1>z>0)?

(2.2) Show that, as R_t has to be equal between the sectors, γ_{P_I} =0 must hold.

(2.2) Explain intuitively why, in equilibrium, the following relationship has to hold:

$$r_t = (R_t - \delta P_{It} + \dot{P}_{It}) / P_{It} .$$

Show that this implies $r_t = \alpha B z^{-(1-\alpha)} - \delta$.

(2.4) Using the household's Euler equation, $\gamma_C = r_t - \rho$ (you do not need to prove it), and other conditions we have shown, derive the equilibrium condition that z must satisfy (you may not be able to obtain an explicit solution in the form "z=..."). Express γ_K as a function of z. (Note: you may need to impose a certain restriction on the parameters to ensure existence of the equilibrium).

(2.5) Set up and solve the social planner's problem and derive the steady state condition for z. Explain intuitively why z (and thus γ_K) is larger than in the competitive equilibrium.

Problem 3

Consider the following dynamic labor demand problem of a firm that produces a product y_t with a production function $y_t = n_t x_t$ and sells it in the competitive product market. Variable x_t is an increasing function with respect to the one-period past employment n_{t-1}, i.e., $x_t = n_{t-1}$. This reflects a learning-by-doing mechanism: the more the one-period past employment, the higher the current labor productivity. The firm optimally hires employment n_t from the competitive labor market at the competitive real wage w_t. The firm maximizes the following present values of expected future profits:

$$E_t \sum_{i=0}^{\infty} \beta^i \{n_{t+i} x_{t+i} - w_{t+i} n_{t+i}\}, \quad 0 < \beta < 1,$$

where β is the discount factor. Assume that the firm's expectation formation is rational; hence E_t is the mathematical expectation operator conditional on the information set on date t. The firm internalizes (i.e., takes into account) the effect of learning-by-doing when solving its profit maximization problem.

(a) Write the Bellman equation for the firm's problem. Derive the first-order necessary condition and the envelope condition (i.e., the Benveniste and Scheinkman condition).

(b) Derive the Euler equation (i.e., the intertemporal optimality condition) with respect to employment n_t. Discuss the economic intuition behind the Euler equation.

Suppose that there are a large number of the identical firms in the competitive labor market. The aggregate labor demand N_t is given as $N_t = n_t$. Suppose also that there are a large number of identical households in the competitive labor market. The aggregate labor supply L_t is given as an increasing function of the competitive real wage

$$w_t = a_t + AL_t,$$

where A is a positive constant and a_t is an aggregate labor supply shock following a stochastic process given below.

(c) What is the rational expectations equilibrium (REE) in this competitive labor market? Define the REE paths of employment and wage, $\{n_{t+i}\}_{i=0}^{\infty}$ and $\{w_{t+i}\}_{i=0}^{\infty}$.

(d) Suppose that $A = 1 + \beta$. Given the initial condition n_{t-1}, derive the REE path of labor demand $\{n_{t+i}\}_{i=0}^{\infty}$ under a suitable transversality condition.

(e) Suppose that the stochastic aggregate labor supply shock follows an AR(1) process, $a_t = \rho a_{t-1} + \epsilon_t$, where $0 < \rho < 1$ and ϵ_t is an i.i.d. shock. Derive the REE policy function of the current employment as a linear function of the state variables n_{t-1} and a_t. Show how the REE employment responds to a positive aggregate labor supply shock. Discuss the economic intuition behind the response.

Problem 4

Suppose that the stochastic discount factor of a habit-forming consumer is given as

$$M_{t+1} = \beta \left(\frac{C_{t+1} H_{t+1}}{C_t H_t} \right)^{-\gamma}, \quad 0 < \beta < 1, \quad \gamma > 0$$

where $H_t \equiv C_{t-1}^{-\eta}$ describes the degree of habit formation with $\eta > 0$.

