GLOBAL ENVIRONMENTAL MANAGEMENT AND BARGAINING: STRATEGIC PUBLIC ABATEMENT INVESTMENT INCENTIVES[†)]

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Abstract. When future international agreement for global environmental control is anticipated, decisions for controlling current carbon gas emissions by improving the country's abatement capabilities are strongly affected by the likelihood of and the likely outcome of such agreements. We construct a two-period two-country model where the quality of the atmospheric environment is a global public capital, and countries invest in abatement investments in the first period and engage in production activities in the second period. Applying the incomplete contract approach to this model where (re)negotiation with or without side payment may take place in the second period, we examine the following questions. What are the characteristics of the country that make its bargaining position more advantageous, what are the cause of distortions in ex ante capital investments as well as in ex post incentives for environmental improvement, and what are the characteristics of countries which are prone to these distortions? Our newly proposed method for compartive statics in Nash bargaining models play a crucial role for the analysis.

1. Introduction

The problem of global warming is a universal concern for the entire humankind. It will affect not only high-tech firms in industrialized world but also people living in an arid area of developing countries, and actions carried out by our generation will significantly affect the welfare of all future generations. Despite its universality of the consequence of our decisions about how to control global environment and, thereby, achieving a sustainable growth, there are heterogeneous, conflicting and often diametrically opposite views about how we should actually do for this global cause. For example, some people advocate for severe reduction of carbon gas emission, while others oppose to it. Even among advocates, some argue for uniform taxation which is enforced by tradable permits allocated to each country, while others argue

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for non-uniform taxation whose rates should positively related with the country's GDP.[1] The present paper is aimed at analyzing theoretically why these heterogeneous views appear and what accounts we should take into when a future international agreement for global environmental control is in sight, but not yet agreed because of such heterogeneity.

Compared with a decade ago, the problem of global environmental control has become much more exposed and the public's environmental consciousness has increased drastically. Despite such exposures and the public's concern, however, the effort for international agreement to contain global warming as well as individual country's attempts for reducing environmental destruction are slow to come. This seems to be quite a contrast compared with an increasing effort of individual companies. For example, the cost of carbon energy consumption is still very low in the US, and it has not changed drastically in the last few decades. The cost in Europe and Japan is relatively high, contributing somewhat to the reduction of the carbon gas. However, high cost in Japan is mainly the result of steep oil price increases in the 1970's, not reflecting a recent increase in public's awareness of global environment. Why, then, is that an increase in public's concern as well as an increase in visibility of global environment does not induce spontaneous efforts to contain environmental destruction of major countries?

We view that one of the reasons for unwillingness of several major governments in actively pursuing the control of global environment lies in the very fact that a future international agreement comes into their view. When governments get together and a negotiation takes place in order to design an international agreement, an outcome will be significantly affected by the bargaining power of each country. Unfortunately, the magnitude of each country's bargaining power will depend negatively upon how much stakes the country will have in the bargaining outcome. Countries with larger stakes will become more desperate to sign a contract, sacrificing some of its possible gains. Similarly, those countries which can control pollution with relatively little cost cannot credibly argue for larger share, being forced to accept small bargaining gain. In contrast, those countries who must bear a larger cost in improving the global environment, but care little about the environment, will resist any agreement. Because such non-cooperation will be viewed credible by other negotiation partners, the negotiation is likely to be concluded with the countries with the latter characteristics benefiting more at the cost of those countries with the former characteristics.

This means that, anticipating a future international bargaining, countries may try to refrain from investments for controlling environmental destruction, because doing so only deteriorates the country's future bargaining position. Anticipating a future bargaining, countries may try to improve their strategic positions by investing less for energy saving and, in an extreme case, by further deteriorating its own environmental situation.

In this paper, we analyze such a possibility using a simple two-period two-country model. A country emits carbon gas as a by-product of economic activity in the second period. The

[1]Uzawa [6] is an example of proposal for non-uniform taxation.

amount of carbon gas per GDP is assumed to depend upon production function, reflecting the country's industrial structure, and upon efficiency of pollution abatement. Abatement efficiency, in turn, is assumed to depend upon the first period investment activity as well as its ex ante efficiency which it inherited from the past.

We compare several scenarios. In one scenario, two countries choose their actions in both periods non-cooperatively. That is, the solution concept we use will be the simple two-period subgame perfect Nash equilibrium. In the other scenario, we assume that two countries will sign a binding international agreement in period 2. For this scenario, we use two alternative solution concepts for the cooperative outcome, the Nash bargaining solution with side-payments and that without side-payments, with the assumption that the associated non-cooperative outcome will be realized if the negotiation breaks down. With the Nash bargaining solution concept, the agreement will provide exactly one half of the gains from an agreement (i.e., the aggregate gains of achieving efficient outcome compared with the non-cooperative outcome) to each country, in addition to the payoff it would have obtained had the non-cooperative outcome prevailed. In period 1, non-cooperative game will be played anticipating this cooperative outcome to prevail in period 2.

We propose a new method of comparative statics for such ex-post bargaining based on the standard duality theory and explore the strategic investment incentive of each country before the bargaining. It clarifies the two effects of pre-bargaining investment on the outcome that affects each country's investment incentive compared with when there is no ex-post bargaining. They are the *bargaining-frontier expansion effect* (working regardless of transfer availability at the second-period bargaining) and the *strategic valuation effect* (working only in the absence of transfers). The former represents the size of an increase in the total bargaining surplus made by more investment and the latter the associated change in the shadow valuation of each country's bargaining surplus. We find that when more investments increases the total bargaining surplus and lowers the own relative shadow valuation a country has an incentive to invest more on abatement than in the absence of bargaining.

The rest of the paper is organized as follows. We shall present our model in section 2, and analyze non-cooperative equilibrium in the second period and analyze sub-game perfect equilibrium when there is no possibility of international agreement in section 3. Section 4 gives theoretical100 model formulation of bargaining with and without transfers and propose a new method of their comparative statics using the maximum bargaining-surplus function and the minimum bargaining-cost function. In section 5, we apply the method to delineate the two strategic effects of pre-bargaining investment and measure them. Section 7 concludes the paper.

2. Model Set-up

We consider a world consisting of two countries, 1 and 2. Country i ($i = 1$, and 2) produces a single final good, which can be used either for consumption or investment, while emitting carbon gas as its by-product. The final goods produced by the two countries are perfect substitutes. Thus if they are traded freely in the world market, then their prices should become equal.

The (reduced form) production function for the final good, which is assumed to be the same for the two countries, [2] is denoted as:

$$y_i = f(z_i, \bar{a}_i, x_i), \tag{1}$$

where y_i is country i's level of real produced national income, z_i its level of carbon gas emission, x_i the initial endowment of the ordinary factors, which we may call "labor" throughout the paper, [3] and $\bar{a}_i$ a parameter representing its efficiency in environmental control, which we call the abatement efficiency. The abatement efficiency is measured by the sum of the abatement investment costs undertaken by the government in the past. And the investment cost is measured in terms of the final good.[4]

The production function is assumed to satisfy $f_z > 0$, $f_{zz} < 0$, and $f_a > 0$.[5] In view of the differences in the initial labor endowment, we often express country i's production function by $f^i(z_i, a_i)$ by suppressing the labor endowment.

Each country's emission of carbon gas aggravates the quality of global environment and damages the welfare of both countries. Such damage depends upon the world total emission of carbon gas, z_T, which is defined by

$$z_T = \sum_{\ell} z_\ell, \tag{2}$$

while the world damage is expressed by the function $D(z_w)$.

Thus country i's welfare net of the abatement investment cost is expressed by:

[2]This assumption of common production technology does not affect the succeeding results at all.

[3]The initially endowed factors of production other than carbon gas emissions as unpaid factor are assumed to be fully employed.

[4]More generally the abatement investment may require combination of several distinct factors of production. For taking this into account, we have to incorporate more than two endowed factors and to see how much the abatement investment decreases the factors available for final-good production. But this makes the analysis too much complicated, so that we employ the assumption in the text.

[5]Even we exclude the investment costs, some type of abatement may lower both the real produced income ($f_a < 0$) and the marginal productivity of the emission ($f_{az} < 0$). We will discuss how the results will change for this case after completing the present case in the concluding section.

$$u^i(\mathbf{z}, a_i, \theta_i) = f^i(z_i, a_i) - \theta_i D\left(\sum_\ell z_\ell\right) - a_i, \tag{3}$$

where θ_i is the (constant) marginal value of global environment for the country i. That is, this parameter represents the country's valuation of global environment or its perception of the world environment damage as its own in terms of its GDP, which we call the *environmental consciousness.*

Assumption 1. *The components in the individual country's net welfare function (3) satisfies the following two conditions.*
(A 1-1) The production function $f^i(z_i, a_i)$ is strictly increasing, twice-continuously differentiable, and strictly concave in (z_i, a_i).
(A 1-2) The world damage function $D(z_T)$ is strictly increasing, twice-continuously differentiable and convex in z_T.

We often discuss the level of the world net welfare at each possible equilibrium in the succeeding analysis. It is defined as below:

$$u^w(\mathbf{z}, \mathbf{a}, \theta_w) = \sum_\ell f^\ell(z_\ell, a_\ell) - \theta_w D\left(\sum_\ell z_\ell\right) - \sum_\ell a_\ell, \tag{4}$$

where $\theta_w := \sum_\ell \theta_\ell$ represents the world environment consciousness. For making the concept of world optimum sensible enough, we assume:

Assumption 2. *The world net welfare function (4) is strictly concave in $(\mathbf{z}, \mathbf{a})$.*

There is one remark in order here on abatement investment undertaken only by the government. Although the levels of carbon gas emission and abatement investment are determined by the private sector given the tax-cum-subsidy policy of the government, we assume, for simplifying the analysis, that each country's government directly controls them. And even under this assumption, there are several issues of interest in theory and practice concerning strategic policy interactions between the countries.

