Time inconsistency and impure benevolent planner*

Takashi Hayashi†, Noriaki Kiguchi‡and Norio Takeoka§

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Abstract

As shown in Zuber [34] and Jackson and Yariv [23], in a society of individuals with exponential discounting, the only way to make the Pareto condition compatible with a social planner with exponential discounting is the dictatorship. Unlike the previous studies, we investigate the compatibility of the Pareto condition with an impure social planner who has a dynamically consistent self-control utility function characterized by Gul and Pesendorfer [15] and Noor [30]. We require that a social planner is tempted to adopt the majority's opinion when there is a conflict of opinions among individuals. Under the Pareto condition and the temptation condition described above, we show the following two ways of avoiding the dictatorship result: Either we accept a social planner with temptation from immediate consumption and avoid dictatorship by allowing the temptation utility to reflect the static tastes of multiple individuals, or instead of adopting a more far-sighted social planner with temptation from consumption streams, we allow the preferences of at most two individuals, including their time preferences, to be reflected into the social planner's preference.

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†The corresponding author. Adam Smith Business School, University of Glasgow. email: `Takashi.Hayashi@glasgow.ac.uk`

‡Graduate School of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo, 186-8601, Japan. email: `ed225004@g.hit-u.ac.jp`

§Graduate School of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo, 186-8601, Japan. email: `norio.takeoka@r.hit-u.ac.jp`

1 Introduction

How the society should discount future benefits and costs over time is critical in the policy arguments on issues with long-term consequences, such as environmental conservation, climate change and also inequality between families/classes. This paper studies how a benevolent social planner should form social preference by aggregating the preferences of individuals with different opinions about tastes over per-period outcomes and trade-offs among different time periods. Our starting point is the impossibility result regarding existence of a social planner as assumed in the standard dynamic economic analysis which is common in studying policy issues with long-term consequences. Zuber [34] and Jackson and Yariv [23] have shown that when individuals differ in time discounting as well as in tastes over per-period social outcomes, only dictatorship can meet the following two requirements: the social planner's objective function satisfies the standard stationary discounted utility theory and the Pareto condition stating that if everybody prefers a stream of social outcomes over another so should the society.

There have been two approaches to escape from the dictatorship result. One is to weaken the Pareto condition, which weakens the requirement that every physical individual is perfectly responsible for his/her time preference (Feng and Ke [12], Hayashi and Lombardi [19], Billot and Qu [3]). The other is to give up or weaken stationary/exponential/geometric discounting at the social level (Millner and Heal [27], Millner [26]). It is controversial, however, if the planner can commit to the maximization solution for such a non-stationary objective function.

Although stationarity is not immediately the logical equivalent of dynamic consistency, practically it is: if you say "today is special, so give me a favour" today, it is natural to expect that you repeat saying the same thing tomorrow and onward. Or, in other words, if the social planner is such a person to accept the claim "today is special, so give me a favour" today, it is natural to expect that he/she is such a person to repeat the same thing tomorrow and onward. Under such empirically natural assumption, called time invariance (Millner and Heal [27]), stationarity is indeed equivalent to dynamic consistency. When the planner is expected not to behave in a time-consistent manner, it is left unclear what objective is being accomplished or implemented eventually, because what we have there is a *sequence* of social welfare functions rather than a single one and the current society has to play a game against successive future societies.[1]

In a dynamic environment, it is rare to be able to commit in advance to a particular sequence of social outcomes; rather, choices are usually made sequentially. In such cases, it is essential for the decision maker to rank among sets of future options, called *menus*, (for instance, intertemporal budget sets determined by the current decision on savings or capital accumulation). The advantage of assuming the social planner's preference as an exponential discounting model is that one can construct a recursive (thus, dynamically consistent) value function over menus and carry out sequential policy decisions based on it. As the menu preference literature (for example, Gul and Pesendorfer [15]) uncovers, assuming an exponential discounting model on consumption streams is not the only way to obtain a recursive model over menus. The purpose of this paper is to study the compatibility of the social planner's utility function with various Pareto criteria by adopting a general setting in which a social planner has a recursive utility function over menus, unlike the setting in previous studies in which a social planner has preference over consumption streams.

A social planner will face several conflicts in the process of aggregating the preferences of individuals with diverse opinions about social outcomes and trade-offs among different time periods. Even if one group's opinion is adopted for some normative reason when considering the ranking of alternatives, it may be difficult to consistently ignore the opinions of other groups. The literature on social preferences (e.g., Dillenberger and Sadowski [10] and Saito [32]) has shown that people are subject to social pressure to behave altruistically in public situations that are observed by others when making social allocation decisions.

[1] To illustrate the point, consider the following example which is common in the macroeconomics literature regarding existence of representative agent under heterogeneous time preferences (see Pakhnin [31] for a survey). There is 1 unit of consumption good at each period, and there are two individuals, A and B. For each $i = A, B$, his/her preference over lifetime consumption streams is represented in the form $\sum_{t=1}^{\infty} \beta_i^{t-1} \ln c_{it}$. Then the solution to maximize a social welfare defined by $\sum_{t=1}^{\infty} \beta_A^{t-1} \ln c_{At} + \sum_{t=1}^{\infty} \beta_B^{t-1} \ln c_{Bt}$ is given as

$$c_{At} = \frac{\beta_A^{t-1}}{\beta_A^{t-1} + \beta_B^{t-1}}, \quad c_{Bt} = \frac{\beta_B^{t-1}}{\beta_A^{t-1} + \beta_B^{t-1}}, \quad \forall t = 1, 2, 3, \cdots$$

However, once period 1 passes and we come to period 2, the planner will again optimize the social welfare by maximizing $\sum_{t=2}^{\infty} \beta_A^{t-2} \ln c_{At} + \sum_{t=2}^{\infty} \beta_B^{t-2} \ln c_{Bt}$, which changes the solution to

$$c_{At} = \frac{\beta_A^{t-2}}{\beta_A^{t-2} + \beta_B^{t-2}}, \quad c_{Bt} = \frac{\beta_B^{t-2}}{\beta_A^{t-2} + \beta_B^{t-2}}, \quad \forall t = 2, 3, 4, \cdots$$

Even without altruism, when only the opinions of some groups are reflected in social choices, concern about possible backlash from other groups may result in a more equitable social decision that takes into account the opinions of both groups as a result of compromise.

To accommodate such internal conflicts and resulting compromises, we assume that a social planner has a recursive self-control utility function over menus. Thus, we consider a problem of aggregating individual preferences over *streams* into a social ranking over *menus* of social alternatives. We adopt the future temptation (FT) model proposed by Noor [30] as a recursive self-control utility function. In this model, the decision maker has a normative stationary discounted utility but is tempted by a consumption stream that maximizes a different stationary discounted utility. Such a formulation allows us to consider an "impure" benevolent decision making in which the social planner forms a normative discounted utility that reflects the opinions of one group, but is tempted to maximize a discounted utility that reflects the opinions of another group, resulting in choosing from each menu by maximizing a linear combination of the two. The characteristic that distinguishes impure benevolent social planners from purely benevolent social planners is that they use commitment whenever they can avoid temptation from the opinions of other groups.[2]

We consider three types of axioms on preference aggregation. The first is the Pareto condition applied to commitment streams of social outcomes. The second is that the planner should not be tempted by Pareto-inferior streams, which is natural since we intend to capture temptation arising solely because of disagreements of individuals' opinions. The third is that the social planner should be tempted to accommodate majority's opinion, which illustrates the nature of temptation a benevolent planner should face. The more a smaller minority's opinion is adopted as the social choice, the greater will be the social pressure and the backlash from the other majority that the social planner faces.

We provide axiomatic characterizations in three folds. First one presents the most permissive kind of aggregation, which involves no intertemporal trade-offs. When the planner only take individuals' per-period utility into account, the individuals are divided into two groups: per-period utility functions in one group reflect to the planner's commitment per-period utility function and ones in the other group reflect to its temptation per-period utility. Second one presents aggregation in which the planner's temptation utility takes the individual's time preferences into account. When imposing Pareto for commitment

[2]See Dillenberger and Sadowski [10] and Saito [32] for impure altruism.

paths, for the same reason as Zuber [34] and Jackson and Yariv [23], the planner's commitment utility function can incorporate only one individual's time preference into account. We call such an individual a "commitment dictator". Also, because the temptation utility function in the FT model satisfies stationarity, a "temptation dictator" arises, which contrasts with a commitment dictator. In the third characterization, we assume the case where the social planner is tempted only by immediate consumption allocations such as in Gul and Pesendorfer [15].[3] This assumption prevents the social planner's temptation utility from including the individuals' time preferences, but instead allows its temptation per-period utility function to reflect the opinions of multiple individuals. It thereby avoids the existence of a temptation dictator.

Finally, we give some remarks on other possible settings in which menu preferences are taken into account. As explained above, we consider the problem of aggregating individuals' discounted utilities, defined on consumption streams, to the social planner's preference, defined on menus. If, instead of this setting, individuals can also express opinions about their menu preferences, we may consider an alternative aggregation problem, such as aggregating individuals' preferences over menus into the social planner's menu preference.

We adopt the current framework for two reasons. One is that each individual cannot evaluate a menu without knowing which one in it is chosen by the society, which depends again on how we aggregate preferences. In single-person cases, an individual without any self-control issue simply evaluates a menu as the best element in it.[4] In a multi-person setting, however, different individuals cannot do this in one society, since they in general have different best elements in the same menu. One may still consider a version of Pareto condition, which states that if everybody prefers a menu over another according to the above-stated sense then so should the society. Such unanimity may be "spurious" in the sense that different individual have different best ex-post choices in mind, the social ranking may exhibit preference for flexibility which supports a menu based on mutually conflicting reasons (Kreps [24], Dekel, Lipman, and Rustichini [8]). In fact, under a reasonable "non-delusional" condition for the social ranking (including temptation-driven preferences

[3]In the FT model, if the discount factor of the temptation discounted utility is zero, the decision maker faces the temptation to consume all the wealth in the present period. This special case corresponds to Gul and Pesendorfer [15].

[4]Such a standard decision making is called strategic rationality and axiomatized in terms of menu preference by Kreps [24].

such as self-control preferences), only dictatorship can satisfy such Pareto condition (see Proposition 1).

Second is that we aim to emphasize the self-control problem that arises *specifically* at the level of collective decision. More precisely, we are considering here the temptation that the social planner suffers only when there are disagreements of opinions among individuals, such as the social pressure that arises from the other groups with the opposite opinions when only one group's opinion is adopted. It is in the same spirit as in the voting model considered by Lizzeri and Yariv [25], which arises even when nobody has any self-control issue at an individual level.

The paper is organized as follows. Section 1.1 provides an overview of the existing literature. Section 2 introduces the choice setting, the individuals' utility functions, and the social planner's utility function. Section 3 proposes the consistency between the individuals' preferences and the social planner's preference. We introduce three types of Pareto conditions and axioms that restrict the direction of the social planner's temptation. Section 4 presents characterization theorems on the possible preference aggregation under each set of axioms. Using the optimal growth model as an example, section 5 discusses the social welfare implications of the optimal growth path when the social planner sequentially optimizes a self-control utility function characterized in the previous section. Then Section 6 concludes.

