Optimal Disclosure of Public Information with Endogenous Acquisition of Private Information*

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Abstract

A public authority provides agents with public information, but each agent also acquires his own private information, and they play a linear quadratic Gaussian game. More provision of public information induces less acquisition of private information, yet this effect attenuates as the elasticity of marginal cost of information acquisition increases. The main result of this paper characterizes the optimal disclosure of public information in terms of an arbitrary quadratic welfare function, where the elasticity of marginal cost plays an essential role. To this end, we obtain a necessary and sufficient condition for welfare to increase with public information. We find that the welfare effect of public information is determined by a linear combination of the two extreme cases with zero and infinite elasticities of marginal cost.

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*Preliminary.

1 Introduction

Decision making under uncertainty depends upon information, but information choice is also an important aspect of decision making. What information agents acquire depends upon what information they have beforehand. Thus, by providing agents with public information, a public authority can have an influence on agents' information acquisition and the resulting outcome. It has at least three effects. First, more provision of public information induces less acquisition of private information, which is referred to as the crowding-out effect. Next, agents take more correlated actions because they share more information. Finally, agents' cost of information acquisition decreases.

To understand the total effect on welfare, we consider the following three-period model of information acquisition studied by Colombo et al. (2014), which generalizes the seminal works of Vives (1988) and Li et al. (1987) on information acquisition in linear quadratic Gaussian games.[1] In period 0, a public authority chooses the precision of public information. In period 1, each agent chooses the precision of private information given that of public information, where the cost function is an isoelastic function of the precision of private information. In period 2, each agent observes private and public signals and chooses an action in a linear quadratic Gaussian game studied by Angeletos and Pavan (2007).

Our measure of welfare is the expected value of an arbitrary quadratic function of actions minus total cost of information acquisition. The total expected net payoff considered by Colombo et al. (2014) is a special case. We represent welfare as a linear combination of the variance of a common term in an equilibrium strategy and that of an idiosyncratic term, which follows Ui and Yoshizawa (2015) who study the case of exogenous private information. These variances are referred as the common variance and the idiosyncratic variance of actions, respectively. The common variance equals the covariance of actions. Thus, it increases with public information because more precise information causes more correlated actions. In contrast, the idiosyncratic variance equals the difference between the variance and covariance of actions. Thus, it decreases with public information because a higher correlation of actions brings the covariance and variance closer.

In our main result, we characterize the optimal precision of public information. In so doing, we give a necessary and sufficient condition for welfare to increase with public in-

[1]See Vives (2008) for more details. Recent studies on information acquisition in linear quadratic Gaussian games have focused on a beauty contest game of Morris and Shin (2002), including Colombo and Femminis (2008), Wong (2008), Hellwig and Veldkamp (2009), Myatt and Wallace (2012), and Ui (2014).

formation. Key parameters in our condition are not only the coefficients of the common variance and the idiosyncratic variance but also the elasticity of marginal cost of information acquisition. The elasticity of marginal cost determines the strength of the crowding-out effect. The crowding-out effect is the largest when the elasticity is zero (i.e. a linear cost function) and decreases with the elasticity. In the limit as the elasticity goes to infinity, the crowding-out effect disappears, where agents do not change the precision of their private information even if a public authority increases the precision of public information. We find that the welfare effect of public information is determined by a linear combination of the two extreme cases with zero and infinite elasticities of marginal cost.

Given the elasticity of marginal cost, suppose that the coefficients of the common and idiosyncratic variances in welfare are positive. If the coefficient of the common variance is relatively large, welfare necessarily increases with public information, so the optimal precision of public information is the highest precision. If the coefficient of the common variance is relatively small, welfare decreases with public information if the precision is low and increases if the precision is high, so the optimal precision of public information is either the lowest or the highest precision. On the other hand, if the coefficients of the common and idiosyncratic variances in welfare are negative, the optimal precision of public information is either the lowest precision or a strictly positive finite value.

This paper builds on the model of Colombo et al. (2014), who incorporate information acquisition considered by Vives (1988) and Li et al. (1987) into the linear quadratic Gaussian game of Angeletos and Pavan (2007). Adopting the total expected net payoff as a measure of welfare, Colombo et al. (2014) compare the social value of public information with endogenous private information and that with exogenous private information. They give a sufficient condition guaranteeing that the former is positive whenever the latter is positive based upon a comparison between the equilibrium strategy and the socially optimal strategy profile, which follows Angeletos and Pavan (2007).

In contrast to Colombo et al. (2014), we adopt a general quadratic welfare function and give a complete characterization of the social value of public information with endogenous private information using the coefficients of the common and idiosyncratic variances in welfare and the elasticity of marginal cost. Ui and Yoshizawa (2015) study the case of exogenous private information using the same coefficients. These coefficients play an essential role in both cases of endogenous and exogenous private information, the latter of which can be understood as the extreme case with the infinite elasticity of marginal cost.

This paper is organized as follows. Section 2 introduces the model and Section 3 calculates welfare in the equilibrium. We give a necessary and sufficient condition for welfare to increase with public information in Section 4 and characterize the optimal precision of public information in Section 5. Section 6 is devoted to an application to a Cournot game.

