In Section 3 of “Self-Control Games” by Norio Takeoka and Takashi Ui, peer effects affect only the strength of temptation through $\kappa_i(m_{-i}) \geq 0$. The purpose of this online appendix is to provide an axiomatic foundation for a special class of self-control preferences in which a normative payoff function is independent of the opponents’ actions and a temptation payoff function is multiplicatively separable from the opponents’ actions. That is, player $i$’s normative and temptation payoff functions are of the form

$$u_i(m) = f_i(m_i), \quad v_i(m) = \kappa_i(m_{-i}) g_i(m_i)$$

for all $m \in \Delta(A)$, \hspace{1cm} (1)

where $f_i : \Delta(A_i) \to \mathbb{R}$, $g_i : \Delta(A_i) \to \mathbb{R}$, and $\kappa_i : \prod_{j \neq i} \Delta(A_j) \to \mathbb{R}^+$ are continuous linear functions. Then, his payoff of choosing $M_i \in M_i$ is

$$U_i(M_i, m_{-i}) = \max_{m_i \in M_i} \left( f_i(m_i) - \kappa_i(m_{-i}) \left( \max_{m_i' \in M_i} g_i(m_i') - g_i(m_i) \right) \right)$$

and his choice correspondence is

$$C_i(M_i, m_{-i}) = \arg \max_{m_i \in M_i} (f_i(m_i) + \kappa_i(m_{-i}) g_i(m_i)).$$

Let $\succeq_i$ be player $i$’s preference relation on $\mathcal{K}(\Delta(A))$. In addition to GP’s axioms, we use the following axioms.

**Axiom 1 (Indifferent Commitment).** For all $m_{-i}, m'_{-i} \in \prod_{j \neq i} \Delta(A_j)$, there exist $m_i, m'_i \in \Delta(A_i)$ such that

$$\{ (m_i, m_{-i}) \} \sim_i \{ (m_i, m'_{-i}) \} \not\succeq_i \{ (m'_i, m_{-i}) \} \sim_i \{ (m'_i, m'_{-i}) \}.$$
This axiom requires that, for any pair of the opponents’ action profiles, there exist at least two actions to which full commitment yields the same payoffs.

For each \( m_{-i} \in \prod_{j \neq i} \Delta(A_j) \), let \( \succeq_{i,m_{-i}} \) denote a conditional preference relation on \( \mathcal{K}(\Delta(A_i)) \) defined by
\[
M_i \succeq_{i,m_{-i}} M'_i \iff M_i \times \{m_{-i}\} \succeq_{i} M'_i \times \{m_{-i}\} \iff U_i(M_i,m_{-i}) \geq U_i(M'_i,m_{-i}),
\]
where \( M_i \times \{m_{-i}\} = \{ x \in \prod_{j \neq i} \Delta(A_j) \mid x_i \in M_i, x_j = m_j \text{ for } j \neq i \} \in \mathcal{K}(\Delta(A)) \). The opponents’ action \( m_{-i} \) is said to be nondegenerate (with respect to the induced conditional preference relation) if there exist \( m_i, m'_i \in \Delta(A_i) \) with \( \{m_i\} \succeq_{i,m_{-i}} \{m_i, m'_i\} \); that is, \( m'_i \) is normatively less preferred and more tempting than \( m_i \). Note that the nondegeneracy of \( m_{-i} \) requires the existence of a tempting action.

**Axiom 2** (Identical Tempting Actions). For all nondegenerate \( m_{-i}, m'_{-i} \in \prod_{j \neq i} \Delta(A_j) \) and for all \( m_i, m'_i \in \Delta(A_i) \), if \( \{m_i\} \succeq_{i,m_{-i}} \{m_i, m'_i\} \) then \( \{m_i\} \succeq_{i,m'_{-i}} \{m_i, m'_i\} \).

This axiom requires that if an action is tempting given \( m_{-i} \), then it is also given any \( m'_{-i} \).

The next axiom requires that any pair of the opponents’ actions can be ordered by a player’s attitude toward commitment according to Definition 1.

**Definition 1.** For all \( m_{-i}, m'_{-i} \in \prod_{j \neq i} \Delta(A_j) \), player \( i \) is more willing to make a commitment at \( m'_{-i} \) than at \( m_{-i} \) if \( \{m_i\} \succeq_{i,m_{-i}} M_i \) implies \( \{m_i\} \succeq_{i,m'_{-i}} M_i \) for all \( M_i \in \mathcal{K}(\Delta(A_i)) \) and \( m_i \in \Delta(A_i) \). We write \( m'_{-i} \succeq^C_i m_{-i} \) if this condition holds.