In fact, there are two issues for theoretical discussion. First, how does incomplete international cooperation in environmental regulations affect the global environment quality? In the most non-cooperative case, each country decides on both levels of carbon gas emission and abatement investment independently, while in the most cooperative one both countries jointly decide each policy variable of the individual country and maximize the joint benefits. And in the intermediate case the two countries coordinate either of the two policy variables only.

Second, how does the sequential decision structure of the individual country's environment regulation affect the quality of the world environment? In other words, it is a problem of pre-commitment to environment regulations. The most natural scenario showing the importance of policy pre-commitment is the one in which the two countries first decide on their abatement investment independently and simultaneously, and after observing their investments they coordinate their choices over the levels of carbon gas emission.

To explore these issues, we first delineate the properties of non-cooperative equilibria as the reference state.

3. Non-Cooperative Environment Policy Game

Let us first consider a non-cooperative game in which the government chooses both carbon-gas emission and abatement investment independently and simultaneously. And we then compare the associated equilibrium with the one in which the governments pre-commit to abatement investments before, again non-cooperatively, choosing the carbon-gas emission level.

3.1. **One-shot Decision on Emission and Abatement Investment.** Let $\mathbf{z}^{NO} = (z_1^{NO}, z_2^{NO})$ (or $\mathbf{a}^{NO} = (a_1^{NO}, a_2^{NO})$) denote the equilibrium profile of emissions (or abatement investments) for the present non-cooperative one-shot game. Then the profile $(\mathbf{z}^{NO}, \mathbf{a}^{NO})$ should satisfy:

$$0 = \frac{\partial u^i(\mathbf{z}^{NO}, a_i^{NO}, \theta_i)}{\partial z_i} = f_z^i(z_i^{NO}, a_i^{NO}) - \theta_i D'(\sum_{\ell} z_\ell^{NO}), \tag{5}$$

$$0 = \frac{\partial u^i(\mathbf{z}^{NO}, a_i^{NO}, \theta_i)}{\partial a_i} = f_a^i(z_i^{NO}, a_i^{NO}) - 1. \tag{6}$$

Let us take the symmetric case as our reference state for inquiry, i.e., $\theta = \theta_i$ for $i = 1, 2$. Insofar as we confine ourselves to a symmetric equilibrium, the above equilibrium conditions are reduced to

$$0 = u_z^I(z, a, \theta) := f_z(z, a) - \theta D'(2z), \tag{7}$$

$$0 = u_a^I(z, a, \theta) := f_a(z, a) - 1. \tag{8}$$

The symmetric equilibrium is given a graphical representation as shown in Figures 1 and 2. Figure 1 shows the case for $f_{za}(z, a) > 0$ where the carbon-gas emission and abatements are complements in production of the final good, while Figure 2 shows the case for $f_{za}(z, a) < 0$ where they are substitutes.[6] In each figure, the curve named $u_z^I = 0$ shows the individual country's best choice of carbon gas emission given the own abatement investment, and the curve named $u_a^I = 0$ its best choice of the abatement investment given the own carbon gas

[6]As with the structural factors determining sgn $\{f_{az}(z, a)\}$, see the discussion in Appendix A.

emission. We may call the former the *abatement-constrained individual emission curve* and the latter the *emission-constrained individual abatement curve*. The intersection E_{NO} of the two curves gives the symmetric Nash equilibrium for our non-cooperative one-shot game.

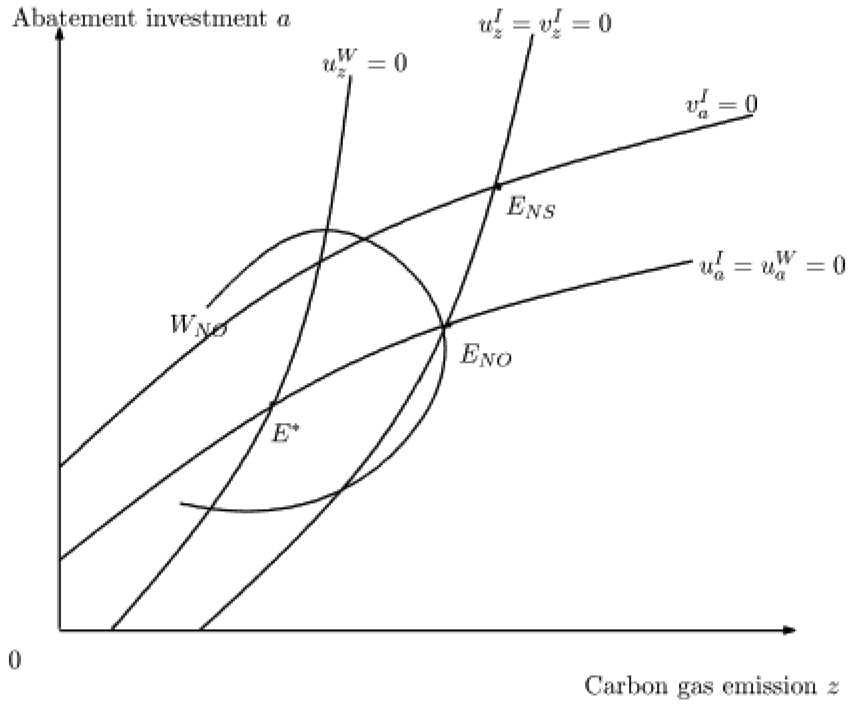

FIGURE 1. Symmetric Equilibrium for the One-Shot Non-cooperative Game – Case of Complements ($f_{za}(z, a) > 0$)

One remark is in order here. Although we assume $f_a(z, a) > 0$ in the present paper, i.e., that more abatement investment increases the real national income exclusive of the investment cost, the condition may not hold in actuality. When $f_a(z, a) < 0$ holds instead, the one-shot simultaneous-move game equilibrium requires neither country to undertake any abatement investment. However even under the present assumption there arise several interesting issues from the view-points of theory and practice as we will discuss below.

3.2. **Strategic Abatement Investment Game.** Let us now consider what if the two countries can precommit to their abatement investments before deciding their carbon-gas emission levels. More specifically, this is the game in which the two governments first decide on the abatement investments simultaneously and independently, and after observing their investments they further choose their levels of carbon gas emission simultaneously. The solution concept in use here is of subgame perfection.

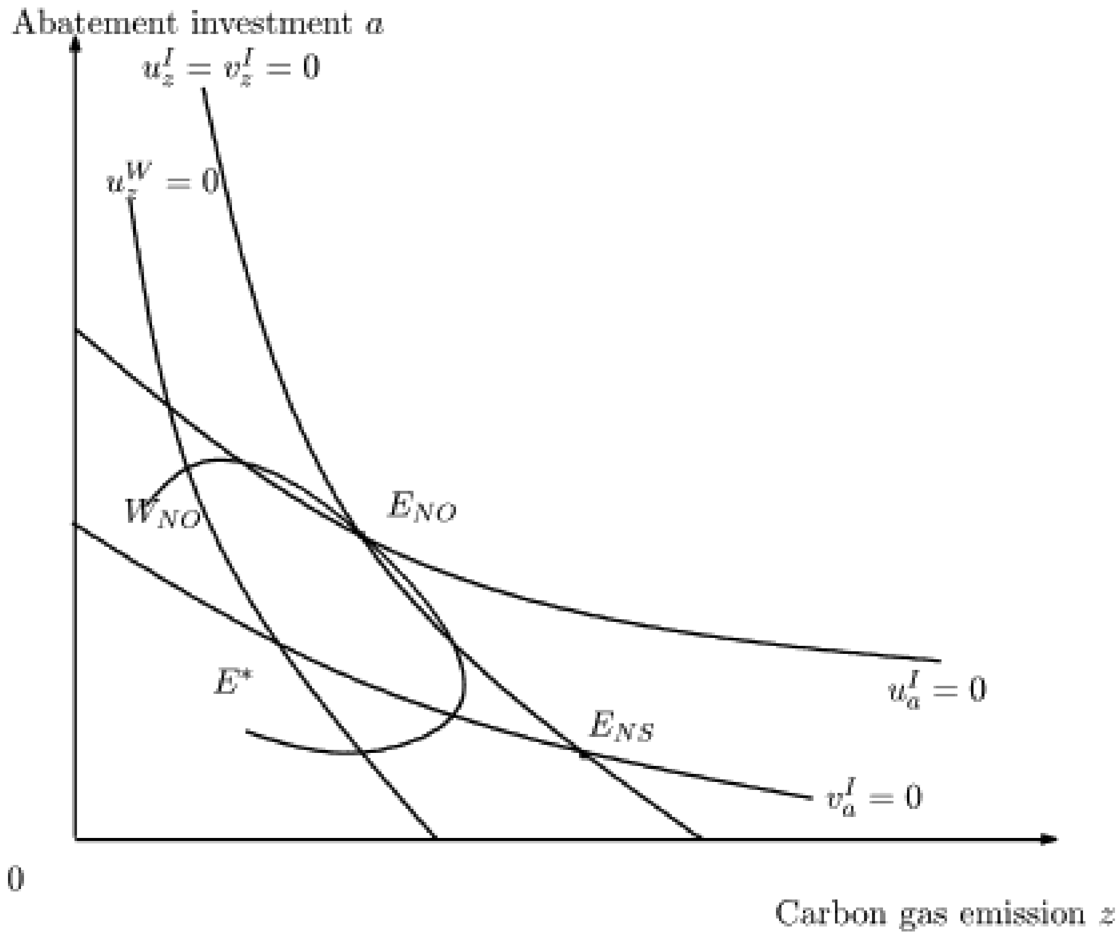

FIGURE 2. Symmetric Equilibrium for the One-Shot Non-Cooperative Game – Case of Substitutes ($f_{za}(z,a) < 0$)