1.1 Related literature

1.1.1 Escaping from the impossibility

As shown by Zuber [34] and Jackson and Yariv [23], the impossibility theorem states that, assuming a group of individuals with stationary discounted utility, the only social preference that satisfies Pareto efficiency and stationary discounted utility can be dictatorship in the sense that it can only coincide with a particular preference in the group. There have been two main approaches to escape from this dictatorship result. One direction is to relax Pareto efficiency with keeping the stationarity of the social preference, while the other is to relax the stationarity with keeping Pareto efficiency.

Feng and Ke [12] propose an axiom that treats successive selves of an identical individual as different individuals and that if all such selves prefer a stream over another so should the society. Their axiom allows that the initial self of an individual is not fully responsible for his/her life course planning, even if his/her preference is time-consistent. For example, a future self of an impatient individual may not be responsible for having only little saving left to him/her. By taking all those successive selves' preferences into account, they characterize bounds on the social planner's discount factor.

Hayashi and Lombardi [19] propose an axiom in which each individual's lifetime discounted utility is calculated in different ways by using all the individuals' discount factors, not just his/her own discount factor. This axiom is still strong enough to imply that the society has to adopt exactly one individual's discount factor, meaning that averaging is ruled out, while we can take weighted sum of per-period utilities. The selection of such "discounting dictator" can be reasonably done, since the apparent dictatorship is only for a fixed profile, and adding two variable-profile axioms (anonymity and continuity) characterizes selection of generalized median (order statistic). Billot and Qu [3] propose an even weaker axiom that limits attention to restricted class of comparisons of streams and obtain a permissive result which allows averaging of discount factors.

As in the above literature, considering a social preference defined on streams implies that the most preferable option in terms of the social preference is eventually chosen among all possible streams that can be chosen in a given choice problem, whether it is under commitment or in a sequential choice environment. Therefore, if the social preference on streams satisfies the Pareto condition, then its maximization solution from any choice problem satisfies Pareto efficiency. In contrast, under self-control preferences, a choice from menus is determined by a compromise between normative and temptation utilities. As explained in detail in Section 3, although we will assume both normative and temptation utilities to respect the Pareto condition in the usual sense (if all individuals prefer one stream to another, so does from the social perspective), this axiom is not strong enough to guarantee that a Pareto-efficient stream is eventually chosen in all choice problems or menus. Thus, as in the above literature, our study can be categorized into a group of studies that avoid the dictatorship by weakening Pareto efficiency.

Giving up or weaken stationary/exponential/geometric discounting at the social level allows us to maintain efficiency. Millner and Heal [27], Millner [26] propose time-dependent social welfare function with non-stationary discounting. As we have discussed, it is debatable if the planner can commit to the maximization solution for such a non-stationary objective function, which is the main point of our paper. One may argue, however, that we can allow non-stationary discounting at a social level while maintaining time-consistency, since it is a normative argument and we may achieve it through deliberate discussions.

Commitment to follow the maximization solution for a non-stationary social welfare function requires an extra restriction, in any case, which is beyond the mere fact that the solution was already decided in the past period. Hayashi [18] proposes a meta axiom stating that if the planner bases welfare judgment at one period on some normative reason the same reason must apply to welfare judgment in the next period as well.

Another possible direction to escape from the dictatorship result is to weaken the completeness axiom at a social level. Chambers and Echenique [6] characterize the Pareto ranking that is induced when the individuals have discounted utility preferences and show that it has a tractable structure.

Positive analysis of what would happen if the society cannot commit to future actions is also a natural direction. Heal and Millner [22] show that the planning solution is not time-consistent under lack of commitment, but it is renegotiation-proof. In the setting of accumulation of public capital where agents vote over current savings without commitment to future, Borissov, Pakhnin and Puppe [4] and Borissov, Hanna and Lambrecht [5] consider a class of voting strategies as linear functions of current capital amount, and establish a recursive voting equilibrium in which the agent with median discount factor is decisive. Hayashi and Lombardi [20] show that such equilibrium exists without the assumption of linear strategy, but show that the assumption of common per-period utility function is indispensable for the existence of equilibrium.

1.1.2 Self-control preferences

A self-control utility is first axiomatized by Gul and Pesendorfer [14] as a utility representation of preference over menus. In this model, the rankings over menus are determined through conflicts between the normative utility, corresponding to the commitment utility, and the temptation utility. Gul and Pesendorfer [15] extend the self-control utility to a recursive domain applicable to infinite-horizon dynamic decision making. Salient features of this model are that (1) the decision maker is tempted only from immediate consumption and (2) the representation is dynamically consistent. This model can explain anomalies such as the preference reversal due to present bias for choices from a menu. To apply the self-control model for addictive behavior, Gul and Pesendorfer [16] consider a generalization in which the intensity of temptation depends on past consumption history.

Noor [30] generalizes the assumption of being tempted only from immediate consumption and axiomatizes the future temptation (FT) model in which the decision maker may be tempted from the entire consumption stream. Here, both normative utility and temptation utility follow a stationary discount model.

Ahn, Iijima, and Sarver [1] establish a recursive representation with naiveté. Although their primary concern is a recursive model allowing for naiveté, as a special case, they axiomatize a model, called a sophisticated quasi-hyperbolic discounting representation. As in Noor [30], the decision maker may be tempted from the entire consumption streams. More specifically, the continuation value function of the temptation utility is the same as that of the normative utility with the current discount factor being specified as in quasi-hyperbolic discounting. Thus, the decision maker exhibits the quasi-hyperbolic discounting only when succumbed to this temptation.

Hayashi and Takeoka [21] are motivated by the projection bias and consider a decision maker who correctly anticipates his/her future preference by considering the effect of habits, while he/she is also tempted to ignore such a habit formation. The decision maker exerts self-control for resisting such a self-deception. As in Noor [30], their decision maker is also tempted to the entire consumption streams.

2 The setting and decision models

Time is discrete and infinite. Let C be the set of social outcomes per period, which is assumed to be compact metric. Given a compact metric space Y, let $\Delta(Y)$ be the set of lotteries (Borel probability measures) over Y, which is a compact metric space with respect to the Prokhorov metric and is also a convex set of a linear space. Given a compact metric space Y, let $\mathcal{K}(Y)$ be the set of compact subsets of L, which is a compact metric space with respect to the Hausdorff metric.

We consider the domain of infinite-horizon choice problems $\mathcal{Z}$, which satisfies the recursive formula[5]

$$\mathcal{Z} \simeq \mathcal{K}(\Delta(C) \times \mathcal{Z}).$$

A generic element is denoted by $z \in \mathcal{Z}$ and called a menu. Note that $\mathcal{Z}$ includes $\Delta(C)^\infty$, the subdomain of sequences of lotteries over social outcomes. Given $l \in \Delta(C)^\infty$, let $\{l\}$ denote the choice problem in which there is no choice other than committing to $\{l\}$ once and for all.[6]

Let $I = \{1, \cdots, |I|\}$ be the set of individuals. For each individual $i \in I$, let $\succsim_i$ denote his/her preference over $\Delta(C)^\infty$, which is represented in the exponential discounted utility form

$$U_i(l) = \sum_{t=1}^{\infty} u_i(l_t)\beta_i^{t-1},$$

where $u_i : \Delta(C) \to \mathbb{R}$ is a continuous and mixture-linear function and $\beta_i \in (0, 1)$ is a discount factor.

We assume that the social planner has a preference $\succsim_0$ over menus $\mathcal{Z}$, which admits a future temptation (FT) utility representation (Noor [30]): There exist continuous expected utility functions $u_0, v_0 : \Delta(C) \to \mathbb{R}$ (u is non-constant), discount factors $\beta_0 \in (0, 1)$, $\gamma_0 \in [0, 1)$, and a parameter $\kappa_0 > 0$, which captures the intensity of temptation, such that $\succsim_0$ is represented by

$$W_0(z) = \max_{(m,z')\in z} \left\{ u_0(m) + \beta_0 W_0(z') - \kappa_0 \left(\max_{(n,y')\in z} V_0(n, y') - V_0(m, z') \right) \right\}, \tag{1}$$

where

$$V_0(m, z') = v_0(m) + \gamma_0 \max_{(m',z'')\in z'} V_0(m', z'').$$

A basic structure of the FT representation $W_0(z)$ is the same as the self-control representation of Gul and Pesendorfer [14]. Two component functions $U_0(m, z') := u_0(m)+\beta_0 W_0(z')$ and $V_0(n, y')$ are interpreted as normative and temptation utility functions, respectively,

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[5] This domain is smaller than the one established in Gul and Pesendorfer [15] and Noor [30], say $\mathcal{Z}^*$, which satisfies the recursive formula $\mathcal{Z}^* \simeq \mathcal{K}(\Delta(C \times \mathcal{Z}^*))$. It includes the domain of probability trees (Epstein and Zin [11], Chew and Epstein [7]) satisfying the recursive formula $\mathcal{D} \simeq \Delta(C \times \mathcal{D})$. We adopt the current formulation for simplicity of presentation, however, because compound lotteries are not playing any role here. The recursive formula $\mathcal{Z} \simeq \mathcal{K}(\Delta(C) \times \mathcal{Z})$ can be shown directly or by taking a suitable restriction of $\mathcal{Z}^* \simeq \mathcal{K}(\Delta(C \times \mathcal{Z}^*))$.

[6] It is a shorthand and imprecise notation of $\{(l_1, \{(l_2, \{(l_3, \{(l_4, \cdots)\})\})\})\}$, which actually has to involve infinitely many brackets and parentheses.

as in Gul and Pesendorfer [14]. The non-negative term $(\max_{(n,y')\in z} V_0(n,y') - V_0(m,z'))$ is regarded as self-control costs, which are opportunity costs in terms of temptation utilities.

Note that W_0 admits a recursive form, and hence, the FT representation satisfies dynamic consistency. The temptation utility V_0 also has a stationary recursive form. If v_0 is constant, V_0 is also constant, and hence, W_0 is reduced to a standard stationary discounted utility. If $\gamma_0 = 0$, W_0 is reduced to the dynamic self control (DSC) model by Gul and Pesendorfer [15],

$$W_0(z) = \max_{(m,z')\in z} \{u_0(m) + v_0(m) + \beta_0 W_0(z')\} - \kappa_0 \max_{(n,y')\in z} v_0(n),$$

in which the the planner is tempted only by immediate consumption allocations.

The FT representation (1) suggests which element in z is chosen in the ex post stage. The choice from z should achieve the value function $W_0(z)$, which is equivalent to a maximizer for the problem $\max_{(m,z')\in z} U_0(m,z') + \kappa_0 V_0(m,z')$. Hence, self-control from temptation will result in a compromise choice between the normative utility and the temptation utility.