**Axiom 3** (Comparative Commitment Attitude). The binary relation \( \succeq^C_i \) is complete; that is, for all \( m_{-i}, m'_{-i} \in \prod_{j \neq i} \Delta(A_j) \), either \( m'_{-i} \succeq^C_i m_{-i} \) or \( m_{-i} \succeq^C_i m'_{-i} \) holds.

When \( m'_{-i} \succeq^C_i m_{-i} \), if player \( i \) prefers to make a commitment to \( m_i \) rather than to have a menu \( M_i \) given \( m_{-i} \), he also does given \( m'_{-i} \). Presumably, this is because player \( i \) anticipates a stronger peer effect toward temptation under \( m'_{-i} \). The above axiom requires that any pair \( m_{-i}, m'_{-i} \in \prod_{j \neq i} \Delta(A_j) \) can be ordered by their strength of peer effects.

The following is our characterization result.

**Theorem 1.** Let \( \succeq_i \) be player \( i \)’s self-control preference relation on \( \mathcal{K}(\Delta(A)) \) represented by self-control utility
\[
U_i(M_i,m_{-i}) = \max_{m_i \in M_i} \left( u_i(m_i,m_{-i}) - \max_{m_i' \in M_i} (v_i(m_i',m_{-i}) - v_i(m_i,m_{-i})) \right). \tag{3}
\]
The following statements are equivalent.

(i) \( \succeq_i \) satisfies Indifferent Commitment, Identical Tempting Actions, and Comparative Commitment Attitude.
(ii) For all $m_{-i} \in \prod_{j \neq i} \Delta(A_j)$, the conditional preference relation $\succeq_{i, m_{-i}}$ is represented by $U_i(M_i, m_{-i})$ of the form (2) such that $\kappa_i(m'_{-i}) \succeq \kappa_i(m_{-i})$ if and only if $m'_{-i} \succeq^C m_{-i}$, where a strict inequality holds if $\succeq_{i, m_{-i}} \not\succeq_{i, m'_{-i}}$.

The above theorem guarantees that the strength of temptation $\kappa_i(m_{-i})$ can be captured by the order $\succeq^C_i$.

In a different context, Gul and Pesendorfer (2007) axiomatize a self-control representation similar to (2). They consider an infinite horizon extension of the GP model with habit formation in consumption and study a model of addiction. As a special case, they axiomatize a recursive self-control representation in which past consumption affects only the strength of temptation. The role of past consumption in their representation corresponds to that of peer effects in our representation (2). Gul and Pesendorfer (2007) consider the difference between a normative choice derived from normative utility and an actual choice in the second stage derived from a compromise between normative and temptation utilities, and show that the larger difference implies stronger temptation. On the other hand, we define Comparative Commitment Attitude to measure the strength of temptation.

In the remainder of this online appendix, we give a proof of Theorem 1. It is straightforward to check that (ii) implies (i). We show that (i) implies (ii). We first show that a normative payoff function is independent of the opponents’ actions.

**Lemma 1.** If $\succeq_i$ satisfies Indifferent Commitment and Comparative Commitment Attitude, then there exists $f_i : \Delta(A_i) \rightarrow \mathbb{R}$ such that $u_i(m_i, m_{-i}) = f_i(m_i)$ for all $m_i \in \Delta(A_i)$ and $m_{-i} \in \prod_{j \neq i} \Delta(A_j)$.

**Proof.** Take arbitrary $m_{-i}, m'_{-i} \in \prod_{j \neq i} \Delta(A_j)$. By Comparative Commitment Attitude, we can assume $m_{-i} \succeq^C_i m'_{-i}$ without loss of generality. If $\{x_i\} \succeq_{i, m_{-i}} \{y_i\}$ then $\{x_i\} \succeq_{i, m'_{-i}} \{y_i\}$, which implies that if $\{x_i\} \sim_{i, m_{-i}} \{y_i\}$ then $\{x_i\} \sim_{i, m'_{-i}} \{y_i\}$.