2 The model

There are a continuum of agents indexed by $i \in [0,1]$ and a public authority. A public authority knows the state θ and provides agents with public information about θ, but each agent also acquires his own private information. We consider the following three-period setting. In period 0, a public authority chooses the precision of public information. In period 1, each agent chooses the precision of private information given that of public information. In period 2, each agent observes private and public signals and chooses an action in a Bayesian game.[2]

Agent i's private signal is $x_i = \theta + \varepsilon_i$ and a public signal is $y = \theta + \varepsilon_0$, where ε_i, ε_0, and θ are independently and normally distributed with

$$E[\theta] = \bar{\theta},\ E[\varepsilon_i] = E[\varepsilon_0] = 0,\ \mathrm{var}[\theta] = \tau_\theta^{-1},\ \mathrm{var}[\varepsilon_i] = \tau_i^{-1},\ \mathrm{var}[\varepsilon_0] = \tau_y^{-1}.$$

We refer to τ_i and τ_y as the precision of private information and that of public information, respectively. A public authority chooses τ_y in period 0 with no cost. Agent i chooses τ_i in period 1 with a cost $C(\tau_i) = c\tau_i^{\rho+1}/(\rho+1)$, where $c > 0$ is a constant and $\rho \geq 0$ is the elasticity of marginal cost.

In a Bayesian game, agent i's action is a real number $a_i \in \mathbb{R}$. We write $a = (a_i)_{i\in[0,1]}$ and $a_{-i} = (a_j)_{j\neq i}$. Agent i's payoff function is

$$u_i(a,\theta) = -a_i^2 + 2\alpha a_i \int_0^1 a_j dj + 2\beta\theta a_i + h(a_{-i},\theta), \tag{1}$$

where $\alpha, \beta \in \mathbb{R}$ are constants and $h(a_{-i},\theta)$ is a measurable function. Note that agent i's best response is determined by α and β. This game exhibits strategic complementarity if $\alpha > 0$ and strategic substitutability if $\alpha < 0$. We assume $\alpha < 1$, by which a unique symmetric Bayesian Nash equilibrium exists when $\tau_i = \tau_j$ for all $i \neq j$ (see Lemma 1). We also assume $\beta > 0$ without loss of generality.

[2]The earliest papers on information acquisition in linear quadratic Gaussian games are Li et al. (1987) and Vives (1988), who study Cournot games.

A welfare function is

$$v(a,\theta) - \int C(\tau_j)dj, \tag{2}$$

where $v(a,\theta)$ is a symmetric and quadratic function of a and θ; that is,

$$v(a,\theta) = c_1 \int_0^1 a_j^2 dj + c_2 \left(\int_0^1 a_j dj\right)^2 + c_3\theta \int_0^1 a_j dj + c_4 \int_0^1 a_j dj + c_5. \tag{3}$$

A public authority chooses τ_y to maximize the expected welfare in period 0.

In the model of Colombo et al. (2014), each agent's payoff function is quadratic in a and θ, i.e.,

$$h(a_{-i},\theta) = \kappa \int_0^1 a_j^2 dj + \lambda \left(\int_0^1 a_j dj\right)^2 + \mu\theta \int_0^1 a_j dj + \nu \int_0^1 a_j dj + f(\theta),$$

and a public authority's payoff function is the total net payoff of agents, i.e.,

$$\int (u_i(a,\theta) - C(\tau_i))\, di.$$

In this case, a welfare function is written as (2) with (3) given by

$$c_1 = \kappa - 1,\ c_2 = 2\alpha + \lambda,\ c_3 = 2\beta + \mu,\ c_4 = \nu.$$

3 The expected welfare

Colombo et al. (2014) show that if $\alpha < 1$ then there exists a unique symmetric equilibrium of the game in periods 1 and 2 given τ_y. In this section, we obtain the expected welfare in period 0 using the equilibrium strategy in periods 1 and 2 and represent it as a linear combination of the common variance and the idiosyncratic variance.

Period 2

Angeletos and Pavan (2007) study the game in period 2 with $\tau_i = \tau_j$ for all $i \neq j$ and show the following result.[3]

Lemma 1. *Assume that $\alpha < 1$. Then, there exists a unique symmetric equilibrium of the Bayesian game in period 2 with $\tau_i = \tau_x$ for all i. Agent i's strategy is $\sigma_i(x_i, y) = b_x(x_i - \bar{\theta}) + b_y(y - \bar{\theta}) + \beta\bar{\theta}/(1-\alpha)$, where*

$$b_x = \frac{\beta}{(1-\alpha)\tau_x + \tau_y + \tau_\theta} \cdot \tau_x,\ b_y = \frac{\beta}{(1-\alpha)\tau_x + \tau_y + \tau_\theta} \cdot \frac{\tau_y}{1-\alpha}.$$

[3]This result is also implied by Radner (1962, Theorem 5). See Ui and Yoshizawa (2013).

An equilibrium strategy is linear in private and public signals. The ratio of their coefficients is $b_x/b_y = (1-\alpha)\tau_x/\tau_y$. Thus, if α is close to one or τ_x/τ_y is small, the relative weight of a public signal is large, and if α is small or τ_x/τ_y is large, that of a public signal is large.