Consider the goods market clearing condition for the economy with a representative household, $C_t = Y_t$, where Y_t is the endowment. Suppose that the log of the endowment follows a random walk with drift:

$$\log Y_t = \mu + \log Y_{t-1} + \sigma_y \epsilon_t,$$

where μ and σ_y are constants and ϵ_t is an i.i.d. standard normal random variable. Notice that Y_t/Y_{t-1} follows a log-normal distribution.[1]

Using the property of the log-normal distribution of Y_t/Y_{t-1}, calculate the log of the risk-free rate in equilibrium, $\log R_{f,t}$, which satisfies the fundamental asset pricing equation $E_t M_{t+1} R_{f,t} = 1$. Discuss then how and why the current level of the endowment growth rate, $\Delta \log Y_t$, affects the log of the risk-free rate in equilibrium.

[1]For your information, a log-normal random variable X_{t+1} is defined as follows. If $\ln X_{t+1} \sim N(E_t \ln X_{t+1}, Var_t(\ln X_{t+1}))$, then $E_t X_{t+1} = \exp(E_t \ln X_{t+1} + \frac{1}{2} Var_t(\ln X_{t+1}))$.

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以上

Microeconomics

September 9, 2016

There are three problems. Answer all problems either in Japanese or in English.

Problem 1

There are two subproblems.

1. [Slutsky Equations]

There are L commodities. A price vector is denoted by $p = (p_1, \cdots, p_l, \cdots, p_L)$. Let $m \in \mathbb{R}_+$ be a consumer's income level and $v \in \mathbb{R}$ be a consumer's utility level. The consumer's demand function and compensated (Hicksian) demand function are denoted by $x(p, m) = (x_1(p, m), \cdots, x_l(p, m), \cdots, x_L(p, m))$ and $h(p, v) = (h_1(p, v), \cdots, h_l(p, v), \cdots, h_L(p, v))$, respectively. Answer the following questions.

(1) By using the duality between the utility maximization and expenditure minimization, derive the Slutsky equation of commodity l with respect to price p_k.

From now on, assume $L = 2$. Assume also that the consumer's preference is represented by a utility function $u(x_1, x_2) = x_1 + \sqrt{x_2}$.

(2) Derive a demand function for each commodity. (You can assume an interior solution for the utility maximization problem.)

(3) Derive the Slutsky matrix explicitly.

2. [Pareto Efficiency and Walrasian Equilibrium]

Consider a pure exchange economy with two consumers $i = 1, 2$ and two commodities x and y. Consumer i's consumption vector is denoted by (x_i, y_i). Two consumers have identical preferences, which are represented by $u(x_i, y_i) = \sqrt{x_i y_i}$, $i = 1, 2$. The aggregated endowment of commodities is denoted by $\bar{x}$, $\bar{y} > 0$. Answer the following questions.

(1) Formulate the following Pareto programming problem: Find an allocation that maximizes consumer 1's utility level subject to meeting some reservation utility level, denoted by v, of consumer 2 and the feasible condition of resources.

(2) Given the level of $v > 0$, derive a solution to the Pareto programming problem. (You can assume an interior solution.)

(3) To achieve the above solution as a Walrasian equilibrium allocation in a private ownership economy, how should the initial endowment be distributed across consumers? Derive also the relative price associated with the Walrasian equilibrium.

Problem 2

Answer all questions below in a clear, logical and readable manner. Your way of academic writing is evaluated as well as mathematics.

(1) Player $i \in \{1, \ldots, n\}$ chooses a real number $a_i \in \mathbb{R}$. Let

$$a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

denote a strategy profile. Player i's payoff function is

$$u_i(a) = -q_{ii}a_i^2 - 2\sum_{j \neq i} q_{ij}a_i a_j + 2\theta_i a_i,$$

where q_{ij} and θ_i are constant for $i, j \in N$. Obtain a necessary and sufficient condition for the existence of a unique Nash equilibrium and calculate the equilibrium. In answering the question, you can use the following notations:

$$Q = \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{n1} & \cdots & q_{nn} \end{pmatrix} \text{ and } \theta = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix}.$$

(2) Firm $i \in \{1, \ldots, n\}$ produces $a_i \geq 0$ units of a homogenous product. The inverse demand function is $1 - \sum_{i=1}^{n} a_i$. For each firm, the production cost is assumed to be zero. Consider the following game. First, k firms simultaneously decide how much to produce. Next, after observing their decisions, the remaining $n-k$ firms simultaneously decide how much to produce.