We show that if $\{x_i\} \succ_{i, m_{-i}} \{y_i\}$ then $\{x_i\} \succ_{i, m'_{-i}} \{y_i\}$. Seeking a contradiction, suppose that $\{x_i\} \sim_{i, m'_{-i}} \{y_i\}$. Let $\bar{x}_i, \bar{y}_i \in \Delta(A_i)$ be such that $\{\bar{x}_i\} \succ_{i, m'_{-i}} \{\bar{y}_i\}$, which exist by Indifferent Commitment. Then, $\{\lambda \bar{x}_i + (1 - \lambda)\bar{y}_i\} \succ_{i, m'_{-i}} \{\lambda x_i + (1 - \lambda)\bar{x}_i\}$ for all $\lambda \in (0, 1)$ by linearity of $u_i$, but $\{\lambda x_i + (1 - \lambda)\bar{x}_i\} \succ_{i, m_{-i}} \{\lambda y_i + (1 - \lambda)\bar{y}_i\}$ for sufficiently large $\lambda \in (0, 1)$ by continuity of $u_i$, which implies $\{\lambda x_i + (1 - \lambda)\bar{x}_i\} \succeq_{i, m'_{-i}} \{\lambda y_i + (1 - \lambda)\bar{y}_i\}$ because $m_{-i} \succeq^C_i m'_{-i}$, a contradiction.

We have shown that $\{x_i\} \succeq_{i, m_{-i}} \{y_i\}$ if and only if $\{x_i\} \succeq_{i, m'_{-i}} \{y_i\}$; that is, $\succeq_{i, m_{-i}}$ and $\succeq_{i, m'_{-i}}$ restricted to singletons are identical. Since the two identical preference relations on $\Delta(A_i)$ are represented by $u_i(\cdot, m_{-i})$ and $u_i(\cdot, m'_{-i})$ respectively, we must have $u_i(\cdot, m_{-i}) = au_i(\cdot, m'_{-i}) + \beta$ with $a > 0$ and $\beta \in \mathbb{R}$. By Indifferent Commitment, there exist $m_i, m'_i \in \Delta(A_i)$ such that $\{(m_i, m_{-i})\} \sim_i \{(m'_i, m'_{-i})\}$, or equivalently,
By Lemma 1, there exists $v_i$ which is taken from Lemma 0 of Gul and Pesendorfer (2001).

Lemma 2. For each $M_j \in \mathcal{K}(\Delta(A_i))$, there exists a sequence of finite subsets \( \{ M_j^k \subseteq M_j \}_{k}^{\infty} \) such that $M_j^k \rightarrow M_j$ as $k \rightarrow \infty$ in the Hausdorff metric.

Lemma 3. Suppose that $m_{-i} \in \prod_{j \neq i} \Delta(A_j)$ is nondegenerate; that is, there exist $x_i, y_i \in \Delta(A_i)$ satisfying \( \{ x_i \} \succ_{i,m_{-i}} \{ x_i, y_i \} \). Then, for $x_i, y_i \in \Delta(A_i)$ with \( \{ x_i \} \succ_{i,m_{-i}} \{ y_i \} \), $v_i(y_i, m_{-i}) > v_i(x_i, m_{-i})$ if and only if $\{ x_i \} \succ_{i,m_{-i}} \{ x_i, y_i \}$.

Proof. Suppose that \( \{ x_i \} \succ_{i,m_{-i}} \{ y_i \} \), i.e., $u_i(x_i, m_{-i}) > u_i(y_i, m_{-i})$. Then, $\{ x_i \} \succ_{i,m_{-i}} \{ x_i, y_i \}$ if and only if

\[
U_i(\{ x_i \}, m_{-i}) = u_i(x_i, m_{-i}) \\
> U_i(\{ x_i, y_i \}, m_{-i}) \\
= \max_{\{ x_i, y_i \}} (u_i(\cdot, m_{-i}) + v_i(\cdot, m_{-i})) - \max_{\{ x_i, y_i \}} v_i(\cdot, m_{-i}).
\]

If $v_i(x_i, m_{-i}) \geq v_i(y_i, m_{-i})$, then $U_i(\{ x_i, y_i \}, m_{-i}) = u_i(x_i, m_{-i})$, which is a contradiction. Thus, we must have $v_i(y_i, m_{-i}) > v_i(x_i, m_{-i})$. Conversely, if $v_i(y_i, m_{-i}) > v_i(x_i, m_{-i})$, then

\[
U_i(\{ x_i, y_i \}, m_{-i}) = \max\{ u_i(x_i, m_{-i}) + v_i(x_i, m_{-i}) - v_i(y_i, m_{-i}), u_i(y_i, m_{-i})\} \\
< u_i(x_i, m_{-i}) = U_i(\{ x_i \}, m_{-i}),
\]

which implies that $\{ x_i \} \succ_{i,m_{-i}} \{ x_i, y_i \}$.

We are ready to establish the separability of a temptation payoff function.