For later use, we obtain the expected welfare when agents follow the above equilibrium strategy. To this end, it is useful to rewrite the equilibrium strategy as

$$\begin{aligned}\sigma_i(x_i, y) =& b_x(\theta + \varepsilon_i - \bar{\theta}) + b_y(\theta + \varepsilon_0 - \bar{\theta}) + \beta\bar{\theta}/(1-\alpha) \\ =& b_x\varepsilon_i + \big(b_y\varepsilon_0 + (b_x + b_y)\theta\big) + \big(\beta/(1-\alpha) - (b_x + b_y)\big)\bar{\theta},\end{aligned}$$

where $b_x\varepsilon_i$ is an idiosyncratic random term and $b_y\varepsilon_0+(b_x+b_y)\theta$ is a common random term. Ui and Yoshizawa (2015) refers to the variances of these terms, $\mathrm{var}[b_x\varepsilon_i]$ and $\mathrm{var}[b_y\varepsilon_0 + (b_x+b_y)\theta]$, as the idiosyncratic variance and the common variance of actions, respectively. Because ε_i, ε_0, and θ are independent, it holds that

$$\mathrm{var}[b_y\varepsilon_0 + (b_x + b_y)\theta] = \mathrm{cov}[\sigma_i, \sigma_j],\ \mathrm{var}[b_x\varepsilon_i] = \mathrm{var}[\sigma_i] - \mathrm{cov}[\sigma_i, \sigma_j].$$

That is, the common variance equals the covariance of actions and the idiosyncratic variance equals the difference between the variance and covariance of actions.[4] We write the common variance and the idiosyncratic variance as functions of τ_x and τ_y,

$$CV(\tau_x, \tau_y) = \mathrm{cov}[\sigma_i, \sigma_j],\ IV(\tau_x, \tau_y) = \mathrm{var}[\sigma_i] - \mathrm{cov}[\sigma_i, \sigma_j],$$

respectively.

The next lemma due to Ui and Yoshizawa (2015) represents the expected welfare as a linear combination of the common variance and the idiosyncratic variance minus the cost of information acquisition. Their coefficients play an essential role in our characterization of the optimal disclosure of public information.

Lemma 2. *The ex ante expected welfare in the unique symmetric equilibrium equals*

$$W(\tau_x, \tau_y) = E[w((\sigma_i)_{i\in[0,1]}, \theta) - C(\tau_x)] = \zeta IV(\tau_x, \tau_y) + \eta CV(\tau_x, \tau_y) - C(\tau_x) + k, \quad (4)$$

where $\zeta = c_1 + c_3/\beta$, $\eta = c_1 + c_2 + (1-\alpha)c_3/\beta$, and k is a constant independent of (τ_x, τ_y).

[4] Bergemann and Morris (2013) consider the variance of the average action $\int a_j dj$ and that of the idiosyncratic difference $a_i - \int a_j dj$ and refer to them as volatility and dispersion, respectively. The common variance equals the volatility and the idiosyncratic variance equals the dispersion. See Ui and Yoshizawa (2015) for more details.

Period 1

The next lemma due to Colombo et al. (2014) gives the first order condition for the equilibrium precision in period 1.

Lemma 3. *When all the opponents follow the unique symmetric equilibrium strategy of the Bayesian game in period 2 with $\tau_j = \tau_x$ for all $j \neq i$, agent i's marginal benefit of choosing τ_i in period 1 evaluated at $\tau_i = \tau_x$ is*

$$\frac{d}{d\tau_i} E[u_i(a,\theta)]\bigg|_{\tau_i=\tau_x} = -b_x^2 \frac{\partial \operatorname{var}[x_i]}{\partial \tau_x} = \frac{\beta^2}{\big((1-\alpha)\tau_x + \tau_y + \tau_\theta\big)^2}.$$

Thus, the first order condition for the precision in a symmetric equilibrium is

$$\frac{\beta^2}{\big((1-\alpha)\tau_x + \tau_y + \tau_\theta\big)^2} = C'(\tau_x). \tag{5}$$

Note that the marginal benefit is strictly decreasing in τ_x and τ_y, whereas the marginal cost is increasing in τ_x (see Figure 1). The equilibrium precision is the unique value of τ_x solving (5) if $C'(0) \leq \beta^2/(\tau_y + \tau_\theta)^2$ and it is zero if $C'(0) > \beta^2/(\tau_y + \tau_\theta)^2$. We denote the equilibrium precision by $\phi(\tau_y)$ as a function of τ_y. Then, the expected welfare in period 0 is $W(\phi(\tau_y), \tau_y)$.

For example, suppose that $C(\tau_x) = c\tau_x$, i.e., $\rho = 0$. Then,

$$\phi(\tau_y) = \begin{cases} \big(\beta/\sqrt{c} - \tau_y - \tau_\theta\big)/(1-\alpha) & \text{if } c < \beta^2/(\tau_y+\tau_\theta)^2, \\ 0 & \text{if } c \geq \beta^2/(\tau_y+\tau_\theta)^2. \end{cases} \tag{6}$$

Only in the linear case, we can obtain $\phi(\tau_y)$ in a closed form. Colombo and Femminis (2008) study this case in the context of beauty contest games of Morris and Shin (2002). Li et al. (1987) and Vives (1988) obtain a similar formula in Cournot games.

By (5), an increase in the precision of public information results in a decrease in the precision of private information as shown by Colombo et al. (2014); that is, $\phi'(\tau_y) < 0$. We refer to this effect as the crowding-out effect of public information on private information.[5]

Figure 1 illustrates the equilibrium precision and the crowding-out effect, where the horizontal axis is the τ_x-axis (the precision of private information). The equilibrium precision is the intersection of the downward sloping marginal benefit curve and the upward sloping marginal cost curve. An increase in the precision of public information shifts the

[5]This effect is also found in Colombo and Femminis (2008), Wong (2008), Hellwig and Veldkamp (2009), and Myatt and Wallace (2012).