(i) Obtain a subgame perfect equilibrium of this game.

(ii) Obtain the integer k that minimizes the total output $\sum_{i=1}^{n} a_i$.

(iii) Obtain the integer k that maximizes the total output $\sum_{i=1}^{n} a_i$.

Problem 3

Suppose that in an international economy, there are two nations, A and B; two commodities, 1 and 2; and two primary factors, capital K and labour L. Let $f^i : \mathbb{R}^2_+ \to \mathbb{R}_+$ be a production technology to produce commodity $i = 1, 2$ by inputting the two factors such as: for each $(V_K, V_L) \in \mathbb{R}^2_+$, $f^i(V_K, V_L) = x_i \in \mathbb{R}_+$, where x_i represents an amount of commodity $i = 1, 2$; and V_j represents an amount of the primary factor $j = K, L$. Assume that for each $i = 1, 2$, f^i is continuous, strongly monotonic, concave, homogeneous of degree one, and $f^i(0, 0) = 0$. Assume also that the production of commodity 1 is relatively more intensive in capital than the production of commodity 2.

Let each nation $\nu \in \{A, B\}$ be endowed with one unit of labour and $\omega^\nu_K (> 0)$ amount of capital. Suppose that $\omega^A_K > \omega^B_K$. Assume that both nations have a common preference relation $\succsim$ on the common consumption space $C = \mathbb{R}^2_+$ such that which is complete and transitive, and continuous, strongly monotonic, convex, and homothetic on C. Let $u : C \to \mathbb{R}$ be a numerical representation of the preference relation $\succsim$.

Suppose that the two factors are not mobile across nations and each nation is a price taker with respect to the international prices of the two commodities. Given such an international economy, let

$$\left(p, (r^\nu, w^\nu)_{\nu=A,B}\, ; (x^\nu_1, x^\nu_2)_{\nu=A,B}\, ; (c^\nu_1, c^\nu_2)_{\nu=A,B}\right)$$

be an international free trade equilibrium, where $p = (p_1, p_2) \in \mathbb{R}^2_{++}$ is an international equilibrium price vector of commodities; (r^ν, w^ν) is nation ν's domestic equilibrium price vector of the two factors, where r^ν is an equilibrium interest rate and w^ν is an equilibrium wage rate in the domestic factor markets of nation ν; (x^ν_1, x^ν_2) is nation ν's supply of commodities; and (c^ν_1, c^ν_2) is nation ν's consumption demand of commodities. Assume that $(x^\nu_1, x^\nu_2) > 0$ holds for both nations $\nu = A, B$. Then:

(i) prove that in this free trade equilibrium, the factor prices are equalized between these two nations.

(ii) prove that in this free trade equilibrium, nation A exports commodity 1 and imports commodity 2, while nation B exports commodity 2 and imports commodity 1.

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以上

Statistics · Econometrics

Answer both problems 1 and 2 either in Japanese or English.

Problem 1. Answer EITHER problem 1-1 or problem 1-2.

1-1. (Probability and Statistics) Answer EITHER problem (a) or problem (b).

(a) Let $U_1, \cdots, U_n$ and $V_1, \cdots, V_n$ be mutually independent i.i.d. $N(0,1)$ random variables. Define

$$X_i = U_i, \quad Y_i = \rho U_i + (1-\rho^2)^{1/2} V_i, \quad \text{for } i = 1, \cdots, n,$$

where $|\rho| < 1$. Answer questions i-iv.

i. Find the joint distribution of (X_i, Y_i).

ii. Prove that

$$R_n = \frac{\sum_{i=1}^n X_i Y_i}{\sqrt{\sum_{i=1}^n X_i^2}\sqrt{\sum_{i=1}^n Y_i^2}}$$

is a consistent estimator of ρ.

iii. Prove that the limiting distribution as $n \to \infty$ of

$$T_n = \frac{1}{\sqrt{n}} \sum_{i=1}^n \left(X_i Y_i - \frac{\rho}{2} X_i^2 - \frac{\rho}{2} Y_i^2 \right)$$

is $N(0, (1-\rho^2)^2)$.

iv. Using the results in question iii, find the limiting distribution of $\sqrt{n}(R_n - \rho)$.