Lemma 4. If $\succeq_{i}$ satisfies Indifferent Commitment, Identical Tempting Actions, and Comparative Commitment Attitude, then there exist $\kappa_i : \prod_{j \neq i} \Delta(A_j) \rightarrow \mathbb{R}_{+}$ and $g_i : \Delta(A_i) \rightarrow \mathbb{R}$ such that $v_i(m_i, m_{-i}) = \kappa_i(m_{-i})g_i(m_i)$ for all $m_i \in \Delta(A_i)$ and $m_{-i} \in \prod_{j \neq i} \Delta(A_j)$.

Proof. By Lemma 1, there exists $f_i : \Delta(A_i) \rightarrow \mathbb{R}$ such that $u_i(\cdot, m_{-i}) = f_i(\cdot)$ for all $m_{-i}$. Note that $f_i(m_i) \geq f_i(m_i')$ if and only if $\{ m_i \} \succeq_{i,m_{-i}} \{ m_i' \}$ for all $m_{-i}$.

Consider degenerate $m_{-i} \in \prod_{j \neq i} \Delta(A_j)$ such that $\{ x_i \} \sim_{i,m_{-i}} \{ x_i, y_i \}$ for all $x_i, y_i \in \Delta(A_i)$ with $f_i(x_i) \geq f_i(y_i)$. For such $x_i, y_i \in \Delta(A_i)$,

\[
U_i(\{ x_i \}, m_{-i}) = U_i(\{ x_i \}, m_{-i}) = f_i(x_i).
\]
Then, we can show that

$$U_i(M_i, m_{-i}) = \max_{M_i} f_i(\cdot)$$

(4)

for all $M_i \in \mathcal{K}(\Delta(A_i))$. This is true if $|M_i| = 2$. Suppose that (4) holds if $|M| = k$ with $2 \leq k \leq n$. For $M_i$ with $|M_i| = n + 1$, let $x^*_i \in \text{arg max}_{M_i} f_i(\cdot)$. Then, for any $x_i \in M_i \setminus \{x^*_i\}$, Set Betweenness implies \(\{x^*_i, x_i\} \succ_{i,m_{-i}} M_i \succ_{i,m_{-i}} M_i \setminus \{x_i\}\) or \(\{x^*_i, x_i\} \succ_{i,m_{-i}} M_i \succ_{i,m_{-i}} \{x^*_i, x_i\}\), but \(\{x^*_i\} \sim_{i,m_{-i}} \{x^*_i, x_i\} \sim_{i,m_{-i}} M_i \setminus \{x_i\}\) by the induction hypothesis. Hence, we must have $x^*_i \sim_{i,m_{-i}} M_i$. By induction, (4) holds for any finite $M_i$, and by Lemma 2 and continuity of $U_i$, (4) holds for all $M_i \in \mathcal{K}(\Delta(A_i))$. Therefore, if $\kappa_i(m_{-i}) = 0$ then we have the representation (2) for any $g_i$.

Let $m_{-i}, m'_{-i} \in \prod_{j \neq i} \Delta(A_j)$ be nondegenerate. By Identical Tempting Actions, \(\{x_i\} \succ_{i,m_{-i}} \{x_i, y_i\}\) and \(\{x_i\} \succ_{i,m'_{-i}} \{x_i, y_i\}\) are equivalent, and such $x_i$ and $y_i$ exist by nondegeneracy. Thus, for $x_i, y_i \in \Delta(A_i)$, with $f_i(x_i) > f_i(y_i)$, we have

$$v_i(y_i, m_{-i}) > v_i(x_i, m_{-i}) \iff v_i(y_i, m'_{-i}) > v_i(x_i, m'_{-i}).$$

(5)

Furthermore, as we show below, (5) holds for all $x_i, y_i \in \Delta(A_i)$. Accordingly, there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that $v_i(\cdot, m'_{-i}) = \alpha v_i(\cdot, m_{-i}) + \beta$, where we can choose $\beta = 0$ since only the difference of temptation payoffs matter. By setting $g_i(\cdot) = v_i(\cdot, m_{-i})$ for some nondegenerate $m_{-i}$, we have $v_i(\cdot, m'_{-i}) = \kappa_i(m'_{-i})g_i(\cdot)$ for all $m'_{-i}$.