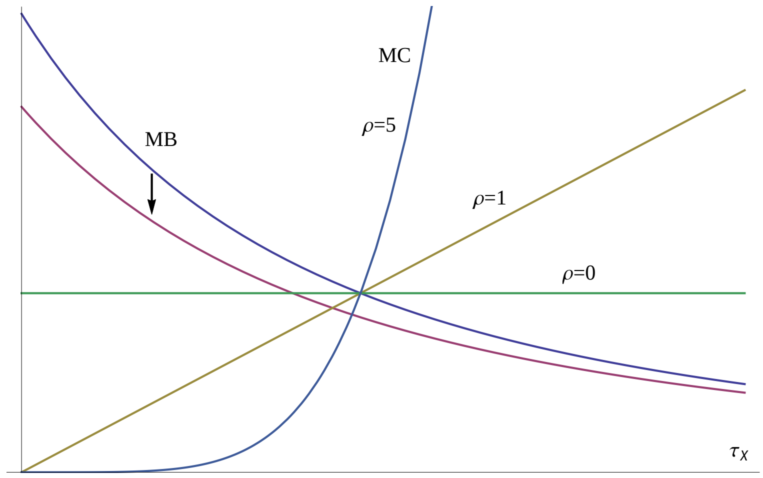

Figure 1: The marginal benefit curve and the marginal cost curve. An increase in τ_y shifts the marginal benefit curve down, by which the equilibrium precision decreases.

marginal benefit curve down, by which the equilibrium precision decreases. As Figure 1 shows, the crowding-out effect measured by $|\phi'(\tau_y)|$ is the largest when the elasticity of marginal cost ρ is zero, i.e., the cost function is linear. Moreover, the crowding-out effect becomes small as the elasticity of marginal cost ρ increases. The next lemma formally states this observation, which will help us to understand our characterization of the optimal disclosure of public information.

Lemma 4. *Let $\phi_\rho(\tau_y)$ be the equilibrium precision when the elasticity of marginal cost is ρ. If $\phi_{\rho_1}(\tau_y) = \phi_{\rho_2}(\tau_y) > 0$ and $\rho_1 > \rho_2 \geq 0$, then $0 > \phi'_{\rho_2}(\tau_y) > \phi'_{\rho_1}(\tau_y)$.*

Proof. We can prove this lemma by direct calculation using the fact that $\phi' = (d\phi^{-1}/d\tau_x)^{-1}$ and that $\phi^{-1}(\tau_x) = -(1-\alpha)\tau_x + \beta/\sqrt{C'(\tau_x)} - \tau_\theta$ by (5). □

4 The welfare effects of public information

When agents follow the equilibrium strategy given the precision of public information τ_y, the expected welfare is $W(\phi(\tau_y), \tau_y)$. To obtain the optimal precision of public information, we study under what condition $W(\phi(\tau_y), \tau_y)$ is increasing in τ_y.

Colombo et al. (2014) study a related issue, but their analysis differs from ours in the following way. Colombo et al. (2014) ask under what condition $\partial W(\phi(\tau_y), \tau_y)/\partial \tau_y > 0$ implies $dW(\phi(\tau_y), \tau_y)/d\tau_y > 0$. In contrast, we study a necessary and sufficient condition for $dW(\phi(\tau_y), \tau_y)/d\tau_y > 0$.

We start with a benchmark case, where the precision of private information does not change even if that of public information changes. In other words, there is no crowding-out effect. As we will see later, this case corresponds to the limit of the general case as $\rho \to \infty$.

Proposition 1. *Suppose that* $\rho > 0$. *For* $\tau_x = \phi(\tau_y)$,

$$\frac{\partial W(\tau_x, \tau_y)}{\partial \tau_y} \gtrless 0 \Leftrightarrow \Delta_\infty(\tau_x) \gtrless 0,$$

where

$$\Delta_\infty(\tau_x) \equiv \frac{\eta}{1-\alpha}\left(1 + \frac{\beta}{2(1-\alpha)\tau_x\sqrt{C'(\tau_x)}}\right) - \zeta = -\frac{\partial W(\tau_x, \tau_y)}{\partial \tau_y} \Big/ \frac{\partial IV(\tau_x, \tau_y)}{\partial \tau_y}. \tag{7}$$

Proof. By (4),

$$\frac{\partial W(\tau_x, \tau_y)}{\partial \tau_y} = \frac{\beta^2\left(-(1-\alpha)\tau_x(2(1-\alpha)\zeta - 3\eta) + \eta\left(\tau_\theta + \tau_y\right)\right)}{(1-\alpha)^2\left((1-\alpha)\tau_x + \tau_y + \tau_\theta\right)^3}. \tag{8}$$

By (5), $\tau_y = \phi^{-1}(\tau_x) = -(1-\alpha)\tau_x + \beta/\sqrt{C'(\tau_x)} - \tau_\theta$. By plugging this into (8), we obtain

$$\frac{\partial W(\tau_x, \tau_y)}{\partial \tau_y} = \Delta_\infty(\tau_x) \times 2\tau_x(C'(\tau_x))^{2/3}/\beta,$$

which implies that

$$\frac{\partial IV(\tau_x, \tau_y)}{\partial \tau_y} = -2\tau_x(C'(\tau_x))^{2/3}/\beta.$$

Thus, (7) holds. □

Proposition 1 says that welfare increases with public information if the coefficient of the common variance η is relatively large, but welfare decreases with public information if the coefficient of the idiosyncratic variance ζ is relatively large.

