

ECONOMIC WELFARE AND COMPETITION IN OLIGOPOLICTIC MARKETS

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## SUMMARY

This paper examines the functional relationship between economic welfare and competition in a quasi-Cournot market. In the analysis, conjectural variation and a firm's profit are adopted as measures of competition, which also specify the state of the market. Conjectural variation indicates the extent of interdependence of firms and thus that of intra-industry competition. On the other hand, a firm's profit indicates the extent of entry barrier and thus that of inter-industry competition. We view economic welfare as a function of these parameters and induce conditions for increasing it. Our result asserts that the welfare function is not necessarily a monotone function of competition, though the most competitive market attains maximum welfare, which can be regarded as a generalization of the excess entry theorem by Suzumura and Kiyono (1987).

## 1. INTRODUCTION

Recent studies in theoretical industrial organization literature have uncovered several instances which are incompatible with the widely held belief in the welfare-enhancing effect of increasing competition. The excess entry theorem by Suzumura and Kiyono (1987) is one such instance, which insists that welfare can be increased by reducing the number of firms under some conditions. In the meantime, what the instances really imply, contrasted with the meaning of a fundamental theorem of welfare economics, is not so apparent, because only a rather limited aspect of the relationship between economic welfare and competition has been focused on.

In these circumstances, based on a more general framework, this paper examines the functional relationship between economic welfare and competition in a quasi-Cournot market, trying to throw light on the essential aspect of the excess entry theorem.

In the analysis, conjectural variation and a firm's profit are adopted as measures of competition, which also specify the state of the market. Conjectural variation indicates the extent of the interdependence of firms and can be regarded as a parameter for intra-industry competition. On the other hand, a firm's profit indicates the extent of entry barriers and can be regarded as a parameter for inter-industry competition. We view

economic welfare as a function of these parameters and induce conditions for increasing welfare. Our result asserts that the welfare function is not necessarily a monotone function of a two-dimensional parameter of competition, which means more competition does not necessitate more welfare, though the most competitive market attains maximum welfare. This can be regarded as a generalization of the excess entry theorem.

## 2. MODEL

This paper concerns a quasi-Cournot market model. In the model, all firms are assumed to be identical both in terms of technology and behavior and produce a homogeneous single good. The number of firms in the industry is denoted by  $n$ , which will be treated as a positive real number. The output of the  $i$ -th firm is denoted by  $q_i$  ( $i=1, 2, \dots, n$ ), and  $Q = \sum q_i$  represents the total output of the industry. The inverse demand function and cost function of each firm are represented by  $f(Q)$  and  $c(q_i)$ , respectively. The profit of the  $i$ -th firm is denoted by  $\pi_i \equiv q_i f(Q) - c(q_i)$ , and conjectural variation by  $\lambda_i \equiv \partial Q / \partial q_i$ , where  $\pi_i \geq 0$  and  $0 \leq \lambda_i \leq 1$  are assumed.

Throughout this paper, only symmetric quasi-Cournot equilibrium is examined, thus any suffix for firms will be omitted. Therefore, for example,  $q=q_i$  and  $Q=nq$ .

Concerning  $f$  and  $c$ , the following is assumed:

A1:  $f(Q) > 0, f'(Q) < 0$  for  $\forall Q \geq 0$  ;

A2:  $c(q) > 0, c'(q) > 0, c''(q) > 0$  for  $\forall q \geq 0$  ;

A3:  $f' + qf'' < 0$  .

The right side of A3 is concerned with the strategic substitutability (see Suzumura (1993), for example) and the following inequalities are obtained:

(1)  $\alpha \equiv \alpha(q, n, \lambda) = 2f'\lambda + qf''\lambda^2 - c'' < 0$  ,

(2)  $\beta \equiv \beta(q, n, \lambda) = f'\lambda + qf''\lambda^2 < 0$  .

The condition for maximizing firm's profit is:

(3)  $\frac{\partial \pi}{\partial q} = f(nq) - c'(q) + qf'(nq)\lambda = 0$  ,

(4)  $\frac{\partial^2 \pi}{\partial q^2} = 2f'(nq)\lambda + qf''(nq)\lambda^2 - c''(q) < 0$  ,

where (4) holds from A3.