We prove that (5) is true for all $x_i, y_i \in \Delta(A_i)$. Note that (5) is true for all $x_i, y_i \in \Delta(A_i)$ with $f_i(x_i) > f_i(y_i)$. Seeking a contradiction, suppose otherwise. Then, there exist $x_i, y_i \in \Delta(A_i)$ such that $f_i(x_i) = f_i(y_i)$, $v_i(x_i, m_{-i}) \geq v_i(y_i, m_{-i})$, and $v_i(x_i, m'_{-i}) < v_i(y_i, m'_{-i})$. Let $x'_i, y'_i \in \Delta(A_i)$ be such that \(\{y'_i\} \succ_{i,m_{-i}} \{x'_i, y'_i\}\). Then, $f_i(y'_i) > f_i(x'_i)$, $v_i(x'_i, m_{-i}) > v_i(y'_i, m_{-i})$, and $v_i(x'_i, m'_{-i}) > v_i(y'_i, m'_{-i})$ by Lemma 3. Thus, for sufficiently small $\lambda \in (0, 1)$,

$$v_i(\lambda x'_i + (1 - \lambda)x_i, m_{-i}) > v_i(\lambda y'_i + (1 - \lambda)y_i, m_{-i}),$$

(6)

$$v_i(\lambda x'_i + (1 - \lambda)x_i, m'_{-i}) < v_i(\lambda y'_i + (1 - \lambda)y_i, m'_{-i}).$$

(7)

Since $f_i(y_i) = f_i(x_i)$, we have $f_i(\lambda y'_i + (1 - \lambda)y_i) > f_i(\lambda x'_i + (1 - \lambda)x_i)$ for all $\lambda \in (0, 1)$. Then, by (6) and Lemma 3, \(\{\lambda y'_i + (1 - \lambda)y_i\} \succ_{i,m_{-i}} \{\lambda x'_i + (1 - \lambda)x_i, \lambda y'_i + (1 - \lambda)y_i\}\) for all $\lambda \in (0, 1)$. By Identical Tempting Actions, \(\{\lambda y'_i + (1 - \lambda)y_i\} \succ_{i,m'_{-i}} \{\lambda x'_i + (1 - \lambda)x_i, \lambda y'_i + (1 - \lambda)y_i\}\), and $v_i(\lambda x'_i + (1 - \lambda)x_i, m'_{-i}) > v_i(\lambda y'_i + (1 - \lambda)y_i, m'_{-i})$ for all $\lambda \in (0, 1)$ by Lemma 3. This contradicts to (7) for sufficiently small $\lambda \in (0, 1)$. □

To complete the proof, we show that $\kappa_i(m'_{-i}) \geq \kappa_i(m_{-i})$ if and only if $m'_{-i} \succeq m_{-i}$.

The next lemma establishes the “only if” part.

**Lemma 5.** Suppose that $U_i$ is given by (2). If $\kappa_i(m'_{-i}) \geq \kappa_i(m_{-i})$, then $m'_{-i} \succeq m_{-i}$.  

5
Proof. Suppose that \( \{x_i\} \succeq_{i,m_{-i}} M_i \). If \( \kappa_i(m'_i) \geq \kappa_i(m_{-i}) \), then

\[
U_i(\{x_i\}, m_{-i}) = f_i(x_i) \\
\geq \max_{m_i \in M_i} \left( f_i(m_i) - \kappa_i(m_{-i}) \left( \max_{m'_i \in M_i} g_i(m'_i) - g_i(m_i) \right) \right) \\
\geq \max_{m_i \in M_i} \left( f_i(m_i) - \kappa_i(m'_i) \left( \max_{m'_i \in M_i} g_i(m'_i) - g_i(m_i) \right) \right).
\]

Since \( U_i(\{x_i\}, m'_i) = f_i(x_i) \), we have \( \{x_i\} \succeq_{i,m'_i} M_i \), which implies \( m'_i \succeq_i m_{-i} \). \( \square \)

If \( m_{-i} \) is not nondegenerate, then \( \kappa_i(m_{-i}) \) is characterized as follows.

**Lemma 6.** Suppose that \( U_i \) is given by (2). Let \( m_{-i} \in \prod_{j \neq i} \Delta(A_j) \) be nonempty and closed by continuity of \( f_i \). The following sets are nonempty and closed by continuity of \( U_i \) and disjoint since \( \{x_i\} \succeq_{i,m_{-i}} \left( \{x_i\} \succeq_{i,m_{-i}} M_i \right) \), which implies \( m'_i \succeq_i m_{-i} \).

Proof. By the argument in the proof of Lemma 4, we can set \( \kappa_i(m_{-i}) = 0 \). This implies that \( \kappa_i(m'_i) \geq \kappa_i(m_{-i}) \) and \( m'_i \succeq_i m_{-i} \) for all \( m'_i \in \Delta(m_{-i}) \).