Second-stage equilibrium. The second-stage equilibrium with choices over carbon gas emissions depends on the abatement investment profile **a**. Given **a**, each government unilaterally sets the level of carbon gas emission so as to maximize the own national welfare. Her optimal emissions of carbon gas should satisfy the following first-order condition for welfare maximization given the other country's emissions of carbon gas

$$0 = \frac{\partial u^i(\mathbf{z}, a_i, \theta_i)}{\partial z_i} = f_z^i(z_i, a_i) - \theta_i D'(z_i + z_j), \tag{9}$$

which can be viewed as

$$0 = f_z^i(z_i, a_i) - \theta_i D'(z_T).$$

The above equation defines the unilaterally optimal carbon-gas emissions of country i as a function of the world total emissions z_T as well as the own abatement investment a_i and the environment consciousness θ_i. We express this relation by $r^i(z_T, a_i, \theta_i)$ and call country i's quasi-reaction function.[7] This function preserves the strategic substitution and complementarity defined over the reaction function defined in a standard fashion. Its properties are summarized as below.

$$
\begin{aligned}
(i)\ r^i_z(z_T, a_i, \theta_i) &:= \frac{\partial r^i(z_T, a_i, \theta_i)}{\partial z_T} = \frac{\theta_i D"(z_T)}{f^i_{zz}(z_i, a_i)} < 0, \\
(ii)\ r^i_a(z_T, a_i, \theta_i) &:= \frac{\partial r^i(z_T, a_i, \theta_i)}{\partial a_i} = -\frac{f^i_{za}(z_i, a_i)}{f^i_{zz}(z_i, a_i)} \propto f^i_{za}(z_i, a_i), \\
(ii)\ r^i_\theta(z_T, a_i, \theta_i) &:= \frac{\partial r^i(z_T, a_i, \theta_i)}{\partial \theta_i} = \frac{D'(z_T)}{f^i_{zz}(z_i, a_i)} < 0,
\end{aligned}
$$

where $z_i = r^i(z_T, a_i)$ and use was made of Assumption 1. The three relations have the following implications.

(i) shows that an increase in the world total carbon-gas emissions decreases each country's emissions, This is because an increase in the other country's emissions raises the marginal environmental damage (i.e, $D"(\cdot) > 0$), leading to the less incentive to emit the own carbon gas. Put another way, each country's carbon-gas emissions are mutually strategic substitutes. (ii) means that improvement in the abatement efficiency increases the best-response carbon-gas emission if and only if it is of Type H technical progress. And the last (iii) represents that when the country gets more environment conscious in the sense of greater θ_i, its best-response gas emission decreases, for it raises the marginal environment damage perceived by the country.

For the later reference, we summarize these results as below:

Lemma 1. *Each country's reaction function in the second stage, given by $r^i(z_T, a_i, \theta_i)$, satisfies:*
(i) Each country's carbon-gas emission is a strategic substitute to the other's, i.e., $r^i_z(z_T, a_i, \theta_i) < 0$.
(ii) Improvement in the abatement efficiency increases the individual best-response carbon-gas emission if and only if the associated technical progress is of type H, i.e., $sgn\left\{r^i_a(\cdot)\right\} = sgn\left\{f^i_{za}(\cdot)\right\}$.
(iii) An increase in the environment consciousness decreases the individual best-response carbon-gas emission, i.e., $r^i_\theta(\cdot) < 0$.

The second-stage equilibrium carbon-gas profile (z^N_1, z^N_2) coupled with the world total emissions z^N_T, then, should be a solution to the following set of equations:

[7] This name is used by Suzumura.

$$
(10) \qquad \begin{aligned} z_T^N &= \textstyle\sum_k r^k(z_T^N, a_k, \theta_k), \\ z_i^N &= r^i(z_T^N, a_i, \theta_i) \ (i = 1, 2). \end{aligned}
$$

The second-stage equilibrium carbon gas emission profile depends on the the abatement-efficiency profile $\mathbf{a} = (a_1, a_2)$ and the environment-consciousness profile $\theta = (\theta_1, \theta_2)$, the relation of which we express by $z_i^N = z^{iN}(\mathbf{a}, \theta)$ for $i = 1, 2$. We also express the associated equilibrium world total emissions by $z_T^N = z^{TN}(\mathbf{a}, \theta) \left(:= \sum_{i=1,2} z^{iN}(\mathbf{a}, \theta)\right)$. Exercise of standard comparative statics with respect to (10) based on Lemma 1 leads to:

$$
\begin{aligned}
\frac{\partial z^{TN}(\mathbf{a}, \theta)}{\partial m_i} &:= \frac{r_m^i(z_T^N, a_i, \theta_i)}{\Delta} \propto r_m^i(\cdot) \\
\frac{\partial z^{iN}(\mathbf{a}, \theta)}{\partial m_i} &:= \frac{1 - r_z^j(z_T^N, a_j, \theta_j)}{\Delta} \times r_m^i(z_T^N, a_i, \theta_i) \propto r_m^i(\cdot), \\
\frac{\partial z^{jN}(\mathbf{a}, \theta)}{\partial m_i} &:= \frac{\partial z^{TN}(\cdot)}{\partial m_i} \times r_z^j(z_T^N, a_j, \theta_j),
\end{aligned}
$$

where $m = a, \theta$ and $\Delta := 1 - \sum_k r_z^k \left(z^{TN}(\mathbf{a}, \theta), a_k, \theta_k\right) > 0$ by virtue of Lemma 1. Thus we have established:

Proposition 1. *The second-stage non-cooperative equilibrium of carbon-gas regulation game has the following properties:*
(i) Improvement in a country's abatement efficiency in the form of Type H, i.e., $f_{za}(z, a) > 0$ (or Type L, i..e, $f_{za}(z, a) < 0$) increases (or decreases) the own equilibrium carbon-gas emission but decreases (or increases) the other's.
(ii) An increase in a country's environment consciousness decreases the own equilibrium carbon-gas emission and increases the other's.

First-stage equilibrium. Using the second-stage equilibrium carbon-gas emission profile, the equilibrium welfare of country i in the second stage is given by:

$$
(11) \qquad v_i^N(\mathbf{a}) := f^i\left(z^{iN}(\mathbf{a}, \theta), a_i\right) - \theta_i D\left(\sum_\ell z^{\ell N}(\mathbf{a}, \theta)\right) - a_i,
$$

where $\theta = (\theta_1, \theta_2)$ is suppressed for simplicity of exposition and the function $v(\cdot)$ is employed rather than $u(\cdot)$ to distinguish from the equilibrium payoff for the one-shot simultaneous-move game in the previous section.

In the first stage, each government decides on the abatement investment so as to maximize the own national welfare given by (11). It takes into account the effect of the own investment on the second-stage emission profile $\mathbf{z}^{TN}(\mathbf{a}, \theta)$. The associated first-order condition for welfare maximization by country i's government is:

$$0=\frac{\partial v_i^N(\mathbf{a})}{\partial a_i}=f_a^i\left(z^{iN}(\mathbf{a},\theta),a_i\right)-1-\theta_i D'\left(z^{TN}(\mathbf{a},\theta)\right)\frac{\partial z^{jN}(\mathbf{a},\theta)}{\partial a_i}, \tag{12}$$

where use was made of the envelope theorem in view of (9).

Let a_i^{NS} denote the equilibrium abatement investment by country i and $z_i^{NS} := z^{iN}(\mathbf{a}^{NS},\theta)$ its associated second-stage carbon-gas emission in the present non-cooperative strategic abatement-investment game. We also let $\mathbf{a}^{NS} := (a_1^{NS}, a_2^{NS})$ the equilibrium abatement investment profile. Then comparison between (6) and (12) reveals the additional third term in (12) showing the strategic incentive to alter the first-stage equilibrium more favorable for country i herself. In view of Lemma **??**, this strategic term is positive if and only if the improvement in abatement efficiency is of type H technical progress. Thus we have established:

Proposition 2. *Each country has an incentive to invest more to improve the own abatement efficiency in the strategic abatement investment game than in the one-shot game if and only if the improvement in abatement efficiency is of type H technical progress, i.e.,* $f_{za}(z,a)>0$.

The strategic investment incentive is more easily captured in a graphical fashion for the reference symmetric case. The sub-game perfect equilibrium outcome denoted by (z_{NS}, a_{NS}) is a solution to the following set of equations

$$0=v_a^I(z,a):=f_z(z,a)-\theta D'(2z), \tag{13}$$

$$0=v_a^I(z,a):=f_a(z,a)-1-\theta D'(2z)r_z^j(z,a)\frac{\partial \hat{z}^{iN}(a,a)}{\partial a_i}. \tag{14}$$

Comparison between (7) and (8) governing the symmetric one-shot game equilibrium gives rise to

$$v_z^I(z,a)=u_z^I(z,a),$$
$$v_a^I(z,a)-u_a^I(z,a)=-\theta D'(2z)r_z^j(z,a)\frac{\partial z^{iN}(a,a)}{\partial a_i}\propto f_{za}\left(z^{iN}(a,a),a\right),$$

where the right-hand side of the second equation is positive if and only if the improvement in abatement efficiency is of type H technical progress. That is, each country has an incentive to strategically increase (or decrease) the own abatement investment under the type H (or L) technical progress so as to reduce the other country's emission of carbon gas and make the second-stage equilibrium more favorable to herself.