Axiomatic characterizations of the above decision model are established in Gul and Pesendorfer [15] (the case of $\gamma_0 = 0$) and Noor [30] (the case of $\gamma_0 > 0$). Note that when we restrict attention to commitment plans, (1) reduces to the standard discounted expected utility model, that is,

$$W_0(\{l\}) = \sum_{t=1}^{\infty} u_0(l_t)\beta_0^{t-1}.$$

As mentioned above, the parameter κ_0 captures the intensity of temptation. From (1), we can see that a higher κ_0 implies greater self-control costs, which yields a smaller value of utility representation $W_0(z)$ for all menus z. A behavioral implication is that assuming the other parameters identical, the FT representation with κ_0^1, denoted by W_0^1, exhibits more desires for commitment than that with $\kappa_0^2 < \kappa_0^1$, denoted by W_0^2 that is, for all $l \in \Delta(C)^\infty$ and all menus z,[7]

$$W_0^2(\{l\}) \geq W_0^2(z) \implies W_0^1(\{l\}) \geq W_0^1(z).$$

This condition states that whenever the representation with κ_0^2 prefers a commitment to $\{l\}$, the representation with a higher parameter κ_0^1 does so.

[7]Since $W_0^1(\{l\}) = W_0^2(\{l\})$ for all $l \in \Delta(C)^\infty$, $W_0^2(\{l\}) \geq W_0^2(z)$ implies $W_0^1(\{l\}) = W_0^2(\{l\}) \geq W_0^2(z) \geq W_0^1(z)$, as desired.

Note also that if κ_0 is extremely higher, self-control costs become prohibitively greater. Then, the social planner is likely to yield to the temptation, and a choice from menus tends to be governed by V_0. At the limit, the FT representation converges to the Strotz representation (Strotz [33]) as $\kappa_0 \to \infty$.

3 The axioms on aggregation

This section describes the normative axioms that the individuals' discount utility functions and the social planner's self-control utility function should satisfy, as well as the descriptive axioms for characterizing the social planner's temptation.

Throughout, we assume the following Richness Condition. Given $m \in \Delta(C)$, let $m^\infty \in \Delta(C)$ denote the constant sequence of m.

Richness Condition

There exists $\underline{m} \in \Delta(C)$ such that for all $i \in I$, there exists $m_i \in \Delta(C)$ with $m_i^\infty \succ_i \underline{m}^\infty$ and $m_i^\infty \sim_h \underline{m}^\infty$ for all $h \neq i$.

Note that the condition implies $\left(\sum_{h \in I} \frac{1}{|I|} m_h\right)^\infty \succ_i \underline{m}^\infty$ holds for all $i \in I$. The richness condition technically states that the utility possibility set defined over per-period outcomes is full-dimensional and spanned by linearly independent vectors. In particular, this property excludes the possibility that all individuals have the same preference on $\Delta(C)$.

The condition is natural in the context of allocating consumption in the dynamic general equilibrium environment in which each individual has selfish preference over own private consumption. We can take $\underline{m}$ to be the lottery that is degenerated to the allocation which gives everybody zero consumption. For each $i \in I$, we can take m_i to be the lottery that is degenerated to an allocation which gives positive consumption only to i. Note that the condition puts no restriction on preferences over private consumption lotteries once we restrict attention to selfish ones, hence it does not exclude for example the case that everybody has identical taste over private consumption.

3.1 Pareto conditions

First, we present the most permissive Pareto condition in which no intertemporal trade-offs are involved.

Constant Commitment Pareto: For all $m, n \in \Delta(C)$, if $m^\infty \succ_i n^\infty$ for all $i \in I$, then $\{m^\infty\} \succ_0 \{n^\infty\}$.

Since comparisons between m^∞ and n^∞ do not involve intertemporal concerns, the axiom imposes the Pareto condition on static risk preferences.

The next is the Pareto condition stating that if everybody prefers one stream of social outcomes over another so should the society. To emphasize that the condition is only about preference over commitment plans, we call it *Commitment Pareto*.

Commitment Pareto: For all $l, l' \in \Delta(C)^\infty$, if $l \succ_i l'$ for all $i \in I$, then $\{l\} \succ_0 \{l'\}$.

When the social planner's decision model is a traditional one in which no self-control issue arises, Commitment Pareto simply implies that the resulting optimization solution is Pareto-efficient in *any* choice problem, since the decision model does not distinguish between commitment problems and non-commitment problems. More formally, for each menu z, say that $l = (l_1, l_2, \cdots) \in \Delta(C)^\infty$ is feasible in z if there exists a sequence of menus $(z_t)_{t=1}^\infty$ such that $(l_1, z_1) \in z$ and $(l_t, z_t) \in z_{t-1}$ for all $t \geq 2$. A traditional way to evaluate menus, called strategic rationality in Kreps [24], requires that each menu z admits a feasible stream $l^z \in z$ such that $\{l^z\} \succsim \{l\}$ for all feasible streams $l \in z$ and

$$z \sim_0 \{l^z\}. \tag{2}$$

That is, each menu z is evaluated according to its best element in terms of the commitment ranking among all feasible streams in z. Under this criterion, given any recursive menu $z \in \mathcal{Z}$, an optimal stream in terms of the commitment preferences will be chosen through the sequential decision making. Thus, if the commitment preference respects the Pareto condition, for any recursive menu, a stream eventually chosen through sequential decision is Pareto efficient.

However, a self-control preference is not consistent with the strategic rationality. Indeed, it is easy to see that condition (2) implies that for all menus z and z',

$$z \succsim_0 z' \implies z \sim_0 z \cup z',$$

which is not consistent with either preference for commitment such as $z \succ_0 z \cup z'$ or a self-control preference. Consequently, even under Commitment Pareto, the social planner's choice from a menu may not be Pareto efficient. For example, consider a feasible stream $l^z = (l_1^z, l_2^z, \cdots)$ for a menu z satisfying (2). Let z_1 be a continuation menu after the current outcome l_1^z, which is necessary to choose the stream l^z. If the social planner is tempted from the immediate consumption, for instance, the planner may choose some different $(l_1', z_1') \in z$ over (l_1^z, z_1) when the current outcome l_1' is more tempting than l_1^z. Thus, l^z is not chosen from z.

Although Commitment Pareto appears to be an identical statement to the existing Pareto condition, it is indeed a weaker requirement in the sense that it requires the Pareto condition only on the commitment preference but not on the ex post choice from menus, which may end up with a Pareto-inferior stream at the end of sequential choice.

Since Commitment Pareto is a weaker requirement imposed only on the commitment preference, one might think of extending the Pareto condition to the entire domain of choice problems without commitment. Since each individual has a preference $\succsim_i$ over consumption streams, we first extend it to the corresponding preference $\succsim_i^*$ over menus $\mathcal{Z}$ by the procedure of strategic rationality, that is,

$$z \sim_i^* \{l^z\}$$

for a feasible $l^z \in z$ such that $l^z \succsim_i l$ for all feasible $l \in z$. An attempt to extend a Pareto condition to the whole domain of menus is as follows:

Non-Commitment Pareto: For all $z, z' \in \mathcal{Z}$, if $z \succ_i^* z'$ for all $i \in I$, then $z \succ_0 z'$.

By definition, Non-Commitment Pareto is stronger than Commitment Pareto.

As mentioned in the introduction, the evaluation of menus via strategic rationality may not be valid in terms of individual's level when menus consist of social outcomes. Unlike the evaluation of consumption streams, the evaluation of menus crucially depends on beliefs about in what way options will be chosen from menus. Evaluating a menu of social outcomes via strategic rationality implies that the individual has an optimistic belief that his/her best option in the menu will be always a consequence of social choice. Moreover, such too optimistic beliefs are in general mutually incompatible across individuals.

For an illustration, consider two individuals A and B and three alternatives l, l', l'' such that $l \succ_A l'' \succ_A l'$ and $l' \succ_B l'' \succ_B l$. Under strategic rationality, since $\{l, l'\} \succ_A^* \{l''\}$ and $\{l, l'\} \succ_B^* \{l''\}$, Non-Commitment Pareto concludes $\{l, l'\} \succ_0 \{l''\}$. This must involve a delusion, since the reason why A prefers $\{l, l'\}$ is that A hopes l is chosen ex-post and the reason why B prefers $\{l, l'\}$ is that B hopes l' is chosen ex-post, and one of them must be wrong eventually.[8]

In fact, the claim below shows generally that Non-Commitment Pareto leads to dictatorship unless the planner exhibits preference for flexibility based on mutually contradicting reasons like above.

Proposition 1 Let X be a compact metric space in which a mixture operation is defined. Let $\mathcal{K}(X)$ be the set of compact subsets of X endowed with the Hausdorff metric, in which the corresponding set-mixture operation is defined.

Let $\{\succsim_i\}_{i\in I}$ be individual preferences over X which satisfy completeness, transitivity, continuity, and mixture independence: $a \succsim_i b$ implies $\lambda a + (1-\lambda)c \succsim_i \lambda b + (1-\lambda)c$ for all $a, b, c \in X$ and $\lambda \in (0,1)$. Assume, moreover, that there exist $\underline{a} \in X$ such that for all $i \in I$ there exists $a_i \in X$ with $a_i \succ_i \underline{a}$ and $a_i \sim_h \underline{a}$ for all $h \neq i$.

Let $\succsim_0$ be the social ranking over $\mathcal{K}(X)$ which satisfies completeness, transitivity, continuity and the following two axioms:

- Set-Mixture Independence: $A \succsim_0 B$ implies $\lambda A + (1-\lambda)C \succsim_0 \lambda B + (1-\lambda)C$ for all $A, B, C \in \mathcal{K}(X)$ and $\lambda \in (0,1)$;
- Weak Set-Betweenness: $A \sim_0 B$ implies $A \cup B \sim_0 A \sim_0 B$ for all $A, B, C \in \mathcal{K}(X)$.

For each $i \in I$, let $\succsim_i^*$ be the extension of $\succsim_i$ to $\mathcal{K}(X)$ defined as in (2). Then, the profile $\succsim_0$ and $\{\succsim_i^*\}_{i\in I}$ satisfies Non-Commitment Pareto if and only if there exists $i^* \in I$ such that $\succsim_0 = \succsim_{i^*}^*$.

In the proof (Section A.1), we show that the conditions and axioms except for Weak Set-Betweenness imply a non-negative additive aggregation of individuals' utility functions over menus having the form of strategic rationality. This welfare function has the same form of representations consistent with preference for flexibility (Kreps [24] and Dekel, Lipman, and Rustichini [8]). Weak Set-Betweenness is generally incompatible with preference for

[8]This argument against strategic rationality on the social choice environment and Non-Commitment Pareto is similar to the critique to the Pareto condition in choice under uncertainty, called spurious unanimity. See Mongin [28] and Gilboa, Samet and Schmeidler [13] for the impossibility of Paretian aggregation under heterogeneous beliefs and weakening of the Pareto condition, and also Mongin [29] for philosophical arguments on the idea of spurious unanimity.

flexibility and does not allow for positive weights to be placed on multiple individuals. Therefore, by adding this axiom, the weight vector is degenerate on one individual, leading to the dictatorship. Since the self-control preference satisfies an even stronger axiom, called Set Betweenness,[9] Proposition 1 implies that the dictatorship is the only possibility in our setting if Non-Commitment Pareto is assumed.