We distinguish the following two types of nondegeneracy. The opponents’ action profile \( m_{-i} \in \prod_{j \neq i} \Delta(A_j) \) is said to be regular if it is nondegenerate and there exist \( x_i, y_i \in \Delta(A_i) \) such that \( \{x_i\} \succ_{i,m_{-i}} \{x_i, y_i\} \succ_{i,m_{-i}} \{y_i\} \); \( m_{-i} \) is said to be nonregular if it is nondegenerate and \( \{x_i, y_i\} \sim_{i,m_{-i}} \{y_i\} \) for all \( x_i, y_i \in \Delta(A_i) \) with \( \{x_i\} \succ_{i,m_{-i}} \{x_i, y_i\} \). Nonregularity is equivalent to the following stronger condition.

**Lemma 7.** The opponents’ action profile \( m_{-i} \in \prod_{j \neq i} \Delta(A_j) \) is nonregular if and only if it is nondegenerate and \( \{x_i, y_i\} \sim_{i,m_{-i}} \{y_i\} \) for all \( x_i, y_i \in \Delta(A_i) \) with \( \{x_i\} \succ_{i,m_{-i}} \{y_i\} \).

Proof. It is enough to show the “only if” part. Let \( m_{-i} \) be nonregular. There exist \( x_i, y_i \in \Delta(A_i) \) such that \( \{x_i\} \succ_{i,m_{-i}} \{x_i, y_i\} \sim_{i,m_{-i}} \{y_i\} \). Seeking a contradiction, suppose that the lemma does not hold. Then, there exist \( x'_i, y'_i \in \Delta(A_i) \) such that \( \{x'_i\} \sim_{i,m_{-i}} \{x'_i, y'_i\} \succ_{i,m_{-i}} \{y'_i\} \). Then, \( \{\lambda x'_i + (1 - \lambda)x_i\} \succ_{i,m_{-i}} \{\lambda y'_i + (1 - \lambda)y_i\} \) for all \( \lambda \in [0, 1] \).

Since \( m_{-i} \) is nonregular, either of the following holds for all \( \lambda \in [0, 1] \):

\[
\{\lambda x'_i + (1 - \lambda)x_i\} \succ_{i,m_{-i}} \{\lambda x'_i + (1 - \lambda)x_i, \lambda y'_i + (1 - \lambda)y_i\} \sim_{i,m_{-i}} \{\lambda y'_i + (1 - \lambda)y_i\}, \\
\{\lambda x'_i + (1 - \lambda)x_i\} \sim_{i,m_{-i}} \{\lambda x'_i + (1 - \lambda)x_i, \lambda y'_i + (1 - \lambda)y_i\} \succ_{i,m_{-i}} \{\lambda y'_i + (1 - \lambda)y_i\}.
\]

The following sets are nonempty and closed by continuity of \( U_i \) and disjoint since \( \{\lambda x'_i + (1 - \lambda)x_i\} \succ_{i,m_{-i}} \{\lambda x'_i + (1 - \lambda)x_i, \lambda y'_i + (1 - \lambda)y_i\} \sim_{i,m_{-i}} \{\lambda y'_i + (1 - \lambda)y_i\} \):

\[
I_1 \equiv \{\lambda \in [0, 1] \mid \{\lambda x'_i + (1 - \lambda)x_i\} \sim_{i,m_{-i}} \{\lambda x'_i + (1 - \lambda)x_i, \lambda y'_i + (1 - \lambda)y_i\}\}, \\
I_2 \equiv \{\lambda \in [0, 1] \mid \{\lambda x'_i + (1 - \lambda)x_i, \lambda y'_i + (1 - \lambda)y_i\} \sim_{i,m_{-i}} \{\lambda y'_i + (1 - \lambda)y_i\}\}.
\]
Since $[0, 1]$ is a connected set, there exists $\lambda \in [0, 1] \setminus (I_1 \cup I_2)$, which satisfies
\[
\{\lambda x'_i + (1 - \lambda)x_i \} >_{i,m_{-i}} \{\lambda x'_i + (1 - \lambda)x_i, \lambda y'_i + (1 - \lambda)y_i\} >_{i,m_{-i}} \{\lambda y'_i + (1 - \lambda)y_i\}.
\]
This implies that $m_{-i}$ is regular, a contradiction. □

If $m_{-i}$ is nonregular, then $\kappa_i(m_{-i})$ is characterized as follows.