The results are summarized in Figures 1 and 2. The strategic abatement-investment incentive locus showing $v_a^I(z,a)=0$ is located above (or below) the one-shot one showing $u_a^I(z,a)=0$ for type H (or L) technical progress, while the carbon-gas emission incentive loci are the same for the two games, as is shown in Figures 1 and 2.

Regardless of the types in technical progress, the world gets worse off in the strategic abatement-investment game than in the one-shot decision game. This can be demonstrated in a straightforward fashion for the symmetric case as follows. Let us consider the following set of equations giving the emission-abatement pair as a function of a single parameter γ:

$$\begin{aligned} f_z(z,a) - \theta D'(2z) &= 0, \\ f_a(z,a) - 1 &= \gamma A, \end{aligned}$$

where $A := \theta D'(2z_{NS}) r_z^j(2z_{NS}, a_{NS}) \frac{\partial z^{TN}(a_{NS}, a_{NS})}{\partial a_i}$. We represent the solution by $(\tilde{z}(\gamma), \tilde{a}(\gamma))$. Undertake the comparative statics with regard to a change in γ, and obtain:

$$\begin{pmatrix} f_{zz}(z,a) - 2\theta D"(2z) & f_{za}(z,a) \\ f_{az}(z,a) & f_{aa}(z,a) \end{pmatrix} \begin{pmatrix} \tilde{z}'(\gamma) \\ \tilde{a}'(\gamma) \end{pmatrix} = \begin{pmatrix} 0 \\ A \end{pmatrix}.$$

Note that the solution for $\gamma = 0$ coincides with the equilibrium for the one-shot decision game, while the one for $\gamma = 1$ is the one for the strategic abatement-investment game. Then the above comparative statics yields:

$$\begin{aligned} \tilde{\Delta}\tilde{z}'(\gamma) &= -f_{za}(\cdot)A > 0, \\ \tilde{\Delta}\tilde{a}'(\gamma) &= A\left\{f_{zz}(\cdot) - 2\theta D"(\cdot)\right\} \propto f_{za}(\cdot), \end{aligned}$$

where $\tilde{\Delta} := (f_{zz}(\cdot) - 2\theta D"(\cdot))(f_{aa}(\cdot)) - (f_{za})^2 > 0$, by virtue of Assumption 2. We then calculate the change in the representative country's welfare:

$$\begin{aligned} & \frac{du\left(\tilde{z}(\gamma), \tilde{a}(\gamma)\right)}{d\gamma} \\ =& (f_z - 2\theta D')\tilde{z}'(\gamma) + (f_a - 1)\tilde{a}'(\gamma) \\ =& -\theta D'\tilde{z}'(\gamma) + \gamma A\tilde{a}'(\gamma) < 0. \end{aligned}$$

Proposition 3. *For the symmetric case, the world welfare is worse at the equilibrium for the strategic abatement-investment game than at the equilibrium for the one-shot decision game. At the strategic abatement-investment game, there hold two properties.*

(i) Each country's carbon gas emission is larger than in the one-shot decision game.

(ii) Each country's abatement investment is larger than in the one-shot decision game if and only if the improvement in abatement efficiency is of type H technical progress.

Again there is a remark for the case of $f_a(z,a) < 0$. As we have discussed at the end of subsection 3.1, neither country invests on abatement for the one-shot decision game. Since the strategic effect to expand the abatement investment investment works even under $f_a(z,a) < 0$, it is also possible that the world gets better off at the strategic abatement-investment game equilibrium, for each country may be willing to do such an investment desired for the world efficiency.

4. Policy Coordination for Bargaining

We have now fully characterized both ex ante and ex post non-cooperative equilibria. In this and the next sections, using the Nash bargaining solution, we shall analyze the outcome of ex post cooperation when an enforceable international agreement in the second period is possible. To describe the second-stage cooperation phase, we now define the Nash bargaining solution for the case when side payments are allowed and the case when they are not.

4.1. **Bargaining Solutions with and without Transfers.** There are two distinctly different possibilities for international agreements. First, two countries may negotiate over their respective domestic regulations without any international income transfer. Agreement reached through such a negotiation may be described by the Nash bargaining solution for the game without transfers (i.e., side payments). Alternatively, two countries may negotiate over domestic regulations with transfer of incomes as an additional term in negotiation. For example, the negotiation may be carried out over total amount of and its initial distribution of tradable permits. In this case, transfer payment will be realized in the form of either receipts from the sales of or expenditures for the purchase of permits. The corresponding solution concept will be the Nash bargaining solution for the game with side payments.

Whether transfers are available or not, we may discuss the basic structure of the bargaining problem using Figure 3. The area $F_1 0 F_2$ shows the set of all payoff allocations that would be achieved by an international coordination without side payments given the abatement investment profile $\mathbf{a}$, i.e.,

$$\mathcal{F}(\mathbf{a}) := \left\{ (u_1, u_2) |\, u_i = f^i(z_i, a_i) - \theta_i D\left(\sum_k z_k\right) - a_i \right\}. \tag{15}$$

Since each country's welfare function is strictly concave in the emission profile, it is straightforward to prove that the feasible set $\mathcal{F}(\mathbf{a})$ is also strictly convex. And its outer boundary F_1F_2 , which is strictly concave towards the origin, represents the set of Pareto-efficient payoff pairs given the abatement investment profile, and we call it the *before-transfer Utility Possibility Frontier (UPF).* Let $w_i = u_i - v_i^N(\mathbf{a})$ represent country i's net gains from participating in the bargaining, which we call country *i's bargaining surplus.* Since any possible bargaining should assign each country the payoff level feasible and not smaller what she gets at the disagreement point, the second-stage equilibrium bargaining surplus of each country should belong to the following acceptable set given the abatement investment profile $\mathbf{a}$.

$$\mathcal{A}(\mathbf{a}) := \left\{ (w_1, w_2) |\, w_i = u_i - v_i^N(\mathbf{a}), (u_1, u_2) \in \mathcal{F}(\mathbf{a}) \right\} \cap \Re^2_+. \tag{16}$$

Then the second period bargaining game is then characterized by the following two factors.

(i): Acceptable set $\mathcal{A}(\mathbf{a})$

(ii): Availability of side payments

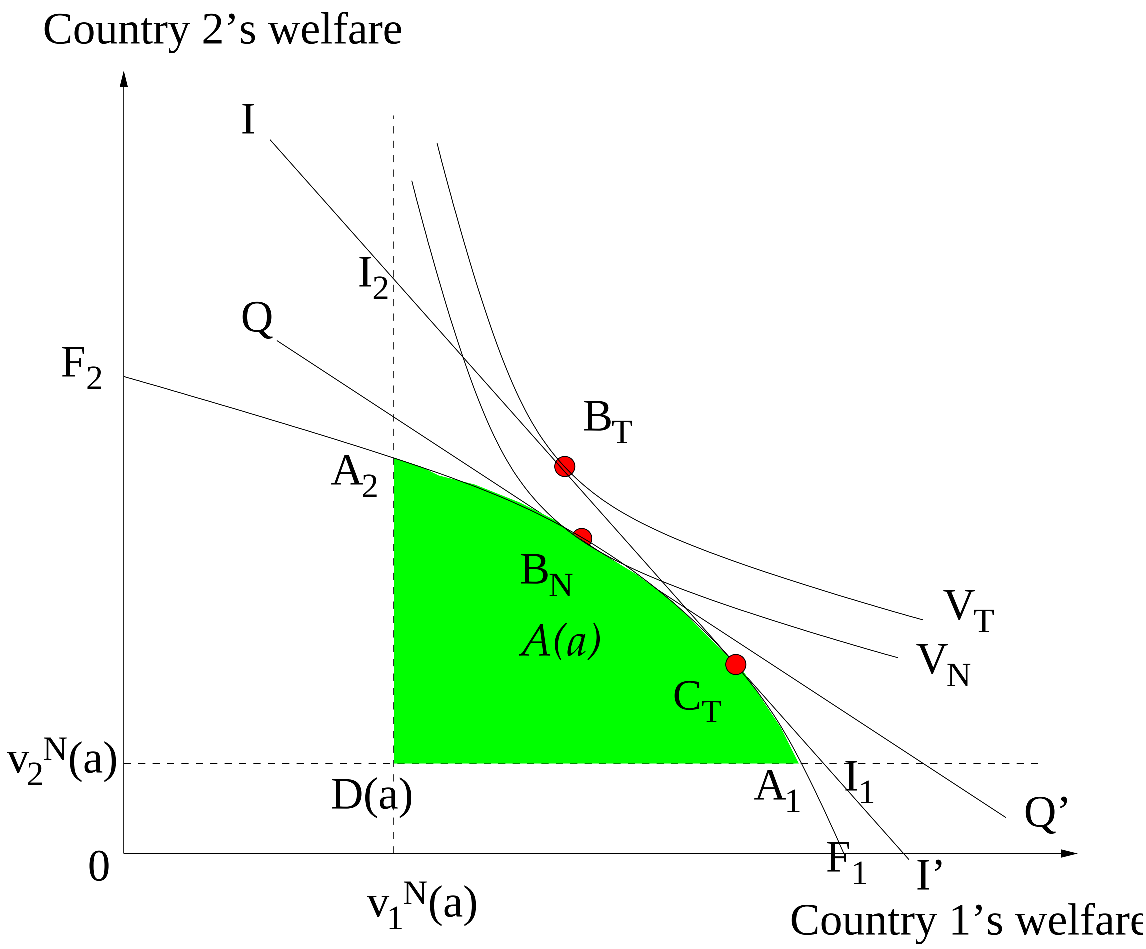

FIGURE 3. Bargaining and Solutions

Then these two factors come to determine the *bargaining set* $\Omega(\mathbf{a})$ given the abatement investment profile which shows the set of the bargaining surpluses of both countries as possible candidates for the bargaining outcome. Given the bargaining set (which we shall specify below for each type of bargaining), the set of axioms for Nash bargaining requires the resulting bargaining surplus pair (w_1^e, w_2^e) should be a solution to

