# Self-Control Games*

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**Abstract**

A majority of self-control models have been considered in the context of single-person decision making. One's propensity to resist temptation, however, may depend on others' decision, as observed in smoking, overeating, and overspending behaviors. As a self-control model with strategic interaction, this paper introduces a normal form game in which players have self-control preferences of Gul and Pesendorfer (2001). As an application, we consider a game in which players purchase a commitment device to cope with temptation in the presence of peer effects. In some equilibrium, players utilize a small amount of the commitment device in a coordinated manner, but in another equilibrium, they purchase an inefficiently large amount of the commitment device. Thus, restricting commitment options can improve welfare, which is never the case in the context of single-person decision making.

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# 1 Introduction

## 1.1 Objective

Since the seminal work of Strotz (1956), decision making in the presence of internal conflicts has been recognized as an important aspect of human behavior. Typical examples are diet or savings problems, where an individual has a particular preference prior to choice, but he may behave differently at the moment of choice because of changing his mind or experiencing temptation. Under such internal conflicts, the individual encounters a self-control problem; that is, he must decide whether to choose a desired option by exercising self-control, or to succumb to the temptation.

Gul and Pesendorfer (2001) (henceforth GP) provide a decision-theoretic model for choice under temptation. Their model, called *self-control preferences*, accommodates a self-control action as described above. In the GP model, a decision maker is dynamically consistent and decides whether to exercise self-control depending on its benefits and costs. Since their work, an increasing number of studies have adopted self-control preferences to examine economic behavior subject to temptation.

In most temptation models, one's preference is assumed to be independent of others' decision. One's propensity to resist temptation, however, may depend on it. For example, consider smoking behavior. Many smokers are unable to quit smoking even though they want to. This phenomenon can be explained by the temptation to smoke and the costly self-control to quit, on which peers' smoking may have a certain influence. In fact, past studies have documented the impact of peer effects on the initiation and cessation of smoking, primarily in young people (Chen et al., 2001; Powell et al., 2005). To explain a recent decrease in smoking behavior, Christakis and Fowler (2008) empirically argue that people have not been quitting by themselves; instead they have been quitting together, in droves.

Similar dependence on others' decisions is found in eating behavior. People sometimes overeat in spite of it being unhealthy, which is particularly true in the presence of others. In fact, it has been shown that one's food intake increases as the number of dining companions increases (de Castro, 2000).[1] In addition, even a light eater eats much more in a group of heavy eaters (Bell and Pliner, 2003). Christakis and Fowler (2007) empirically examine how obesity spreads in a social network and demonstrate that an obese person is more likely to have obese friends, friends of obese friends, and friends of friends of obese friends.

[1] de Castro (2000) finds that people who eat with one other person eat 35 percent more than they do when they are alone; members of a group of four eat 75 percent more; those in groups of seven or more eat 96 percent more.

We address the following question: how do decision makers cope with temptation in the presence of such peer influence? To illustrate it, consider a heavy smoker Bob and his friend Alice. Bob chooses one of the three actions: action $k$ to keep smoking, action $r$ to reduce smoking, and action $q$ to quit smoking. The preference relation prior to choice is as follows: Bob prefers action $q$ to action $k$ because it is bad for his health to smoke many cigarettes, but he prefers action $r$ to action $q$ because it needs a lot of cost and effort to quit smoking abruptly. Bob can make a commitment to choose action $q$ by joining a smoking cessation program prior to the choice. Otherwise, Bob exercises self-control to choose one of the three actions at the moment of choice: if Alice does not smoke, Bob chooses action $r$, but if Alice smokes, Bob chooses action $k$ because of peer influence. Then, whether Bob makes a commitment in advance depends upon whether Alice smokes. If Alice smokes, Bob makes a commitment and chooses action $q$ in order to avoid the worst option $k$, whereas if Alice does not smoke, Bob does not make a commitment and chooses action $r$ by exercising self-control. That is, Bob quits smoking if and only if Alice smokes. If Alice is also a heavy smoker who has the same options and preferences as those of Bob, we can analyze this situation as a game.

The purpose of this paper is to provide a framework to study such strategic interactions of decision makers coping with temptation. In particular, we are interested in whether decision makers choose commitment or self-control (or a combination of them) under strategic interactions and the resulting welfare implications as well. Our framework is a normal form game in which players have self-control preferences, which we call a *self-control game*. There are two stages of decision making. Given the other players' actions, a player chooses a subset of actions, called a menu, from his collection of available menus in the first stage, and chooses an action from the menu in the second stage. Decisions are made according to a player's self-control preference relation over menus, which depends on the other players' action profiles. As in GP, a payoff function in a self-control game consists of a normative payoff function and a temptation payoff function; however, in contrast to GP, we allow them to depend on the other players' action profiles. An equilibrium is defined as a strategy profile in which every player chooses his best response simultaneously. That is, we adopt a Nash equilibrium as an equilibrium concept.

Let us represent the above example as a self-control game played by Alice and Bob. Let $\succsim_a$ denote a player's self-control preference relation over his or her menus when the opponent chooses $a \in \{k, r, q\}$, which is the same for both players. For each $a \in \{k, r, q\}$, we assume

$$\{r\} \succ_a \{q\} \succ_a \{k\}.$$

This is the preference relation over the set of full commitments in the first stage, which

corresponds to the preference relation before facing with temptation. When $a = q$, we assume

$$\{r\} \succ_a \{k, r, q\} \succ_a \{q\} \succ_a \{k\},$$

which implies that when the opponent quits smoking, the temptation to smoke is so weak that the player exercises self-control and chooses $r$ from $\{k, r, q\}$ in the second stage. On the other hand, when $a \neq q$, we assume

$$\{r\} \succ_a \{q\} \succ_a \{k, r, q\} \sim_a \{k\},$$

which implies that when the opponent does not quit smoking, the temptation to smoke is so strong that the player yields to temptation and chooses $k$ from $\{k, r, q\}$ in the second stage. Assume that Alice and Bob have the same set of available menus: either $\{q\}$ or $\{k, r, q\}$. Then, when Bob chooses $\{q\}$ (i.e. makes a commitment to $q$), Alice's best response is to choose $\{k, r, q\}$ in the first stage and $r \in \{k, r, q\}$ in the second stage. Conversely, when Alice chooses $\{k, r, q\}$ in the first stage and $r \in \{k, r, q\}$ in the second stage, Bob's best response is to chooses $\{q\}$. Thus, the strategy profile such that one player chooses $\{q\}$ (to make a commitment) and the other player chooses $\{k, r, q\}$ (to exercise self-control) is an equilibrium, where both players utilize one player's commitment in a coordinated manner.

As a more elaborate application, we study a self-control game where two players purchase two goods in the second stage. One good is called commodity E, which generates a consumption externality, and the other is called commodity N, which does not generate it. For example, a player cares about both consumption and leisure, while he is tempted to consume more when the other player also consumes more. This tendency is known as "keeping up with the Joneses."[2] In this example, consumption is commodity E and leisure is commodity N. Players purchase both commodities in the second stage, but they also have an option to purchase commodity N in the first stage at a higher price. Because of opportunity costs,[3] commodity N in the first stage is more expensive than that in the second stage. Using this option, however, players can reduce their wealth carried over to the second stage and cap the amount of commodity E to purchase in advance. In this sense, commodity N purchased in the first stage serves as a commitment device.

We first show that there are exactly two types of best responses. If $p$ is less than a threshold, a player exercises the commitment option and purchases commodity N solely in the first stage as a commitment device. This strategy is referred to as a commitment strategy. On the other hand, if $p$ is greater than the threshold, a player does not exercise

[2]Most of the literature on consumption externalities such as Dupor and Liu (2003) and Ljungqvist and Uhlig (2000)) does not consider commitment options.

[3]For example, the money spent in the first stage bears no interest. See Section 3.1 for more details.

the commitment option and purchases commodity N solely in the second stage. This strategy is referred to as a self-control strategy. The threshold is shown to be increasing in the intensity of temptation. Thus, a player chooses a commitment strategy when he expects a strong peer effect (with high costs of self-control), whereas a player chooses a self-control strategy when he expects a weak peer effect.

Because two players have two types of best responses, there are three types of pure-strategy equilibria: (i) a commitment equilibrium where both players choose a commitment strategy, (ii) a self-control equilibrium where both players choose a self-control strategy, and (iii) an asymmetric equilibrium in which one player chooses a commitment strategy and the other chooses a self-control strategy.

Our main finding is that different types of equilibria are Pareto ranked if they coexist: more precisely, all players are weakly better off in an equilibrium where more players choose a self-control strategy and do not purchase the commitment device. In other words, coordination failure is driven by "overcommitment." For example, if a self-control equilibrium and a commitment equilibrium coexist, then the former Pareto dominates the latter, in which players purchase an inefficiently large amount of the commitment device. Thus, restricting the commitment option can eliminate the inefficient equilibrium, thus improving welfare. It should be noted that restricting the commitment option never improves welfare in single-person decision making.

We demonstrate that if the demand for commodity E in a commitment strategy is strictly increasing in $p$ (i.e., commodity E is substitute for the commitment device), then a self-control equilibrium and a commitment equilibrium can in fact coexist when $p$ is high. A self-control equilibrium exists because players are more likely to avoid the expensive cost of making a commitment. To see why a commitment equilibrium also exists, imagine that both players choose a commitment strategy. Because $p$ is high and the demand for commodity E is increasing in $p$, players consume a large amount of commodity E. Therefore, they expect a strong peer effect and thus optimally choose a commitment strategy. In this way, both players may purchase an inefficiently large amount of the commitment device.

In contrast, if the demand for commodity E in a commitment strategy is weakly decreasing in $p$ (i.e., commodity E is complementary for the commitment device), then a self-control equilibrium and a commitment equilibrium cannot coexist for all $p$. More specifically, a commitment equilibrium is a unique equilibrium if $p$ is low and a self-control equilibrium is a unique equilibrium if $p$ is high.

Even in this case, however, by considering the mixed extension of the self-control game, we can find similar coordination failure driven by overcommitment. That is, in the middle range of $p$, an asymmetric equilibrium and a symmetric mixed-strategy equilibrium coexist, and the former Pareto dominates the latter. In an asymmetric equilibrium, one

player chooses a commitment strategy and the other player chooses a self-control strategy. The former player consumes a small amount of commodity E by making a commitment and the latter player consumes a large amount of commodity E by exercising self-control. Thus, the former player expects a strong peer effect and thus optimally chooses a commitment strategy, whereas the latter player expects a weak peer effect and thus optimally chooses a self-control strategy. In this way, two players efficiently utilize one player's commitment device in a coordinated manner, which is analogous to the example discussed above. Contrastingly, in a mixed-strategy equilibrium, a player randomly chooses either a self-control strategy or a commitment strategy. That is, both players can simultaneously choose a commitment strategy and purchase an inefficiently large amount of the commitment device with a strictly positive probability. Therefore, restricting the commitment option can eliminate the inefficient mixed-strategy equilibrium, thus improving welfare.

The organization of the rest of this paper is as follows. Section 1.2 summarizes related literature. Section 2 introduces self-control games. Section 3 is devoted to an application, where we study the demand for the commitment device under peer effects. We conclude the paper in Section 4.

## 1.2 Related literature

Since GP's seminal work, several generalizations have been considered. Gul and Pesendorfer (2004) extend the GP model to an infinite horizon setting. Gul and Pesendorfer (2007) allow a cost of self-control to depend on histories or habits of past consumption. GP's representation of self-control preferences is axiomatized by four axioms. Dekel et al. (2009) and Stovall (2010) relax one of them, called the Set Betweenness axiom, and allow uncertainty about future temptations. Noor and Takeoka (2010, 2015) consider a non-linear or menu-dependent cost of self-control by relaxing GP's Independence axiom, and accommodate a behavior violating the weak axiom of revealed preference and several experimental findings about choice under risk and over time. See Lipman and Pesendorfer (2011) for a survey of the recent development.

There are several studies on multi-person decision making under internal conflicts. Battaglini et al. (2005) consider an agent with self-control preferences who has incomplete information about his ability to resist temptation and tries to infer it from his own past actions. In such a situation, an agent is concerned with self-reputation, as studied in Bénabou and Tirole (2004). Battaglini et al. (2005) assume that agents' characteristics are correlated, and hence, an agent can infer information about his own ability to resist temptation by observing not only his own past actions but also other players' actions. Their study focuses on how and why peer effects arise. In contrast, we assume peer effects a priori and explore their implications in a strategic setting.

By adopting the Strotz model (in particular, hyperbolic discounting consumers), Ludmer (2004), Herings and Rohde (2006), Luttmer and Mariotti (2006, 2007), and Malin (2008) consider general equilibrium models that do not admit a representative agent (without peer effects). Herings and Rohde (2006) and Luttmer and Mariotti (2006) study the existence of a competitive equilibrium[4] and Luttmer and Mariotti (2007) discuss its efficiency. Ludmer (2004) and Malin (2008) consider a more specific model in which illiquid assets work as commitment devices. In their model, consumers adopt an extreme decision rule where either a full commitment or no commitment is optimal. In Section 3, we provide a condition under which player's best response is obtained as a similar extreme decision rule also in our model with self-control preferences.

# 2 Self-control games

## 2.1 Setup

In GP's model of choice under temptation, an agent has a particular normative preference prior to choice, but he may behave differently at the moment of choice experiencing temptation. The agent may exercise self-control and choose an option in favor of a normative preference, or may yield to temptation and choose an inferior option. GP's self-control preferences explain such decision making.

We consider multiple agents who have self-control preferences, where the set of outcomes is an action space. Let $\mathcal{I} = \{1, \ldots, I\}$ be a set of players. Player $i \in \mathcal{I}$ has an action set $A_i$, which is a compact metric space. The set of mixed actions is denoted by $\Delta(A_i)$, which is the set of all Borel probability measures on $A_i$ with the weak convergence topology. We regard $a_i \in A_i$ as a mixed action placing probability one on $a_i$.

The action space $A = \prod_{i \in \mathcal{I}} A_i$ is endowed with the product metric. We denote the set of all Borel probability measures on $A$ by $\Delta(A)$, which is endowed with the weak convergence topology. Note that both $A$ and $\Delta(A)$ are compact metric spaces. Let $\mathcal{K}(\Delta(A))$ be the set of all nonempty compact subsets of $\Delta(A)$ with the Hausdorff metric.[5] Throughout this paper, we use the symbols $\Delta(\cdot)$ and $\mathcal{K}(\cdot)$ with the same meanings as those in the above: for a compact metric space $X$, $\Delta(X)$ is the set of all Borel probability measures on $X$ and $\mathcal{K}(X)$ is the set of all nonempty compact subsets of $X$.