Economic welfare is defined as the net market surplus, the sum of the consumers' surplus and the producers' surplus:

$$W(n, q) = \int_0^{nq} f(x) dx - nc(q) .$$

The necessary condition for maximizing  $W$  is:

$$\left( \frac{\partial W}{\partial n} \right)_q = qf - c = 0 ,$$

$$\left( \frac{\partial W}{\partial q} \right)_n = n(f - c') = 0 .$$

Before the analysis, it is necessary to parametrize the space of market equilibrium in order to construct a welfare function. Several types of parametrization are possible: for example, for given  $(n, \lambda)$ ,  $q$  and  $\pi$  are obtained by solving equation (3) and  $\pi = qf(nq) - c(q)$ , which means market equilibrium is specified by  $(n, \lambda)$ .

This paper adopts  $(\lambda, \pi)$  as a parameter for market equilibrium. For given  $(\lambda, \pi)$ ,  $q$  and  $n$  are obtained by solving equation (3) and  $\pi = qf(nq) - c(q)$ , which means that market equilibrium is specified by  $(\lambda, \pi)$ .  $(\lambda, \pi)$  can also be regarded directly as an indicator for competition. Conjectural variation  $\lambda$  indicates the extent of the interdependence of firms and thus that of intra-industry competition;  $\lambda = 0$  represents perfect competition and a larger  $\lambda$  implies lower intra-industry competition. On the other hand, the firm's profit indicates the extent of

entry barriers and thus that of inter-industry competition;  $\pi=0$  represents competition with no entry barriers, which is named free-entry competition in this paper, and a larger  $\pi$  implies lower inter-industry competition.

Before using this parametrization for analysis, we must show several relationships between  $(\lambda, \pi)$  and other variables, especially  $n$  and  $q$ , since welfare  $W$  is defined as the function of  $n$  and  $q$ .

First, let us consider the relationship between  $q$  and  $(n, \lambda)$ . It can be naturally inferred that more firms will reduce the product of each as will their greater interdependence, as is shown in the following lemma.

**Lemma 1**

$$\frac{\partial q(n, \lambda)}{\partial n} = \left( \frac{\partial q}{\partial n} \right)_{\lambda} = \frac{-\beta/\lambda}{\alpha + (n/\lambda - 1)\beta} q < 0 ,$$

$$\frac{\partial q(n, \lambda)}{\partial \lambda} = \left( \frac{\partial q}{\partial \lambda} \right)_{n} = \frac{-f'}{\alpha + (n/\lambda - 1)\beta} q < 0 .$$

**Proof:**

Suppose  $q=q(n, \lambda)$ . By differentiating (3) with respect to  $n$ , we obtain

$$\left(\frac{\partial q}{\partial n}\right)_\lambda \{f'\lambda - c'' + n(f' + qf''\lambda)\} + (qf' + q^2f''\lambda) = 0 ,$$

and, by invoking A3,

$$\frac{\partial q(n, \lambda)}{\partial n} = \left(\frac{\partial q}{\partial n}\right)_\lambda = \frac{-\beta/\lambda}{\alpha + (n/\lambda - 1)\beta} q < 0 .$$

Then, by differentiating (3) with respect to  $\lambda$ , we obtain

$$\left(\frac{\partial q}{\partial \lambda}\right)_n \{f'\lambda - c'' + n(f' + qf''\lambda)\} + qf' = 0 ,$$

and, by invoking A3,

$$\frac{\partial q(n, \lambda)}{\partial \lambda} = \left(\frac{\partial q}{\partial n}\right)_n = \frac{-f'}{\alpha + (n/\lambda - 1)\beta} q < 0 . \quad \text{Q.E.D.}$$

Next, let us consider  $n(\lambda, \pi)$  and  $q(\lambda, \pi)$ . By definition,

$$(5) \quad q(\lambda, \pi) f(nq(\lambda, \pi)) - c(q(\lambda, \pi)) - \pi = 0 ,$$

$$(6) \quad q(\lambda, \pi) = q(n(\lambda, \pi), \lambda) .$$

It is also naturally inferred that the greater interdependence of firms will enable more to gain constant profit and reduce the product of each, and that more

profit will lead to a decrease in the number of firms and increase in the product of each. This is described and proved in the following lemma.