$$\max_{\{w_1,w_2\}} V := \sum_{i=1,2} \ln w_i \quad \text{subject to} \quad (w_1, w_2) \in \Omega(\mathbf{a}), \tag{17}$$

where

$$V = \sum_{i=1,2} \ln w_i, \tag{18}$$

represents what we may call the *Nash bargaining-value* function, equivalent to the familiar Nash product function $w_1 w_2$.[8]

Since we have characterized the bargaining solutions, let us make more specific inquiry into the equilibrium for each type of bargaining. As the bargaining with side payments is already familiar and easier to characterize, we discuss it first.

4.2. **Bargaining with Transfers.** The bargaining solution with side payments requires each country to pay transfers to the other. Let t_i denote country i's transfer payment, and w_i' its bargaining surplus after the bargaining with transfers. Then coordination on the emissions coupled with transfer payments yields:

$$w_i' = w_i - t_i,$$

where the world budget constraint on transfer payments requires

$$t_1 + t_2 = 0.$$

The two equations yields the following single feasible constraint for the bargaining:

$$\sum_i w_i' = \sum_i w_i,$$

for each $(w_1, w_2) \in \mathcal{A}(a)$. Thus the bargaining set for the bargaining with transfers, denoted by $\Omega_T(\mathbf{a})$ is given by

$$\Omega_T(\mathbf{a}) := \left\{ (w_1', w_2') \Big| \sum_{i=1,2} w_i' = \sum_{i=1,2} w_i, (w_1, w_2) \in \mathcal{A}(\mathbf{a}) \right\}.$$

The pair of bargaining surpluses as the equilibrium outcome should then be a solution to

$$\max_{\{w_1', w_2'\}} \left\{ \sum_{i=1,2} \ln w_i' \middle| \left(w_1', w_2'\right) \in \Omega_T(\mathbf{a}) \right\},$$

or equivalently

$$\max V = \sum_{i=1,2} \ln w_i' \text{ subject to } (i) \ \sum_{i=1,2} w_i' = \sum_{i=1,2} w_i, \ (ii) \ (w_1, w_2) \in \mathcal{A}(\mathbf{a}).$$

The solution is easily obtained by using the familiar two steps in Figure 3. First, maximize the total bargaining surplus $\sum_i w_i$ over the acceptable set $\mathcal{A}(\mathbf{a})$. This is achieved at point

[8]This function is sometimes called the Nash social-welfare function.

C_T where the iso-total-bargaining-surplus curve named II' with slope minus unity touches the acceptable set. Given the maximized value of the total bargaining surplus, we obtain the region $I_1 D(\mathbf{a}) I_2$ as the set of bargaining with transfers, Ω_T. The second step is to choose a bargaining surplus pair along the maximized total surplus curve II' to maximize the Nash bargaining-value function. The point is shown by point B_T where the line II' touches the highest iso-bargaining-value curve V_T.

One should note that this solution of bargaining with transfers has very appealing features for characterization. That is, when one views each country's bargaining surplus as consumption of good called "country i", then the Nash bargaining-value function serves as the standard individual utility function in a Cobb-Douglas form. Availability of transfers makes the bargaining constraint just the same as the standard budget line with equal prices of unity where the maximized total surplus serves as the total available income. Then the bargaining solution coincides with the optimal consumption point, so that the expenditure for each good is just a half of the total income, i.e., the bargaining makes each country get just a half of the total bargaining surplus.

4.3. **Bargaining without Transfers.** When transfers are not available, then the bargaining set for the second stage, denoted by $\Omega_N(\mathbf{a})$, coincides with the acceptable set, i.e.,

$$\Omega_N(\mathbf{a}) = \mathcal{A}(\mathbf{a}). \tag{19}$$

Thus our bargaining problem is described by

$$\max_{\{w_1', w_2'\}} V = \sum_{i=1,2} \ln w_i' \text{ subject to } \left(w_1', w_2'\right) \in \Omega_N(\mathbf{a}) \left(= \mathcal{A}(\mathbf{a})\right).$$

The resulting bargaining outcome, denoted by $\left(w_1^N, w_2^N\right)$ is easy to obtain in Figure 3. It is described as point B_N where the iso-bargaining value curve named V_N touches the acceptable set.

4.4. **Features of Bargaining Solutions.** What we are interested in is how each government decides on her abatement investment when facing each type of bargaining at the second stage. Rather than discussing comparative statics for each bargaining problem separately, we propose a new unified method of comparative statics for bargaining in the next section.

The point is how we characterize the equilibrium outcome for each type of bargaining. As is shown by the equilibrium outcome B_N for the bargaining without transfers in Figure 3, the outcome is characterized by the bargaining frontier touching the iso-bargaining value curve. The slope of the common tangent line QQ' determines the valuation of country 1's bargaining surplus relative to country 2's. This plays the role of price in deciding on the equilibrium bargaining outcome. In fact, once we notice the role of the relative valuation on the countries' bargaining surpluses, it becomes greatly easier to trace the properties governing each type of bargaining game.

Shadow Valuation of Bargaining Surplus

Let q_i denote what we may call the *shadow valuation* of country i's bargaining surplus, $q_r := \frac{q_1}{q_2}$ measure the shadow valuation of country 1's bargaining surplus relative to country 2's, and $\mathbf{q} := (q_1, q_2)$ the shadow valuation vector. What we note are the following features underlying our bargaining outcomes.

Total Weighted Bargaining Surplus & Bargaining Contributions

First, whether transfers are available or not, the equilibrium bargaining outcome requires both countries to coordinate their carbon-gas emissions so as to maximize the world total bargaining value weighted by the shadow valuations. More specifically, given the shadow valuation vector, let $\sum_{i=1,2} q_i w_i$ represent the *total weighted bargaining surplus*. In Figure 3, the line II' with $\mathbf{q} = \mathbf{1}\,(= (1,1))$ shows an *iso-total-weighted-bargaining-surplus curve* for the bargaining with transfers and the line QQ' the one for the bargaining without transfers where the relative shadow valuation of country 1's bargaining surplus is equal to its slope. The coordinated actions between the countries first realizes the point on the before-transfer utility possibility frontier F_1F_2 that maximizes the weighted total bargaining surplus, i.e., point C_T for the bargaining with transfers and point B_N for the one without transfers. We call the associated bargaining surplus of each country her *bargaining contribution*, for it decides the size of the pie (= the weighted total bargaining surplus) divided between the two countries at the table of bargaining.

Bargaining Rewards

Second, given the maximized weighted total bargaining surplus and the shadow valuation vector, the Nash bargaining rule proposes the bargaining-surplus pair, as the final bargaining outcome, that maximizes the Nash bargaining value. We call this final bargaining surplus assigned to each country her *bargaining reward*, for it represents what she finally obtains from policy coordination through bargaining. The outcome is shown by point B_T in the presence of transfers, and the difference between the bargaining contribution C_T and the bargaining reward B_T shows the required transfers between the two countries. And when transfers are unavailable, point B_N shows the bargaining rewards. What is crucial here is the role of adjustment in the shadow valuations for the bargaining without transfers in determining each country's bargaining reward. Since transfers are not allowed, i.e., the contributions should be the same as the rewards for both countries, so must be adjusted the shadow valuations.

Any change in either country's abatement investment alters not only the disagreement point D but also the before-transfer utility possibility frontier, leading to a change in the acceptable set for the second-period bargaining. As we will show in the next section, the associated comparative statics is easier by applying the duality approach in international trade. Let us now formalize the present approach.

5. Duality Approach to Nash Bargaining

We first describe the bargaining game more formally within the present framework of the model. The following two functions play critical roles in the following analysis.

5.1. **Maximum Bargaining Surplus Function.** The first is the *maximum bargaining surplus function.* It is defined as the maximum total weighted bargaining surplus given the shadow valuation $\mathbf{q} = (q_1, q_2)$ and the abatement investment profile $\mathbf{a} = (a_1, a_2)$ as follows.

$$S(\mathbf{q}, \mathbf{a}) := \max_{\{\mathbf{w}\}} \sum_i q_i w_i \text{ subject to } (w_1, w_2) \in \mathcal{A}(\mathbf{a}).$$

or alternatively

$$S(\mathbf{q}, \mathbf{a}) := \max_{\{\mathbf{z}\}} \sum_i q_i \left(f(z_i, a_i) - \theta D \left(\sum_j z_j \right) - a_i - v_i^N(\mathbf{a}) \right), \tag{20}$$

which serves as the GDP function in the standard trade theory.