**Lemma 8.** Suppose that $U_i$ is given by (2). Let $m_{-i} \in \prod_{j \neq i} \Delta(A_j)$ be nonregular. Then, we can set $\kappa_i(m_{-i}) = 1$. Furthermore, $\kappa_i(m_{-i}) \geq \kappa_i(m'_{-i})$ and $m_{-i} \succeq m'_{-i}$ for all $m'_{-i} \in \prod_{j \neq i} \Delta(A_j)$.

**Proof.** By Lemma 7, $\{x_i, y_i\} >_{i,m_{-i}} \{y_i\}$ for all $x_i, y_i \in \Delta(A_i)$ with $\{x_i\} >_{i,m_{-i}} \{y_i\}$. By Set Betweenness, $\{x_i\} >_{i,m_{-i}} \{y_i\}$ if and only if $\{x_i, y_i\} >_{i,m_{-i}} \{y_i\}$, which is equivalent to $v_i(y_i, m_{-i}) > v_i(x_i, m_{-i})$ by Lemma 3. Thus, $f_i(x_i) > f_i(y_i)$ if and only if $v_i(y_i, m_{-i}) > v_i(x_i, m_{-i})$. Since $v_i(\cdot, m_{-i})$ and $-f_i(\cdot)$ represent the same ranking over $\Delta(A_i)$, there exists $\alpha > 0$ and $\beta \in \mathbb{R}$ such that $v_i(\cdot, m_{-i}) = -\alpha f_i(\cdot) + \beta$, where we can choose $\beta = 0$ since only the difference of temptation payoffs matter. Then, by (2),
\[
U_i(M_i, m_{-i}) = \max_{M_i} (1 - \alpha) f_i(\cdot) + \alpha \min_{M_i} f_i(\cdot).
\]
Thus, if $\{x_i\} >_{i,m_{-i}} \{y_i\}$ (i.e. $f_i(x_i) > f_i(y_i)$), then
\[
U_i(\{x_i, y_i\}, m_{-i}) = \max_{\{x_i, y_i\}} (1 - \alpha) f_i(\cdot) + \alpha \min_{\{x_i, y_i\}} f_i(\cdot) = f_i(y_i)
\]
since $\{x_i, y_i\} >_{i,m_{-i}} \{y_i\}$. Therefore, $\alpha \geq 1$ and $U_i(M_i, m_{-i}) = \min_{M_i} f_i(\cdot)$, whereby we can set $\kappa_i(m_{-i}) = 1$ and $g_i(\cdot) = -f_i(\cdot)$ for all nonregular $m_{-i} \in \prod_{j \neq i} \Delta(A_j)$. For $m'_{-i} \neq m_{-i}$, we must have $v_i(\cdot, m'_{-i}) = \kappa_i(m'_{-i}) g_i(\cdot) = -\kappa_i(m'_{-i}) f_i(\cdot)$, and by (2),
\[
U_i(M_i, m_{-i}) = \max_{M_i} (1 - \kappa_i(m_{-i})) f_i(\cdot) + \kappa_i(m_{-i}) \min_{M_i} f_i(\cdot).
\]
If $m'_{-i}$ is nonregular, we can set $\kappa_i(m'_{-i}) = 1$; otherwise, we must have $\kappa_i(m'_{-i}) < 1$ because $U_i(M_i, m_{-i}) = \min_{M_i} f_i(\cdot)$ implies nonregularity. Accordingly, $1 = \kappa_i(m_{-i}) \geq \kappa_i(m'_{-i})$ for all $m'_{-i} \in \prod_{j \neq i} \Delta(A_j)$. Then, Lemma 5 implies that $m_{-i} \succeq m'_{-i}$ for all $m'_{-i} \in \prod_{j \neq i} \Delta(A_j)$. □

To characterize $\kappa_i(m_{-i})$ with regular $m_{-i}$, we use the following lemma.

**Lemma 9.** Suppose that $U_i$ is given by (2). If $m'_{-i} \succeq m_{-i}$ then $U_i(M_i, m_{-i}) \geq U_i(M_i, m'_{-i})$ for all $M_i \in \mathcal{K}(\Delta(A_i))$.

**Proof.** By (2), there exist $\tilde{x}_i, \tilde{x}_j \in \Delta(A_i)$ such that $f_i(\tilde{x}_i) \geq U_i(M_i, m_{-i}) \geq f_i(x_j)$. Since $f_i$ is continuous, there exists $x_i \in \Delta(A_i)$ such that $f_i(x_i) = U_i(M_i, m_{-i})$, i.e., $\{x_i\} >_{i,m_{-i}} M_i$. Since $m'_{-i} \succeq m_{-i}$, we have $\{x_i\} \succeq_{i,m'_{-i}} M_i$, which implies that $U_i(M_i, m_{-i}) = f_i(x_i) = U_i(\{x_i\}, m'_{-i}) \geq U_i(M, m'_{-i})$. □
If \( m_{-i} \) is regular, then \( \kappa_i(m_{-i}) \) is characterized as follows.