**Lemma 2**

$$\frac{\partial n(\lambda, \pi)}{\partial \lambda} = \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} = \frac{(n-\lambda) f'}{\alpha} > 0 ,$$

$$\frac{\partial n(\lambda, \pi)}{\partial \pi} = \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} = \frac{\alpha + (n/\lambda - 1) \beta}{\alpha q^2 f'} < 0 ,$$

$$\frac{\partial q(\lambda, \pi)}{\partial \lambda} = \left( \frac{\partial q}{\partial \lambda} \right)_{\pi} = \frac{-q f'}{\alpha} < 0 ,$$

$$\frac{\partial q(\lambda, \pi)}{\partial \pi} = \left( \frac{\partial q}{\partial \pi} \right)_{\lambda} = \frac{-\beta/\lambda}{\alpha q f'} > 0 .$$

**Proof:**

By differentiating (5) with respect to  $\lambda$ ,

$$(7) \quad (f - c') \left\{ \left( \frac{\partial q}{\partial \lambda} \right)_n + \left( \frac{\partial q}{\partial n} \right)_{\lambda} \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} \right\} + q f \left[ q \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} + n \left\{ \left( \frac{\partial q}{\partial \lambda} \right)_n + \left( \frac{\partial q}{\partial n} \right)_{\lambda} \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} \right\} \right] = 0 .$$

Using Lemma 1, A1, and A3,

$$(8) \quad \frac{\partial n(\lambda, \pi)}{\partial \lambda} = \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} = \frac{(n-\lambda) f'}{\alpha} > 0$$

is yielded.

By differentiating (5) with respect to  $\pi$ ,

$$(9) \quad (f-c') \left( \frac{\partial q}{\partial n} \right)_{\lambda} \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} + q f' \left\{ q \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} + n \left( \frac{\partial q}{\partial n} \right)_{\lambda} \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} \right\} - 1 = 0 .$$

Using Lemma 1, A1, and A3,

$$(10) \quad \frac{\partial n(\lambda, \pi)}{\partial \pi} = \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} = \frac{\alpha + (n/\lambda - 1) \beta}{\alpha q^2 f'} < 0$$

is yielded.

Then, by differentiating (6) with respect to  $\lambda$  and  $\pi$ ,

$$(11) \quad \frac{\partial q(\lambda, \pi)}{\partial \lambda} = \left( \frac{\partial q}{\partial \lambda} \right)_{\pi} = \left( \frac{\partial q}{\partial \lambda} \right)_{n} + \left( \frac{\partial q}{\partial n} \right)_{\lambda} \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} = \frac{-q f'}{\alpha} < 0 ,$$

$$(12) \quad \frac{\partial q(\lambda, \pi)}{\partial \pi} = \left( \frac{\partial q}{\partial \pi} \right)_{\pi} = \left( \frac{\partial q}{\partial n} \right)_{\lambda} \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} = \frac{-\beta/\lambda}{\alpha q f'} > 0$$

hold by (8) and (10).

Q.E.D.

At the end of this section, we paraphrase the excess entry theorem using our notation, which will be proved and examined later within our framework.

**Excess entry theorem (Suzumura and Kiyono (1987))**

The number of firms when a firm's profit equals 0 (free-entry competition) exceeds the number of firms when welfare is maximized (first-best excess entry theorem).

$$n(\lambda, 0) > n(\lambda^*, \pi^*) \text{ for } (\lambda^*, \pi^*) = \arg \max_{\lambda, \pi} W(\lambda, \pi)$$

The number of firms when a firm's profit equals 0 exceeds the number of firms when welfare is maximized under constant conjectural variation (second-best excess entry theorem).

$$n(\lambda, 0) > n(\lambda, \pi^{**}) \text{ for } \pi^{**} = \arg \max_{\pi} W(\lambda, \pi) .$$

In addition, the following inequality holds (excess entry theorem at the margin).

$$\left( \frac{\partial W}{\partial n} \right)_{\pi} < 0 .$$

### 3. ECONOMIC WELFARE AND COMPETITION

Using the model presented above, we analyze the relationship between economic welfare and competition, i.e.  $W(\lambda, \pi) = W(n(\lambda, \pi), q(\lambda, \pi))$  is examined.