(b) Answer questions i and ii. Prove your answer carefully.

i. Let $X_1, X_2, \cdots$ be a sequence of i.i.d. random variables such that $E|X_1| < \infty$. Let $S_n = \sum_{i=1}^n X_i, n \geq 1$. Find $E[X_1|\sigma(S_n, S_{n+1}, S_{n+2}, \cdots)]$.

ii. Let X and Y be random variables such that $E|X| < \infty$ and $E|Y| < \infty$. Suppose $E[X|\sigma(Y)] = Y$ a.s. and $E[Y|\sigma(X)] = X$ a.s. Show that $X = Y$ a.s.

1-2. (Econometrics) Answer EITHER problem (a) or problem (b).

(a) The data is generated from the linear model:

$$y = X_1\beta + X_2\gamma + u, \tag{1}$$

where y is an $n \times 1$ vector of dependent variable, X_1 and X_2 are $n \times k$ matrices of regressors where $X = [X_1 \ X_2]$ has full column rank and

$$\frac{1}{n} X'X \xrightarrow{p} \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}.$$

The coefficients β and γ are $k \times 1$ vectors and u is an $n \times 1$ vector of $i.i.d.$ errors with $E(u) = 0$ and $Var(u) = \sigma^2 I_n$. Model (1) may have endogeneity so that $E(X_1'u) \neq 0$. Denote the ordinary least squares (OLS) coefficient estimate of (1) by $\hat{\beta}$ and $\hat{\gamma}$. We consider the regression model with omitted variables

$$y = X_1\beta + error. \tag{2}$$

Answer questions i-v.

i. Show that the OLS estimator $\tilde{\beta}_{OLS}$ of (2) satisfies

$$\tilde{\beta}_{OLS} = \hat{\beta} + (X_1'X_1)^{-1}X_1'X_2\hat{\gamma}. \tag{3}$$

ii. Explain all the conditions that X_2 becomes valid instrumental variables (IV) to consistently estimate β using (2).

iii. Show that the IV estimator $\tilde{\beta}_{IV}$ of (2) using X_2 as instruments satisfies

$$\tilde{\beta}_{IV} = \hat{\beta} + (X_2'X_1)^{-1}X_2'X_2\hat{\gamma}. \tag{4}$$

iv. Suppose that the conditions in question ii are all satisfied. Under $E(X_1'u) = 0$, show

$$AVar(\tilde{\beta}_{IV} - \tilde{\beta}_{OLS}) = AVar(\tilde{\beta}_{IV}) - AVar(\tilde{\beta}_{OLS}),$$

where $AVar$ denotes the asymptotic variance.

v. Suppose that the conditions in question ii are all satisfied. Construct the Hausman test statistic for the null hypothesis $H_0 : E(X_1'u) = 0$. It is important to express your final answer only by data y, X_1, and X_2.

(b) Let us consider the following AR(2) model:

$$y_t = \mu + \phi y_{t-2} + u_t, \quad \text{for } t = 1, 2, \cdots, T, \tag{5}$$

where $|\phi| < 1$ and u_t are independent and identically distributed with mean 0 and variance σ^2 with $E(u_t^4) < \infty$. Answer questions i-iv.

i. Derive the (unconditional) mean of y_t.

ii. Let $\gamma(h) = Cov(y_t, y_{t-h})$ for $h = 0, \pm 1, \pm 2, \cdots$ be the autocovariances of y_t. Derive $\gamma(0)$, $\gamma(1)$, and $\gamma(2)$.

iii. Suppose that (5) is the true model but we estimate the following AR(1) model:

$$y_t = \mu' + \phi' y_{t-1} + u_t'.$$

Derive the probability limit of the OLS estimator of ϕ'.

iv. Suppose that model (5) includes an exogenous regressor x_t as follows:

$$y_t = \mu + \beta x_t + \phi y_{t-2} + u_t.$$

Derive the short-run and long-run effects (the short-run and long-run multipliers) of x on y.