Let us denote by $\mathbf{z}^C(\mathbf{q}, \mathbf{a}) := \left(z_1^C(\mathbf{q}, \mathbf{a}), z_2^C(\mathbf{q}, \mathbf{a}) \right)$ the solution to the above maximization problem and by $w^C(\mathbf{q}, \mathbf{a}) := \left(w_1^C(\mathbf{q}, \mathbf{a}), w_2^C(\mathbf{q}, \mathbf{a}) \right)$ the associated bargaining surplus profile. By definition, we have

$$w_i^C(\mathbf{q}, \mathbf{a}) = f(z_i^C(\mathbf{q}, \mathbf{a}), a_i) - \theta D \left(z_T^C(\mathbf{q}, \mathbf{a}) \right) - a_i - v_i^N(\mathbf{a}), \tag{21}$$

where $z_T^C(\mathbf{q}, \mathbf{a}) := \sum_{i=1,2} z_i^C(\mathbf{q}, \mathbf{a})$ denotes the world total emissions associated with $\mathbf{z}^C(\mathbf{q}, \mathbf{a})$. Now it is straightforward to establish that $\mathbf{w}^C(\mathbf{q}, \mathbf{a})$ satisfies

$$w_i^C(\mathbf{q}, \mathbf{a}) = S_i(\mathbf{q}, \mathbf{a}) \left(:= \frac{\partial S(\mathbf{q}, \mathbf{a})}{\partial q_i} \right) \ (i = 1, 2), \tag{22}$$

$$\frac{\partial w_i^C(\mathbf{q}, \mathbf{a})}{\partial q_i} = S_{ii}(\mathbf{q}, \mathbf{a}) \left(:= \frac{\partial^2 S(\mathbf{q}, \mathbf{a})}{\partial q_i^2} \right) > 0 \ (i = 1, 2), \tag{23}$$

i.e., $w_i^C(\mathbf{q}, \mathbf{a})$ serves just the same as the competitive output supply function, where the first result is nothing but Shephard's lemma.[9] And in the present framework of bargaining theory we call it country i's bargaining-contribution function, for it represents what she can contribute through the coordination in emissions for those at the bargaining table.

5.2. **Minimum Bargaining Cost Function.** The second tool is what we may call the *minimum bargaining-cost function*, a counterpart for the minimum expenditure function in the consumer theory. Let V the Nash bargaining value. Then the *minimum bargaining cost function* $E(\mathbf{q}, V)$ where $\mathbf{q} = (q_1, q_2)$ is defined as below:

[9]Note that all these properties hold by virtue of the strict convexity of the acceptable set $\mathcal{A}(\mathbf{a})$.

$$E(q,V) := \min_{\{\mathbf{w}\}} \left\{ \sum_i q_i w_i \,\middle|\, \sum_i \ln w_i \geq V \right\}, \tag{24}$$

which shows the minimum total weighted valuations, given the shadow valuation of each country's bargaining surplus, ensuring at least the Nash bargaining value V for those at the bargaining table. It is straightforward to ascertain that the function is given the following specific form

$$E(q,V) = 2\exp\left(\frac{1}{2}\left\{V + \sum_i \ln q_i\right\}\right). \tag{25}$$

This minimum bargaining-cost function shares all the same properties with the standard minimum expenditure cost function associated with the symmetric Cobb-Douglas utility function. Let $w^R(\mathbf{q},V) := \left(w_1^R(\mathbf{q},V), w_2^R(\mathbf{q},V)\right)$ denote the solution to the above constrained expenditure minimization problem. Then it shows the bargaining surplus assigned to country i so as to yield the Nash bargaining value as much as V at the bargaining table, which in this sense represents her bargaining reward $w_i^R(\mathbf{q},V)$ defined in the previous section. By virtue of the standard duality theory coupled with (25), it is straightforward to establish

$$w_i^R(\mathbf{q},V) = E_i(\mathbf{q},V)\left(:= \frac{\partial E(\mathbf{q},V)}{\partial q_i}\right) = \frac{E(\mathbf{q},V)}{2q_i} \; (i=1,2), \tag{26}$$

$$\frac{\partial w_i^R(\mathbf{q},V)}{\partial q_i} = E_{ii}(\mathbf{q},V) < 0, \tag{27}$$

$$\frac{\partial w_i^R(\mathbf{q},V)}{\partial V} = E_{iV}(\mathbf{q},V) > 0, \tag{28}$$

$$E_V(\mathbf{q},V) = \frac{1}{2}E(\mathbf{q},V). \tag{29}$$

5.3. **General Forms of Bargaining Solution.** Using the two functions above, let us characterize the properties governing each bargaining solution. Let $v_i^{Bk}(\mathbf{a})(k=T,N)$ denote country i's welfare achieved by the bargaining where T represents the bargaining with transfers and N the one without transfers. Then, as discussed in the previous section, the bargaining outcomes are easily characterized as below.

♡ **Bargaining solution $\mathbf{v}^{BT}(\mathbf{a}) = \left(v_1^{BT}(\mathbf{a}), v_2^{BT}(\mathbf{a})\right)$ with transfers:**

$$v_i^{BT}(\mathbf{a}) = E_i(\mathbf{1}, V_{BT}) + v_i^N(\mathbf{a}) \; (i=1,2), \tag{30}$$

$$E(\mathbf{1}, V_{BT}) = S(1,\mathbf{a}), \tag{31}$$

where V_{BT} shows the equilibrium Nash bargaining value for the bargaining with transfers. (31) determines first V_{BT}, and then each country's welfare through (30). Put differently, given the equal shadow valuations, each country's bargaining contribution is determined so as to maximize the total weighted bargaining surplus. Given this weighted surplus as well the equal shadow valuations each country's reward is assigned so as to maximize the Nash bargaining value. The difference between the contribution and the reward represents the country's net transfer to the other. Since the equilibrium Nash bargaining value depends on the abatement investment profile $\mathbf{a}$, we make its explicit expression by $V^{BT}(\mathbf{a})$.

$\diamondsuit$ **Bargaining solution $\mathbf{v}^{BN}(\mathbf{a}) = \big(v_1^{BN}(\mathbf{a}), v_2^{BN}(\mathbf{a})\big)$ without transfers:**

$$v_i^{BN}(\mathbf{a}) = E_i(\mathbf{q}^{BN}, V_{BN}) + v_i^N(\mathbf{a}) \ (i = 1, 2), \tag{32}$$

$$E_i(\mathbf{q}^{BN}, V_{BN}) = S_i(\mathbf{q}^{BN}, \mathbf{a}) \ (i = 1, 2), \tag{33}$$

$$E(\mathbf{q}^{BN}, V_{BN}) = S(\mathbf{q}^{BN}, \mathbf{a}), \tag{34}$$

where $\mathbf{q}^{BN} := (q_1^{BN}, q_2^{BN})$ represents the equilibrium shadow valuation vector and V_{BN} the equilibrium Nash bargaining value for the present bargaining. The third condition (34) shows the same as the budget constraint facing the two countries at the bargain given the shadow valuation vector $\mathbf{q}^{BN}$. The shadow valuation vector is adjusted to equate each country's bargaining reward $E_i(\mathbf{q}^{BN}, V_{BN})$ with the own contribution $S_i(\mathbf{q}^{BN}, \mathbf{a})$, i.e., the second condition (**??**). At a first glance the two equations contain three unknowns, q_1^{BN}, q_2^{BN} and V_{BN}. However as is clear from the construction of the minimum bargaining-cost function (24) and the maximum bargaining-surplus function (20), the two functions are linear homogeneous in the shadow valuations, so that only the relative valuation q_1/q_2 matters. Thus the two conditions can be solved for the relative valuation $q_r^{BN} = q_1^{BN}/q_2^{BN}$ and the Nash bargaining value V_{BN}, which depends again on the abatement investment profile. We express the relations by $q_r^{BN}(\mathbf{a})$ and $V^{BN}(\mathbf{a})$.

We have completed all the necessary formulation for comparative statics of the bargaining outcomes with respect to the change in the abatement investment profile. Let us advance to this exercise first for the bargaining with transfers.

5.4. **Strategic Decomposition of Pre-Bargaining Investment Effects.** Since the two types of bargaining games differ only with respect to the requirement $q_1 = q_2 = 1$, we may first analyze the bargaining game without transfers and reinterpret the results given the equal shadow-valuation requirement. Then our task is to undertake comparative statics for the set of equations (**??**) and (34) with respect to a change in country i's abatement investment. Since the maximum bargaining-surplus function and the minimum bargaining-cost function are linear-homogeneous in the shadow valuations, we may set $q_j = 1$ throughout the analysis without loss of generality. Fortunately, we can do our task step by step, first obtaining from

(34) the change in the equilibrium Nash bargaining value and then deriving from (**??**) the change in the relative shadow valuation of country i, i.e.,q_i.