Our main result is the following theorem, which is obtained by using Lemma 2.

#### Theorem 1

$$(13) \quad \lambda=0, \pi=0 \quad \Rightarrow \quad \left(\frac{\partial W}{\partial \lambda}\right)_{\pi} = 0, \left(\frac{\partial W}{\partial \pi}\right)_{\lambda} = 0 ;$$

$$(14) \quad \lambda \neq 0, \pi = \epsilon < \epsilon' \text{ for } \exists \epsilon' > 0 \quad \Rightarrow \quad \left(\frac{\partial W}{\partial \lambda}\right)_{\pi} < 0, \left(\frac{\partial W}{\partial \pi}\right)_{\lambda} > 0 ;$$

$$(15) \quad \lambda = \epsilon < \epsilon', \pi \neq 0 \text{ for } \exists \epsilon' > 0 \quad \Rightarrow \quad \left(\frac{\partial W}{\partial \lambda}\right)_{\pi} > 0, \left(\frac{\partial W}{\partial \pi}\right)_{\lambda} < 0 .$$

#### Proof:

By differentiating  $W$  with respect to  $n$  and  $q$  respectively,

$$(16) \quad \left(\frac{\partial W}{\partial n}\right)_q = qf - c = \pi ,$$

$$(17) \quad \left(\frac{\partial W}{\partial q}\right)_n = n(f - c') = -nqf'/\lambda .$$

Therefore,

$$\begin{aligned}
 (18) \quad \left(\frac{\partial W}{\partial \lambda}\right)_{\pi} &= \left(\frac{\partial W}{\partial n}\right)_{\mathfrak{q}} \left(\frac{\partial n}{\partial \lambda}\right)_{\pi} + \left(\frac{\partial W}{\partial \mathfrak{q}}\right)_{\mathfrak{n}} \left(\frac{\partial \mathfrak{q}}{\partial \lambda}\right)_{\pi} \\
 &= \pi \left(\frac{\partial n}{\partial \lambda}\right)_{\pi} - n\mathfrak{q}f' / \lambda \left(\frac{\partial \mathfrak{q}}{\partial \lambda}\right)_{\pi} \\
 &= \pi \left(\frac{\partial n}{\partial \lambda}\right)_{\pi} + \lambda \left\{ -n\mathfrak{q}f' / \left(\frac{\partial \mathfrak{q}}{\partial \lambda}\right)_{\pi} \right\} ,
 \end{aligned}$$

$$\begin{aligned}
 (19) \quad \left(\frac{\partial W}{\partial \pi}\right)_{\lambda} &= \left(\frac{\partial W}{\partial n}\right)_{\mathfrak{q}} \left(\frac{\partial n}{\partial \pi}\right)_{\lambda} + \left(\frac{\partial W}{\partial \mathfrak{q}}\right)_{\mathfrak{n}} \left(\frac{\partial \mathfrak{q}}{\partial \pi}\right)_{\lambda} \\
 &= \pi \left(\frac{\partial n}{\partial \pi}\right)_{\lambda} - n\mathfrak{q}f' / \lambda \left(\frac{\partial \mathfrak{q}}{\partial \pi}\right)_{\lambda} \\
 &= \pi \left(\frac{\partial n}{\partial \pi}\right)_{\lambda} + \lambda \left\{ -n\mathfrak{q}f' / \left(\frac{\partial \mathfrak{q}}{\partial \pi}\right)_{\lambda} \right\} .
 \end{aligned}$$