3.2 Disagreement and temptation

We introduce axioms regarding the temptation of the social planner. As we consider the benevolent social planner, the only source of temptation here is disagreement among individuals. Thus, it is appealing to assume that the temptation to choose a Pareto-inferior option over an unanimously preferred option would not arise.

Based on such a hypothesis, the next two axioms state that a Pareto-inferior item should not be tempting for the planner. Such condition is reasonable given the scope of this paper, since the source of temptation for the planner is political tension due to disagreements between individuals. We can state this in two ways. One is weaker in the sense that no intertemporal trade-offs are involved in the definition. The other is stronger in the sense that intertemporal trade-offs are involved.

No Temptation from Pareto Dominated Outcomes (NTPDO): For all $m, n \in \Delta(C)$, if $m^\infty \succ_i n^\infty$ for all $i \in I$ and $\{m^\infty\} \succ_0 \{n^\infty\}$, then $\{m^\infty\} \sim_0 \{m^\infty, n^\infty\}$.

No Temptation from Pareto Dominated Paths (NTPDP): : For all $l, l' \in \Delta(C)^\infty$, if $l \succ_i l'$ for all $i \in I$ and $\{l\} \succ_0 \{l'\}$, then $\{l\} \sim_0 \{l, l'\}$.

The presumption of NTPDO states that all individuals prefer a lottery m to n, and so does the social planner (presumably because of the Pareto condition). Since there is no conflict among individuals' opinions, the planner can choose a normatively preferred m^∞ from $\{m^\infty, n^\infty\}$ without any temptation. The same interpretation is applicable for NTPDP with replacing lotteries with streams.

The final axiom states that the benevolent social planner should be indeed tempted to accommodate majority when there are disagreements.

[9]Set Betweenness requires that $A \succsim_0 B$ implies $A \succsim_0 A \cup B \succsim_0 B$ for all $A, B \in \mathcal{K}(X)$. See Gul and Pesendorfer [14] for more details.

Temptation to Accommodate Majority (TAM): For all $i \in I$, for all $m, n \in \Delta(C)$, if $m^\infty \succ_i n^\infty$, $m^\infty \prec_h n^\infty$ for all $h \neq i$ and $\{m^\infty\} \succ_0 \{n^\infty\}$, then $\{m^\infty\} \succ_0 \{m^\infty, n^\infty\}$.

The presumption of the axiom states that all but one individual prefer n^∞ over m^∞ as a commitment path but the planner ranks m^∞ over n^∞ for some reason. The conclusion states that then the planner prefers to avoid an opportunity to choose from m^∞ and n^∞ ex-post, since he/she expects to get tempted to choose n^∞ over m^∞ against his/her choice over commitment plans, which is painful.

4 Aggregation characterization

4.1 Aggregation of per-period-outcome preferences

Let us start with the most permissive type of aggregation, in which no time preference is involved. That is, we restrict attention to preferences over constant sequences.

Proposition 2 The profile $\succsim_0$ and $\{\succsim_i\}_{i \in I}$ satisfy Constant Commitment Pareto if and only if there exists $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ such that

$$u_0 = \sum_i \widetilde{\alpha}_h u_h.$$

Since Constant Commitment Pareto is interpreted as the Pareto condition on static lotteries, Proposition 2 is the same as Harsanyi's aggregation theorem.

Next, we impose axioms regarding the social planner's temptation and characterize their implications on the temptation risk preference.

Theorem 1 The profile $\succsim_0$ and $\{\succsim_i\}_{i \in I}$ satisfy Constant Commitment Pareto and NT-PDO if and only if there exist $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ and $(\alpha_h) \in \mathbb{R}_+^{|I|}$ such that

$$u_0 = \sum_h \widetilde{\alpha}_h u_h, \text{ and } v_0 = \sum_h \alpha_h u_h.$$

Moreover, $\succsim_0$ and $\{\succsim_i\}_{i \in I}$ satisfy TAM in addition if and only if $(\alpha_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ with $\widetilde{\alpha}_h \alpha_h = 0$ for all h.

Theorem 1 shows that an implication of NTPDO is to deliver a non-negative additive aggregation of individuals' utility functions also for the temptation risk preference. Since

NTPDO means that the temptation risk preference respects the Pareto condition, again, by the Harsanyi's aggregation-type argument, it must be written as a non-negative (possibly zero vector) additive aggregation of individuals' utility functions.

Moreover, under TAM, (α_h) is ensured to be non-zero, which implies that v_0 is not constant. A more important characterization of TAM is the complementarity between the welfare weights associated with normative and temptation utilities. Individuals who are evaluated with positive weights in the normative utility do not have positive welfare weights in terms of the temptation utility. Thus, under TAM, the population of individuals can be divided into three categories: individuals who are evaluated only through the normative utility, individuals who are evaluated only through the temptation utility, and individuals who are evaluated in neither.

The intuition of the sufficiency of TAM is as follows: If the normative utility is written as a weighted sum of individuals' utilities, we can always find alternatives such that only one individual has a different ranking, as in the premise of TAM, but normative utility favors that one individual. Indeed, according to the Richness condition, by choosing, for each individual, two lotteries such that only that individual is preferred and the other individuals are indifferent, and by properly randomizing over them, we can find two alternatives such that only one individual has the opposite ranking from the other and the utility difference between these alternatives for the other individuals is infinitely small. Since the improvement of the rest of the individuals can be infinitely close to zero, the opinions of them will have no impact in the normative utility under the additive aggregation. Nevertheless, TAM requires that the social planner is tempted by such an alternative, which leads to the property that the temptation utility is not constant and that individuals with positive weights in the normative utility must have zero weights in the temptation utility.

4.2 Taking time preferences into account through temptation utility

In this subsection, we consider the class of social preference represented with the FT form with $\gamma_0 > 0$. That is, evaluation of temptation utility function strictly involves intertemporal trade-offs.

First, we consider the case that all individuals agree on discounting.

Proposition 3 Assume $\beta_i = \beta$ for all i. Assume also $\gamma_0 > 0$. The profile $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy Commitment Pareto, NTPDP, and TAM if and only if there exist $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ and $(\alpha_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ with $\alpha_h \widetilde{\alpha}_h = 0$ for all h such that

$$u_0 = \sum_h \widetilde{\alpha}_h u_h, \ v_0 = \sum_h \alpha_h u_h, \text{ and } \beta_0 = \gamma_0 = \beta.$$

Since Commitment Pareto implies Constant Commitment Pareto and NTPDP implies NTPDO, the aggregation result about risk preferences is obtained as a corollary of Theorem 1. When there is no disagreement among individuals' time preferences, the benevolent social planner also adopts this common discount factor both as normative and temptation discount factors. In particular, the latter property is ensured by NTPDP. This result can be regarded as a "possibility" result because multiple individuals can have a positive welfare weight either via the normative or the temptation risk preference.

Now we consider the case where individuals disagree about intertemporal trade-offs. The next result states that when the individuals differ in time discounting as well as in tastes, there exist two kinds of dictators, one determines the commitment discounted utility function and the other determines the temptation discounted utility function. Thus we would call them "commitment dictator" and "temptation dictator," respectively.

Theorem 2 Assume $\beta_i \neq \beta_j$ for all i and j. Assume also $\gamma_0 > 0$. The profile $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy Commitment Pareto, NTPDP, and TAM if and only if there exist $i^*, j^* \in I$ with $i^* \neq j^*$ such that

$$u_0 = u_{i^*}, \ \beta_0 = \beta_{i^*}, v_0 = u_{j^*}, \text{ and } \gamma_0 = \beta_{j^*}.$$

The intuition about the roles of axioms is simple. The existence of the commitment dictator follows from the dictatorship result under Commitment Pareto as in Zuber [34], Jackson and Yariv [23]. Since NTPDP implies that the temptation discounted utility also respects the Pareto condition, a temptation dictator must exist by exactly the same argument as above. Finally, since TAM requires the complementarity between the normative and temptation utilities as mentioned in Section 4.1, the commitment dictator and the temptation dictator must be different.

Under disagreement about time preferences among individuals, Theorem 2 states that at most two individuals can have a positive welfare weight from the planner's viewpoint. Thus, in most economies where the population of individuals is more than three, this theorem is interpreted as impossibility.

4.3 Escaping from temptation dictatorship

Section 4.2 shows that under a Pareto condition and reasonable axioms about temptation, other than the commitment dictator, only one individual's preference including discount factor can be reflected into the temptation utility of the social planner. In this subsection, we show one way of avoiding from this temptation dictatorship result.

Assume $\gamma_0 = 0$ hereafter, that is, we assume that the social preference $\succsim_0$ follows the self-control utility of Gul and Pesendorfer [15].

First, we consider the case where all individuals agree on time preference and show the counterpart of Proposition 3.

Proposition 4 Assume $\beta_i = \beta$ for all i. Assume also $\gamma_0 = 0$. Then, the profile $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy Commitment Pareto, NTPDO, and TAM if and only if there exist $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ and $(\alpha_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ satisfying $\widetilde{\alpha}_h \alpha_h = 0$ for all h such that

$$u_0 = \sum_h \widetilde{\alpha}_h u_h, \ \beta_0 = \beta, \text{ and } v_0 = \sum_h \alpha_h u_h.$$

As in Proposition 3, the aggregation result about risk preferences is obtained as a corollary of Theorem 1. The result about time preference is a consequence of Commitment Pareto.

Second, we consider the case where the individuals differ in time discounting as well as in tastes.

Theorem 3 Assume $\beta_i \neq \beta_j$ for all i and j. Assume also $\gamma_0 = 0$. Then, the profile $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy Commitment Pareto, NTPDO, and TAM if and only if there exist $i^* \in I$ and $(\alpha_h) \in \mathbb{R}_+^{|I|-1} \setminus \{0\}$ such that

$$u_0 = u_{i^*}, \ \beta_0 = \beta_{i^*}, \text{ and } v_0 = \sum_{h\neq i^*} \alpha_h u_h.$$

As in Theorem 2, the commitment dictatorship follows from Commitment Pareto. On the other hand, unlike Theorem 2, the temptation utility term is allowed to reflect multiple individuals' per-period utilities. However, this seemingly more benevolent result does not come for free because we have to accept a social planner with short-sighted temptation.

Consequently, we face a trade-off. Either we accept a social planner with a present-biased temptation and let the temptation per-period utility reflect only the tastes of non-commitment dictators, or instead of adopting a social planner with a more long-run perspective in the sense of the FT model, we allow at most only two individuals' opinions including their time preferences to be reflected into the social planner's preference. We need to choose either one of them.