Let $q_i^B(\mathbf{a})$ denote the equilibrium relative valuation of country i's bargaining surplus and $V^B(\mathbf{a})$ the equilibrium Nash bargaining value. Total differentiation of (34) yields

$$(S_i - E_i)\frac{\partial q^B(\mathbf{a})}{\partial a_i} - E_V\frac{\partial V^B(\mathbf{a})}{\partial a_i} + \frac{\partial S}{\partial a_i} = 0, \tag{35}$$

which is reduced to

$$\frac{\partial V^B(\mathbf{a})}{\partial a_i} = \frac{2}{S}\frac{\partial S\left(\mathbf{q}^B(\mathbf{a}), \mathbf{a}\right)}{\partial a_i}, \tag{36}$$

where the first term in (35) vanished whether transfers are available or not, [10]and use was made of (29) and (34). Thus (36) gives rise to

Proposition 4. *Irrespective of transfer availability at the bargaining, an increase in country i's abatement investment increases the equilibrium Nash bargaining value if and only if it increases the maximum bargaining-surplus evaluated at the initial shadow valuations.*

When the Nash bargaining value increases along with country i's abatement investment, it implies that the investment expand the bargaining frontier at least locally around the initial bargaining outcome. For this reason, the effect on the Nash bargaining value expressed by (36) may be called the *bargaining-frontier expansion effect.* Proposition 4 implies that country i's investment increase enhances the potential gains from bargaining if and only if it brings the positive frontier expansion effect.

By virtue of the bargaining-frontier expansion effect, (**??**) gives rise to

$$(S_{ii} - E_{ii})\frac{\partial q_i^B(\mathbf{a})}{\partial a_i} = \frac{E}{4q_i^B(\mathbf{a})}\frac{\partial V^B(\mathbf{a})}{\partial a_i} - \frac{\partial^2 S\left(\mathbf{q}^B(\mathbf{a}), \mathbf{a}\right)}{\partial a_i \partial q_i}.$$

In view of (22) and (26), the above equation can be rewritten as

$$\frac{\partial q_i^B(\mathbf{a})}{\partial a_i} = \frac{1}{S_{ii} - E_{ii}}\left\{\frac{1}{2q_i^B(\mathbf{a})}\frac{\partial S\left(\mathbf{q}^B(\mathbf{a}), \mathbf{a}\right)}{\partial a_i} - \frac{\partial^2 S\left(\mathbf{q}^B(\mathbf{a}), \mathbf{a}\right)}{\partial a_i \partial q_i}\right\}, \tag{37}$$

where use was made of (36). Since the bargaining-surplus function is linear homogeneous in the shadow valuations, it should satisfy

[10] $S_i - E_i = 0$ holds by virtue of (**??**) in the absence of transfers, while $\partial q_i^B(\mathbf{a})/\partial a_i = 0$ in their presence by virtue of the requirement $q_1 = q_2 = 1$.

$$\frac{\partial S(\mathbf{q},\mathbf{a})}{\partial a_i} = \sum_{k=1,2} q_k \frac{\partial^2 S(\mathbf{q},\mathbf{a})}{\partial a_i \partial q_k}.$$

By virtue of this relation, one can rewrite (37) as below.

$$(38)\quad \frac{\partial q_i^B(\mathbf{a})}{\partial a_i} = \frac{1}{2q_i^B(\mathbf{a})\left(S_{ii}-E_{ii}\right)} \left\{ q_j^B(\mathbf{a}) \frac{\partial^2 S\left(\mathbf{q}^B(\mathbf{a}),\mathbf{a}\right)}{\partial a_i \partial q_j} - q_i^B(\mathbf{a}) \frac{\partial^2 S\left(\mathbf{q}^B(\mathbf{a}),\mathbf{a}\right)}{\partial a_i \partial q_i} \right\},$$

which establishes

Proposition 5. *More investment by country i before the bargaining lowers the own relative valuation if and only if the associated own increase in the bargaining contribution is outweighed by the other country's.*

Analogy from the effects of growth in trade, the meaning of the above proposition should be clear.

Since the denominator on the right hand side is always strictly positive, country i's relative valuation increases along with an increase in her investment if and only if the numerator is positive. Thus when her bargaining reward increases over her own bargaining contribution, her valuation will be raised. Since the efficient bargaining requires the smaller bargaining surplus assigned to the country with the greater valuation, the country will obtain smaller net gains from bargaining when her valuation becomes larger. Put differently, each country has an incentive to strategically alter the own relative valuation and make the bargaining outcome more favorable to herself. For this reason, we call the effect expressed by (37) the *strategic valuation effect.*

Using the two effects, we may finally obtain the change in country i's welfare. Total differentiation of (32) gives rise to

$$(39)\quad \frac{\partial v_i^B(\mathbf{a})}{\partial a_i} = \frac{\partial v_i^N(\mathbf{a})}{\partial a_i} + \frac{1}{2q_i^B(\mathbf{a})} \frac{\partial S\left(q^B(\mathbf{a}),\mathbf{a}\right)}{\partial a_i} + E_{ii}\left(q^B(\mathbf{a}), V^B(\mathbf{a})\right) \frac{\partial q_i^B(\mathbf{a})}{\partial a_i},$$

where use was made of (29) and (36).

The effect of an increase in the abatement investment can thus be decomposed into three factors. First, the non-cooperative strategic effect (the first term on the right hand side), second the bargaining-frontier expansion effect, and lastly the strategic valuation effect. Of course, the last strategic valuation effect vanishes for the bargaining with transfers. Note that the above decomposition of the unilateral investment increase is in fact very general and can be applied to other bargaining problems. Thus we summarize it in the following theorem.

Theorem 1. *Each country has the greater incentive of abatement investment than in the strategic investment game without the second-stage bargaining when her investment expands* *the bargaining-frontier and lowers the own relative valuation. More specifically the following is the sufficient condition for country i's pre-bargaining investment to exceed its level in the absence of bargaining.*

$$\begin{array}{lll} (i) & \frac{\partial S\left(q^B(\mathbf{a}),\mathbf{a}\right)}{\partial a_i} & > 0, \\ (ii) & q_i^B(\mathbf{a})\frac{\partial^2 S\left(\mathbf{q}^B(\mathbf{a}),\mathbf{a}\right)}{\partial a_i \partial q_i} & > q_j^B(\mathbf{a})\dfrac{\partial^2 S\left(\mathbf{q}^B(\mathbf{a}),\mathbf{a}\right)}{\partial a_i \partial q_j} \end{array}$$

Since the relative valuation is fixed constant for the bargaining with transfers, the following corollary is straightforward.

Corollary 1. *Insofar as we confine ourselves to symmetric equilibria, compared with the case in the absence of the ex-post bargaining, country i has an incentive to make greater abatement investment if and only if the bargaining-frontier expansion effect is strictly positive, i.e.,* $\frac{\partial S(\mathbf{1},\mathbf{a})}{\partial a_i} > 0$.

The present decomposition for the bargaining outcome reminds us of Miyazaki's Slutky decomposition of the bargaining solution. Our analysis extends his result further. Let us now inquire into the size of each effect and their sum.

5.5. **Bargaining-Frontier Expansion Effect.** Although we have made clear the effects of strategic pre-bargaining investment, what we are rather interested in is the sign of each effect governing the total effect on each country's welfare. So let us first discuss the size of the bargaining-frontier expansion effect.

In view of (20), the effect is further rewritten as

$$\begin{aligned} \frac{\partial S(\mathbf{q},\mathbf{a})}{\partial a_i} &= q_i \left\{ f_a\left(z^{iC}(\mathbf{q},\mathbf{a}),a_i\right) - f_a\left(z^{iN}(\mathbf{a}),a_i\right)\right\}, \\ & \quad +D'\left(z^{TN}(\mathbf{a})\right)\left\{ q_i\theta_i \frac{\partial z^{jN}(\mathbf{a})}{\partial a_i} + q_j\theta_j\frac{\partial z^{iN}(\mathbf{a})}{\partial a_i}\right\} \end{aligned}$$

which, in the case of symmetric equilibrium, is further reduced to

$$(40) \qquad \frac{\partial S(\mathbf{1},\mathbf{a})}{\partial a_i} = \left\{ f_a\left(z^{iC}(\mathbf{1},\mathbf{a}),a_i\right) - f_a\left(z^{iN}(\mathbf{a}),a_i\right)\right\} + \theta D'\left(z^{TN}(\mathbf{a})\right)\frac{\partial z^{TN}(\mathbf{a})}{\partial a_i}.$$

where $z^{iC}(\mathbf{a})$ (or $z^{iN}(\mathbf{a})$) is the carbon-gas emissions of country i at the symmetric bargaining outcome (or the second-stage symmetric non-cooperative equilibrium) and θ denotes the environment consciousness common to both countries.

The first bracketed term shows the *productivity enhancement* effect of country i's investment due to the coordination in carbon-gas emissions at the second-stage bargaining. The second term shows the world marginal external damage caused by country i's investment.

Its addition to the bargaining surplus means that the bargaining at the second stage internalizes those damages. For this reason, we may call it the *externality internalization* effect of bargaining. Since there holds $z^{iC}(\mathbf{1}, \mathbf{a}) < z^{iN}(\mathbf{a})$, i.e., excessive emission of carbon gas at the non-cooperative equilibrium, the first productivity enhancement effect is negative but the second coordination effect positive if and only if $f_{az}(z, a) > 0$. Thus the sign of the left hand side is generally ambiguous.

Fortunately one can rewrite each term on the right hand side of(40) and gain its further implication of the relation. For this purpose, we first define the function $z^\star(t, a, \theta)$ as a solution to $f_z\left(z^\star(t, a, \theta), a\right) = t\theta D'(2z^\star(t, a, \theta))$and we denote its elasticity with respect to t by $\eta(t, a, \theta) := -\frac{\partial \ln z^\star(t,a,\theta)}{\partial \ln t}$.