Using Lemma 2 and A1, the following inequalities hold:

$$(20) \quad \left(\frac{\partial n}{\partial \lambda}\right)_{\pi} > 0, \quad -n\mathfrak{q}f' / \left(\frac{\partial \mathfrak{q}}{\partial \lambda}\right)_{\pi} < 0 ,$$

$$(21) \quad \left(\frac{\partial n}{\partial \pi}\right)_\lambda < 0, \quad -nqf'\left(\frac{\partial q}{\partial \pi}\right)_\lambda > 0 .$$

Therefore,

$$(22) \quad \lambda=0, \pi=0 \Rightarrow \left(\frac{\partial W}{\partial \lambda}\right)_\pi = 0, \left(\frac{\partial W}{\partial \pi}\right)_\lambda = 0 ,$$

$$(23) \quad \lambda \neq 0, \pi=0 \Rightarrow \left(\frac{\partial W}{\partial \lambda}\right)_\pi < 0, \left(\frac{\partial W}{\partial \pi}\right)_\lambda > 0 ,$$

$$(24) \quad \lambda=0, \pi \neq 0 \Rightarrow \left(\frac{\partial W}{\partial \lambda}\right)_\pi > 0, \left(\frac{\partial W}{\partial \pi}\right)_\lambda < 0$$

are yielded and the continuity of the functions proves this theorem. Q.E.D.

The implication of the theorem is as follows. At first, it should be noted that, concerning (13),  $W$  is maximized when  $(\lambda, \pi) = (0, 0)$ ; perfect and free-entry competition endorses the most efficient market with respect to welfare. In other words, welfare is necessarily increased when competition comes near enough to perfect and free-entry competition.

On the other hand, (14) indicates that more intra-industry competition (lower interdependence of firms) may

bring about less welfare in the neighborhood of  $(\lambda, 0)$ . Likewise, (15) indicates that more inter-industry competition (higher entry barriers) may bring about less welfare in the neighborhood of  $(0, \pi)$ .

The detailed relationship between  $(d\lambda, d\pi)$  and  $dW$  can be derived as follows. Using total differential representation,

$$(25) \quad dW = \left( \frac{\partial W}{\partial \lambda} \right)_{\pi} d\lambda + \left( \frac{\partial W}{\partial \pi} \right)_{\lambda} d\pi$$

is obtained and thus

$$(26) \quad dW > 0 \Leftrightarrow \left( \frac{\partial W}{\partial \lambda} \right)_{\pi} d\lambda + \left( \frac{\partial W}{\partial \pi} \right)_{\lambda} d\pi > 0$$

holds. By solving the inequality (26), Table 1 is derived, which insists that increases in both intra-industry competition and inter-industry competition do not necessitate more welfare in the neighborhood of  $(\lambda, 0)$  and  $(0, \pi)$ .

The relationship shown in Table 1 contradicts the widely held belief in the welfare enhancing effect of increasing competition in the sense that welfare may diminish even if the market becomes more competitive under certain conditions. However, the result does not contradict the belief in the sense that welfare

necessarily increases when the market becomes competitive enough (nearly perfect and free-entry competition).

#### 4. ECONOMIC WELFARE AND THE NUMBER OF FIRMS

The relationship between  $(d\lambda, d\pi)$  and  $dW$  is derived above. In this section, the relationship between  $dn$  and  $dW$  is examined based on the above result.

First, concerning the relationship between  $dn$  and  $(d\lambda, d\pi)$ ,

$$(27) \quad dn \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} d\lambda + \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} d\pi \begin{matrix} > \\ < \end{matrix} 0$$

holds. Thus, by solving (27) and inequalities concerning  $(d\lambda, d\pi)$  and  $dW$  simultaneously, the relationship between  $dn$  and  $dW$  will be yielded. In this context, the following lemma is useful.