Player $i \in \mathcal{I}$ has a self-control preference relation $\succsim_i$ on $\mathcal{K}(\Delta(A))$ represented by a

[4]Gabrieli and Ghosal (2011) demonstrate by a robust example with a representative agent that a competitive equilibrium may not exist even if consumer's demand correspondence is convexified.

[5]For example, see Aliprantis and Border (2006).

self-control utility function

$$\max_{x \in M} \left( u_i(x) - \left( \max_{y \in M} v_i(y) - v_i(x) \right) \right) \text{ for all } M \in \mathcal{K}(\Delta(A)), \tag{1}$$

where $u_i : \Delta(A) \to \mathbb{R}$ and $v_i : \Delta(A) \to \mathbb{R}$ are continuous linear functions. We refer to $u_i$ and $v_i$ as a normative payoff function and a temptation payoff function, respectively, for the reason explained later. GP show that $\succsim_i$ has the above representation if and only if it satisfies four axioms called Order, Continuity, Independence, and Set Betweenness.[6]

Player $i \in \mathcal{I}$ is endowed with a set of available opportunity sets $\mathcal{M}_i \subseteq \mathcal{K}(\Delta(A_i))$, which is assumed to be compact. We call each element $M_i \in \mathcal{M}_i$ a menu. Decision making consists of two stages: player $i$ chooses a menu $M_i \in \mathcal{M}_i$ in the first stage and chooses a mixed action $m_i \in M_i$ in the second stage. Player $i$'s strategy is $(M_i, m_i)$ with $m_i \in M_i \in \mathcal{M}_i$. Let $S_i$ denote player $i$'s set of strategies:

$$S_i \equiv \{(M_i, m_i) \in \mathcal{M}_i \times \Delta(A_i) \mid m_i \in M_i\}.$$

The set $S_i$ is a compact subset of $\mathcal{M}_i \times \Delta(A_i)$ (see Lemma F.1 in Appendix F). We call $\Gamma = (\mathcal{I}, (\mathcal{M}_i, u_i, v_i)_{i \in \mathcal{I}})$ *a self-control game*.

## 2.2 An equilibrium

We analyze a self-control game by regarding it as a normal-formal game. Thus, each player is assumed to choose his strategy based on his belief about the opponents' strategy profiles. To be more specific, consider player $i \in \mathcal{I}$ who believes that the opponents' strategy profile is $((M_j, m_j))_{j \neq i} \in \prod_{j \neq i} S_j$. Since player $i$ believes that $m_{-i} = (m_j)_{j \neq i}$ is eventually chosen, he chooses $M_i \in \mathcal{M}_i$ in the first stage by maximizing a self-control payoff function

$$U_i(M_i, m_{-i}) = \max_{m_i \in M_i} \left( u_i(m_i, m_{-i}) - \left( \max_{m_i' \in M_i} v_i(m_i', m_{-i}) - v_i(m_i, m_{-i}) \right) \right). \tag{2}$$

The normative payoff function $u_i$ captures a normative preference, which is consistent with commitment ranking in the first stage because $U_i(\{m_i\}, m_{-i}) = u_i(m_i, m_{-i})$. The temptation payoff function $v_i$ captures a temptation preference, whereby the term

$$\max_{m_i' \in M_i} v_i(m_i', m_{-i}) - v_i(m_i, m_{-i})$$

[6]The first three axioms are standard and can be viewed as a natural counterpart of the VNM axioms for the theory of choice under risk. The fourth axiom, Set Betweenness, plays a central role in characterizing the model. See Gul and Pesendorfer (2001) for more details.

is interpreted as a self-control cost of choosing $m_i$ and resisting the maximum temptation $\arg\max_{m_i' \in M_i} v_i(m_i', m_{-i})$. Note that the self-control utility is defined as the indirect utility function from maximizing normative payoffs minus self-control costs.

After choosing $M_i \in \mathcal{M}_i$ in the first stage, player $i$ chooses $m_i \in M_i$ in the second stage by maximizing $u_i(m_i, m_{-i}) - (\max_{m_i' \in M_i} v_i(m_i', m_{-i}) - v_i(m_i, m_{-i}))$. We denote player $i$'s choice correspondence in the second stage as

$$\begin{aligned} C_i(M_i, m_{-i}) &\equiv \arg\max_{m_i \in M_i} \Big( u_i(m_i, m_{-i}) - (\max_{m_i' \in M_i} v_i(m_i', m_{-i}) - v_i(m_i, m_{-i})) \Big) \\ &= \arg\max_{m_i \in M_i} \Big( u_i(m_i, m_{-i}) + v_i(m_i, m_{-i}) \Big). \end{aligned} \tag{3}$$

In summary, player $i$'s best response to $((M_j, m_j))_{j \neq i}$ is $(M_i, m_i)$ satisfying the following conditions.

$$M_i \in \arg\max_{M_i' \in \mathcal{M}_i} U_i(M_i', m_{-i}), \tag{4}$$

$$m_i \in C_i(M_i, m_{-i}). \tag{5}$$

We define an equilibrium of $\Gamma$ as a strategy profile in which all players choose their best responses each other. Note that all players make a two-stage decision simultaneously, so a self-control game can be understood as a normal-form game.

**Definition 1.** A strategy profile $((M_i, m_i))_{i \in \mathcal{I}}$ is an equilibrium of $\Gamma$ if (4) and (5) hold for all $i \in \mathcal{I}$.

For illustration, recall the example discussed in the introduction.

**Example.** There are two players, who are heavy smokers. For each $i \in \{1, 2\}$, the set of actions is $A_i = \{k_i, r_i, q_i\}$: $k_i$ is to keep smoking, $r_i$ is to reduce smoking, and $q_i$ is to quit smoking. The menu set is given by $\mathcal{M}_i = \{\{q_i\}, \{k_i, r_i, q_i\}\}$, where $\{q_i\}$ is to make a commitment to $q_i$ and $\{k_i, r_i, q_i\}$ is to make no commitment. We assume the following:

$$u_i(r_i, a_j) > u_i(q_i, a_j) > u_i(k_i, a_j) \text{ for each } a_j \in A_j, \tag{6}$$

$$v_i(k_i, a_j) > v_i(r_i, a_j) > v_i(q_i, a_j) \text{ for each } a_j \in A_j, \tag{7}$$

$$u_i(r_i, a_j) - u_i(q_i, a_j) > v_i(k_i, a_j) - v_i(r_i, a_j) \text{ if } a_j = q_j, \tag{8}$$

$$u_i(k_i, a_j) + v_i(k_i, a_j) > u_i(r_i, a_j) + v_i(r_i, a_j) \text{ if } a_j \neq q_j. \tag{9}$$

The condition (6) implies that, in terms of a normative preference, $r_i$ ranks at the top and $k_i$ ranks at the bottom. The condition (7) implies that, in terms of a temptation preference, $k_i$ ranks at the top and $q_i$ ranks at the bottom. By (8), when the opponent quits smoking, the cost of self-control to reduce smoking $v_i(k_i, a_j) - v_i(r_i, a_j)$ is not so high. By (9),

when the opponent does not quit smoking, $k_i$ is preferred to $r_i$ in the second stage. Then, by (6), (7) and (8),

$$U_i(\{k_i, r_i, q_i\}, q_j) > U_i(\{q_i\}, q_j) \text{ and } r_i = C_i(\{k_i, r_i, q_i\}, q_j).$$

On the other hand, by (6), (7) and (9),

$$U_i(\{q_i\}, r_j) > U_i(\{k_i, r_i, q_i\}, r_j) \text{ and } k_i = C_i(\{k_i, r_i, q_i\}, r_j).$$

Thus, it is straightforward to see that there are two pure-strategy equilibria,

$$((\{q_1\}, q_1), (\{k_2, r_2, q_2\}, r_2)) \text{ and } ((\{k_1, r_1, q_1\}, r_1), (\{q_2\}, q_2)).$$

That is, one player exercises self-control to reduce smoking, whereas the other player makes a commitment to quit smoking.

Although we relegate a detailed discussion on the existence of equilibria to Appendix F, it is worth pointing out here that even under standard assumptions on action spaces and payoff functions, a pure-strategy equilibrium does not necessarily exist. To see this, assume that $A_i$ is a convex set and that both $u_i$ and $v_i$ are concave. When players choose only pure actions (i.e., $\mathcal{M}_i \subseteq \mathcal{K}(A_i)$), (2) is rewritten as

$$U_i(M_i, a_{-i}) = \max_{a_i \in M_i} (u_i(a_i, a_{-i}) + v_i(a_i, a_{-i})) - \max_{a_i' \in M_i} v_i(a_i', a_{-i}).$$

Then, $\max_{M_i}(u_i + v_i)$ and $\max_{M_i} v_i$ are concave in $M_i$, but their difference $U_i$ is not necessarily quasi-concave, and hence, its best response may not be convex-valued.[7] In Appendix F, we provide a sufficient condition for the existence of pure-strategy equilibria. In Appendix G, we define a mixed-strategy equilibrium and establish the existence.

We discuss some properties of pure-strategy equilibria. The next lemma shows that to see whether $m_i \in \Delta(A_i)$ is a mixed action in some equilibrium, it is sufficient to consider a minimal menu including $m_i$.

**Lemma 1.** *If $((M_i, m_i))_{i \in \mathcal{I}}$ is an equilibrium of $\Gamma$, then for any $M_i' \in \mathcal{M}_i$ with $m_i \in M_i' \subseteq M_i$, $((M_i', m_i))_{i \in \mathcal{I}}$ is also an equilibrium.*

*Proof.* Since $m_i \in M_i' \subseteq M_i$ and $m_i \in C_i(M_i, m_{-i})$, we have

$$\max_{x_i \in M_i'} \Big(u_i(x_i, m_{-i}) + v_i(x_i, m_{-i})\Big) = \max_{x_i \in M_i} \Big(u_i(x_i, m_{-i}) + v_i(x_i, m_{-i})\Big),$$

which implies that $m_i \in C_i(M_i', m_{-i})$. Therefore,

$$\begin{aligned} U_i(M_i', m_{-i}) &= \max_{x_i \in M_i'} \Big(u_i(x_i, m_{-i}) + v_i(x_i, m_{-i})\Big) - \max_{y_i \in M_i'} v_i(y_i, m_{-i}) \\ &\geq \max_{x_i \in M_i} \Big(u_i(x_i, m_{-i}) + v_i(x_i, m_{-i})\Big) - \max_{y_i \in M_i} v_i(y_i, m_{-i}) = U_i(M_i, m_{-i}), \end{aligned}$$

[7] We discuss such an example in Section 3.

which implies that $U_i(M_i', m_{-i}) = U_i(M_i, m_{-i}) = \max_{M_i'' \in \mathcal{M}_i} U_i(M_i'', m_{-i})$. Therefore, $((M_i', m_i))_{i \in \mathcal{I}}$ is also an equilibrium. □

In the next two lemmas, we consider equilibria in two extreme cases. At one extreme, assume that every player can make a full commitment to each action in the first stage. Then, an equilibrium of $\Gamma$ is reduced to a Nash equilibrium for normative payoff functions.

**Lemma 2.** *Assume that $\mathcal{M}_i \supseteq \{\{m_i\} : m_i \in \Delta(A_i)\}$ for all $i \in \mathcal{I}$. If $((M_i, m_i))_{i \in \mathcal{I}}$ is an equilibrium of $\Gamma$, then $((\{m_i\}, m_i))_{i \in \mathcal{I}}$ is also an equilibrium of $\Gamma$. In addition, $((\{m_i\}, m_i))_{i \in \mathcal{I}}$ is an equilibrium of $\Gamma$ if and only if $m = (m_i)_{i \in \mathcal{I}}$ is a Nash equilibrium of a game in which player i's payoff function is $u_i$.*

*Proof.* The first half of the lemma is implied by Lemma 1. We show the second half. If $((\{m_i\}, m_i))_{i \in \mathcal{I}}$ is an equilibrium of $\Gamma$, then

$$U_i(\{m_i\}, m_{-i}) = u_i(m_i, m_{-i}) \geq u_i(m_i', m_{-i}) = U_i(\{m_i'\}, m_{-i})$$

for all $\{m_i'\} \in \mathcal{M}_i$ and $i \in \mathcal{I}$. This implies that $m$ is a Nash equilibrium of a game in which player $i$'s payoff function is $u_i$. Conversely, if $m$ is a Nash equilibrium of a game in which player $i$'s payoff function is $u_i$,

$$\begin{aligned} U_i(\{m_i\}, m_{-i}) &= u_i(m_i, m_{-i}) \\ &\geq u_i(x_i, m_{-i}) \\ &\geq u_i(x_i, m_{-i}) - \left( \max_{y_i \in M_i'} v_i(y_i, m_{-i}) - v_i(x_i, m_{-i}) \right) \end{aligned}$$

for all $x_i \in \Delta(A_i)$, $M_i' \in \mathcal{M}_i$, and $i \in \mathcal{I}$. Therefore, $U_i(\{m_i\}, m_{-i}) \geq U_i(M_i', m_{-i})$ for all $M_i' \in \mathcal{M}_i$. Since $m_i \in C_i(\{m_i\}, m_{-i})$, $((\{m_i\}, m_i))_{i \in \mathcal{I}}$ is an equilibrium of $\Gamma$. □

At the other extreme, assume that every player can make no commitment in the first stage. Then, an equilibrium of $\Gamma$ is reduced to a Nash equilibrium for compromised payoff functions, i.e., the sum of normative and temptation payoff functions.