**Lemma 10.** Suppose that \( U_i \) is given by (2). Let \( m_{-i}, m_{-i} \in \prod_{j \neq i} \Delta(A_j) \) be regular. If \( m'_{-i} \geq_i m_{-i} \), then \( \kappa_i(m'_{-i}) \geq \kappa_i(m_{-i}) \).

**Proof.** Suppose that \( m'_{-i} \geq_i m_{-i} \). Since \( m_{-i} \) is regular, there exist \( x_i, y_i \in \Delta(A_i) \) such that \( \{x_i\} >_{i,m_{-i}} \{x_i, y_i\} >_{i,m_{-i}} \{y_i\} \). Thus, \( f_i(x_i) = U_i(\{x_i, y_i\}, m_{-i}) > U_i(\{y_i\}, m_{-i}) = f_i(y_i) \) and \( g_i(y_i) > g_i(x_i) \) by Lemma 3. Since

\[
U_i(\{x_i, y_i\}, m_{-i}) = \max_{\{x_i, y_i\}} (f_i(\cdot) + \kappa_i(m_{-i})g_i(\cdot)) - \max_{\{x_i, y_i\}} \kappa_i(m_{-i})g_i(\cdot)
\]

\[
= \max\{f_i(x_i) - \kappa_i(m_{-i})(g_i(y_i) - g_i(x_i)), f_i(y_i)\}
\]

we have \( U_i(\{x_i, y_i\}, m_{-i}) = f_i(x_i) - \kappa_i(m_{-i})(g_i(y_i) - g_i(x_i)) \). By Lemma 9,

\[
f_i(x_i) - \kappa_i(m'_{-i})(g_i(y_i) - g_i(x_i)) \leq \max\{f_i(x_i) - \kappa_i(m'_{-i})(g_i(y_i) - g_i(x_i)), f_i(y_i)\}
\]

\[
= U_i(\{x_i, y_i\}, m'_{-i})
\]

\[
\leq U_i(\{x_i, y_i\}, m_{-i})
\]

\[
= f_i(x_i) - \kappa_i(m_{-i})(g_i(y_i) - g_i(x_i)).
\]

The above is reduced to \( (\kappa_i(m'_{-i}) - \kappa_i(m_{-i}))(g_i(y_i) - g_i(x_i)) \geq 0, \) and thus \( \kappa_i(m'_{-i}) - \kappa_i(m_{-i}) \geq 0 \) since \( g_i(y_i) - g_i(x_i) > 0 \). \( \Box \)

We are ready to establish the “if” part.

**Lemma 11.** Suppose that \( U_i \) is given by (2). If \( m'_{-i} \geq_i m_{-i} \), then \( \kappa_i(m'_{-i}) \geq \kappa_i(m_{-i}) \). A strict inequality holds if \( \geq_{i,m_{-i}} \neq \geq_{i,m'_{-i}} \).

**Proof.** By Lemma 6, if \( m_{-i} \) is not nondegenerate, then \( m'_{-i} \geq_i m_{-i} \) for all \( m'_{-i} \in \prod_{j \neq i} \Delta(A_j) \) and \( \kappa_i(m'_{-i}) \geq \kappa_i(m_{-i}) = 0 \).

By Lemma 8, if \( m'_{-i} \) is nonregular, then \( m'_{-i} \geq_i m_{-i} \) for all \( m_{-i} \in \prod_{j \neq i} \Delta(A_j) \) and \( 1 = \kappa_i(m'_{-i}) \geq \kappa_i(m_{-i}) \).

In the other case, both \( m_{-i} \) and \( m'_{-i} \) are regular. By Lemma 10, if \( m'_{-i} \geq_i m_{-i} \), then \( \kappa_i(m'_{-i}) \geq \kappa_i(m_{-i}) \). It should be noted that if there is no nonregular opponent’s action profile, then \( \kappa_i(m_{-i}) \leq 1 \) is not necessarily true.

Finally, by (2), if \( \kappa_i(m'_{-i}) = \kappa_i(m_{-i}) \) then \( \geq_{i,m_{-i}} \neq \geq_{i,m'_{-i}} \), which implies that a strict inequality holds if \( \geq_{i,m_{-i}} \neq \geq_{i,m'_{-i}} \). \( \Box \)
References