##### Lemma 3

When  $\lambda \neq 0$  and  $\pi = 0$ ,

$$(28) \quad dW \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \left( \frac{\partial \alpha}{\partial \lambda} \right)_{\pi} d\lambda + \left( \frac{\partial \alpha}{\partial \pi} \right)_{\lambda} d\pi \begin{matrix} > \\ < \end{matrix} 0 .$$

When  $\lambda = 0$  and  $\pi \neq 0$ ,

$$(29) \quad dW \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} d\lambda + \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} d\pi \begin{matrix} > \\ < \end{matrix} 0 .$$

**Proof:**

Using (18), (19), and Theorem 1,

$$\lambda \neq 0, \pi = 0 \Rightarrow \left( \frac{\partial W}{\partial \lambda} \right)_{\pi} = \left( \frac{\partial W}{\partial q} \right)_{n} \left( \frac{\partial q}{\partial \lambda} \right)_{\pi} < 0, \left( \frac{\partial W}{\partial \pi} \right)_{\lambda} = \left( \frac{\partial W}{\partial q} \right)_{n} \left( \frac{\partial q}{\partial \pi} \right)_{\lambda} > 0 ,$$

$$\lambda = 0, \pi \neq 0 \Rightarrow \left( \frac{\partial W}{\partial \lambda} \right)_{\pi} = \left( \frac{\partial W}{\partial n} \right)_{q} \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} > 0, \left( \frac{\partial W}{\partial \pi} \right)_{\lambda} = \left( \frac{\partial W}{\partial n} \right)_{q} \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} < 0 .$$

Therefore, when  $\lambda \neq 0$  and  $\pi = 0$ ,

$$dW \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \left( \frac{\partial W}{\partial q} \right)_{n} \left( \frac{\partial q}{\partial \lambda} \right)_{\pi} d\lambda + \left( \frac{\partial W}{\partial q} \right)_{n} \left( \frac{\partial q}{\partial \pi} \right)_{\lambda} d\pi \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \left( \frac{\partial q}{\partial \lambda} \right)_{\pi} d\lambda + \left( \frac{\partial q}{\partial \pi} \right)_{\lambda} d\pi \begin{matrix} > \\ < \end{matrix} 0$$

holds, and when  $\lambda = 0$  and  $\pi \neq 0$ ,

$$dW \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \left( \frac{\partial W}{\partial n} \right)_{q} \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} d\lambda + \left( \frac{\partial W}{\partial n} \right)_{q} \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} d\pi \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} d\lambda + \left( \frac{\partial n}{\partial \pi} \right)_{\lambda} d\pi \begin{matrix} > \\ < \end{matrix} 0$$

holds

Q.E.D.

In order to solve inequalities (27), (28) and (29), the following lemma concerning the coefficient of (28) and (29) is important.

Lemma 4

$$-\left(\frac{\partial q}{\partial \lambda}\right)_{\pi} / \left(\frac{\partial q}{\partial \pi}\right)_{\lambda} > -\left(\frac{\partial n}{\partial \lambda}\right)_{\pi} / \left(\frac{\partial n}{\partial \pi}\right)_{\lambda} .$$

Proof:

$$\left\{ -\left(\frac{\partial q}{\partial \lambda}\right)_{\pi} / \left(\frac{\partial q}{\partial \pi}\right)_{\lambda} \right\} - \left\{ -\left(\frac{\partial n}{\partial \lambda}\right)_{\pi} / \left(\frac{\partial n}{\partial \pi}\right)_{\lambda} \right\}$$

$$= -\frac{q^2 f'^2 \lambda}{\beta} + \frac{(n-\lambda) q^2 f'^2}{\alpha + (n/\lambda - 1) \beta}$$

$$= \frac{-\alpha q^2 f'^2 \lambda}{\beta \{ \alpha + (n/\lambda - 1) \beta \}} > 0 .$$

Q.E.D.

Using Lemma 4, inequalities (27), (28), and (29) are solved as shown in Table 2. The result is the expansion of the excess entry theorem at the margin. The most interesting point here is that when  $\lambda > 0$  and  $\pi = 0$ , a reduction in the number of firms is a necessary condition for an increase in economic welfare, and when  $\lambda = 0$  and  $\pi > 0$ , an increase in the number of firms is a necessary and sufficient condition for an increase in welfare. This is summarized in the following theorem:

## Theorem 2

If  $\lambda > 0$ ,  $\pi = 0$ , and  $d\pi \geq 0$ , then  $dW > 0 \Rightarrow dn < 0$ .