5 Implication to resource allocation

Here we present positive implications of the social welfare functions as characterized above to resource allocation problems. We look at two dynamic resource allocation problems, one is in which resource amount is fixed at each period and there is no intertemporal trade-off, the other is an optimal growth problem in which resource is saved and reproduced over time.

In these problems, the standard Pareto efficiency condition has a sharp implication under heterogeneous discounting: only the most patient individual receives positive consumption in the long-run and all the others' consumption paths converge to zero (Becker [2]).

Even under heterogeneous discounting among individuals, below we obtain solutions of the social planner's problem in which multiple individuals receive positive consumption amounts in the long-run by sacrificing Pareto-efficiency. We see this feature is similar to the one obtained from stationary social welfare functions which violate the Pareto condition (Hayashi and Lombardi [19]), in the sense that both respond to distributive concerns by sacrificing intertemporal efficiency. The mechanism of doing so is different, however. In the Hayashi-Lombardi model, the distributive property is delivered directly by violating Commitment Pareto. On the other hand, in the current model, impure benevolence allows the planner to deliver the distributive property through ex-post choice under no commitment, even though Commitment Pareto is imposed.

5.1 Fixed resource

Assume that there are e units of a single good at every time period. This initial resource will be allocated to the consumers in the economy. The set of per-period social outcomes is the set of allocations across consumers and taken as $C = \mathbb{R}^n_+$. There is no technology to carry over a resource to the next period. Thus, the social planner faces the same menu every period, which is given as

$$z(e) = \left\{ (c, z(e)) \,\middle|\, \sum_{i \in I} c_i = e \right\}.$$

Since the continuation menu for the next period onward is always equal to $z(e)$, the social planner is faced with a static problem such as determining only consumption allocation for each period.

If the planner's value function, denoted by $W_0(z)$, admits a temptation dictator, it is written as

$$W_0(z(e)) = \max_{c \in z(e)} \left\{ u_{i^*}(c_{i^*}) + \beta_{i^*} W_0(z(e)) + \kappa_0 \left[u_{j^*}(c_{j^*}) + \beta_{j^*} V_0(z(e)) \right] \right\} - \kappa_0 V_0(z(e))$$

with

$$V_0(z(e)) = \max_{c \in z(e)} \left\{ u_{j^*}(c_{j^*}) + \beta_{j^*} V_0(z(e)) \right\},$$

or if the planner is tempted only be immediate consumption allocations,

$$W_0(z(e)) = \max_{c \in z(e)} \left\{ u_{i^*}(c_{i^*}) + \kappa_0 \sum_{h \neq i^*} \alpha_h u_h(c_h) + \beta_{i^*} W_0(z(e)) \right\} - \kappa_0 \max_{c' \in z(e)} \sum_{h \neq i^*} \alpha_h u_h(c'_h).$$

Since e stays constant over time, $W_0(z(e))$ and $V_0(z(e))$ are constant over time. Since there is effectively no intertemporal choice in this setting, the solution is to provide a time-constant sequence in both of the above two cases. More precisely, in the two-dictatorship rule, it is the solution to $\max_{c \in z(e)} \left\{ u_{i^*}(c_{i^*}) + \kappa_0 u_{j^*}(c_{j^*}) \right\}$, where only two consumers, the commitment dictator and the temptation dictator, can receive positive consumption allocations. In the case of the temptation from immediate consumption, it is the solution to $\max_{c \in z(e)} \left\{ u_{i^*}(c_{i^*}) + \kappa_0 \sum_{h \neq i^*} \alpha_h u_h(c_h) \right\}$. Thus, this impure social planner chooses a consumption allocation that maximizes a weighted sum of all individuals' per-period utility functions, and, consequently, behaves like a benevolent social planner.

Such a benevolent allocation is not obtained for free. It is worth emphasizing that a consumption stream chosen by the impure benevolent social planner is generally not Pareto efficient. As suggested in Sections 3.1 and 3.2, the impure benevolent social planner who respects Commitment Pareto and NTPDP will choose a Pareto efficient allocation from menus of streams, but not necessarily from other recursive menus. For an illustration, suppose there are only two consumers, $i = 1, 2$, in the economy. Assume that their per-period utilities are given by $u_1(c) = u_2(c) = \log c$. If consumer 1 is a commitment dictator and consumer 2 is a temptation dictator, at each period, the planner chooses

$$\arg\max_{c\in z(e)}\{\log c_1+\kappa_0\log c_2\}=\left(\frac{1}{1+\kappa_0},\frac{\kappa_0}{1+\kappa_0}\right), \tag{3}$$

and so the constant stream $\left(\frac{1}{1+\kappa_0},\frac{\kappa_0}{1+\kappa_0}\right)_{t=1}^{\infty}$ is obtained as a social outcome. However, as shown by Becker [2], if consumers have heterogeneous discount factors, in a Pareto efficient allocation, the most patient consumer will consume all resources in the long run. Hence, if $\beta_1\neq\beta_2$, a constant stream allocating a positive consumption to all consumers, such as (3), is not Pareto efficient.[10]

We compare the stream (3) to a solution in a situation where the social planner can choose a consumption stream with commitment from period 1 onward. Such a situation can be formalized as a menu

$$x=\{(c_1^t,c_2^t)_{t=1}^{\infty}\mid c_1^t+c_2^t=1\ \forall t\}.$$

The planner's value function of this menu is reduced to

$$W_0(x)=\max_{(c_1^t,c_2^t)_{t=1}^{\infty}\in x}\left\{\sum_{t=1}^{\infty}\beta_1^{t-1}\log c_1^t+\kappa_0\sum_{t=1}^{\infty}\beta_2^{t-1}\log c_2^t\right\},$$

which implies that the optimal stream from x is given by

$$\begin{aligned}&\arg\max_{(c_t^1,c_t^2)_{t=1}^{\infty}\in x}\left\{\sum_{t=1}^{\infty}\beta_1^{t-1}\log c_t^1+\kappa_0\sum_{t=1}^{\infty}\beta_2^{t-1}\log c_t^2\right\}\\&=\left(\frac{\beta_1^{t-1}}{\beta_1^{t-1}+\kappa_0\beta_2^{t-1}},\frac{\kappa_0\beta_2^{t-1}}{\beta_1^{t-1}+\kappa_0\beta_2^{t-1}}\right)_{t=1}^{\infty}.\end{aligned} \tag{4}$$

Since (4) is obtained by maximizing a weighted sum of all consumers' utility functions under the resource constraints, (4) is Pareto efficient. Indeed, if $\beta_1\neq\beta_2$, or for instance, $\beta_1>\beta_2$, (4) converges to $(1,0)$ in the long-run, which is consistent with the property of Pareto efficient allocations characterized in Becker [2]. In a situation requiring sequential choice, such as in the menu $z(e)$, the impure benevolent social planner cannot make a commitment to a Pareto-efficient allocation (4) and end up with an inefficient allocation (3) due to the stationary nature of the representation W_0.

[10] Compared to an allocation, where resources are divided in a fixed proportion each period, a Pareto improvement is achieved through a trade such that a more impatient consumer consumes more in earlier periods and a more patient consumer consumes more in later periods.

5.2 Optimal growth

Unlike the previous sub-section, which ignores intertemporal aspect of decision making, this sub-section considers a situation in which capital accumulation is possible. Capital accumulation can be interpreted as the choice of a continuation menu from the next period onward. In choosing from a given menu, benevolent choices are made as a result of a compromise between the normative and temptation utilities, but an impure social planner avoids temptation from the other group's opinion, so the subsequent menu choice tends to be distorted.

We incorporate a production technology into the setting of Section 5.1. There is one good at each period, which can be consumed or used as input for reproduction under strictly concave production function f. Let k_0 be the initial capital. Given the current capital amount k, the social planner faces the constraint

$$\sum_{i \in I} c_i + k' = f(k),$$

where k' denotes the capital amount to be carried over to the next period. Formally, given a current level of capital k, the corresponding menu is written as

$$z(k) = \left\{ (c, z(k')) \,\middle|\, \sum_{i \in I} c_i + k' = f(k) \right\}.$$

For notational simplicity, $W_0(z(k))$ and $V_0(z(k))$ are denoted as $W_0(k)$ and $V_0(k)$, respectively.

5.2.1 The planner tempted from immediate consumption

Suppose that the social planner is tempted from immediate consumption only. For simplicity, consider a symmetric situation that $u_i = u$ for all $i \in I$ and $\kappa_0 = n - 1$, $\alpha_i = 1/(n-1)$ for all $i \neq i^*$. Assume u is differentiable, strictly increasing, and concave.

Then the social planner's dynamic programming problem is formulated as

$$W_0(k) = \max_{\sum_{i \in I} c_i + k' = f(k)} \left\{ \sum_{i \in I} u(c_i) + \beta_{i^*} W_0(k') \right\} - \max_{\sum_{i \in I} c_i + k' = f(k)} \sum_{h \neq i^*} u(c_h).$$

Let $c_i(k)$ be the consumption function for each i. Then, the first-order condition for the ex post choice is given by

$$u'(c_i(k)) - \beta_{i^*} W_0' \left(f(k) - \sum_{h=1}^{n} c_h(k) \right) = 0$$

for every $i \in I$. Hence it holds $c_i(k) = c(k)$ for all i, where c is seen as the per-individual consumption function.

On the other hand, since $\max_{\sum_{i \in I} c_i + k' = f(k)} \sum_{h \neq i^*} u(c_h) = (|I| - 1) u \left(\frac{f(k)}{|I|-1} \right)$, we obtain

$$W_0(k) = |I| u(c(k)) + \beta_{i^*} W_0 (f(k) - |I| c(k)) - (|I| - 1) u \left(\frac{f(k)}{|I| - 1} \right).$$

By taking the derivative of both sides of this and combining with the first-order condition, we obtain the envelope condition

$$W_0'(k) = \left[u'(c(k)) - u' \left(\frac{f(k)}{|I| - 1} \right) \right] f'(k).$$

By combining with the first-order condition again, we obtain the Euler equation

$$u'(c(k)) = \beta_{i^*} \left[u'(c(f(k) - |I| c(k))) - u' \left(\frac{f(f(k) - |I| c(k))}{|I| - 1} \right) \right] f'(f(k) - |I| c(k)).$$

Let $k^\star$ be the steady-state capital level for the above Euler equation. Then it must hold $(f(k^\star) - |I| c(k^\star) = k^\star$ and

$$u'(c(k^\star)) = \beta_{i^*} \left[u'(c(k^\star)) - u' \left(\frac{f(k^\star)}{|I| - 1} \right) \right] f'(k^\star).$$

Since $u' > 0$ and $f' > 0$, for the steady-state condition to be met, it is necessary that $c(k^\star) < \frac{f(k^\star)}{|I|-1}$ and $\beta_{i^*} f'(k^\star) > 1$ hold.