**Lemma 3.** *Assume that $\mathcal{M}_i = \{\Delta(A_i)\}$ for all $i \in \mathcal{I}$. Then, $((\Delta(A_i), m_i))_{i \in \mathcal{I}}$ is an equilibrium of $\Gamma$ if and only if $m = (m_i)_{i \in \mathcal{I}}$ is a Nash equilibrium of a game in which player i's payoff function is $u_i + v_i$.*

*Proof.* A strategy profile $((\Delta(A_i), m_i))_{i \in \mathcal{I}}$ is an equilibrium of $\Gamma$ if and only if $m_i \in C_i(\Delta(A_i), m_{-i})$ for all $i \in \mathcal{I}$, i.e.,

$$u_i(m_i, m_{-i}) + v_i(m_i, m_{-i}) \geq u_i(m_i', m_{-i}) + v_i(m_i', m_{-i})$$

for all $m_i' \in \Delta(A_i)$ and $i \in \mathcal{I}$. This implies that $m$ is a Nash equilibrium of a game in which player $i$'s payoff function is $u_i + v_i$. □

# 3 Application: demand for commitment device

## 3.1 Setup

Overspending behavior caused by a consumption externality is known as "keeping up with the Joneses." That is, people may be tempted to consume more for a higher standard of living if their neighbors have the same tendency. In this section, we model this situation as a self-control game and investigate welfare implications of a commitment option.

We consider two players, each of whom possesses an initial wealth $w > 0$ and purchases two goods in the second stage at price 1. One good is called commodity E, which generates a consumption externality, and the other is called commodity N, which does not generate it. For example, a player cares about both consumption and leisure, while he is tempted to consume more when the other player also consumes more. In this case, consumption is commodity E and leisure is commodity N.

In the first stage, a player has an option to purchase commodity N at a higher price $p \geq 1$. According to the money spent on commodity N in the first stage, a player can reduce the maximum money spent on commodity E in the second stage. Thus, commodity N purchased in the first stage serves as a commitment device. Notice that commodity N as a commitment device is more expensive than commodity N in the second stage. This can be interpreted as an opportunity cost. For example, the money spent in the first stage bears no interest. Alternatively, there is a possibility of consuming "better" commodity N in the second stage, while players forgo the flexibility of purchasing a better commodity later by purchasing commodity N in the first stage. Otherwise, we can also assume that the price of commodity N is the same for both stages, but the initial wealth $w$ is available only in the second stage, so players must borrow money with an interest rate $p - 1 \geq 0$ in the first stage to purchase the commitment device.

An action of player $i$ is a consumption bundle $a_i = (x_i, y_i) \in \mathbb{R}^2_+$, where $x_i$ is the consumption of commodity E and $y_i$ is that of commodity N. When player $i$ purchases $c_i \in [0, w/p]$ units of commodity N in the first stage, his consumption bundle $(x_i, y_i)$ with $y_i \geq c_i$ must satisfy the budget constraint $x_i + (y_i - c_i) = w - pc_i$. This implies that the corresponding menu of actions is

$$M(c_i) \equiv \{(x_i, y_i) \in \mathbb{R}^2_+ \mid y_i \geq c_i,\ x_i + y_i = w - (p-1)c_i\}$$

and the set of all available menus is

$$\mathcal{M}_i \equiv \{M(c_i) \mid 0 \leq c_i \leq w/p\}.$$

Player $i$'s strategy $(M(c_i), (x_i, y_i))$ is abbreviated to $(c_i, x_i, y_i)$. We call $(c_i, x_i, y_i)$ a commitment strategy if $c_i = y_i$, in which player $i$ purchases commodity N solely in the

first stage and makes a commitment to choose the consumption bundle $(w - pc_i, c_i)$. We call $(c_i, x_i, y_i)$ a self-control strategy if $c_i = 0$, in which player $i$ does not purchase commodity N in the first stage and exercises self-control to choose a consumption bundle $(x_i, y_i)$.

Throughout this section, we adopt the following specification of normative and temptation payoff functions.

**Assumption 1.** A normative payoff function of player $i$ is $u(x_i) + u(y_i)$, and a temptation payoff function is $\kappa(x_j)v(x_i)$. Both $u$ and $v$ are twice differentiable with $u', v' > 0$, $u'', v'' < 0$, and $\lim_{x\to 0} u'(x) = \lim_{x\to 0} v'(x) = \infty$. Moreover, $\kappa(x_j) \geq 0$ is strictly increasing and continuous in $x_j$ with $j \neq i$.

The normative payoff function values both commodities equally, whereas the temptation payoff function values solely commodity E. The intensity of temptation is captured by $\kappa(x_j)$ and increases with the other's consumption level of commodity E. Note that peer effects operate only on the intensity of temptation $\kappa(x_j)$.[8]

Given the other's consumption level $x_j$, player $i$ evaluates a menu $M(c_i)$ by a self-control payoff function

$$\begin{aligned} V(c_i) &\equiv \max_{(x_i,y_i)\in M_i(c_i)} \Big(u(x_i) + u(y_i) - \kappa_j \left(v(w - pc_i) - v(x_i)\right)\Big) \\ &= \max_{(x_i,y_i)\in M_i(c_i)} (u(x_i) + \kappa_j v(x_i) + u(y_i)) - \kappa_j v(w - pc_i), \end{aligned} \tag{10}$$

where $\kappa_j \equiv \kappa(x_j)$. Thus, the best response is $(c_i, x_i, y_i)$ satisfying

$$c_i \in \arg\max_{c_i'\in[0,w/p]} V(c_i') \text{ and } (x_i, y_i) \in \arg\max_{(x_i',y_i')\in M_i(c_i)} (u(x_i') + \kappa_j v(x_i') + u(y_i')).$$

Note that, because a self-control cost is non-negative, a self-control payoff function $V(c_i)$ is less than the maximum normative payoff: for each $c_i \in [0, w/p]$,

$$V(c_i) \leq \max_{x_i+y_i\leq w} u(x_i) + u(y_i) = u(x_i^*) + u(y_i^*), \tag{11}$$

where $(x_i^*, y_i^*) = \arg\max_{x_i+y_i\leq w} u(x_i) + u(y_i)$ is the consumption bundle that achieves the maximum normative payoff over all consumption bundles satisfying the budget constraint.

## 3.2 Single-agent decision

We start with obtaining an optimal strategy in a single-agent decision problem by assuming that $\kappa_j$ is an exogenously given constant.

[8]In the online appendix, we provide an axiomatic foundation for such a class of self-control games in which a normative payoff function is independent of the opponents' actions and a temptation payoff function is multiplicatively separable from the opponents' actions.

### The case of the lowest price or prohibitively high intensity of temptation

If the price of the commitment device is lowest ($p = 1$) or the intensity of temptation is prohibitively high ($\kappa_j = \infty$), it is easy to show that an optimal strategy is a commitment strategy.

First, consider the case of $p = 1$, where the price of commodity N is identical across the two stages. Then, player $i$ can achieve the maximum normative payoff in (11) by purchasing $y_i^*$ units of commodity N in the first stage and $x_i^* = w - y_i^*$ units of commodity E in the second stage; that is,

$$\max_{c_i \in [0, w/p]} V(c_i) = V(y_i^*) = u(x_i^*) + u(y_i^*).$$

Thus, a commitment strategy is optimal.

Next, consider the limiting case as $\kappa_j \to \infty$, where the temptation of purchasing commodity E is overwhelming. Note that

$$V(c_i) = \max_{(x_i, y_i) \in M_i(c_i)} \Big( u(x_i) + u(y_i) - \kappa_j \left( v(w - pc_i) - v(x_i) \right) \Big) \geq u(w - pc_i) + u(c_i).$$

GP show that, as $\kappa_j \to \infty$, $V(c_i) \to u(w - pc_i) + u(c_i)$; that is, player $i$ always chooses a commitment strategy in the limit.[9] This is because otherwise a self-control cost $\kappa_j \left( v(w - pc_i) - v(x_i) \right)$ goes to infinity as $\kappa_j \to \infty$. Thus, a commitment strategy is optimal.

### The general case

We show that player $i$ chooses a commitment strategy if $p$ is sufficiently low or $\kappa_j$ is sufficiently large; otherwise, player $i$ chooses a self-control strategy.

First, we obtain $V(c_i)$ in (10). If the solution to (10) is a corner solution $(x_i, y_i) = (w - pc_i, c_i)$, we have

$$V(c_i) = V_1(c_i) \equiv u(w - pc_i) + u(c_i).$$

If the solution to (10) is an interior solution, where the constraint $y_i \geq c_i$ is not binding, we have

$$V(c_i) = V_0(c_i) \equiv \max_{x_i + y_i = w - (p-1)c_i} u(x_i) + \kappa_j v(x_i) + u(y_i) - \kappa_j v(w - pc_i). \tag{12}$$

Let $(\xi(c_i), \eta(c_i))$ denote the interior solution, which is the unique value of $(x, y)$ satisfying

$$u'(x) + \kappa_j v'(x) = u'(y), \;\; x + y = w - (p-1)c_i. \tag{13}$$

The next lemma shows that the solution is an interior solution if and only if $c_i$ is sufficiently small.

[9] GP relate this result to the Strotz model (Strotz, 1956).

**Lemma 4.** *Let $\bar{c}$ be the unique value of $c$ solving $u'(w - pc) + \kappa_j v'(w - pc) = u'(c)$. Then, the solution to* (10) *is*

$$(x_i, y_i) = (\min\{\xi(c_i), w - pc_i\}, \max\{\eta(c_i), c_i\}) = \begin{cases} (\xi(c_i), \eta(c_i)) & \text{if } c_i \leq \bar{c}, \\ (w - pc_i, c_i) & \text{if } c_i \geq \bar{c}, \end{cases}$$

*and*

$$V(c_i) = \begin{cases} V_0(c_i) & \text{if } c \leq \bar{c}, \\ V_1(c_i) & \text{if } c \geq \bar{c}. \end{cases}$$

*Proof.* This is an immediate consequence of the following KKT conditions

$$u'(x_i) + \kappa_j v'(x_i) = \lambda,\ u'(y_i) = \lambda + \mu,\ \mu(y_i - c_i) = 0,\ \mu \geq 0,$$

where $\lambda$ and $\mu$ are KKT multipliers for $x_i + y_i = w - (p-1)c_i$ and $y_i \leq c_i$, respectively. □

If player $i$ purchases a large amount of the commitment device, i.e., $c_i > \bar{c}$, he spends all the remaining money on commodity E. If player $i$ purchases a small amount of the commitment device, i.e., $c_i < \bar{c}$, he eventually chooses the consumption bundle that maximizes a compromise utility $u(x_i) + \kappa_j v(x_i) + u(y_i)$ subject to the budged constraint $x_i + y_i = w - (p - 1)c_i$.

We are ready to consider the optimal commitment level that maximizes $V(c_i)$. To simplify the analysis, we assume the following single crossing property of the marginal benefit $V_0'(c_i)$.

**Assumption 2.** The equation $V_0'(c_i) = 0$ has at most one solution on $[0, w/p]$.

Note that, in Assumption 2, we regard $V_0(c_i)$ as a function on $[0, w/p]$ (rather than on $[0, \bar{c}_i]$), which is an imaginary payoff to purchasing $c_i$ units of commodity N in the first stage when player $i$ could not only purchase but also sell commodity N in the second stage. We can verify that Assumption 2 holds if $u = v$ and it exhibits constant relative risk aversion. Note that

$$V_0'(c_i) = -(p - 1)(u'(\xi(c_i)) + \kappa_j v'(\xi(c_i))) + p\kappa_j v'(w - pc_i)$$

by the envelop theorem. When $c_i$ increases, the wealth carried over to the second stage $w - (p - 1)c_i$ and the self-control cost $\kappa_j v(w - pc_i)$ decreases. The former has a negative effect $-(p - 1)(u'(\xi(c_i)) + \kappa_j v'(\xi(c_i))) < 0$ and the latter has a positive effect $p\kappa_j v'(w - pc_i) > 0$ on the utility. We require that the total effect $V_0'(c_i)$ satisfy the above single crossing property.

In the next theorem, we characterize an optimal strategy, where player $i$ purchases commodity N either solely in the first stage or solely in the second stage.

**Theorem 1.** *Under Assumptions 1 and 2, there exists a strictly increasing unbounded function $\theta(\kappa_j)$ with $\theta(0) = 1$ such that player i's optimal strategy is*

$$(c_i, x_i, y_i) = \begin{cases} ((y^c(p), x^c(p), y^c(p)) & \text{if } p \leq \theta(\kappa_j), \\ (0, x^s(\kappa_j), y^s(\kappa_j)) & \text{if } p \geq \theta(\kappa_j), \end{cases} \tag{14}$$

*where $(x^c(p), y^c(p))$ is the unique value of $(x, y)$ solving $u'(x) = u'(y)/p$ and $y = (w - x)/p$, and $(x^s(\kappa_j), y^s(\kappa_j)) = (\xi(0), \eta(0))$ is the unique value of $(x, y)$ solving $u'(x) + \kappa_j v'(x) = u'(y)$ and $x + y = w$. The optimal demand for the commitment device $c_i$ is weakly decreasing in $p$ and weakly increasing in $\kappa_j$. On the other hand, the optimal consumption bundle $(x_i, y_i)$ is not monotone in $\kappa_j$.*

*Proof.* See Appendix A. □

By Theorem 1, an optimal decision depends on the relative size of the price $p$ and the intensity $\kappa_j$, which are associated with costs of making a commitment and exercising self-control, respectively.

If the price is low relative to the intensity, that is, $p \leq \theta(\kappa_j)$, player $i$ purchases commodity N solely in the first stage and makes a commitment to choose the consumption bundle maximizing a normative utility $u(x_i) + u(y_i)$ subject to $x_i + py_i = w$, of which solution is $(x^c(p), y^c(p))$. This is a commitment strategy. Because $(x^c(p), y^c(p))$ does not depend on $\kappa_j$, the payoff to a commitment strategy

$$u(x^c(p)) + u(y^c(p)) \tag{15}$$

is independent of $\kappa_j$. Note that the optimal demand for the commitment device $c_i = y^c(p)$ is decreasing in $p$ and independent of $\kappa_j$.

If the price is high relative to the intensity, that is, $p \geq \theta(\kappa_j)$, player $i$ purchases commodity N solely in the second stage and exercises self-control to choose the consumption bundle maximizing a compromise utility $u(x_i) + \kappa_j v(x_i) + u(y_i)$ subject to $x_i + y_i = w$, of which solution is $(x^s(\kappa_j), y^s(\kappa_j))$. This is a self-control strategy. Whenever player $i$ optimally chooses a self-control strategy, his payoff is greater than the payoff to a commitment strategy (15). Note that the optimal demand for the commitment device is independent of $p$ and $\kappa_j$ because it equals zero.