If  $\lambda = 0$ ,  $\pi > 0$ , and  $d\lambda \geq 0$ , then  $dW > 0 \Leftrightarrow dn < 0$ .

At the end of this section, we prove first-best and second-best excess entry theorems within our framework and consider the implication.

The first-best excess entry theorem insists

$$n(\lambda, 0) > n(\lambda^*, \pi^*) \text{ for } (\lambda^*, \pi^*) = \arg \max_{\lambda, \pi} W(\lambda, \pi) .$$

Since  $(\lambda^*, \pi^*) = (0, 0)$ , the theorem is paraphrased as  $n(\lambda, 0) > n(0, 0)$  for  $\lambda > 0$ , which holds by

$$\left( \frac{\partial n}{\partial \lambda} \right)_{\pi} < 0$$

in Lemma 2. In the meantime, considering that  $(\lambda^*, \pi^*) = (0, 0)$  is more competitive than  $(\lambda, 0)$  and  $W(0, 0) > W(\lambda, 0)$ , the first-best excess entry theorem does not necessarily contradict belief in the welfare enhancing effect of increasing competition.

On the other hand, the second-best excess entry theorem insists

$$n(\lambda, 0) > n(\lambda, \pi^{**}) \text{ for } \pi^{**} = \arg \max_{\pi} W(\lambda, \pi) .$$

According to (19) and (21),

$$\pi \leq 0 \Rightarrow \left( \frac{\partial W}{\partial \pi} \right)_\lambda > 0$$

holds and thus  $\pi^{**} > 0$ . Therefore, the second-best excess entry theorem holds by

$$\left( \frac{\partial n}{\partial \pi} \right)_\lambda < 0$$

in Lemma 2. In the meantime, considering that  $(\lambda, 0)$  is more competitive than  $(\lambda, \pi^{**})$  but  $W(\lambda, \pi^{**}) > W(\lambda, 0)$ , the second-best excess entry theorem may contradict belief in the welfare enhancing effect of increasing competition.

## 5. CONCLUDING REMARKS

This paper has shown that, with respect to the two dimensional parameter for competition, welfare is not a monotone function. As concluding remarks, we reexamine the above obtained result.

First, our result is obtained by introducing the two dimensional parameter with respect to intra-industry competition and inter-industry competition. As is known, order relation defined in a one-dimensional space cannot naturally be extended to that in two-dimensional space. In this sense, it is natural that the obtained

relationship between economic welfare and competition is rather unexpected.

Second, some may think that the above result overly depends on parametrization. In fact, the space of market equilibrium can be parametrized differently. However, considering the process in proving Theorem 1, we can understand that the essence lies in Lemma 2, which is naturally inferred instinctively as mentioned. Therefore, it can be concluded that the choice of the parametrization is not crucial for the essence of the result.

Third, therefore, our result strongly suggests that the relationship between competition and economic welfare is much more complicated than generally assumed. Thus, from the viewpoint of economic policy, welfare analysis is strongly recommended before promoting competition.

From a theoretical viewpoint, our result also suggests that further investigation of multidimensional competition or a new concept of competition is necessary to make advances in this field.