Problem 2. Answer EITHER problem 2-1 or problem 2-2.

2-1. (Probability and Statistics) Answer BOTH problem (a) and problem (b).

(a) Let $X_1, \cdots, X_n$ be i.i.d. $N(\theta, \theta^2), \theta > 0$, random variables,where $\theta > 0$ is unknown. Answer questions i-iii.

i. Find a minimal sufficient statistic for θ.

ii. Prove that no sufficient statistic for this problem is complete.

iii. Consider unbiased estimators of θ^2;

$$\delta_1 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$

and

$$\delta_2 = \frac{\sum_{i=1}^n X_i^2}{2n}.$$

Compare the variance of δ_1 and δ_2.

(b) Answer questions i and ii.

i. State the assertions of Borel-Cantelli lemmas.

ii. Give an example of random variables X_n and real numbers $0 < b_n \uparrow \infty$ such that $P\{X_n \in (a, b]\} > 0$, for any $0 \leq a < b < \infty$ and $n \in \mathrm{N}$, $P\{X_n > 2b_n \text{ i.o.}\} = 0$ and $P\{X_n > b_n \text{ i.o.}\} = 1$.

2-2. (Econometrics) Answer BOTH problem (a) and problem (b).

(a) Let us consider the following simple structural change model:

$$y_t = \begin{cases} \mu_1 + u_t, & \text{for} \quad t = 1, \cdots, T_b^0 \\ \mu_2 + u_t, & \text{for} \quad t = T_b^0 + 1, \cdots, T, \end{cases} \tag{6}$$

where u_t are independent and identically distributed with mean 0 and variance σ^2 with $E(u_t^4) < \infty$. We would like to test if a constant term changes from μ_1 to μ_2. That is, the testing problem is given by

$$H_0: \ \mu_1 = \mu_2 \quad \text{vs.} \quad H_1: \ \mu_1 \neq \mu_2. \tag{7}$$

Suppose that we can specify the break date T_b^0 (the known break point case) and that $T_b^0/T \to \lambda_0$ as $T \to \infty$ $(0 < \lambda_0 < 1)$. Answer questions i-v.

i. Let $\hat{\mu}_1$ and $\hat{\mu}_2$ be the OLS estimators of μ_1 and μ_2. Derive the limiting distribution of $[\sqrt{T}(\hat{\mu}_1 - \mu_1), \sqrt{T}(\hat{\mu}_2 - \mu_2)]'$.

ii. Let $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^T \hat{u}_t^2$ where $\hat{u}_t$ are the OLS residuals. Write down the Wald test statistic for the testing problem (7) using $\hat{\mu}_1$, $\hat{\mu}_2$, and $\hat{\sigma}^2$.

iii. Derive the limiting distribution of the Wald test statistic in question ii under the null hypothesis.

iv. Show that the Wald test statistic in question ii diverges to infinity under the alternative hypothesis.

v. Suppose that we cannot specify the break date T_b^0 (the unknown break point case). Explain how to test for (7) in this case (you do not have to derive the limiting distribution).

(b) Consider the linear regression model:

$$y_i = x_i\beta + u_i, \quad \text{for } i = 1, ..., n, \tag{8}$$

where y_i is the dependent variable, x_i is a scalar of deterministic regressor and β is a coefficient. The errors u_i are independent and identically distributed random variables with mean zero and variance σ^2. We are interested in testing for the null hypothesis $H_0 : \beta = 0$ against the alternative hypothesis $H_1 : \beta \neq 0$. Answer questions i-iv.

i. Write down the t-test statistic by using the OLS estimator of β in (8).

For questions ii-iv, let the true value of β be a sequence of alternative hypotheses $\beta = c/\sqrt{n}$ where c is a non-zero constant.

ii. Derive the limit distribution of the t-test statistic.

iii. Notice that β converges to zero as n goes to infinity. Explicitly derive the probability of the test rejecting the null hypothesis $H_0 : \beta = 0$ in the limit.

iv. Explain how the probability in question iii varies when c becomes larger in absolute value.