To summarize, an optimal strategy is a commitment strategy if $p$ is sufficiently low or $\kappa_j$ is sufficiently large, and it is a self-control strategy if $p$ is sufficiently high or $\kappa_j$ is sufficiently small. This implies that the optimal demand for the commitment device $c_i$ is weakly decreasing in $p$ and weakly increasing in $\kappa_j$ because the demand is zero when a player chooses a self-control strategy. On the other hand, the optimal consumption bundle $(x_i, y_i)$ is not monotone in $\kappa_j$. To see this, note that if $\kappa_j < \theta^{-1}(p)$ then $x_i = x^s(\kappa_j)$ is

increasing in $\kappa_j$ by Theorem 1. When $\kappa_j = \theta^{-1}(p)$, $x_i$ changes from $x^s(\kappa_j)$ to $x^c(p)$, but we can verify that $x^s(\kappa_j) > x^c(p)$; that is, $x_i$ jumps down. In fact, by Lemma 4, $y^c(p) \geq \bar{c}$. Thus, $x^c(p) \leq w - p\bar{c} = \xi(\bar{c}) < \xi(0) = x^s(\kappa_j)$, where the latter inequality holds because each good is a normal good.

## 3.3 Pure-strategy equilibrium

Given Theorem 1, we consider strategic interactions by focusing on pure-strategy equilibria.

### 3.3.1 Three types of equilibria with a welfare comparison

Each player chooses either a commitment strategy or a self-control strategy depending on the relative size of $p$ and $\kappa_j = \kappa(x_j)$ by Theorem 1. Thus, in a pure-strategy equilibrium, exactly one of the following holds: (i) both players choose a commitment strategy; (ii) both players choose a self-control strategy; (iii) one player chooses a commitment strategy and the other player chooses a self-control strategy. We refer to them as a commitment equilibrium, a self-control equilibrium, and an asymmetric equilibrium, respectively.

The first and most important observation is that different types of equilibria are Pareto ranked if they coexist.

**Proposition 1.** *Under Assumptions 1 and 2, the following holds if two types of equilibria coexist.*

(i) *Each player's payoff in a self-control equilibrium is greater than or equal to that in an asymmetric equilibrium or a commitment equilibrium.*

(ii) *Each player's payoff in an asymmetric equilibrium is greater than or equal to that in a commitment equilibrium.*

This is because the payoff to a commitment strategy (15) is independent of the opponent's strategy. This implies that a player can always secure this payoff level by choosing a commitment strategy. Thus, if a player chooses a self-control strategy in an equilibrium, the payoff to a self-control strategy must be greater than or equal to that to a commitment strategy, as discussed before.

Proposition 1 implies that all players are weakly better off in an equilibrium where more players choose a self-control strategy and do not purchase the commitment device. In other words, less provision of the commitment device can eliminate inefficient equilibria, thus improving welfare. For example, suppose that there exist multiple equilibria and one of them is a self-control equilibrium. Imagine that a social planner reduces the supply of

the commitment device to zero. Then, commitment or asymmetric equilibria cannot exist any more, and a self-control equilibrium is a unique equilibrium. That is, a reduction in the commitment device removes inefficient equilibria.

It should be noted that, although restricting commitment options in a self-control game can improve welfare, it is always harmful to welfare in single-person decision making.

### 3.3.2 Existence

We study under what condition each type of an equilibrium exists and demonstrate that different types of equilibria can coexist.

A strategy profile $((c_i, x_i, y_i))_{i\in\{1,2\}}$ is a commitment equilibrium if and only if

$$(c_i, x_i, y_i) = \big(y^c(p), x^c(p), y^c(p)\big) \text{ for each } i \in \{1, 2\}, \tag{16}$$

$$p \leq \theta(\kappa(x_1)) = \theta(\kappa(x_2)). \tag{17}$$

Therefore, there exists a commitment equilibrium if and only if

$$p \in P_c \equiv \{p \in [1, \infty) \mid p \leq \theta(\kappa(x^c(p)))\}. \tag{18}$$

For example, if $1 \leq p \leq \theta(\kappa(0))$, then $p \in P_c$ because $\theta(\kappa(\cdot)) \geq 1$ is strictly increasing and thus $p \leq \theta(\kappa(0)) < \theta(\kappa(x^c(p)))$. This means that $P_c$ is nonempty.

A strategy profile $((c_i, x_i, y_i))_{i\in\{1,2\}}$ is a self-control equilibrium if and only if

$$(c_i, x_i, y_i) = (0, x^s(\kappa(x_j)), y^s(\kappa(x_j))) \text{ for } i \neq j,$$
$$\max\{\theta(\kappa(x_1)), \theta(\kappa(x_2))\} \leq p.$$

If $x_i > x_j$, then $x_j = x^s(\kappa(x_i)) > x^s(\kappa(x_j)) = x_i$ because $x^s(\kappa(\cdot))$ is strictly increasing, a contradiction. Thus, we must have $x_1 = x_2 = x^*$ and the above condition is rewritten as

$$(c_i, x_i, y_i) = (0, x^s(\kappa(x^*)), y^s(\kappa(x^*))) \text{ for each } i \in \{1, 2\}, \tag{19}$$

$$x^* = x^s(\kappa(x^*)), \tag{20}$$

$$\theta(\kappa(x^*)) \leq p. \tag{21}$$

If equation (20) admits a unique solution, a self-control equilibrium is unique, but in general, there may exist multiple self-control equilibria. Let $\underline{x}^* \in [0, w]$ be the minimum solution of (20), which exists because $0 \leq x^s(\kappa(\cdot)) \leq w$ and $x^s(\kappa(\cdot))$ is continuous. Then, there exists a self-control equilibrium if and only if

$$p \in P_s \equiv \{p \in [1, \infty) \mid \theta(\kappa(\underline{x}^*)) \leq p\}. \tag{22}$$

A strategy profile $((c_i, x_i, y_i))_{i\in\{1,2\}}$ is an asymmetric equilibrium if and only if

$$(c_i, x_i, y_i) = (y^c(p), x^c(p), y^c(p)), \tag{23}$$

$$(c_j, x_j, y_j) = (0, x^s(\kappa(x_i)), y^s(\kappa(x_i))), \tag{24}$$

$$\theta(\kappa(x_i)) \leq p \leq \theta(\kappa(x_j)) \tag{25}$$

for $i \neq j$. The inequality $\theta(\kappa(x_i)) \leq p$ guarantees the optimality of player $j$'s self-control strategy and $p \leq \theta(\kappa(x_j))$ guarantees the optimality of player $i$'s commitment strategy. Therefore, there exists an asymmetric equilibrium if and only if

$$p \in P_a \equiv \{p \in [1, \infty) \mid \theta(\kappa(x^c(p))) \leq p \leq \theta(\kappa(x^s(\kappa(x^c(p)))))\}. \tag{26}$$

The next theorem summarizes the above argument.

**Theorem 2.** *Under Assumptions 1 and 2, the following holds.*

(i) *There exists a commitment equilibrium given by* (16) *if and only if* $p \in P_c$.

(ii) *There exists a self-control equilibrium given by* (19) *and* (20) *if and only if* $p \in P_s$.

(iii) *There exists an asymmetric equilibrium given by* (23) *and* (24) *if and only if* $p \in P_a$.

As discussed above, a self-control equilibrium exists if $p$ is sufficiently high and a commitment equilibrium exists if $p$ is sufficiently low. However, in general, a commitment equilibrium can exist even if $p$ is very high. That is, different types of equilibria can coexist. For example, consider square-root utility $u(z) = v(z) = \sqrt{z}$ and an isoelastic peer effect $\kappa(x) = x^\alpha$ with $\alpha > 0$, where $\alpha$ is a parameter for the intensity of peer effects. The best response is (14) with

$$\begin{aligned}
(x^c(p), y^c(p)) &= (pw/(1+p), w/(p+p^2)), \\
(x^s(\kappa_j), y^s(\kappa_j)) &= ((1+\kappa_j)^2 w/(1+(1+\kappa_j)^2), w/(1+(1+\kappa_j)^2)), \\
\theta(\kappa_j) &= 1/(((1+(1+\kappa_j)^2)^{1/2} - \kappa_j)^2 - 1)
\end{aligned}$$

by Theorem 1. Using the above together with (18), (22), and (26), we can obtain $P_c$, $P_s$, and $P_a$ and show that $P_c \cap P_s$ is nonempty; that is, a commitment equilibrium and a self-control equilibrium can coexist.

To see why, notice that $x^c(p) = pw/(1+p)$ is increasing in $p$; that is, commodity E is a substitute good for the commitment device. Then, even if $p$ is very high, $p \leq \theta(\kappa(x^c(p)))$ holds and a commitment equilibrium exist by (18). The intuition is as follows. Suppose that $p$ is very high. When a player chooses a commitment strategy, he consumes a large amount of commodity E because $x^c(p)$ is increasing in $p$. Thus, when a player believes that the opponent chooses a commitment strategy, he expects a very strong peer effect, so

his best response is a commitment strategy. A self-control equilibrium also exists when $p$ is very high by (22). Therefore, a commitment equilibrium and a self-control equilibrium can coexist. The next proposition confirms this observation.

**Proposition 2.** *Suppose that* $u(z) = v(z) = \sqrt{z}$, $\kappa(x) = x^{\alpha}$ *with*

$$\alpha > \bar{\alpha} \equiv \inf_{p>1/(w-1)} \frac{\log \theta^{-1}(p)}{\log(pw/(1+p))} > 0$$

*and* $5/4 < w < (\sqrt{5}-1)^2$. *Then, there exists* $p > 1/(w-1)$ *such that both a commitment equilibrium and a self-control equilibrium exist. On the other hand, if* $\theta(1) < p < 1/(w-1)$, *then a self-control equilibrium exists, but a commitment equilibrium does not exist.*

*Proof.* Recall that a commitment equilibrium exists if and only if $p \leq \theta(\kappa(x^c(p)))$. For $\alpha'$ with $\bar{\alpha} < \alpha' < \alpha$, let $p > 1/(w-1)$ be such that

$$\alpha' = \frac{\log \theta^{-1}(p)}{\log(pw/(1+p))},$$

which exists by the definition of $\bar{\alpha}$. Then,

$$p = \theta\left(\left(\frac{pw}{1+p}\right)^{\alpha'}\right) < \theta\left(\left(\frac{pw}{1+p}\right)^{\alpha}\right) = \theta(\kappa(x(p))),$$

so a commitment equilibrium exists.

If $\theta(1) < p < 1/(w-1)$, then

$$\theta(\kappa(x(p))) = \theta\left(\left(\frac{pw}{1+p}\right)^{\alpha}\right) < \theta(1) < p,$$

so a commitment equilibrium does not exist.

Recall that a self-control equilibrium exists if and only if $\theta(\kappa(\underline{x}^*)) \leq p$, where $\underline{x}^*$ is the minimum solution to (20). Because $x^s(\kappa(0)) = w/2 < 1$ and $x^s(\kappa(1)) = 4w/5 > 1$, we must have $\underline{x}^*(k) < 1$. Therefore,

$$\theta(\kappa(\underline{x}^*)) \leq \theta(\kappa(1)) = \theta(1) = 1/((\sqrt{5}-1)^2 - 1) < 1/(w-1),$$

which establishes the existence of a self-control equilibrium. □

This proposition asserts that a commitment equilibrium does not exist if $\theta(1) < p < 1/(w-1)$, but it does exist for some $p > 1/(w-1)$. That is, the demand for the commitment device can be greater when the price is higher. In this sense, the commitment device is a Giffen good and the law of demand is violated. This is a stark contrast with the case of single-agent decision, in which the demand for the commitment device is weakly decreasing in the price by Theorem 1.

### 3.3.3 Uniqueness

We study under what condition different types of equilibria do not coexist. In the case of square-root utility, commodity E is a substitute good, which generates multiplicity of equilibria. Hence, to obtain uniqueness, it is natural to assume that commodity E is a complementary good.

**Assumption 3.** The consumption of commodity E in a commitment strategy $x^c(p)$ is non-increasing in $p$. That is, commodity $E$ is a complementary good for the commitment device.

This assumption requires that the negative income effect dominates the positive substitution effect in terms of normative utility. For example, Assumption 3 holds if $u$ exhibits constant relative risk aversion and the coefficient of relative risk aversion is greater than or equal to 1.

We show that, under Assumption 3, the price ranges $P_c$, $P_a$, and $P_s$ are intervals whose intersections are null sets.

**Theorem 3.** *Under Assumptions 1, 2, and 3, we have*

$$P_c = [1, p_1],\ P_a = [p_1, p_2],\ P_s = [p_3, \infty),$$

*where* $1 < p_1 < p_2 < p_3$ *holds and* $p_1$, $p_2$, *and* $p_3$ *are given by* $p_1 = \theta(\kappa(x^c(p_1)))$, $p_2 = \theta(\kappa(x^s(\kappa(x^c(p_2)))))$, *and* $p_3 = \theta(\kappa(\underline{x}^*))$, *respectively.*

*Proof.* See Appendix B. □

A unique type of a pure-strategy equilibrium is a commitment equilibrium if $p < p_1$, a self-control equilibrium if $p \geq p_3$, and an asymmetric equilibrium if $p_1 < p \leq p_2$, whereas there is no pure-strategy equilibrium if $p_2 < p < p_3$.

To see why players cannot choose the same strategy if $p_1 < p \leq p_2$, imagine that both players choose a commitment strategy. Then, they consume a small amount of commodity E, inducing a weak peer effect to each other. In this case, the price of the commitment device $p > p_1$ is too expensive, whereby a deviation to a self-control strategy is profitable. Next, imagine that both players choose a self-control strategy. Then, they consume a large amount of commodity E, inducing a strong peer effect to each other. Thus, the price of the commitment device $p \leq p_2$ is affordable, whereby a deviation to a commitment strategy is profitable. On the other hand, when one player chooses a commitment strategy and the other player chooses a self-control strategy, the former player consumes a small amount of commodity E and the latter player consumes a large amount of commodity E. Therefore, the former player optimally chooses a commitment strategy expecting a strong

peer effect, and the latter player optimally chooses a self-control strategy expecting a weak peer effect.

By Theorem 3, a pure-strategy equilibrium cannot be a Pareto-dominated equilibrium under Assumption 3 because different types of equilibria do not coexist.[10] In addition, there is no violation of the law of demand. Indeed, the demand for the commitment device is $2y^c(p)$ if $p < p_1$, $y^c(p)$ if $p_1 \leq p \leq p_2$, and zero if $p \geq p_3$, which is weakly decreasing in $p$.