Let us compare the performance of the above growth solution with the one obtained from another objective function. Here we consider the objective function as characterized by Hayashi and Lombardi [19], in which the planner maintains the stationary discounted utility model by adopting one individual's discount factor and taking weighted sum of individuals' per-period utilities at each period. Again we take the symmetric sum of per-period utilities for simplicity and the "discounting dictator" is the same as the "commitment dictator." More precisely, the social planner's dynamic programming problem is

$$\widetilde{W}_0(k) = \max_{\sum_{i \in I} c_i + k' = f(k)} \left\{ \sum_{i \in I} u(c_i) + \beta_{i^*} \widetilde{W}_0(k') \right\}.$$

As shown by Hayashi and Lombardi [19], this social objective function violates Commitment Pareto, while it satisfies a weaker Pareto condition.

Let $\widetilde{c}$ be the consumption function per individual, then the Euler equation is obtained as

$$u'(\widetilde{c}(k)) = \beta_{i^*} u'(\widetilde{c}(f(k) - |I|\widetilde{c}(k)))f'(f(k) - |I|\widetilde{c}(k)).$$

and the steady-state capital level $\widetilde{k}$ is given by

$$1 = \beta_{i^*} f'(\widetilde{k}).$$

Since $\beta_{i^*} f'(k^\star) > 1$ and f is strictly concave, we see that $k^\star < \widetilde{k}$. Also, since $(f(k)-k)' = f'(k) - 1 > \frac{1}{\beta_{i^*}} - 1 > 0$ for all $k \in (0, \widetilde{k}]$, we have

$$c(k^\star) = \frac{f(k^\star) - k^\star}{|I|} < \frac{f(\widetilde{k}) - \widetilde{k}}{|I|} = c(\widetilde{k}),$$

meaning that the consumption path converging to $c(k^\star)$ is strictly dominated by the path converging to $c(\widetilde{k})$ in the long-run.

Imposing Commitment Pareto and leaving distributive issues to the planner's impure benevolence results in low growth and low consumption path in the long-run, when compared to the one obtained from an objective function that violates Commitment Pareto. This is because carrying over more resource to future creates larger self-control cost and the planner prefers to avoid it in the long-run.

Comparing streams of allocations chosen by the above two different objective functions, note that even though one may Pareto-dominate the other, starting from a sufficiently long time period onward, there may be no Pareto-dominance relationship between the two streams when considering the entire period, including the present and the near future. Indeed, the impure benevolent social planner is tempted by the majority's opinion to increase consumption in the present and tends to increase consumption in the short run at the expense of lower steady state in the long run.

5.2.2 The two-dictator case

Let i be the commitment dictator and j be the temptation dictator. The planner's utility representation is given as

$$W_0(k) = \max_{c_i + c_j + k' = f(k)} \{u_i(c_i) + \beta_i W_0(k') + \kappa_0 [u_j(c_j) + \beta_j V_0(k')]\} - \kappa_0 V_0(k),$$

and

$$V_0(k) = \max_{c_j + k' = f(k)} \{u_j(c_j) + \beta_j V_0(k')\}.$$

Note that consumers other than i and j are always assigned zero consumption in the solution of this optimization problem.

Let $\widehat{c}_j$ denote the consumption function when j is supposed to behave as a dictator. Then the envelope condition

$$V_0'(k) = u_j'(\widehat{c}_j(k))f'(k)$$

follows from the standard argument.

Let $c_i(\cdot)$ and $c_j(\cdot)$ denote the consumption functions. Then the first-order condition is

$$\begin{aligned} u_i'(c_i(k)) &= \kappa_0 u_j'(c_j(k)) \\ &= \beta_i W_0'(f(k) - c_i(k) - c_j(k)) + \kappa_0 \beta_j V_0'(f(k) - c_i(k) - c_j(k)). \end{aligned}$$

Since

$$W_0(k) = u_i(c_i(k)) + \beta_i W_0(f(k) - c_i(k) - c_j(k)) + \kappa_0 \left[u_j(c_j(k) + \beta_j V_0(f(k) - c_i(k) - c_j(k)))\right] - \kappa_0 V_0(k),$$

by taking derivatives of both sides and combining with the first-order condition and the envelope condition for V_0, we obtain the envelope condition for W_0, which has the form

$$W_0'(k) = \left[u_i'(c_i(k)) - \kappa_0 u_j'(\widehat{c}_j(k))\right] f'(k).$$

By plugging the two envelope conditions to the first-order condition, we obtain the Euler equation

$$\begin{aligned} & u_i'(c_i(k)) = \kappa_0 u_j'(c_j(k)) \\ = \; & \beta_i u_i'(c_i(f(k) - c_i(k) - c_j(k)))f'(f(k) - c_i(k) - c_j(k)) \\ & -\kappa_0(\beta_i - \beta_j) u_j'(\widehat{c}_j(f(k) - c_i(k) - c_j(k)))f'(f(k) - c_i(k) - c_j(k)). \end{aligned}$$

Let $k^\star$ denote the steady-state capital level, then it satisfies

$$\begin{aligned} & u_i'(c_i(k^\star)) = \kappa_0 u_j'(c_j(k^\star)) \\ = \; & \beta_i u_i'(c_i(k^\star))f'(k^\star) - \kappa_0(\beta_i - \beta_j) u_j'(\widehat{c}_j(k^\star))f'(k^\star). \end{aligned} \tag{5}$$

Note that $\widehat{c}_j(\cdot)$ is a known function. When $\beta_i = \beta_j = \beta$, the condition reduces to

$$1 = \beta f'(k^\star),$$

in which the steady state allocation coincides with the optimal growth solution given by maximizing $\sum_{t=1}^{\infty} \beta^{t-1} \{u_i(c_{it}) + \kappa_0 u_j(c_{jt})\}$ under the constraint $z(k_0)$. Hence the existence of impure benevolence affects distribution alone and does not affect intertemporal allocation in the long-run.

When $\beta_i \neq \beta_j$, (5) is written as

$$\frac{\kappa_0 u_j'(\widehat{c}_j(k^\star))}{u_i'(c_i(k^\star))} = \frac{u_j'(\widehat{c}_j(k^\star))}{u_j'(c_j(k^\star))} = \frac{1}{\beta_i - \beta_j}\left(\beta_i - \frac{1}{f'(k^\star)}\right). \tag{6}$$

When $\beta_i > \beta_j$, that is, the temptation dictator is more impatient than the commitment dictator, (6) requires $f'(k^\star) > 1/\beta_i$. As in Section 5.2.1, compare this with the optimal growth solution given by maximizing the commitment dictator rule (Hayashi and Lombardi [19]) $\sum_{t=1}^{\infty} \beta_i^{t-1} \{u_i(c_{it}) + \kappa_0 u_j(c_{jt})\}$, and let $\widetilde{c}_i(\cdot)$ and $\widetilde{c}_j(\cdot)$ be the corresponding consumption functions and $\widetilde{k}$ be the steady state capital level. Then it holds $f'(\widetilde{k}) = 1/\beta_i$. Since $f'(k^\star) > 1/\beta_i$, we have $k^\star < \widetilde{k}$. Also, since $(f(k) - k)' = f'(k) - 1 > \frac{1}{\beta_i} - 1 > 0$ for all $k \in (0, \widetilde{k}]$, we have

$$c_i(k^\star) + c_j(k^\star) = f(k^\star) - k^\star < f(\widetilde{k}) - \widetilde{k} = \widetilde{c}_i(\widetilde{k}) + \widetilde{c}_j(\widetilde{k}).$$

Consider for simplicity that $\kappa_0 = 1$ and $u_i(\cdot) = u_j(\cdot)$. Then $c_i(\cdot) = c_j(\cdot) = c(\cdot)$ and $\widetilde{c}_i(\cdot) = \widetilde{c}_j(\cdot) = \widetilde{c}(\cdot)$. From the above inequality, $c(k^\star) < \widetilde{c}(\widetilde{k})$, meaning that the consumption path converging to $c(k^\star)$ is strictly dominated by the path converging to $\widetilde{c}(\widetilde{k})$ in the long-run. This is a similar observation as in Section 5.2.1. As two consumers having heterogeneous discount factors receive positive consumption amounts in the long-run, both allocations obtained through our impure benevolent planner and the discounting dictator (Hayashi and Lombardi [19]) are Pareto-inefficient.

Next consider the case, $\beta_i < \beta_j$, that is, the temptation dictator is more patient than the commitment dictator. From (6), $f'(k^\star) < 1/\beta_i$, which implies that the capital amount is over-accumulated in the long-run in terms of the commitment dictator i. When the intensity of temptation κ_0 is close to zero, the commitment dictator i's opinion is almost adopted by the social planner. Since $c_j(k^\star)$ becomes close to zero, the corresponding marginal utility $u_j'(c_j(k^\star))$ is sufficiently large, and hence $f'(k^\star)$ is close to $1/\beta_i$. That is, $k^\star$ is close to the steady-state capital amount obtained when i is the sole dictator. Conversely, when κ_0 is large enough, the social planner almost succumbs to temptation and the ex-post choice $c_j(\cdot)$ tends to be closer to the temptation dictator's optimal choice $\widehat{c}_j(\cdot)$. Hence $f'(k^\star)$ is close to $1/\beta_j$. That is, $k^\star$ is close to the steady-state capital amount obtained when j is the sole dictator. The steady state capital amount in general is thus lying in between, which is again Pareto-inefficient since both individuals are receiving positive consumption amounts.

6 Conclusion

This paper has investigated how we can aggregate individuals' preferences when the planner faces the potential time-inconsistency problem due to heterogeneous discounting, by adopting the decision model of temptation and self-control by Gul and Pesendorfer [14, 15] and Noor [30].

Although the planner can take only one individual's preferences into account when making commitment choices, he may take other individual's preferences into account in the form of temptation utility (impure benevolence, namely) that affects ex-post choice without commitment.

We show that there is a trade-off between the coverage of preference parameters and the coverage of individuals that can be taken into account in the form of temptation utility. When the planner's temptation utility takes individuals' time preferences into account then it can put positive weight only on one individual, who should be called the "temptation dictator." It can put positive weights on more individuals when temptation is assumed to come only from immediate consumption allocations.

We have investigated the positive implications of the models in dynamic resource allocation problems. The solutions allow that all individuals receive positive consumption amounts in the long-run, which contrasts to the implication of the standard Pareto efficiency that only the most individual receive positive consumption amount and all the others' consumption path converge to zero. Impure benevolence allows the planner to respond to distributive concerns through ex-post choice under non-commitment.

In the paper we ruled out the possibility that the planner has a preference for flexibility (Kreps [24], Dekel, Lipman, and Rustichini [8]), because it leads to support "spurious" unanimity arising due to different expectations about ex-post choice from a menu.[11] There may be a context, however, in which it is rather appealing to consider that the planner should have preference for flexibility, and this will be worth investigating.

[11]The idea of spurious unanimity, a unanimity arising due to the existence of multiple kinds of disagreements, was introduced by Mongin [29] in the context of collective decision under heterogeneous beliefs as analyzed in Mongin [28] and Gilboa, Samet and Schmeidler [13].