受験番号	番

平成２９年度 一橋大学大学院経済学研究科博士後期課程編入学試験問題

政 治 経 済 学

実 施 日　平成２８年9月９日（金）
試験時間　９：３０～１２：３０

注意事項

１．「解答はじめ」の指示があるまでは問題冊子を開いてはいけない。

２．問題冊子は１冊**（本文１ページ）、解答用紙は２枚**、下書き用紙は１枚である。試験開始後直ちに確認し、枚数が異なる場合は挙手すること。

３．**すべての解答用紙・下書き用紙、問題冊子の表紙に受験番号を記入せよ（解答用紙の２枚目以降にも忘れずに記入すること）。氏名を記入してはならない。**なお、用紙は一切持ち帰ってはいけない。

４．科目名を、解答用紙の科目欄に明記せよ。

５．解答は横書きとする。解答用紙は裏面も使用できる。

６．解答に際しては、原則として１題ごとに１枚の解答用紙を使用すること。

７．解答用紙の追加配付を希望する受験生には、追加配付を認める。また、解答用紙を汚損した場合、全面的に書き直しを要する場合、解答用紙の交換を認める。解答用紙の追加、交換を求める際には、試験中、静かに挙手すること。

８．辞書その他の持ち込みは許可しない。

以上

政治経済学

次の問(1)から問(5)のうち，２問を選択して解答しなさい。
（解答の冒頭に，選択した問題の番号を明記すること。）

問(1)

マルクス（K. Marx）の『資本論』第１部の商品論・貨幣論の観点から，商品が価格形態を持つことの経済的意義について説明しなさい。ただし，労働生産物が商品として扱われている場合を前提とする。

問(2)

「労働力の価値または価格の労働賃金への転化」の理論的意義について論じなさい。

問(3)

現代経済の分析における「貨幣資本の過剰」の概念の意義について論じなさい。

問(4)

カップ（K.W. Kapp）の社会的費用（social cost）概念には，２種類の定義がある。両定義を示し，それぞれの定義が環境政策のどのような場面で有効となるかを，簡潔に説明しなさい。

問(5)

中東欧・旧ソ連諸国の一部が採用したいわゆる「大衆私有化」（mass privatization）とは，どのような特徴を持つ政策であり，この政策によって民営化された旧国有企業の所有構造や経営体制にいかなる影響が及んだのかを論じなさい。

受験番号	番

平成２９年度 一橋大学大学院経済学研究科博士後期課程編入学試験問題

経 済 史

実 施 日　平成２８年9月9日（金）
試験時間　１４：００～１７：００

注意事項

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８．辞書その他の持ち込みは許可しない。

以上

経済史

　第１題～第３題のうち２題を選択して、それぞれ別紙を用いて解答しなさい（解答用紙の冒頭に、選択した問題番号を明記すること）。

第１題

（１）アブナー･グライフ（岡崎哲二監訳）『比較歴史制度分析』NTT出版（2009）（Greif, Avner (2006) *Institutions and the Path to the Modern Economy: Lessons from Medieval Trade*. Cambridge University Press.）に従って、制度とは何か、また制度変化のメカニズムはどのようなものかを、簡潔に説明しなさい。

（２）上記（１）を利用して、グライフ（前掲書）の10章「個人的関係に依存しない取引の制度的基礎」で取り上げられているヨーロッパにおける共同体責任制の要点を説明しなさい。

第２題

　斎藤修『比較経済発展論』（岩波書店、2008 年）で示されている「スミス的発展」を簡潔に説明するとともに、それに即して、任意の近世社会における経済発展のあり方を論じなさい。

第３題

　ケネス・ポメランツ『大分岐―中国、ヨーロッパ、そして近代世界経済の形成―』（名古屋大学出版会、2015 年）において、ポメランツが定義する「大分岐」の内容を説明するとともに、この概念がグローバル・ヒストリー研究においていかなるインプリケーションを有するのかを論じなさい。