For example, consider log utility $u(z) = v(z) = \log z$ and a linear peer effect $\kappa(x) = \alpha x$ with $\alpha > 0$, where $\alpha$ is a parameter for the intensity of peer effects. The best response is (14) with

$$\begin{aligned}(x^c(p), y^c(p)) &= (w/2, w/(2p)),\\ (x^s(\kappa_j), y^s(\kappa_j)) &= ((1+\kappa_j)w/(2+\kappa_j), w/(2+\kappa_j)),\\ \theta(\kappa_j) &= (2+\kappa_j)^{2+\kappa_j}/(4(1+\kappa_j)^{1+\kappa_j})\end{aligned}$$

by Theorem 1. Notice that $x^c(p) = w/2$ does not depend on $p$, so Assumption 3 holds. Moreover, we can obtain $P_c$, $P_a$, and $P_s$ in a closed form:

$$P_c = [1, p_1(\alpha)],\ P_a = [p_1(\alpha), p_2(\alpha)],\ P_s = [p_3(\alpha), \infty),$$

where $p_1(\alpha) \equiv \theta(\alpha w/2) < p_2(\alpha) \equiv \theta(\alpha x^s(\alpha w/2)) < p_3(\alpha) \equiv \theta(\alpha x^*(\alpha))$ with $x^*(\alpha) = (\alpha w - 2 + \sqrt{\alpha^2 w^2 + 4})/(2\alpha)$. Note that $p_1(\alpha)$, $p_2(\alpha)$, and $p_3(\alpha)$ are increasing in $\alpha$. Figure 1 depicts regions of $(\alpha, p)$ for different types of equilibria on the $(\alpha, p)$-plane, where the boundaries are graphs of $p = p_1(\alpha)$, $p = p_2(\alpha)$, and $p = p_3(\alpha)$.

## 3.4 Mixed-strategy equilibrium

In this subsection, we consider the mixed extension of this game.[11] We show that a mixed-strategy equilibrium and an asymmetric equilibrium can coexist and the former is Pareto-dominated by the latter. That is, even when commodity E is a complementary good for the commitment device, inefficiency can be driven by coordination failure on a commitment.

For each $i \in \{1, 2\}$, let $\sigma_i$ denote player $i$'s mixed strategy, which is a probability distribution over the set of player $i$'s pure strategies. Given player $j$'s mixed strategy $\sigma_j$,

[10] When $p = p_1(\alpha)$, a commitment equilibrium and an asymmetric equilibrium coexist. In the asymmetric equilibrium, however, a player choosing a self-control strategy is indifferent between self-control and commitment strategies. Therefore, the payoffs in the two types of equilibria are the same.

[11] See Appendix G for a general definition of a mixed-strategy equilibrium in self-control games and its existence.

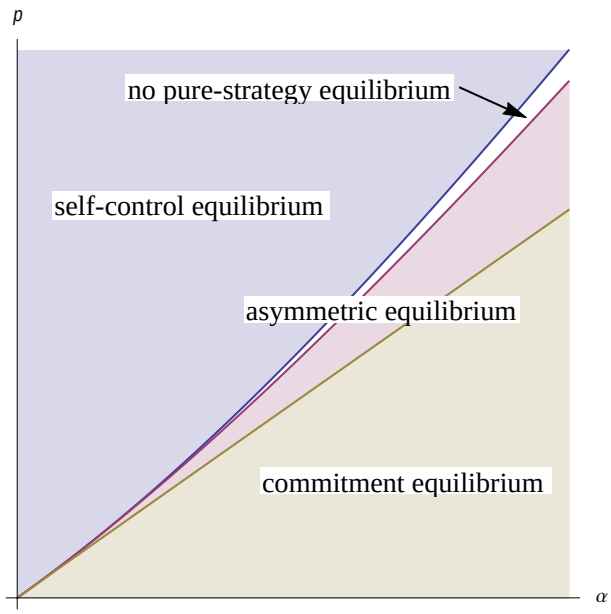

Figure 1: Classification of equilibria on the $(\alpha, p)$-plane

player $i$'s expected payoff of choosing a menu $M(c_i)$ is given by

$$\max_{(x_i,y_i)\in M(c_i)} \Big(u(x_i) + E_{\sigma_j}[\kappa(x_j)]v(x_i) + u(y_i)\Big) - E_{\sigma_j}[\kappa(x_j)]v(w - pc_i),$$

where $E_{\sigma_j}[\kappa(x_j)]$ is the expected value of $\kappa(x_j)$. Note that the above expected payoff is of the same form as (10). Thus, by Theorem 1, player $i$'s best response is either a commitment strategy or a self-control strategy, and if $p = \theta(E_{\sigma_j}[\kappa(x_j)])$ then player $i$ is indifferent between the two strategies. Therefore, a mixed-strategy equilibrium exists if $p = \theta(E_{\sigma_i}[\kappa(x_i)])$ for each $i$ because each player randomizes between the two strategies in a mixed-strategy equilibrium.

The next theorem shows that a unique symmetric mixed-strategy equilibrium exists unless a symmetric pure-strategy equilibrium exists. Thus, a symmetric equilibrium always exists.

**Theorem 4.** *Under Assumptions 1 and 2, if $p \in (P_c \cup P_s)^c$, then there exists a unique symmetric mixed-strategy equilibrium. In this equilibrium, players choose a commitment strategy $(y^c(p), x^c(p), y^c(p))$ with probability*

$$\lambda \equiv \frac{\kappa(x^s(\theta^{-1}(p))) - \theta^{-1}(p)}{\kappa(x^s(\theta^{-1}(p))) - \kappa(x^c(p))} \in (0, 1) \tag{27}$$

*and a self-control strategy $(0, x^s(\theta^{-1}(p)), y^s(\theta^{-1}(p)))$ with probability $1-\lambda$. Conversely, if there exists a symmetric mixed-strategy equilibrium and equation (20) admits a unique solution, then $p \in (P_c \cup P_s)^c$.*

*Proof.* See Appendix C. □

Suppose that Assumptions 1, 2, and 3 hold. By Theorem 3, different types of pure-strategy equilibria do not coexist unless $p = p_1$. However, a mixed-strategy equilibrium and an asymmetric equilibrium coexist if $p \in P_a \cap (P_c \cup P_s)^c = (p_1, p_2)$. In this case, we can show that a mixed-strategy equilibrium is Pareto dominated by an asymmetric equilibrium as a corollary of Proposition 1. The reason is as follows. The payoff in a mixed-strategy equilibrium equals the payoff to a commitment strategy, which equals the payoff in a commitment equilibrium, but a commitment equilibrium is Pareto dominated by an asymmetric equilibrium by Proposition 1.

**Corollary 3.** *Under Assumptions 1 and 2, each player's payoff in an asymmetric equilibrium is greater than or equal to that in a symmetric mixed-strategy equilibrium if both equilibria coexist.*

In an asymmetric equilibrium, a player choosing a commitment strategy consumes a small amount of commodity E, which reduces the other player's self-control cost because of weak peer effects. A player choosing a self-control strategy consumes a large amount of commodity E, which generates no externality because the other player chooses a commitment strategy. Therefore, two players utilize one player's commitment device in a coordinated manner. However, in a symmetric mixed-strategy equilibrium, both players choose a commitment strategy with a positive probability and they purchase a larger amount of the commitment device on average. In this way, asymmetric coordination with less demand for the commitment device is better than symmetric coordination with greater demand.

Finally, we investigate the welfare effect of a price change. In the case of single-agent decision making discussed in Section 3.2, where $\kappa_j$ is exogenous, the optimal payoff is decreasing in $p$ if $p \leq \theta(\kappa_j)$ and independent of $p$ when $p \geq \theta(\kappa_j)$. On the other hand, in the self-control game, $\kappa(x_j)$ is endogenously determined in equilibrium for each $p$. To examine the welfare effect of $p$, we assume that there exists a unique symmetric equilibrium for each $p$ and show that the equilibrium payoff is weakly decreasing in $p$.

**Proposition 4.** *Suppose that Assumptions 1, 2, and 3 hold and that equation* (20) *admits a unique solution. In a unique symmetric equilibrium, the equilibrium payoff is decreasing in $p$; that is, it is strictly decreasing in $p$ if $p < p_3$ and constant if $p \geq p_3$, where $p_1$, $p_2$, and $p_3$ are given in Theorem 3.*

*Proof.* By Theorem 3 and Theorem 4, if $p < p_3$, a symmetric equilibrium is either a commitment equilibrium or a mixed strategy equilibrium. In both cases, the equilibrium payoff equals that to a commitment strategy (15), which is decreasing in $p$. Theorem 4 also implies that the payoff in a mixed-strategy equilibrium is continuous in $p$ and converges to the payoff of a unique self-control equilibrium as $p \to p_3$. If $p \geq p_3$, a

symmetric equilibrium is a self-control equilibrium, where the payoff is independent of $p$. □

Consequently, if we focus on the symmetric equilibrium, all players are weakly better off by a lower price. However, this is not necessarily true when we model a player as a Strotz type, as we discuss in the next subsection.

## 3.5 Comparison with the Strotz model

We consider the Strotz model in the same environment and demonstrate a similarity and a difference between the Strotz model and the GP model under strategic interactions.

In the Strotz model, a player is assumed to consist of different selves at different time periods. We call player $i \in \{1, 2\}$ in the first stage self 1, whose payoff function is the normative payoff function $u(x_i) + u(y_i)$. We call player $i$ in the second stage self 2, whose payoff function is the compromise payoff function $u(x_i) + \kappa(x_j)v(x_i) + u(y_i)$, by which self 2's decision coincides with a GP-type's decision in the second stage.[12] Note that self 2 places a larger weight on the utility of commodity E than does self 1, and such disagreement is reinforced by the peer effect. We study this setting as an extensive form game with four players, regarding each self as a player.

Because self 2's decision coincides with a GP-type's decision in the second stage, his best response is given by Lemma 4. Given self 2's choice $(x_i(c_i), y_i(c_i))$, self 1 solves the following maximization problem:

$$\max_{0 \le c_i \le w/p} u(x_i(c_i)) + u(y_i(c_i)).$$

The next proposition shows that the best response of a Strotz-type player is the same as that of a GP-type player except the threshold price.

**Proposition 5.** *Let $\kappa_j \equiv \kappa(x_j)$. Under Assumption 1, there exists strictly increasing unbounded function $\theta^S(\kappa_j)$ with $\theta^S(0) = 1$ such that player i's best response is*

$$(c_i, x_i, y_i) = \begin{cases} ((y^c(p), x^c(p), y^c(p)) & \text{if } p \le \theta^S(\kappa_j), \\ (0, x^s(\kappa_j), y^s(\kappa_j)) & \text{if } p \ge \theta^S(\kappa_j), \end{cases} \tag{28}$$

*where $(x^c(p), y^c(p))$ and $(x^s(\kappa_j), y^s(\kappa_j))$ are given in Theorem 1. Moreover, $\theta^S(\kappa_j) < \theta(\kappa_j)$ for all $\kappa_j > 0$.*

*Proof.* See Appendix D. □

[12] GP show that when temptation is overwhelming, a self-control preference is represented by the Strotz model, where self 1 has a normative utility function and self 2 has a temptation utility function. Under this identification, an optimal strategy is always a commitment strategy as discussed in Section 3.2.

Proposition 5 requires only Assumption 1, while Theorem 1 requires Assumptions 1 and 2. Thus, the extreme decision rule of choosing either full commitment or no commitment is a more robust phenomenon for Strotz-type players.[13]

A more important difference is $\theta^S(\kappa_j) < \theta(\kappa_j)$; that is, the Strotz-type's threshold price is smaller than the GP-type's threshold price. Thus, whenever a Strotz-type player chooses a commitment strategy, so does a GP-type player, which stems from a self-control cost. When a player chooses a self-control strategy, the payoff in the GP model is

$$u(x^s(\kappa_j)) + u(y^s(\kappa_j)) - \kappa_j(v(w) - v(x^s(\kappa_j))),$$

whereas the payoff of self 1 in the Strotz model is

$$u(x^s(\kappa_j)) + u(y^s(\kappa_j)).$$

The former is smaller due to a self-control cost. On the other hand, when a player chooses a commitment strategy, the payoff in the GP model and that of self 1 in the Strotz model are the same, which equals (15). Therefore, a GP-type player shows a stronger preference for a commitment strategy than a Strotz-type players.[14]

Using Proposition 5 and applying the same argument as that in Theorems 2 and 3, we can characterize a subgame perfect equilibrium in the Strotz model, which is either a commitment equilibrium, an asymmetric equilibrium, or a self-control equilibrium, as stated in the next proposition.

**Proposition 6.** *Under Assumption 1, a pure-strategy equilibrium is either a commitment equilibrium, an asymmetric equilibrium, or a self-control equilibrium. Under Assumptions 1 and 3, for $p_1^S < p_2^S < p_3^S$ given by $p_1^S = \theta^S(\kappa(x^c(p_1^S)))$, $p_2^S = \theta^S(\kappa(x^s(\kappa(x^c(p_2^S)))))$, and $p_3^S = \theta^S(\kappa(\underline{x}^*))$, there exists a commitment equilibrium if and only if $p \le p_1^S$; there exists an asymmetric equilibrium if and only if $p_1^S \le p \le p_2^S$; there exists a self-control equilibrium if and only if $p \ge p_3^S$. Moreover, $p_1^S < p_1$, $p_2^S < p_2$, and $p_3^S < p_3$, where $p_1$, $p_2$, and $p_3$ are the threshold prices of the GP model given by Theorem 3.*

An equilibrium in the Strotz model is qualitatively the same as an equilibrium in the GP model. However, the welfare impact of $p$ may differ between the two models. The following proposition shows that payoffs of self 1 and self 2 may change oppositely in prices.

[13]Ludmer (2004) and Malin (2008) find a similar optimal decision rule of hyperbolic discounting consumers (with no peer effects).

[14]This observation can be also understood from the comparative statics by GP. They show that the benefit of commitment stems from the discrepancy between normative and temptation utilities. In our model, both the GP-type's normative utility and the Strotz-type's self 1 utility are the same as $u(x) + u(y)$, while the GP-type's temptation utility $\kappa_j v(x)$ is more discrepant from $u(x) + u(y)$ than the Strotz-type's self 2 utility $u(x) + \kappa_j v(x) + u(y)$. Therefore, the GP-type has a greater preference for commitment.