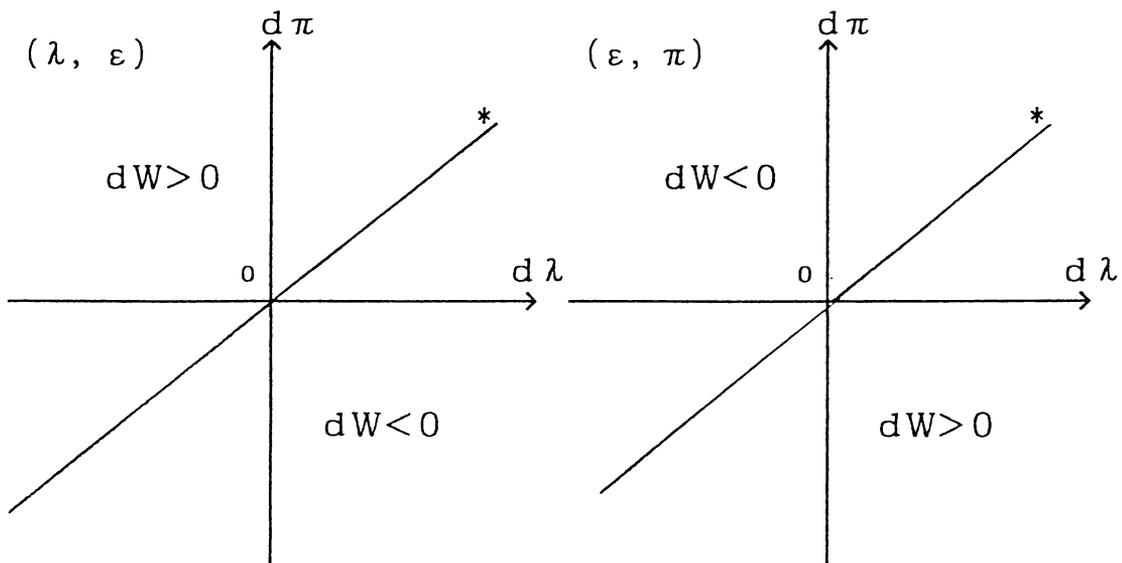
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Table 1 Relationship between  $(d\lambda, d\pi)$  and  $dW$

		$(\lambda, \varepsilon)$ $\lambda \neq 0$ $\varepsilon < \varepsilon'$ for $\exists \varepsilon' > 0$	$(\varepsilon, \pi)$ $\pi \neq 0$ $\varepsilon < \varepsilon'$ for $\exists \varepsilon' > 0$
$d\lambda \geq 0$	$\frac{d\pi}{d\lambda} > - \left( \frac{\partial W}{\partial \lambda} \right)_{\pi} / \left( \frac{\partial W}{\partial \pi} \right)_{\lambda}$	$dW \geq 0$	$dW \leq 0$
$d\pi \geq 0$	$\frac{d\pi}{d\lambda} < - \left( \frac{\partial W}{\partial \lambda} \right)_{\pi} / \left( \frac{\partial W}{\partial \pi} \right)_{\lambda}$	$dW \leq 0$	$dW \geq 0$
$d\lambda \geq 0$ $d\pi \leq 0$	—	$dW \leq 0$	$dW \geq 0$
$d\lambda \geq 0$ $d\pi = 0$	—	$dW \leq 0$	$dW \geq 0$
$d\lambda = 0$ $d\pi \geq 0$	—	$dW \geq 0$	$dW \leq 0$



\* slope  $- \left( \frac{\partial W}{\partial \lambda} \right)_{\pi} / \left( \frac{\partial W}{\partial \pi} \right)_{\lambda}$

Table 2 Relationship between  $(d\lambda, d\pi)$ ,  $dn$  and  $dW$

		$(\lambda, 0)$ $\lambda \neq 0$	$(0, \pi)$ $\pi \neq 0$
$d\lambda > 0$ $d\pi < 0$	—	—	
$d\lambda > 0$ $d\pi = 0$	—	$dn > 0$	$dW < 0$
	$\frac{d\pi}{d\lambda} < A$		
$d\lambda > 0$ $d\pi > 0$	$A < \frac{d\pi}{d\lambda} < B$	$dn < 0$	$dW > 0$
	$B < \frac{d\pi}{d\lambda}$		
$d\lambda = 0$ $d\pi > 0$	—		
$d\lambda < 0$ $d\pi > 0$	—		

$$A = - \left( \frac{\partial n}{\partial \lambda} \right)_{\pi} / \left( \frac{\partial n}{\partial \pi} \right)_{\lambda}, \quad B = - \left( \frac{\partial q}{\partial \lambda} \right)_{\pi} / \left( \frac{\partial q}{\partial \pi} \right)_{\lambda}$$