The intuition is as follows. The common variance increases with public information because it equals the covariance of actions and more precise public information causes more correlated actions. In contrast, the idiosyncratic variance decreases with public information because it equals the difference between the variance and covariance of actions and a higher correlation of actions brings the covariance and variance closer. Therefore, the welfare effect of public information is determined by the relative weights of the common variance and the idiosyncratic variance.

By (7), $\Delta_\infty(\tau_x)$ has the following meaning. When the precision of public information increases so that the idiosyncratic variance decreases by one, then welfare increases by $\Delta_\infty(\tau_x)$, which is analogous to the marginal rate of transformation in producer theory.

Thus, we call $\Delta_\infty(\tau_x)$ the martial rate of transformation of welfare for the idiosyncratic variance.

Next, we consider another benchmark case, where the cost is linear, i.e., $\rho = 0$. In this case, the crowding-out effect is the largest.

Proposition 2. *Suppose that $\rho = 0$. Then, $C(\phi(\tau_y)) = IV(\phi(\tau_y), \tau_y)$ and (4) is reduced to*

$$W(\phi(\tau_y), \tau_y) = (\zeta - 1)IV(\phi(\tau_y), \tau_y) + \eta CV(\phi(\tau_y), \tau_y) + k. \tag{9}$$

If $\tau_y < \beta/\sqrt{c} - \tau_\theta$,

$$\frac{dW(\phi(\tau_y), \tau_y)}{d\tau_y} \gtreqless 0 \iff \Delta_0 \gtreqless 0,$$

where

$$\Delta_0 \equiv \frac{\eta}{1-\alpha} - (\zeta - 1) = -\frac{dW(\phi(\tau_y), \tau_y)}{d\tau_y} \Big/ \frac{dIV(\phi(\tau_y), \tau_y)}{d\tau_y}. \tag{10}$$

If $\tau_y > \beta/\sqrt{c} - \tau_\theta$,

$$\frac{dW(\phi(\tau_y), \tau_y)}{d\tau_y} \gtreqless 0 \iff \eta \gtreqless 0.$$

Proof. The first order condition (5) implies $C(\phi(\tau_y)) = IV(\phi(\tau_y), \tau_y)$ and

$$\frac{dIV(\phi(\tau_y), \tau_y)}{d\tau_y} = C'(\phi(\tau_y))\phi'(\tau_y).$$

Plugging (6) into (9) and differentiate it with respect to τ_y, we obtain

$$\frac{dW(\phi(\tau_y), \tau_y)}{d\tau_y} = \begin{cases} -\Delta_0 \times C'(\phi(\tau_y))\phi'(\tau_y) & \text{if } \tau_y < \beta/\sqrt{c} - \tau_\theta, \\ \eta \times \beta^2/((1-\alpha)(\tau_y + \tau_\theta))^2 & \text{if } \tau_y > \beta/\sqrt{c} - \tau_\theta, \end{cases}$$

which completes the proof. □

The notable property in the linear case is that the total cost of information acquisition equals the idiosyncratic variance. Thus, welfare is represented as a linear combination of the common variance and the idiosyncratic variance, but the coefficient of the idiosyncratic variance is $\zeta - 1$ rather than ζ.

Proposition 2 says that welfare increases with public information if the coefficient of the common variance η is relatively large, but welfare decreases with public information if the coefficient of the idiosyncratic variance $\zeta - 1$ is relatively large. The intuition is essentially the same as that in the case of no crowding-out effect.

We also call Δ_0 the martial rate of transformation of welfare for the idiosyncratic variance on the basis of (10): welfare increases by Δ_0 when the precision of public information increases so that the idiosyncratic variance decreases by one.

Finally, we consider the general case with $\rho > 0$.

Proposition 3. *Suppose that $\rho > 0$. Then,*

$$\frac{dW(\phi(\tau_y),\tau_y)}{d\tau_y} \gtrless 0 \iff \frac{\Delta_0 + \rho\Delta_\infty(\phi(\tau_y))}{1+\rho} \gtrless 0.$$

Moreover,

$$\frac{\Delta_0 + \rho\Delta_\infty(\phi(\tau_y))}{1+\rho} = -\frac{dW(\phi(\tau_y),\tau_y)}{d\tau_y} \Big/ \frac{dIV(\phi(\tau_y),\tau_y)}{d\tau_y}.$$

Proof. By plugging $\phi^{-1}(\tau_x)$ into (4), we obtain

$$W(\tau_x,\phi^{-1}(\tau_x)) = \frac{\eta\left(\beta^2/\tau_z - (1-\alpha)\tau_x C'(\tau_x) - \beta\sqrt{C'(\tau_x)}\right) + \zeta(1-\alpha)^2\tau_x C'(\tau_x)}{(1-\alpha)^2} - C(\tau_x),$$

$$\frac{dW(\tau_x,\phi^{-1}(\tau_x))}{d\tau_x} = -C'(\tau_x)\Delta_0 - \tau_x C''(\tau_x)\Delta_\infty(\tau_x),$$

$$\frac{dIV(\tau_x,\phi^{-1}(\tau_x))}{d\tau_x} = C'(\tau_x) + \tau_x C''(\tau_x).$$