**Proposition 7.** *Under Assumptions 1 and 3, let* $(\underline{x}, \underline{y})$ *be the consumption bundle in a commitment equilibrium when* $p = \underline{p} < p_1^S$ *and let* $(\overline{x}, \overline{y})$ *be that in a self-control equilibrium when* $p = \overline{p} > p_3^S$*. Then, self 1 strictly prefers* $(\underline{x}, \underline{y})$ *to* $(\overline{x}, \overline{y})$*, while self 2 has the opposite ranking.*

*Proof.* See Appendix E □

This proposition implies that we cannot determine whether Strotz-type players are better off with a lower price. This is in clear contrast to Proposition 4 which states that GP-type players are better off with a lower price.

# 4 Concluding remarks

In this paper, we introduce a self-control game, in which every player has a self-control preference relation introduced by GP. Using this game, we can analyze how decision makers cope with temptation when their propensity to resist temptation depends upon others' decision. Typically, a decision maker with a self-control preference relation chooses either to exercise self-control or to make a commitment. Thus, in an equilibrium of a self-control game, the combination of players' commitment and self-control is endogenously determined.

For example, in the commitment device game discussed in Section 3, there are three types of pure-strategy equilibria, a commitment equilibrium in which both players make a commitment, a self-control equilibrium in which both players exercise self-control, and an asymmetric equilibrium in which one player makes a commitment and the other player exercises self-control. On the other hand, in a mixed-strategy equilibrium, both players randomize between making a commitment and exercising self-control.

Our main finding is that, when different types of equilibria coexist, the equilibrium in which the fewest players make a commitment is socially optimal among all the equilibria. In other words, overcommitment can generate coordination failure. This is because the payoff to a commitment strategy is independent of the opponent's strategy and a player can always secure this payoff level by choosing a commitment strategy. This implies that if a player chooses a self-control strategy in an equilibrium, the payoff to a self-control strategy must be greater than or equal to that to a commitment strategy.

The conclusion about coordination failure is robust in the sense that the above discussion relies only on the following three properties.

- Players can make a full commitment incurring no self-control cost.
- The payoff to a full commitment is independent of the opponents' strategies.

- Different types of equilibria can coexist.

Note that these properties may hold in self-control games other than the commitment device game. The first property describes a situation which we are interested in. It is straightforward to see that the second property is satisfied if a normative payoff function is independent of the opponents' actions, i.e., peer effects operate only on the intensity of temptation. To explore the issue of coordination failure driven by overcommitment, it is important to study what nature of strategic interactions generates multiple equilibria in a more general class of self-control games, which would be a topic for future research.

# Appendix

## A Proof of Theorem 1

We first show that a best response is either a commitment strategy or a self-control strategy.

**Lemma A.1.** *Under Assumptions 1 and 2,* $c^* \in \arg\max_{c_i \in [0,w/p]} V(c_i)$ *if and only if*

$$c^* = \begin{cases} 0 \text{ or } y^c(p) & \text{if } y^c(p) \geq \bar{c}, \\ 0 & \text{if } y^c(p) < \bar{c}. \end{cases}$$

*Proof.* Note that $\lim_{c_i \to w/p} V_0'(c_i) = \infty$ by Assumption 1. Thus, by Assumption 2, if $V_0'(0) \geq 0$, then $V_0'(c_i) \geq 0$ for all $c_i \in [0, w/p)$. Moreover, $V_0'(0) < 0$ if and only if there exists $c' \in (0, w/p)$ such that $V_0'(c_i) < 0$ is equivalent to $c_i < c'$.

Suppose that $y^c(p) \geq \bar{c}$. Recall that $y^c(p) \in \arg\max_{0 \leq c_i \leq w/p} u(w - pc_i) + u(c_i) = \arg\max_{0 \leq c_i \leq w/p} V_1(c_i)$. Thus, $\max_{c_i \geq \bar{c}} V_1(c_i) = V_1(y^c(p)) = V(y^c(p)) \geq V_1(\bar{c}) = V(\bar{c})$. If $V_0'(0) \geq 0$, then $V_0'(c_i) \geq 0$ for all $c_i \in [0, w/p)$, so $\max_{c_i \leq \bar{c}} V_0(c_i) = V_0(\bar{c}) = V(\bar{c})$ and

$$\max_{c_i} V(c_i) = \max\{\max_{c_i \leq \bar{c}} V_0(c_i), \max_{c_i \geq \bar{c}} V_1(c_i)\} = V(y^c(p)).$$

If $V_0'(0) < 0$, then $V_0'(c_i) < 0$ if and only if $c_i < c'$, so $\max_{c_i \leq \bar{c}} V_0(c_i) = \max\{V_0(0), V_0(\bar{c})\} = \max\{V(0), V(\bar{c})\}$ and

$$\max_{c_i} V(c_i) = \max\{V(0), V(\bar{c}), V(y^c(p))\} = \max\{V(0), V(y^c(p))\}.$$

Suppose that $y^c(p) < \bar{c}$. In this case, $\max_{c_i \geq \bar{c}} V_1(c_i) = V_1(\bar{c}) = V(\bar{c})$ and $V_1'(\bar{c}) < 0$ by Assumption 1. Applying the envelop theorem to obtain $V'(\bar{c})$, we can show that $V'(\bar{c}) = V_0'(\bar{c}) = V_1'(\bar{c}) < 0$. This implies that $V_0'(c_i) < 0$ for all $c_i \leq \bar{c}$. Thus, we must have $\max_{c_i} V(c_i) = V(0)$. □

Lemma A.1 means that a best response is either a commitment strategy ($c^* = y^c(p)$), where a player purchases commodity N solely in the first stage, or a self-control strategy ($c^* = 0$), where a player purchases commodity N solely in the second stage. To determine which strategy yields a higher payoff, define

$$v^1(p) \equiv \max_{0 \le y \le w/p} u(w - py) + u(y), \tag{A.1}$$

$$v^2(\kappa_j) \equiv \max_{0 \le y \le w} \{u(w - y) + \kappa_j v(w - y) + u(y)\} - \kappa_j v(w), \tag{A.2}$$

where $v^1(p)$ is the maximum payoff in the first stage when a player purchases commodity N solely in the first stage and $v^2(\kappa_j)$ is that when a player purchases it solely in the second stage. Then, a commitment strategy is a best response if and only if $v^1(p) \ge v^2(\kappa_j)$, and a self-control strategy is a best response if and only if $v^2(\kappa_j) \ge v^1(p)$.

The following property of $v^1(p)$ and $v^2(\kappa_j)$ are crucial in establishing Theorem 1.

**Lemma A.2.** *$v^1(p)$ and $v^2(\kappa_j)$ are strictly decreasing, and the range of $v^1(p)$ includes that of $v^2(\kappa_j)$.*

*Proof.* By the envelop theorem,

$$dv^1(p)/dp = -pu'(w - py^c(p)) < 0, \; dv^2(\kappa_j)/d\kappa_j = v(w - y^s(\kappa_j)) - v(w) < 0,$$

where $y^c(p) \in \arg\max_{0 \le y \le w/p} u(w - py) + u(y)$ and $y^s(\kappa_j) \in \arg\max_{0 \le y \le w}(u(w - y) + \kappa_j v(w - y) + u(y))$.

Because $v^1(p)$ and $v^2(\kappa_j)$ are strictly decreasing,

$$\max_{p \ge 1} v^1(p) = v^1(1) = v^2(0) = \max_{\kappa_j \ge 0} v^2(\kappa_j).$$

In addition, since $y^c(p) \to 0$ as $p \to \infty$,

$$\lim_{\kappa_j \to \infty} v^2(\kappa_j) = u(w) + u(0) \ge \lim_{p \to \infty} u(x^c(p)) + u(y^c(p)) = \lim_{p \to \infty} v^1(p).$$

Thus, the range of $v^1$ includes that of $v^2$. □

By this lemma, we can define a strictly increasing function

$$\theta(\kappa_j) \equiv (v^1)^{-1}(v^2(\kappa_j)).$$

Then, $v^1(p) \gtreqless v^2(\kappa_j)$ if and only if $p \lesseqgtr \theta(\kappa_j)$. Thus, a commitment strategy is optimal if and only if $p \le \theta(\kappa_j)$, and a self-control strategy is optimal if and only if $p \ge \theta(\kappa_j)$.

It holds that $\theta(0) = 1$ because $v^1(1) = v^2(0)$. To show that $\theta$ is unbounded, fix $p > 1$ arbitrarily. Since $(x^c(p), y^c(p))$ is a maximizer of $u(x_i) + u(y_i)$ within the budget

constraint $x_i + py_i = w$, we have $u(x^c(p)) + u(y^c(p)) > u(w) + u(0)$. Moreover, since $x^s(\kappa_j)$ is a solution to $u'(x) + \kappa_j v'(x) = u'(w - x)$, we have $x^s(\kappa_j) \to w$ as $\kappa_j \to \infty$. Therefore, for all sufficiently large $\kappa_j$,

$$\begin{aligned} v^1(p) &= u(x^c(p)) + u(y^c(p)) > u(x^s(\kappa_j)) + u(y^s(\kappa_j)) \\ &> u(x^s(\kappa_j)) + u(y^s(\kappa_j)) - \kappa_j(v(w) - v(x^s(\kappa_j))) = v^2(\kappa_j), \end{aligned}$$

which implies that for all $p$, we can find some $\kappa_j$ such that $p < \theta(\kappa_j)$. We can conclude that $\theta$ is unbounded.

# B Proof of Theorem 3

Define $f : [1, \infty) \to \mathbb{R}$ by $f(p) = \theta(\kappa(x^c(p))) - p$. This function is strictly decreasing because $\theta$ is strictly increasing by Theorem 1, $x^c$ is non-increasing by Assumption 3, and $\kappa$ is strictly increasing. Moreover, $f$ is unbounded below and

$$f(1) = \theta(\kappa(x^c(1))) > \theta(\kappa(0)) \geq \theta(0) = 1$$

by Theorem 1. Thus, there exists a unique $p_1 > 1$ such that $f(p_1) = 0$, i.e., $p_1 = \theta(\kappa(x^c(p_1)))$, and thus

$$p \in P_c \;\Leftrightarrow\; p \leq \theta(\kappa(x^c(p))) \;\Leftrightarrow\; p \leq p_1.$$

Next define $g : [1, \infty) \to \mathbb{R}$ by $g(p) = \theta(\kappa(x^s(\kappa(x^c(p))))) - p$. By a similar argument as above, this function is strictly decreasing and there exists a unique $p_2 > 1$ such that $g(p_2) = 0$, i.e., $p_2 = \theta(\kappa(x^s(\kappa(x^c(p_2)))))$. Because

$$x^c(p) \leq x^c(1) = x^s(0) < x^s(\kappa(x^c(p)))$$

and $\theta(\kappa(\cdot))$ is strictly increasing,

$$p_1 = \theta(\kappa(x^c(p_1))) < \theta(\kappa(x^s(\kappa(x^c(p_1))))),$$

so we must have $p_1 < p_2$. Therefore,

$$p \in P_a \;\Leftrightarrow\; \theta(\kappa(x^c(p))) \leq p \leq \theta(\kappa(x^s(\kappa(x^c(p))))) \;\Leftrightarrow\; p_1 \leq p \leq p_2.$$

We already know that $P_s = [p_3, \infty)$. Thus, it remains to show that $p_2 < p_3$. Because

$$x^c(p) \leq x^c(1) = x^s(0) < x^s(\kappa(\underline{x}^*)) = \underline{x}^*$$

and $\theta(\kappa(x^s(\kappa(\cdot))))$ is strictly increasing,

$$p_2 = \theta(\kappa(x^s(\kappa(x^c(p_2))))) < \theta(\kappa(x^s(\kappa(\underline{x}^*)))) = \theta(\kappa(\underline{x}^*)) = p_3.$$

# C Proof of Theorem 4

We first give a necessary and sufficient condition for the existence of a unique symmetric mixed-strategy equilibrium.

**Lemma C.1.** *A unique symmetric mixed-strategy equilibrium exists if and only if* $\kappa(x^c(p)) < \theta^{-1}(p) < \kappa(x^s(\theta^{-1}(p)))$*, or equivalently,* $\theta(\kappa(x^c(p))) < p < \theta(\kappa(x^s(\theta^{-1}(p))))$*. In this equilibrium, players choose a commitment strategy with probability* $\lambda \in (0,1)$ *given by* (27).

*Proof.* Suppose that a symmetric mixed-strategy equilibrium $(\sigma_i)_{i\in\{1,2\}}$ exists. We write $\bar{\kappa} \equiv E_{\sigma_{i|A_i}}[\kappa(x_i)] = E_{\sigma_{j|A_j}}[\kappa(x_j)]$. Then, it holds that $p = \theta(\bar{\kappa})$ by Theorem 1 because players are indifferent between a commitment strategy and a self-control strategy in a mixed-strategy equilibrium. Recall that $\theta(\cdot)$ is strictly increasing, so the inverse function $\theta^{-1}(\cdot)$ exists. Therefore,

$$\theta^{-1}(p) = \bar{\kappa} = \lambda\kappa(x^c(p)) + (1-\lambda)\kappa(x^s(\bar{\kappa})) = \lambda\kappa(x^c(p)) + (1-\lambda)\kappa(x^s(\theta^{-1}(p))),$$

where $\lambda$ is the probability of a commitment strategy. By solving the above, we obtain (27). Note that the denominator of (27) is strictly positive because $x^s(\theta^{-1}(p)) > x^c(p)$ by the argument associated with Theorem 1. Thus, $\lambda \in (0,1)$ implies $\kappa(x^c(p)) < \theta^{-1}(p) < \kappa(x^s(\theta^{-1}(p)))$.