A Proofs

A.1 Proof of Proposition 1

Necessity is obvious.

To show sufficiency, let $W_0 : \mathcal{K}(X) \to \mathbb{R}$ be a continuous and mixture-linear representation of $\succsim_0$, and let $W_i : \mathcal{K}(X) \to \mathbb{R}$ be a continuous and mixture-linear representation of $\succsim_i^*$, for each $i \in I$, which has the form $W_i(z) = \max_{a \in z} u_i(a)$ with $u_i : X \to \mathbb{R}$ being continuous and mixture-linear. Without loss, assume $u_i(\underline{a}) = 0$ for all $i \in I$.

Because the functions $W_0, W_1, \cdots, W_{|I|}$ are mixture-linear and it holds $W_i\left(\frac{1}{|I|}\sum_{h \in I} a_i\right) > W_i(\underline{a})$ for all $i = 1, \cdots, n$, we can apply a version of Harsanyi theorem (Harsanyi [17], De Meyer and Mongin [9]) so that there is a vector $\alpha \in \mathbb{R}_+^{|I|} \setminus \{\mathbf{0}\}$ such that

$$W_0(z) = \sum_{i \in I} \alpha_i W_i(z)$$

holds for all $z \in \mathcal{Z}$.

We show that there are no two distinct i and j such that $\alpha_i > 0$ and $\alpha_j > 0$. Suppose there are.

Without loss, we can take $a_i, a_j \in X$ so that $\alpha_i u_i(a_i) = \alpha_j u_j(a_j) > 0$ and $u_i(a_j) = u_j(a_i) = 0$.

Then it holds

$$W_0(\{a_i\}) = \alpha_i u_i(a_i)$$

and

$$W_0(\{a_j\}) = \alpha_j u_j(a_j).$$

On the other hand, it holds

$$W_0(\{a_i, a_j\}) = \alpha_i u_i(a_i) + \alpha_j u_j(a_j).$$

Thus we obtain

$$W_0(\{a_i, a_j\}) > W_0(\{a_i\}) = W_0(\{a_j\}),$$

which is a violation of Weak Set-Betweenness.

A.2 Proof of Proposition 2

Only-if part: Take any m and n such that $u_i(m) > u_i(n)$ for all i. This implies $m^\infty \succ_i n^\infty$. By Constant Commitment Pareto, $\{m^\infty\} \succ_0 \{n^\infty\}$. Thus, $W_0(\{m^\infty\}) > W_0(\{n^\infty\})$, or equivalently, $u_0(m) > u_0(n)$. By De Meyer and Mongin [9, Proposition 2], there exists $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ such that $u_0 = \sum_h \widetilde{\alpha}_h u_h$.

If part: Assume $u_0 = \sum_i \widetilde{\alpha}_h u_h$ for some $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$. If $m^\infty \succ_i n^\infty$ for all $i \in I$, it implies $u_i(m) > u_i(n)$. Since we have $u_0(m) > u(n)$, $\{m^\infty\} \succ_0 \{n^\infty\}$, as desired.

A.3 Proof of Theorem 1

Only-if part: By Constant Commitment Pareto and Proposition 2, there exists $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ such that $u_0 = \sum_i \widetilde{\alpha}_h u_h$.

Next, we will claim that for all $m, n \in \Delta(C)$, if $u_i(m) \geq u_i(n)$ for all i, then $v_0(m) \geq v_0(n)$. By Richness assumption, for all i, there exists m_i such that $u_i(m_i) > u_i(\underline{m})$ and $u_j(m_i) = u_j(\underline{m})$ for all $j \neq i$. For $\gamma \in (0,1)$, let $m_\gamma = \gamma \sum_h \frac{1}{|I|} m_i + (1-\gamma) m$ and $n_\gamma = \gamma \underline{m} + (1-\gamma) n$. Since $u_i(\sum_h \frac{1}{|I|} m_i) > u_i(\underline{m})$ for all i, $u_i(m_\gamma) > u_i(n_\gamma)$ for all i and γ. Since $u_0 = \sum \widetilde{\alpha}_h u_h$, $u_0(m_\gamma) > u_0(n_\gamma)$. Now we have $m_\gamma^\infty \succ_i n_\gamma^\infty$ for all i and $\{m_\gamma^\infty\} \succ_0 \{n_\gamma^\infty\}$. By NTPDO, $\{m_\gamma^\infty\} \sim_0 \{m_\gamma^\infty, n_\gamma^\infty\}$. From the DSC representation of $\succsim_0$, $v_0(m_\gamma) \geq v_0(n_\gamma)$. Then, $v_0(m) \geq v_0(n)$ as $\gamma \to 0$.

By De Meyer and Mongin [9, Proposition 1], there exists $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|}$ such that $v_0 = \sum_h \widetilde{\alpha}_h u_h$.

Next, suppose that $\succsim_0$ and $\{\succsim_i\}$ satisfy TAM in addition. Since $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$, there exists at least one i with $\widetilde{\alpha}_i > 0$. By Richness assumption, there exists m_i such that $u_i(m_i) > u_i(\underline{m})$ and $u_j(m_i) = u_j(\underline{m})$ for all $j \neq i$. For $\gamma \in (0,1)$, let $m_\gamma = \gamma \sum_h \frac{1}{|I|} m_h + (1-\gamma)\underline{m}$. Then, for all γ, $u_i(m_\gamma) < u_i(m_i)$ and $u_j(m_\gamma) > u_j(m_i)$ for all $j \neq i$. Since $u_0 = \sum \widetilde{\alpha}_h u_h$, for all sufficiently small γ, $u_0(m_i) > u_0(m_\gamma)$. Therefore, for all such γ, we have $m_i^\infty \succ_i m_\gamma^\infty$, $m_\gamma^\infty \succ_j m_i^\infty$ for all $j \neq i$, and $\{m_i^\infty\} \succ_0 \{m_\gamma^\infty\}$. Then, by TAM, $\{m_i^\infty\} \succ_0 \{m_i^\infty, m_\gamma^\infty\}$, which implies $v_0(m_\gamma) > v_0(m_i)$. This means v_0 is not constant. Thus, there exists at least one j such that $\alpha_j > 0$.

Next, we show that if $\widetilde{\alpha}_i > 0$, then $\alpha_i = 0$. Seeking a contradiction, suppose there exists i with $\widetilde{\alpha}_i > 0$ and $\alpha_i > 0$. By the same argument as above, Richness assumption ensures that for all $\gamma \in (0,1)$, $u_i(m_\gamma) < u_i(m_i)$ and $u_j(m_\gamma) > u_j(m_i)$ for all $j \neq i$. Moreover, for all sufficiently small γ, $u_0(m_i) > u_0(m_\gamma)$. Therefore, for all such γ, we have $m_i^\infty \succ_i m_\gamma^\infty$,

$m_\gamma^\infty \succ_j m_i^\infty$ for all $j \neq i$, and $\{m_i^\infty\} \succ_0 \{m_\gamma^\infty\}$. Then, by TAM, $\{m_i^\infty\} \succ_0 \{m_i^\infty, m_\gamma^\infty\}$, which implies $v_0(m_\gamma) > v_0(m_i)$. Thus, we have $v_0(\underline{m}) \geq v_0(m_i)$ as $\gamma \to 0$. On the other hand, since $v_0 = \sum_h \alpha_h u_h$ with $\alpha_i > 0$,

$$v_0(m_i) = \sum_h \alpha_h u_h(m_i) > \sum_h \alpha_h u_h(\underline{m}) = v_0(\underline{m}),$$

which is a contradiction.

If part: Necessity of Constant Commitment Pareto comes from Proposition 2. For NT-PDO, take any $m, n \in \Delta(C)$ such that $m^\infty \succ_i n^\infty$ for all $i \in I$ and $\{m^\infty\} \succ_0 \{n^\infty\}$. Since these conditions imply $u_i(m) > u_i(n)$ for all i, $u_0(m) > u_0(n)$ and $v_0(m) = \sum \alpha_h u_h(m) \geq \sum \alpha_h u_h(n) = v_0(n)$. Then,

$$\begin{aligned} W_0(\{m^\infty, n^\infty\}) =& \max\{u_0(m) + v_0(m) + \beta_0 W_0(\{m^\infty\}), u_0(n) + v_0(n) + \beta_0 W_0(\{n^\infty\})\} \\ &- \max\{v_0(m), v_0(n)\} \\ =& u_0(m) + v_0(m) + \beta_0 W_0(\{m^\infty\}) - v_0(m) \\ =& u_0(m) + \beta_0 W_0(\{m^\infty\}) \\ =& W_0(\{m^\infty\}), \end{aligned}$$

as desired.

To show TAM, take any $m, n \in \Delta(C)$ such that $m^\infty \succ_i n^\infty$, $n^\infty \succ_h m^\infty$ for all $h \neq i$, and $\{m^\infty\} \succ_0 \{n^\infty\}$. These conditions imply $u_i(m) > u_i(n)$, $u_h(n) > u_h(m)$ for all $h \neq i$, and $u_0(m) > u_0(n)$. Since $u_0 = \sum_h \widetilde{\alpha}_h u_h$, we must have $\widetilde{\alpha}_i > 0$. By the assumption, $\alpha_i = 0$, which implies $v_0(n) = \sum_h \alpha_h u_h(n) > \sum_h \alpha_h u_h(m) = v_0(m)$. From the FT representation for $\succsim_0$, we have $\{m^\infty\} \succ_0 \{m^\infty, n^\infty\}$, as desired.

A.4 Proof of Proposition 3

Only if part: Since Commitment Pareto implies Constant Commitment Pareto and NTPDP implies NTPDO, Theorem 1 implies $u_0 = \sum_h \widetilde{\alpha}_h u_h$ for some $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ and $v_0 = \sum_h \alpha_h u_h$ for some $(\alpha_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ with $\alpha_h \widetilde{\alpha}_h = 0$ for all h.