This implies the proposition because

$$\frac{dW(\phi(\tau_y),\tau_y)}{d\tau_y} = \frac{\partial W}{\partial\tau_x}\phi'(\tau_y) + \frac{\partial W}{\partial\tau_y} = \phi'(\tau_y)\left(\frac{\partial W}{\partial\tau_x} + \frac{\partial W}{\partial\tau_y}\cdot\frac{1}{\phi'(\tau_y)}\right) = \phi'(\tau_y)\frac{dW(\tau_x,\phi^{-1}(\tau_x))}{d\tau_x},$$

$$\frac{dIV(\phi(\tau_y),\tau_y)}{d\tau_y} = \phi'(\tau_y)\frac{dIV(\tau_x,\phi^{-1}(\tau_x))}{d\tau_x},$$

$\phi'(\tau_y) < 0$, and $\rho = \tau_x C''(\tau_x)/C'(\tau_x)$. □

Proposition 3 says that the sign of marginal welfare is given by that of a weighted mean of Δ_0 and Δ_∞; that is, $(\Delta_0+\rho\Delta_\infty)/(1+\rho)$. Moreover, $(\Delta_0+\rho\Delta_\infty)/(1+\rho)$ equals the martial rate of transformation of welfare for the idiosyncratic variance. Thus, the martial rate of transformation of welfare equals the weighted average of that with no crowding-out effect and that with the largest crowding-out effect, where the relative weight is determined by the elasticity of marginal cost. The two extreme cases can be interpreted as the limits of the general case as $\rho \to 0$ and $\rho \to \infty$.

5 The optimal disclosure of public information

The next proposition characterizes the optimal precision of public information.

Proposition 4. *Suppose that $\rho \geq 0$ and $(\zeta,\eta) \neq (0,0)$. Define*

$$f(\zeta,\rho) \equiv \frac{2(1-\alpha)\left((\rho+1)\zeta - 1\right)}{3\rho+2},$$

$$\tau_z^* \equiv \frac{(3\rho+2)(f(\zeta,\rho)-\eta)}{2\eta}\cdot\left(\frac{2^\rho(1-\alpha)^\rho\beta^2}{c\left((1-\alpha)((1+\rho)\zeta-1)/\eta-(1+\rho)\right)^2}\right)^{1/(\rho+2)}.$$

Then, the following holds.

(i) *If* $\eta \geq \max\{f(\zeta,\rho),0\}$*, then* $dW(\phi(\tau_y),\tau_y)/d\tau_y > 0$ *for all* τ_y*. Thus,*

$$\sup_{\tau_y} W(\phi(\tau_y),\tau_y) = W(0,\infty).$$

(ii) *If* $0 < \eta < f(\zeta,\rho)$*, then* $dW(\phi(\tau_y),\tau_y)/d\tau_y < 0$ *if* $\tau_y < \tau_z^* - \tau_\theta$ *and* $dW(\phi(\tau_y),\tau_y)/d\tau_y > 0$ *if* $\tau_y > \tau_z^* - \tau_\theta$*. Thus,*

$$\sup_{\tau_y} W(\phi(\tau_y),\tau_y) = \begin{cases} W(0,\infty) & \text{if } \tau_\theta \geq \tau_z^*, \\ \max\{W(0,\infty), W(\phi(0),0)\} & \text{if } \tau_\theta < \tau_z^*. \end{cases}$$

(iii) *If* $\eta \leq \min\{f(\zeta,\rho),0\}$*, then* $dW(\phi(\tau_y),\tau_y)/d\tau_y < 0$ *for all* τ_y*. Thus,*

$$\sup_{\tau_y} W(\phi(\tau_y),\tau_y) = W(\phi(0),0).$$

(iv) *If* $0 > \eta > f(\zeta,\rho)$*, then* $dW(\phi(\tau_y),\tau_y)/d\tau_y > 0$ *if* $\tau_y < \tau_z^* - \tau_\theta$ *and* $dW(\phi(\tau_y),\tau_y)/d\tau_y < 0$ *if* $\tau_y > \tau_z^* - \tau_\theta$*. Thus,*

$$\sup_{\tau_y} W(\phi(\tau_y),\tau_y) = \begin{cases} W(\phi(0),0) & \text{if } \tau_\theta \geq \tau_z^*, \\ W(\phi(\tau_z^* - \tau_\theta), \tau_z^* - \tau_\theta) & \text{if } \tau_\theta < \tau_z^*. \end{cases}$$

Proof. Proposition 2 directly implies the case with $\rho = 0$. We prove the case with $\rho > 0$ using Proposition 3.

Suppose that $\eta = 0$. In this case, $\Delta_0 + \rho\Delta_\infty(\phi(\tau_y))$ is constant, and the above is implied by Proposition 3.

Suppose that $\eta \neq 0$. If $\Delta_0 + \rho\Delta_\infty(\tau_x) = 0$, then

$$\tau_x \sqrt{C'(\tau_x)} = c^{1/2}\tau_x^{(\rho+2)/2} = \frac{\beta\rho}{2(1-\alpha)\left((1-\alpha)\left((1+\rho)\zeta - 1\right)/\eta - (1+\rho)\right)}.$$

Let τ_x^* be the unique solution. Then, $\tau_z^* = \phi^{-1}(\tau_x^*) + \tau_\theta$. Note that if there exists τ_y with $\Delta_0 + \rho\Delta_\infty(\phi(\tau_y)) = 0$, then $\tau_z^* > 0$ must follow.

It holds that $\tau_z^* \leq 0$ if and only if either $\eta \geq \max\{f(\zeta,\rho),0\}$ or $\eta \leq \min\{f(\zeta,\rho),0\}$. In this case, $\Delta_0 + \rho\Delta_\infty(\phi(\tau_y))$ has the same sign for all τ_y. Thus, (i) and (iii) are implied by Proposition 3.