Conversely, if $\kappa(x^c(p)) < \theta^{-1}(p) < \kappa(x^s(\theta^{-1}(p)))$, then it is clear that the above mixed strategy profile characterized by $\lambda \in (0,1)$ is a unique symmetric mixed-strategy equilibrium. □

We are ready to prove the first part of the proposition. Suppose that $p \in (P_c \cup P_s)^c$; that is, $\theta(\kappa(x^c(p))) < p < \theta(\kappa(\underline{x}^*))$. By Lemma C.1, it suffices to show that $p < \theta(\kappa(x^s(\theta^{-1}(p))))$, or equivalently, $\theta^{-1}(p) < \kappa(x^s(\theta^{-1}(p)))$. Because $\kappa(x^c(p)) < \theta^{-1}(p) < \kappa(\underline{x}^*)$ by the assumption, there exists $x \in (x^c(p), \underline{x}^*)$ such that $\kappa(x) = \theta^{-1}(p)$ by the intermediate value theorem. Because $\underline{x}^* > x$ is the minimum value satisfying $\underline{x}^* = x^s(\kappa(\underline{x}^*))$ and $x^s(\kappa(\cdot)) > 0$, we must have $x < x^s(\kappa(x))$ and thus $\theta^{-1}(p) = \kappa(x) < \kappa(x^s(\kappa(x))) = \kappa(x^s(\theta^{-1}(p)))$.

To prove the second part, suppose that equation (20) admits a unique solution $x^*$ and that there exists a symmetric mixed-strategy equilibrium. By Lemma C.1, $\kappa(x^c(p)) < \theta^{-1}(p) < \kappa(x^s(\theta^{-1}(p)))$, so $p \notin P_c$. To show $p \notin P_s$, note that there exists $x \in (x^c(p), x^s(\theta^{-1}(p))$ such that $\kappa(x) = \theta^{-1}(p)$ by the intermediate value theorem. Because

$$\kappa(x) = \theta^{-1}(p) < \kappa(x^s(\theta^{-1}(p))) = \kappa(x^s(\kappa(x))),$$

it follows that $x < x^s(\kappa(x))$. Because $x^*$ is the unique value satisfying $x^* = x^s(\kappa(x^*))$ and $x^s(\kappa(\cdot)) > 0$, we must have $x < x^*$ and thus $p = \theta(\kappa(x)) < \theta(\kappa(x^*))$; that is, $p \notin P_s$.

# D Proof of Proposition 5

The best response of self 2 is the same as that of a GP-type player in the second stage given by Lemma 4 and Theorem 1. When self 2 chooses the best response, the payoff of self 1 choosing $c_i$ is

$$U^S(c_i) = \begin{cases} u(\xi(c_i)) + u(\eta(c_i)) & \text{if } c_i \leq \bar{c}, \\ u(w - pc_i) + u(c_i) & \text{if } c_i \geq \bar{c} \end{cases}$$

by Lemma 4. Note that $\xi(c_i)$ and $\eta(c_i)$ are decreasing in $c_i$ because $(\xi(c_i), \eta(c_i))$ is the solution to

$$\max_{x_i + y_i = w - (p-1)c_i} u(x_i) + \kappa_j v(x_i) + u(y_i)$$

and the income effect is positive in the case of additively separable utility functions. Thus, $u(\xi(c_i)) + u(\eta(c_i))$ is decreasing in $c_i$ and

$$\max_{0 \leq c_i \leq \bar{c}} U^S(c_i) = u(\xi(0)) + u(\eta(0)) = u(x^s(\kappa_j)) + u(y^s(\kappa_j)).$$

On the other hand,

$$\max_{\bar{c} \leq c_i \leq w/p} U^S(c_i) = \begin{cases} u(\xi(\bar{c})) + u(\eta(\bar{c})) & \text{if } y^c(p) \leq \bar{c}, \\ u(x^c(p)) + u(y^c(p)) & \text{if } y^c(p) \geq \bar{c}. \end{cases}$$

Thus, the best response of self 1 given that of self 2 is $c_i = 0$ if $y^c(p) \leq \bar{c}$ and either $c_i = 0$ or $c_i = y^c(p)$ if $y^c(p) > \bar{c}$. Note that $y^c(p)$ is decreasing in $p$ because $(x^c(p), y^c(p))$ is the solution to

$$\max_{x_i + p y_i = w} u(x_i) + u(y_i)$$

and the income effect is positive in the case of additively separable utility functions. Thus, $y^c(p) \gtreqless \bar{c}$ if and only if

$$u(\xi(\bar{c})) + u(\eta(\bar{c})) = u(w - p\bar{c}) + u(\bar{c}) \gtreqless u(w - py^c(p)) + u(y^c(p)) = u(x^c(p)) + u(y^c(p)).$$

Define

$$v^{S1}(p) \equiv u(x^c(p)) + u(y^c(p)),$$
$$v^{S2}(\kappa_j) \equiv u(x^s(\kappa_j)) + u(y^s(\kappa_j)).$$

If $v^{S1}(p) \leq v^{S2}(\kappa_j)$, then $c_i = 0$ is the best response. If $v^{S1}(p) \geq v^{S2}(\kappa_j)$, then $c_i = y^c(p)$ is the best response because

$$u(\xi(\bar{c})) + u(\eta(\bar{c})) < u(\xi(0)) + u(\eta(0)) = u(x^s(\kappa_j)) + u(y^s(\kappa_j)) \leq u(x^c(p)) + u(y^c(p))$$

implies $y^c(p) > \bar{c}$.

**Lemma D.1.** *$v^{S1}(p)$ and $v^{S2}(\kappa_j)$ are strictly decreasing, and the range of $v^{S1}(p)$ includes that of $v^{S2}(\kappa_j)$. Moreover, $v^{S2}(\kappa_j) > v^2(\kappa_j)$, where $v^2(\kappa_j)$ is given by* (A.2).

*Proof.* Note that $v^{S1}(p)$ equals $v^1(p)$ given by (A.1) and thus $v^{S1}(p)$ is strictly decreasing by Lemma A.2.

To show that $v^{S2}(\kappa_j)$ is strictly decreasing, note that

$$dv_S^2(\kappa_j)/d\kappa_j = (u'(x^s(\kappa_j)) - u'(w - x^s(\kappa_j)))(dx^s/d\kappa_j)$$

because $y^s(\kappa_j) = w - x^s(\kappa_j)$. Since $x^s(\kappa_j)$ is strictly increasing, $dx^s/d\kappa_j > 0$. Moreover, since $u'' < 0$ and $x^s(\kappa_j) > x^s(0) = w/2$, we have $u'(x^s(\kappa_j)) < u'(w - x^s(\kappa_j))$. Therefore, $dv_s^2(\kappa_j)/d\kappa_j < 0$.

Because $v^{S1}(p)$ and $v^{S2}(\kappa_j)$ are strictly decreasing,

$$\max_{p\geq 1} v^{S1}(p) = v^{S1}(1) = 2u(w/2) = v^{S2}(0) = \max_{\kappa_j \geq 0} v^{S2}(\kappa_j).$$

In addition, since $y^c(p) \to 0$ as $p \to \infty$,

$$\begin{aligned}\lim_{\kappa_j\to\infty} v^{S2}(\kappa_j) &= \lim_{\kappa_j\to\infty} u(x^s(\kappa_j)) + u(w - x^s(\kappa_j)) = u(w) + u(0)\\ &\geq \lim_{p\to\infty} u(x^c(p)) + u(y^c(p)) = \lim_{p\to\infty} v^{S1}(p).\end{aligned}$$

Thus, the range of $v^{S1}$ includes that of $v^{S2}$.

Finally, by the definition of $(x^s(\kappa_j), y^s(\kappa_j))$,

$$v^2(\kappa_j) = u(x^s(\kappa_j)) + u(y^s(\kappa_j)) - \kappa_j(v(w) - v(x^s(\kappa_j))) < u(x^s(\kappa_j)) + u(y^s(\kappa_j)) = v^{S2}(\kappa_j),$$

which proves the last part of the lemma. □

Let $\theta^S(\kappa_j) \equiv (v^{S1})^{-1}(v^{S2}(\kappa_j))$. By Lemma D.1, $\theta^S(\kappa_j)$ is well-defined, strictly increasing, and $\theta^S(0) = (v^{S1})^{-1}(v^{S2}(0)) = 1$ because $v^{S1}(1) = 2u(w/2) = v^{S2}(0)$. Thus, the best response of self 1 is $c_i = y^c(p)$ if $p \leq \theta^S(\kappa_j)$ and $c_i = 0$ if $p \geq \theta^S(\kappa_j)$ because

$$v^{S1}(p) \gtreqless v^{S2}(\kappa_j) \;\Leftrightarrow\; p \lesseqgtr \theta^S(\kappa_j).$$

Finally, $v^{S2}(\kappa_j) > v^2(\kappa_j)$ implies that

$$(v^1)^{-1}(v^{S2}(\kappa_j)) < (v^1)^{-1}(v^2(\kappa_j)) \Rightarrow (v^{S1})^{-1}(v^{S2}(\kappa_j)) < (v^1)^{-1}(v^2(\kappa_j)) \Rightarrow \theta^S(\kappa_j) < \theta(\kappa_j).$$

Unboundedness of $\theta^S$ can be shown as in the proof of Theorem 1.

# E Proof of Proposition 7

Note that $(\underline{x}, \underline{y}) = (x^c(\underline{p}), y^c(\underline{p}))$ maximizes $u(x_i) + u(y_i)$ subject to $x_i + \underline{p}y_i = w$ and that $(\overline{x}, \overline{y}) = (x^s(\kappa(\overline{x})), y^s(\kappa(\overline{x})))$ maximizes $u(x_i) + \kappa(\overline{x})v(x_i) + u(y_i)$ subject to $x_i + y_i = w$. Because self 2 has the compromise payoff function $u(x_i) + \kappa(x_j)v(x_i) + u(y_i)$, he strictly prefers $(\overline{x}, \overline{y})$ to $(\underline{x}, \underline{y})$.

Consider self 1 with the normative payoff function $u(x_i) + u(y_i)$ and let $\succsim$ denote his preference relation over the consumption bundles. Let $x^* > 0$ be the maximum value of $x$ such that $(\underline{x}, \underline{y}) \sim (x^*, w - x^*)$. Then, $(\underline{x}, \underline{y}) \succ (x, w - x)$ for all $x > x^*$ by the preference's convexity. Thus, it is enough to show that $x^s(\kappa(\overline{x})) > x^*$ because it implies that

$$(\underline{x}, \underline{y}) \succ (x^s(\kappa(\overline{x})), w - x^s(\kappa(\overline{x}))) = (x^s(\kappa(\overline{x})), y^s(\kappa(\overline{x}))) = (\overline{x}, \overline{y}).$$

By Assumption 3, $\underline{x} = x^c(\underline{p}) \leq x^c(1) = x^s(0) < \min\{x^s(\kappa(\underline{x})), x^s(\kappa(\overline{x}))\} \leq \overline{x}$, which implies that $\underline{x} < x^s(\kappa(\underline{x})) < x^s(\kappa(\overline{x}))$. Thus, it is enough to show that $x^s(\kappa(\underline{x})) > x^*$.

Seeking a contradiction, suppose that $x^s(\kappa(\underline{x})) \leq x^*$. Because $(\underline{x}, w - \underline{x}) \succ (\underline{x}, \underline{y}) \sim (x^*, w - x^*)$ by the preference's monotonicity,

$$(x^s(\kappa(\underline{x})), y^s(\kappa(\underline{x}))) = (x^s(\kappa(\underline{x})), w - x^s(\kappa(\underline{x}))) \succ (x^*, w - x^*) \sim (\underline{x}, \underline{y})$$

by the preference's convexity. On the other hand, because self 1's best response is to purchase the commitment device when $p = \underline{p}$, we must have $(\underline{x}, \underline{y}) \succsim (x^s(\kappa(\underline{x})), y^s(\kappa(\underline{x})))$, a contradiction.

# F Existence of pure-strategy equilibria

As shown in Section 3, a pure-strategy equilibrium may not exist. The following theorem provides a sufficient condition for the existence of pure-strategy equilibria in which each menu consists of pure actions. We denote the set of all nonempty compact subsets of $A_i$ by $\mathcal{K}(A_i)$ and regard it as a subset of $\mathcal{K}(\Delta(A_i))$ with some abuse of notation.

**Theorem F.1.** For each $i \in \mathcal{I}$, assume the following conditions:

- $A_i$ is a compact convex metrizable subset of a locally convex Hausdorff topological vector space.
- $\mathcal{M}_i$ is a compact convex subset of $\mathcal{K}(A_i)$. The convexity of $\mathcal{M}_i$ is defined in terms of the mixture operation on $A_i$ and the corresponding Minkowski sum.
- For all $a_{-i} \in \prod_{j \neq i} A_i$, $u_i(a_i, a_{-i}) + v_i(a_i, a_{-i})$ is concave in $a_i$ and $v_i(a_i, a_{-i})$ is convex in $a_i$.

Then, there exists an equilibrium of $\Gamma$.

The third condition[15] implies that $u_i(a)+v_i(a)-\max_{M_i} v_i(\cdot, a_{-i})$ is concave in $(M_i, a_i)$, by which we can apply the fixed point theorem. It requires that the concavity of $u_i(\cdot, a_{-i})$ should dominate the convexity of $v_i(\cdot, a_{-i})$, which means that a normative preference exhibits risk aversion, whereas a temptation preference exhibits risk loving. Though this assumption seems plausible, psychological evidence surveyed in Loewenstein et al. (2001) suggests that the emotional response to risk is that of aversion and dread, which corresponds to the specification that a temptation preference exhibits greater risk aversion than does a normative preference.[16] Under this specification, however, the third condition of Theorem F.1 is violated and an equilibrium may fail to exist, as shown by the example of Section 3.

To prove Theorem F.1, we first show a preliminary lemma.