Next, we want to show $\beta_0 = \beta$. By seeking a contradiction, suppose $\beta_0 \neq \beta$. In particular, assume $\beta_0 > \beta$ (a symmetric argument is applicable also when $\beta_0 < \beta$). By Richness assumption, for each i, there exist $m_i, m_i', n_i, n_i' \in \Delta(C)$ such that $u_i(m_i) > u_i(m_i')$, $u_i(n_i') > u_i(n_i)$, and

$$\beta_0 > \frac{u_i(m_i) - u_i(m_i')}{u_i(n_i') - u_i(n_i)} > \beta. \tag{7}$$

Define $\overline{m} = \sum_i \frac{1}{|I|} m_i$, $\overline{m}' = \sum_i \frac{1}{|I|} m'_i$, $\overline{n} = \sum_i \frac{1}{|I|} n_i$, and $\overline{n}' = \sum_i \frac{1}{|I|} n'_i$. From (7),

$$u_i(\overline{m}) + \beta u_i(\overline{n}) > u_i(\overline{m}') + \beta u_i(\overline{n}'), \text{ and,} \tag{8}$$

$$u_i(\overline{m}) + \beta_0 u_i(\overline{n}) < u_i(\overline{m}') + \beta_0 u_i(\overline{n}') \tag{9}$$

for all i. Moreover, since $u_0 = \sum_i \widetilde{\alpha}_i u_i$, (9) implies

$$u_0(\overline{m}) + \beta_0 u_0(\overline{n}) < u_0(\overline{m}') + \beta_0 u_0(\overline{n}') \tag{10}$$

For any $m_0 \in \Delta(C)$, (8) implies that $(\overline{m}, \overline{n}, m_0, \cdots) \succ_i (\overline{m}', \overline{n}', m_0, \cdots)$. On the other hand, (10) implies $\{(\overline{m}', \overline{n}', m_0, \cdots)\} \succ_0 \{(\overline{m}, \overline{n}, m_0, \cdots)\}$, which contradicts Commitment Pareto.

Finally, we want to show $\gamma_0 = \beta$. By seeking a contradiction, suppose $\gamma_0 \neq \beta$. In particular, assume $\gamma_0 > \beta$ (a symmetric argument is applicable also when $\beta_0 < \beta$). By Richness assumption, for each i, there exist $m_i, m'_i, n_i, n'_i \in \Delta(C)$ such that $u_i(m_i) > u_i(m'_i)$, $u_i(n'_i) > u_i(n_i)$, and

$$\gamma_0 > \frac{u_i(m_i) - u_i(m'_i)}{u_i(n'_i) - u_i(n_i)} > \beta. \tag{11}$$

Define $\overline{m} = \sum_i \frac{1}{|I|} m_i$, $\overline{m}' = \sum_i \frac{1}{|I|} m'_i$, $\overline{n} = \sum_i \frac{1}{|I|} n_i$, and $\overline{n}' = \sum_i \frac{1}{|I|} n'_i$. From (11),

$$u_i(\overline{m}) + \beta u_i(\overline{n}) > u_i(\overline{m}') + \beta u_i(\overline{n}'), \text{ and,} \tag{12}$$

$$u_i(\overline{m}) + \gamma_0 u_i(\overline{n}) < u_i(\overline{m}') + \gamma_0 u_i(\overline{n}') \tag{13}$$

for all i. Moreover, since $v_0 = \sum_i \alpha_i u_i$, (13) implies

$$u_0(\overline{m}) + \gamma_0 u_0(\overline{n}) < u_0(\overline{m}') + \gamma_0 u_0(\overline{n}'). \tag{14}$$

For any fixed $m_0 \in \Delta(C)$, let $l = (\overline{m}, \overline{n}, m_0, \cdots)$ and $l' = (\overline{m}', \overline{n}', m_0, \cdots)$. Then, (12) implies that $l \succ_i l'$ for all i. Moreover, by Commitment Pareto, $\{l\} \succ_0 \{l'\}$. Thus, by NTPDP, $\{l\} \sim_0 \{l, l'\}$. On the other hand, (14) implies $V_0(l') > V_0(l)$. From the FT representation, $\{l\} \succ_0 \{l, l'\}$, which is a contradiction.

If part: TAM follows from Theorem 1. To show Commitment Pareto, suppose $U_i(l) > U_i(l')$ for all i. Then,

$$W_0(\{l\}) = \sum_{t=1}^{\infty} \beta^{t-1} \sum_i \widetilde{\alpha}_i u_i(l_t) = \sum_i \widetilde{\alpha}_i \sum_{t=1}^{\infty} u_i(l_t) \beta^{t-1}$$
$$> \sum_i \widetilde{\alpha}_i \sum_{t=1}^{\infty} u_i(l'_t) \beta^{t-1} = W_0(\{l'\}).$$

Next, to show NTPDP, suppose $U_i(l) > U_i(l')$ for all i and $\{l\} \succ_0 \{l'\}$. Since $\gamma_0 = \beta_i = \beta$,

$$\sum_i \alpha_i U_i(l) = \sum_t \alpha_i \sum_{t=1}^{\infty} \beta_i^{t-1} u_i(l_t) = \sum_{t=1}^{\infty} \beta^{t-1} \sum_t \alpha_i u_i(l_t) = \sum_{t=1}^{\infty} \gamma_0^{t-1} v_0(l_t) = V_0(l).$$

Thus, $U_i(l) > U_i(l')$ implies $V_0(l) > V_0(l')$. By the FT representation, $\{l\} \sim \{l, l'\}$, as desired.

A.5 Proof of Theorem 2

Only if part: Since Commitment Pareto implies Constant Commitment Pareto and NTPDP implies NTPDO, Theorem 1 implies $u_0 = \sum_h \widetilde{\alpha}_h u_h$ for some $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ and $v_0 = \sum_h \alpha_h u_h$ for some $(\alpha_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ with $\alpha_h \widetilde{\alpha}_h = 0$ for all h.

Observe that when we restrict attention to the subdomain of commitment plans, Commitment Pareto implies the existing dictatorship result (Zuber [34], Jackson and Yariv [23], Hayashi and Lombardi [19]). Hence, there exists some $i^* \in I$ such that $\widetilde{\alpha}_{i^*} > 0$, $\widetilde{\alpha}_h = 0$ for all $h \neq i^*$ and $\beta_0 = \beta_{i^*}$. Without loss it holds $u_0 = u_{i^*}$.

Next, we will show $v_0 = u_{j^*}$ and $\gamma_0 = \beta_{j^*}$ for some fixed $j^* \in I$. Since the profile satisfies TAM, v_0 is non-constant by the same argument as in Theorem 1. Together with $\gamma_0 > 0$, V_0 is non-constant.

We will claim that for any $l, l' \in \Delta(C)^\infty$, if $U_i(l) > U_i(l')$ for all i, then $V_0(l) > V_0(l')$. By Commitment Pareto, $\{l\} \succ_0 \{l'\}$. By NTPDP, $\{l\} \sim_0 \{l, l'\}$. From the FT representation of $\succsim_0$, $V_0(l) \geq V_0(l')$. Seeking a contradiction, suppose $V_0(l) = V_0(l')$. Since v_0 is not constant, take any $m, n \in \Delta(C)$ with $v_0(m) > v_0(n)$. For any $\alpha \in (0,1)$, let $l_\alpha = (\alpha l_1 + (1-\alpha)n, l_2, \cdots)$ and $l'_\alpha = (\alpha l'_1 + (1-\alpha)m, l'_2, \cdots)$. Then, $V_0(l_\alpha) < V_0(l'_\alpha)$. On the other hand, by the continuity of utility functions u_i, for all sufficiently small α, $U_i(l_\alpha) > U_i(l'_\alpha)$. By Commitment Pareto, $\{l_\alpha\} \succ_0 \{l'_\alpha\}$. NTPDP implies $\{l_\alpha\} \sim_0 \{l_\alpha, l'_\alpha\}$, while the FT representation implies $\{l_\alpha\} \succ_0 \{l_\alpha, l'_\alpha\}$, a contradiction.

Since $(U_i)_{i \in I}$ and V_0 are discounted utility functions, the existing dictatorship result (Zuber [34], Jackson and Yariv [23], Hayashi and Lombardi [19]) applies. Hence, there exists some $j^* \in I$ such that $\alpha_{j^*} > 0$, $\alpha_h = 0$ for all $h \neq j^*$ and $\gamma_0 = \beta_{j^*}$. Without loss it holds $v_0 = u_{j^*}$. And since $\alpha_h \widetilde{\alpha}_h = 0$ for all h, it holds $i^* \neq j^*$.

If part: Note that for some $i^* \neq j^*$, $W(\{l\}) = U_{i^*}(l)$ and $V_0(l) = U_{j^*}(l)$. So, it is obvious that the profile satisfies Commitment Pareto.

To show NTPDP, suppose $U_i(l) > U_i(l')$ for all i and $\{l\} \succ_0 \{l'\}$. The latter implies $U_{i^*}(l) > U_{i^*}(l')$ and the former implies $U_{j^*}(l) > U_{j^*}(l')$, or equivalently, $V_0(l) > V_0(l')$. By the FT representation, $\{\tilde{l}\} \sim \{\tilde{l}, \tilde{l}'\}$, as desired.

We show that the profile satisfies TAM. Take any m, n such that $u_i(m) > u_i(n)$ for some i, $u_j(n) > u_j(m)$ for all $j \neq i$, and $\{m^\infty\} \succ_0 \{n^\infty\}$. Since $W(\{l\}) = U_{i^*}(l)$, we must have $i = i^*$. Since $j^* \neq i^*$, $u_{j^*}(n) > u_{j^*}(m)$, which implies $V_0(n^\infty) = \frac{u_{j^*}(n)}{1-\beta_{j^*}} > \frac{u_{j^*}(m)}{1-\beta_{j^*}} = V_0(m^\infty)$. From the FT representation, we have $\{m^\infty\} \succ_0 \{m^\infty, n^\infty\}$, as desired.

A.6 Proof of Proposition 4

Only if part: Since Commitment Pareto implies Constant Commitment Pareto, Theorem 1 implies $u_0 = \sum_h \widetilde{\alpha}_h u_h$ for some $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ and $v_0 = \sum_h \alpha_h u_h$ for some $(\alpha_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ with $\alpha_h \widetilde{\alpha}_h = 0$ for all h. By completely the same argument in the proof of Proposition 3, it holds $\beta_0 = \beta$.

If part: NTPDO and TAM follows from Theorem 1. Commitment Pareto follows from completely the same argument in the proof of Proposition 3.

A.7 Proof of Theorem 3

Only if part: Since Commitment Pareto implies Constant Commitment Pareto, together with NTPDO, Theorem 1 implies $u_0 = \sum_h \widetilde{\alpha}_h u_h$ for some $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ and $v_0 = \sum_h \alpha_h u_h$ for some $(\alpha_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ with $\alpha_h \widetilde{\alpha}_h = 0$ for all h.

Observe that when we restrict attention to the subdomain of commitment plans, Commitment Pareto implies the existing dictatorship result (Zuber [34], Jackson and Yariv [23], Hayashi and Lombardi [19]). Hence, there exists some $i^* \in I$ such that $\widetilde{\alpha}_{i^*} > 0$, $\widetilde{\alpha}_h = 0$ for all $h \neq i^*$ and $\beta_0 = \beta_{i^*}$. Without loss it holds $u_0 = u_{i^*}$. Since $\alpha_h \widetilde{\alpha}_h = 0$ for all h and $\widetilde{\alpha}_{i^*} > 0$, $\alpha_{i^*} = 0$. Hence, $v_0 = \sum_{h \neq i^*} \alpha_h u_h$.

If part: Since $u_0 = u_{i^*}$ and $\beta_0 = \beta_{i^*}$, it is obvious that the profile satisfies Commitment Pareto. NTPDO and TAM follow from Theorem 1.

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