It holds that $\tau_z^* > 0$ if and only if either $0 < \eta < f(\zeta,\rho)$ or $0 > \eta > f(\zeta,\rho)$. In this case, $\Delta_0 + \rho\Delta_\infty(\tau_x)$ changes its sign at $\tau_y = \tau_z^* - \tau_\theta$. Thus, (ii) and (iv) are implied by Proposition 3. □

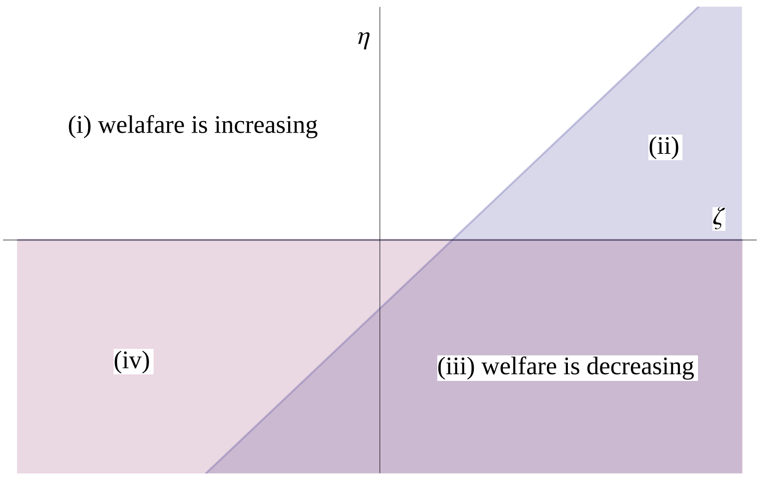

Figure 2: The four cases on the ζ-η plane.

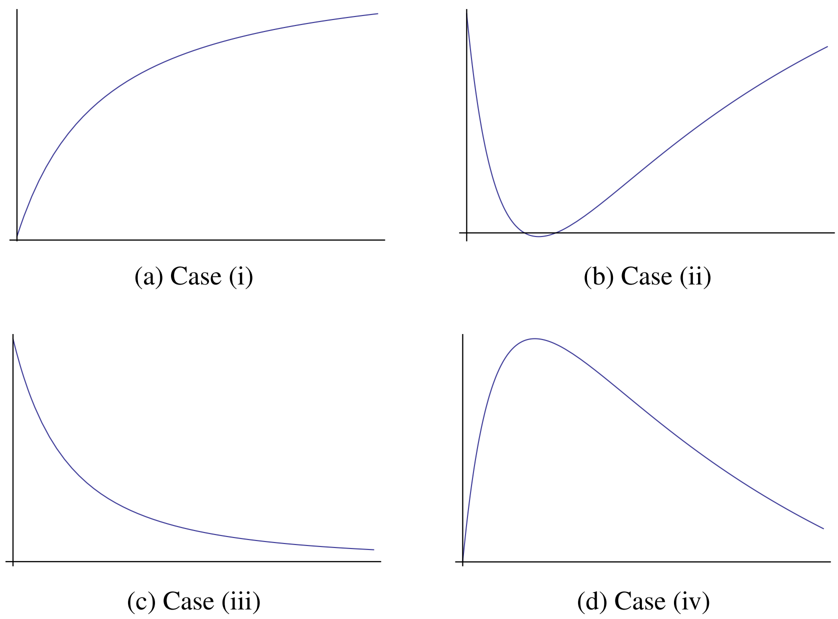

Figure 3: The welfare effects of public information in the four cases.

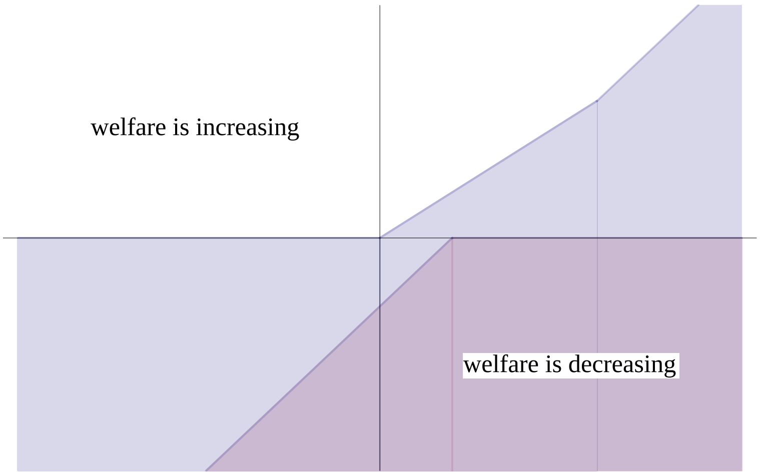

Figure 4: Public information is beneficial (harmful) for any cost function.

There are four cases. Each case is illustrated in the ζ-η plane in Figure 2, where the upward sloping line is a graph of $\eta = f(\zeta, \rho)$. In the region (i) with $\eta \geq \max\{f(\zeta, \rho), 0\}$, welfare necessarily increases with public information (see Figure 3a), so the optimal precision of public information is the highest precision. In the region (ii) with $0 < \eta < f(\zeta, \rho)$, welfare decreases with public information if the precision is low and increases if the precision is high (see Figure 3b), so the optimal precision of public information is either the lowest or the highest precision. In the region (iii) with $\eta \leq \min\{f(\zeta, \rho), 0\}$, welfare necessarily decreases with public information (see Figure 3c), so the optimal precision of public information is the lowest precision. In the region (iv) with $0 > \eta > f(\zeta, \rho)$, welfare increases with public information if the precision is low and decreases if the precision is high (see Figure 3d), so the optimal precision of public information is a strictly positive finite value.