**Lemma F.1.** *A strategy set $S_i$ is a compact subset of $\mathcal{M}_i \times \Delta(A_i)$.*

*Proof.* Since $\mathcal{M}_i \times \Delta(A_i)$ is a compact metric space by assumption, it suffices to show that $S_i$ is closed. Let $d$ denote a metric on $\Delta(A_i)$ and $d_H$ denote the Hausdorff metric on $\mathcal{K}(\Delta(A_i))$. Take any sequence $\{(M_i^n, m_i^n)\}_{n=1}^{\infty} \in S_i$ converging to some point $(M_i^*, m_i^*) \in \mathcal{M}_i \times \Delta(A_i)$. By definition, $m_i^n \in M_i^n$ for all $n$. Seeking a contradiction, suppose that $m_i^* \notin M_i^*$. Since $M_i^*$ is a compact subset of $\Delta(A_i)$, there exists a closed neighborhood $V$ of $m_i^*$ such that $V \cap M_i^* = \emptyset$. Since $m_i^n \to m_i^*$, $m_i^n \in V$ for all sufficiently large $n$. Thus, for such $n$,

$$d_H(M_i^n, M_i^*) \geq \max_{m_i \in M_i^n} \min_{m' \in M_i^*} d(m_i, m_i') \geq \min_{m_i' \in M_i^*} d(m_i^n, m_i') \geq \min_{m_i \in V} \min_{m_i' \in M_i^*} d(m_i, m_i') > 0.$$

This contradicts to $\lim_{n\to\infty} d_H(M_i^n, M_i^*) = 0$. Therefore, $m_i^* \in M_i^*$ must hold. □

A key observation to show the existence is that our equilibrium concept coincides with a Nash equilibrium of the following normal form game $\Gamma^* = (\mathcal{I}, (S)_{i\in\mathcal{I}}, (U_i^*)_{i\in\mathcal{I}})$, where $\mathcal{I}$ is the set of players, $S_i$ is the set of player $i$'s strategies, and $U_i^* : \prod_{j\in\mathcal{I}} S_j \to \mathbb{R}$ is player $i$'s payoff function such that, for all $((M_j, m_j))_{j\in N} \in \prod_{j\in\mathcal{I}} S_j$,

$$U_i^*((M_j, m_j)_{j\in\mathcal{I}}) = u_i(m) + v_i(m) - \max_{x_i \in M_i} v_i(x_i, m_{-i}). \tag{F.1}$$

**Lemma F.2.** *An equilibrium of $\Gamma$ coincides with a Nash equilibrium of $\Gamma^*$.*

[15]This condition is used by Gul and Pesendorfer (2004), who study an infinite-horizon decision making of a representative consumer. It ensures concavity of the value function.

[16]Along with generalizing GP's model to admit menu-dependent self-control, Noor and Takeoka (2015) demonstrate that this specification is useful to explain disparate evidence from experiments on choice under risk and over time.

*Proof.* It is evident that

$$U_i(M_i, m_{-i}) = \max_{m_i \in M_i} U_i^*((M_j, m_j)_{j \in \mathcal{I}}),$$
$$C_i(M_i, m_{-i}) = \arg\max_{m_i \in M_i} U_i^*((M_j, m_j)_{j \in \mathcal{I}}).$$

Therefore, (4) and (5) hold if and only if

$$M_i \in \arg\max_{M_i' \in \mathcal{M}_i} \max_{m_i' \in M_i'} U_i^*(((M_i', m_i'), (M_j, m_j)_{j \neq i})),$$
$$m_i \in \arg\max_{m_i' \in M_i} U_i^*(((M_i, m_i'), (M_j, m_j)_{j \neq i})).$$

This is reduced to

$$(M_i, m_i) \in \arg\max_{(M_i', m_i') \in S_i} U_i^*(((M_i', m_i'), (M_j, m_j)_{j \neq i})),$$

which completes the proof. □

We are ready to prove Theorem F.1.

*Proof of Theorem F.1.* By Lemma F.2, it is enough to show the existence of Nash equilibria of $\Gamma^*$. We write $S = \prod_i S_i$, $S_{-i} = \prod_{j \neq i} S_j$, $(M, a) = (M_j, a_j)_{j \in \mathcal{I}}$, and $(M_{-i}, a_{-i}) = (M_j, a_j)_{j \neq a}$ for all $(M_i, a_i) \in S_i$ and $i \in \mathcal{I}$. Note that $S_i$ is compact by Lemma F.1 and convex since $A_i$ and $\mathcal{M}_i$ are convex.

We first show that $U_i^*(M, a) = u_i(a_i, a_{-i}) + v_i(a_i, a_{-i}) - \max_{x_i \in M_i} v_i(x_i, a_{-i})$ is concave in $(M_i, a_i)$. Since $u_i(a_i, a_{-i}) + v_i(a_i, a_{-i})$ is concave in $a_i$, it is enough to show that $\max_{M_i} v_i(\cdot, a_{-i})$ is convex in $M_i$. For $\lambda \in [0, 1]$, let $\lambda a_i + (1 - \lambda) a_i' \in \lambda M_i + (1 - \lambda) M_i'$ be a maximizer of $\max_{\lambda M_i + (1-\lambda) M_i'} v_i(\cdot, a_{-i})$ with $a_i \in M_i$ and $a_i' \in M_i$. Then, since $v_i(\cdot, a_{-i})$ is a convex function,

$$\begin{aligned}
\max_{\lambda M_i + (1-\lambda) M_i'} v_i(\cdot, a_{-i}) &= v_i(\lambda a_i + (1 - \lambda) a_i', a_{-i}) \\
&\leq \lambda v_i(a_i, a_{-i}) + (1 - \lambda) v_i(a_i', a_{-i}) \\
&\leq \lambda \max_{M_i} v_i(\cdot, a_{-i}) + (1 - \lambda) \max_{M_i'} v_i(\cdot, a_{-i}).
\end{aligned}$$

Hence, $\max_{M_i} v_i(\cdot, a_{-i})$ is convex in $M_i$ and $U_i^*(M, a)$ is concave in $(M_i, a_i)$. In addition, $U_i^*(M, a)$ is continuous in $(M, a)$ since $u_i$ and $v_i$ restricted to $A$ are continuous functions.

For each $i \in \mathcal{I}$, let $B_i : S_{-i} \twoheadrightarrow S_i$ be player $i$'s best response correspondence:

$$B_i(M_{-i}, a_{-i}) = \arg\max_{(M_i, a_i) \in S_i} U_i^*(M, a) \text{ for all } (M_{-i}, a_{-i}) \in S_{-i}.$$

Since $U_i^*(M, a)$ is concave in $(a_i, M_i)$ and continuous in $(M, a)$, $B_i$ is upper hemicontinuous and $B_i(M_{-i}, a_{-i})$ is nonempty, compact, and convex for all $(M_{-i}, a_{-i})$ by the maximum theorem. Now define $\bar{B}_i(a_{-i})$ such that

$$\bar{B}_i(a_{-i}) = \{a_i \in A_i \mid (M_i, a_i) \in B_i(M_{-i}, a_{-i})\}.$$

Then, $\bar{B}_i$ is also upper hemicontinuous and $\bar{B}_i(a_{-i})$ is nonempty, compact, and convex for all $a_{-i}$ since $B_i(M_{-i}, a_{-i})$ does not depend on $M_{-i}$. This implies that $\bar{B} : A \twoheadrightarrow A$ such that $\bar{B}(a) = \prod_{i \in \mathcal{I}} \bar{B}_i(a_{-i})$ is upper hemicontinuous and $\bar{B}(a)$ is nonempty, compact, and convex for all $a$. Since $A$ is a compact convex subset of a locally convex Hausdorff topological vector space by the assumption, we can apply the Kakutani-Glicksberg-Fan fixed point theorem to $\bar{B}$, showing that there exists $a = (a_i)_{i \in \mathcal{I}} \in A$ with $a_i \in \bar{B}_i(a_{-i})$ for all $i \in \mathcal{I}$. Therefore, there exists $(M_i, a_i)$ with $(M_i, a_i) \in B_i(M_{-i}, a_{-i})$, which is a Nash equilibrium of $\Gamma^*$ (we do not apply the fixed point theorem to $B_i$ because we have to prove that $\mathcal{M}$ is a locally convex Hausdorff topological vector space, which is not necessary). □

# G Existence of mixed-strategy equilibria

We define a mixed-strategy equilibrium of $\Gamma$. For each $i \in \mathcal{I}$, let $\Sigma_i$ be the set of all Borel probability measures on $S_i$. We call $\sigma_i \in \Sigma_i$ a mixed strategy of player $i$. Let $\sigma_{i|\Delta(A_i)}$ and $\sigma_{i|\mathcal{M}_i}$ denote the marginal distributions of $\sigma_i$ on $\Delta(A_i)$ and $\mathcal{M}_i$ respectively.

To introduce some notation, consider player $i$ who believes that player $j \neq i$ chooses a mixed strategy $\sigma_j$. Since player $i$'s payoff depends on player $j$'s strategy only through his action eventually chosen in the second stage, player $i$ is only concerned about a probability distribution over the opponent's action set $A_j$ induced by $\sigma_j$. Let $\sigma_{j|A_j}$ denote a probability distribution over $A_j$ induced by $\sigma_j$:

$$\sigma_{j|A_j}(E_j) = \int_{\Delta(A_j)} m_j(E_j)\, \mathrm{d}\sigma_{j|\Delta(A_j)}(m_j),$$

where $E_j$ is a measurable subset of $A_j$. Note that $\sigma_{j|A_j}$ is a lottery reduced from a compound lottery $\sigma_{j|\Delta(A_j)}$.

To define a mixed-strategy equilibrium, consider player $i$ who believes that the opponents' mixed strategy profile is $\sigma_{-i} \equiv (\sigma_j)_{j \neq i} \in \prod_{j \neq i} \Sigma_j$. Since player $i$ knows that the opponents' action profile $a_{-i}$ is randomly chosen according to $\sigma_{-i|A_{-i}} \equiv \prod_{j \neq i} \sigma_{j|A_j} \in \prod_{j \neq i} \Delta(A_j)$, player $i$'s payoff of choosing $M_i \in \mathcal{M}_i$ and his choice correspondence are given by (2) and (3) with $m_{-i} = \sigma_{-i|A_{-i}}$ respectively. Therefore, player $i$'s best response to $\sigma_{-i}$ is $(M_i, m_i) \in S_i$ satisfying (4) and (5) with $m_{-i} = \sigma_{-i|A_{-i}}$. We define a mixed-strategy

equilibrium of $\Gamma$ as a mixed strategy profile such that each player chooses his best response with probability one.

**Definition G.1.** A mixed strategy profile $(\sigma_i)_{i\in\mathcal{I}}$ is an equilibrium of $\Gamma$ if

$$\sigma_i\Big(\Big\{(M_i, m_i) \in S_i \,\Big|\, M_i \in \arg\max_{M_i'\in\mathcal{M}_i} U_i(M_i', \sigma_{-i|A_{-i}}),\ m_i \in C_i(M_i, \sigma_{-i|A_{-i}})\Big\}\Big) = 1$$

for all $i \in \mathcal{I}$.

If $(\sigma_i)_{i\in\mathcal{I}}$ is a mixed-strategy equilibrium and $\sigma_i(\{(M_i, m_i)\}) = 1$ for some $(M_i, m_i) \in S_i$, then, clearly, $((M_i, m_i))_{i\in\mathcal{I}}$ is a pure-strategy equilibrium according to Definition 1.

As in standard normal-form games, if mixed strategies are allowed, the existence of equilibria can be established.

**Theorem G.1.** There exists a mixed-strategy equilibrium of $\Gamma$.

To prove Theorem G.1, we first show that a mixed-strategy equilibrium of $\Gamma$ is a Nash equilibrium of a normal form game $\bar{\Gamma}^* = (\mathcal{I}, (\Sigma_i)_{i\in\mathcal{I}}, (\bar{U}_i^*))_{i\in\mathcal{I}}$ with

$$\bar{U}_i^*(\sigma) = \int_{S_i} \Big(u_i(m_i, \sigma_{-i|A_{-i}}) + v_i(m_i, \sigma_{-i|A_{-i}}) - \max_{m_i'\in M_i} v_i(m_i', \sigma_{-i|A_{-i}})\Big)\mathrm{d}\sigma_i((M_i, m_i))$$

for all $\sigma = (\sigma_j)_{j\in N} \in \prod_{j\in\mathcal{I}} \Sigma_j$.

**Lemma G.1.** *A mixed-strategy equilibrium of* $\Gamma$ *coincides with a Nash equilibrium of* $\bar{\Gamma}^*$.

*Proof.* Let $\sigma^* = (\sigma_i^*)_{i\in\mathcal{I}}$ be a Nash equilibrium of $\bar{\Gamma}^*$; that is, for each $i \in \mathcal{I}$, $\sigma_i^*$ places probability one on the set of strategies

$$\arg\max_{(M_i,m_i)\in S_i} \left(u_i(m_i, \sigma_{-i|A_{-i}}) + v_i(m_i, \sigma_{-i|A_{-i}}) - \max_{m_i'\in M_i} v_i(m_i', \sigma_{-i|A_{-i}})\right).$$

By the same argument as that in the proof of Lemma F.2, the above set coincides with

$$\{(M_i, m_i) \in S_i \mid M_i \in \arg\max_{M_i'\in\mathcal{M}_i} U_i(M_i', \sigma_{-i|A_{-i}}),\ m_i \in C_i(M_i, \sigma_{-i|A_{-i}})\}.$$

Therefore, both equilibria coincide. □

We are ready to prove Theorem G.1.

*Proof of Theorem G.1.* By Lemma G.1, it is enough to show the existence of a mixed-strategy Nash equilibrium of $\bar{\Gamma}^*$. Note that, since $S_i$ is a compact Hausdorff space, $\Sigma_i = \Delta(S_i)$ is a compact convex subset of a locally convex Hausdorff topological vector space as argued by Glicksberg (1952).

Let $B_i : \Sigma_{-i} \twoheadrightarrow \Sigma_i$ be player $i$'s best response correspondence in $\bar{\Gamma}^*$:

$$B_i(\sigma_{-i}) = \arg\max_{\sigma_i \in \Sigma_i} \bar{U}_i^*(\sigma_i, \sigma_{-i}) \text{ for all } \sigma_{-i} \in \Sigma_{-i}.$$

Since $\bar{U}_i^*(\sigma)$ is continuous in $\sigma$ and linear in $\sigma_i$, $B_i$ is upper hemicontinuous and $B_i(\sigma_{-i})$ is nonempty, compact, and convex for all $\sigma_{-i} \in \Sigma_{-i}$. This implies that $B : \Sigma \twoheadrightarrow \Sigma$ such that $B(\sigma) = \prod_{i \in \mathcal{I}} B_i(\sigma_{-i})$ is upper hemicontinuous and $B(\sigma)$ is nonempty, compact, and convex for all $\sigma$. Since $\Sigma$ is a compact convex subset of a locally convex Hausdorff topological vector space, $B$ has a fixed point by the Kakutani-Glicksberg-Fan fixed point theorem, which is a mixed-strategy Nash equilibrium of $\bar{\Gamma}^*$. □

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