The welfare effects of public information depend upon the cost function, but in some cases, welfare necessarily increases with public information for any cost functions. By Proposition 4, welfare necessarily increases with public information for any cost function if and only if $\eta \geq \max\{f(\zeta, \rho), 0\}$ for all ρ. Similarly, welfare necessarily decreases with public information for any cost function if and only if $\eta \leq \min\{f(\zeta, \rho), 0\}$ for all ρ. Both cases are illustrated on the ζ-η plane in Figure 4 and formally stated in the next corollary.

Corollary 5. *$dW(\phi(\tau_y),\tau_y)/d\tau_y > 0$ for all ρ and τ_y if and only if*

$$\eta \geq \begin{cases} 0 & \text{if } \zeta < 0, \\ \lim_{\rho\to\infty} f(\zeta,\rho) = 2(1-\alpha)\zeta/3 & \text{if } 0 \leq \zeta < 2(1-\alpha), \\ f(\zeta,0) = (1-\alpha)(\zeta-1) & \text{if } \zeta > 2(1-\alpha). \end{cases}$$

$dW(\phi(\tau_y),\tau_y)/d\tau_y < 0$ for all ρ and τ_y if and only if

$$\eta \leq \begin{cases} f(\zeta,0) = (1-\alpha)(\zeta-1) & \text{if } \zeta < 1, \\ 0 & \text{if } \zeta \geq 1. \end{cases}$$

6 An application

Using Proposition 4, we study the optimal disclosure of public information in a Cournot game (Vives, 1988) that maximizes the expected net profit. Player i produces a_i units of a homogeneous product. The inverse demand function is $\theta - \delta \int a_j dj$, where $\delta > 0$ is constant and θ is normally distributed, and the cost function is $a_i^2/2$. Then, player i's payoff function is

$$\Big(\theta - \delta \int a_j dj\Big) a_i - a_i^2/2.$$

By normalizing the cost function appropriately, we can apply Proposition 4 and obtain the optimal precision of public information as follows.

Corollary 6. *Consider a Cournot game. The following holds.*

(i) *Suppose that $\rho = 0$ or $\delta \leq (\rho+2)/(2\rho)$ with $\rho > 0$. Then, the expected net profit increases with public information and the optimal precision of public information is $\tau_y^* = \infty$.*

(ii) *Suppose that $\delta > (\rho+2)/(2\rho)$ with $\rho > 0$. Then, the expected net profit decreases with public information if $\tau_y < \tau_z^* - \tau_\theta$ and increases if $\tau_y > \tau_z^* - \tau_\theta$.*

- *If $\delta < (\rho+2)/(\rho)$ or τ_θ is sufficiently large, the optimal precision of public information is $\tau_y^* = \infty$.*
- *If $\delta > (\rho+2)/(\rho)$ and τ_θ is sufficiently small, the optimal precision of public information is $\tau_y^* = 0$.*

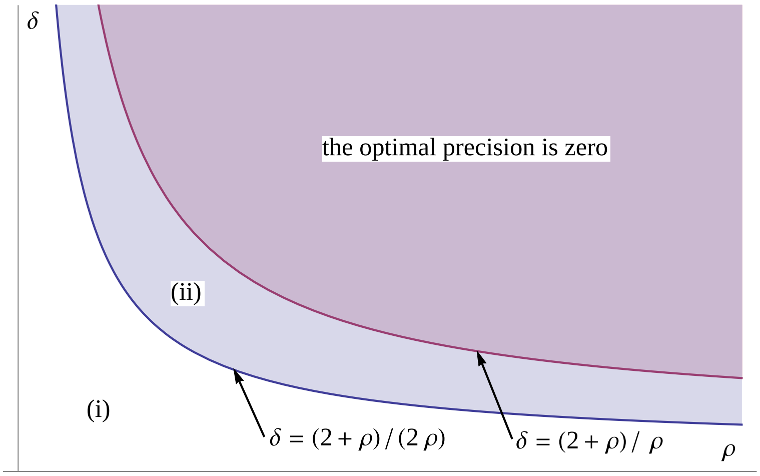

Figure 5: The optimal precision in a large Cournot game.

The above two cases are illustrated on the ρ-δ plane in Figure 5. In the region (i) with $\rho = 0$ or $\delta \leq (\rho+2)/(2\rho)$, where δ or ρ is small, welfare necessarily increases with public information. In particular, this is true for all δ if $\rho = 0$; that is, the cost is linear. In this case, the crowding-out effect of public information is the largest. Thus, an increase in the precision of public information reduces the incentives for acquisition of private information and delivers substantial cost savings enough to compensate any decrease in the expected profit.

In the region (ii) with $\delta > (\rho+2)/(2\rho)$, where δ and ρ are large, welfare can decrease with public information. In particular, if $\delta > (\rho + 2)/(\rho)$ and τ_θ^* is sufficiently small, no provision of public information is optimal. When ρ is large, the crowding-out effect is small and thus an increase in the precision of public information does not deliver cost savings enough to compensate a decrease in the expected profit. When δ is large (i.e., the price elasticity of demand is small), the game exhibits strong strategic substitutability, which induces a large weight on a private signal in the equilibrium strategy and thus a large weight on the idiosyncratic variance in welfare.

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