Note that this function $z^\star(t)$ satisfies $z^\star(1) = z^{iN}(a, a)$ (the symmetric non-cooperative equilibrium emissions) and $z^\star(2) = z^{iC}(1, 1, a, a)$ (the symmetric cooperative equilibrium emissions). Here t serves as the coordinated emission tax rate. At the non-cooperative game equilibrium, each country cares herself only, so that the emission tax rate is set equal to 100% of the marginal environmental damage $\theta D'(z_T)$. However once the two countries bargain to achieve the efficient allocation as a group, each must additionally tax on the emissions. The resulting tax rate on the marginal damage is now 200%. Thus $\eta(t, a, \theta)$ denotes the elasticity of the individual country's emissions with respect to the emission tax rate t.

Using this elasticity $\eta(t, a, \theta)$, one obtains the following alternative expression for the externality internalization effect.[11]

$$(41) \qquad \theta D'\left(z^{TN}(\mathbf{a})\right) \frac{\partial z^{TN}(\mathbf{a})}{\partial a_i} = f_{za}\left(z^{iN}(\mathbf{a}), a\right) z^{iN}(\mathbf{a})\eta(1),$$

which establishes

Lemma 2. *The externality internalization effect is positive if and only if* $f_{za}\left(z^{iN}(\mathbf{a}), a\right) > 0$.

Unlike the externality internalization effect, the first term of productivity enhancement effect is harder to rewrite. However when $f_a(z, a)$ is convex (or concave) in the emissions z, there holds

$$(42)\ f_a\left(z^{iC}(\mathbf{1}, \mathbf{a}), a_i\right) \quad -f_a\left(z^{iN}(\mathbf{a}), a_i\right) \geq \text{ (or } \leq \text{)} \quad \left(z^{iC}(\mathbf{1}, \mathbf{a}) - z^{iN}(\mathbf{a})\right) f_{az}\left(z^{iN}(\mathbf{a}), a_i\right),$$

Particularly, when $f_a(z, a)$ is linear in z, i.e., $f_{azz}(z, a) = 0$ for $\forall(z, a)$, there holds

$$(43) \qquad f_a\left(z^{iC}(\mathbf{1}, \mathbf{a}), a_i\right) - f_a\left(z^{iN}(\mathbf{a}), a_i\right) = \left(z^{iC}(\mathbf{1}, \mathbf{a}) - z^{iN}(\mathbf{a})\right) f_{az}\left(z^{iN}(\mathbf{a}), a_i\right),$$

In view of $z^{iC}(\mathbf{1}, \mathbf{a}) < z^{iN}(\mathbf{a})$, the discussion above establishes

[11] See the proof in Appendix B.

Lemma 3. *The productivity enhancement effect satisfies the following properties.*

(i) Given $f_{azz}(z,a) \geq 0$ for $\forall(z,a)$, the effect is strictly positive if $f_{az}\left(z^{iN}(\mathbf{a}), a_i\right) < 0$.

(ii) Given $f_{azz}(z,a) \leq 0$ for $\forall(z,a)$, the effect is strictly negative if $f_{az}\left(z^{iN}(\mathbf{a}), a_i\right) > 0$.

(iii) When $f_a(z,a)$ is linear in z, i.e., $f_{azz}(z,a) = 0$ for $\forall(z,a)$, the effect is strictly positive (or negative) if there hold both $f_{az}(z,a) <$ (or $>$) 0 and $\eta(1,a,\theta) \geq 1$.

The two lemmas 2 and 3 shows a simpler characterization of the bargaining-frontier expansion effect for the case of $f_a(z,a)$ being linear in z. Since (41) and (43) give rise to

$$\frac{\partial S(\mathbf{1},\mathbf{a})}{\partial a_i} = f_{za}\left(z^{iN}(\mathbf{a}), a\right) z^{iN}(\mathbf{a}) \left(\frac{z^{iC}(\mathbf{1},\mathbf{a})}{z^{iN}(\mathbf{a})} + \eta(1,a,\theta) - 1\right),$$

the following proposition readily holds.

Proposition 6. *Suppose that $f_a(z,a)$ is linear in z, i.e., $f_{azz}(z,a) = 0$ for $\forall(z,a)$. Then given $f_{az}(z,a) >$ (or $<$)0 for $\forall(z,a)$, the bargaining-frontier expansion effect is strictly positive (or negative) when $\eta(1,a,\theta) \geq 1$, i.e., the elasticity of the total emissions with respect to the emission tax rate is not smaller than unity at the non-cooperative second-stage equilibrium.*

Even when $f_a(z,a)$ is not linear in z, one can further specify the conditions governing the sign of the bargaining-frontier expansion effect. For instance, suppose that $f_a(z,a)$ is strictly convex in z. Then (42) implies

$$\begin{aligned} f_a\left(z^{iC}(\mathbf{1},\mathbf{a}), a_i\right) - f_a\left(z^{iN}(\mathbf{a}), a_i\right) &> \left(z^{iC}(\mathbf{1},\mathbf{a}) - z^{iN}(\mathbf{a})\right) f_{az}\left(z^{iN}(\mathbf{a}), a_i\right) \\ &= f_{az}\left(z^{iN}(\mathbf{a}), a_i\right)\left(z^\star(2) - z^\star(1)\right) \\ &= f_{az}\left(z^{iN}(\mathbf{a}), a_i\right) \int_1^2 z_t^\star(t)dt \\ &= -f_{az}\left(z^{iN}(\mathbf{a}), a_i\right) \int_1^2 \frac{z^\star(t)\eta(t,a,\theta)}{t} dt, \end{aligned}$$

which implies

$$\frac{\partial S(\mathbf{1},\mathbf{a})}{\partial a_i} > f_{za}\left(z^{iN}(\mathbf{a}), a_i\right)\left\{z^\star(1)\eta(1,a,\theta) - \int_1^2 \frac{z^\star(t)\eta(t,a,\theta)}{t}dt\right\}. \tag{44}$$

Since the inequality above gets reversed for $f_{azz}(z,a) < 0$ and $z^{iC}(\mathbf{1},\mathbf{a}) = z^\star(2) < z^\star(1) = z^{iN}(\mathbf{a})$, we obtain

Proposition 7. *Suppose that $f_{azz}(z,a) >$ (or $<$)0 for $\forall(z,a)$. Then the bargaining-frontier expansion effect is strictly positive (or negative) when there hold both $f_{az}(z,a) \geq$ (or $\leq$)0 and $\eta(1,a,\theta) \geq \frac{\eta(t,a,\theta)}{t}$ for $\forall t \in [1,2]$.*

As we have already summarize in Corollary 1, each country's abatement investment incentive hinges only on the bargaining-frontier expansion effect in the bargaining with transfers.

Thus we focus our attention on the bargaining without transfers and explore the role of strategic valuation effects.

5.6. **Strategic Effects in the Absence of Transfers.** As with the bargaining without transfers, the solution depends not only on the Nash bargaining-value V_{BN} but also on the shadow valuation vector $\mathbf{q}^{BN}$, both of which depend on the abatement investment profile $\mathbf{a}$. We describe their relations by $\mathbf{q}^{BN}(\mathbf{a})$ and $V^{BN}(\mathbf{a})$. Furthermore, since the solution depends only on the relative shadow valuations, we focus our attention only on the change of q_i given $q_j = 1$. And put (37) into (39), and obtain the following general expression for the pre-bargaining investment incentive over the non-cooperative strategic investment game.

$$\frac{\partial v_i^B(\mathbf{a})}{\partial a_i} - \frac{\partial v_i^N(\mathbf{a})}{\partial a_i} = \frac{1}{S_{ii}(\mathbf{1},\mathbf{a}) - E_{ii}\left(\mathbf{1}, V^B(\mathbf{a})\right)}\left[\frac{S_{ii}(\mathbf{1},\mathbf{a})}{2}\frac{\partial S(\mathbf{1},\mathbf{a})}{\partial a_i} + \{-E_{ii}(\mathbf{1},\mathbf{a})\}\frac{\partial^2 S(\mathbf{1},\mathbf{a})}{\partial a_i \partial q_i}\right]$$

which establishes

Proposition 8. *In the absence of transfers at the second-stage bargaining, each country has the greater abatement investment than in the absence of bargaining when the bargaining-frontier expansion effect is positive and the more investment increases the own bargaining contribution.*

6. Some Examples

To show that the the above propositions are not mere theoretical possibilities, we give two examples giving rise to totally opposite results on the pre-bargaining effect of each country.

6.1. **Example 1.** For simplicity of exposition, assume $\theta_i = 1$ for $i = 1, 2$, and consider the following model

$$f^i(z_i, a_i) = \frac{1}{a_i}\{1 - \exp(-a_i z_i)\},$$
$$D(z_T) = \gamma z_T.$$

Some tedious calculation yields

$$\frac{\partial S(\mathbf{q},\mathbf{a})}{\partial q_i} = \frac{\gamma}{a_i}\left(1 - \frac{\sum_k q_k}{q_i}\right) + \gamma\left(\sum_k \frac{1}{a_k}\right)\ln\left(\frac{\sum q_k}{q_i}\right),$$

so that a the symmetric equilibrium there hold

$$\frac{\partial^2 S(\mathbf{1},\mathbf{a})}{\partial a_i \partial q_i} = -\frac{\gamma}{a^2}(\ln 2 - 1) > 0,$$
$$\frac{\partial^2 S(\mathbf{1},\mathbf{a})}{\partial a_j \partial q_i} = -\frac{\gamma}{a^2}\ln 2 < 0,$$
$$\frac{\partial S(\mathbf{1},\mathbf{a})}{\partial a_i} = -\frac{\gamma}{a^2}(2\ln 2 - 1) < 0.$$

In the presence of transfers, the pre-bargaining investment incentive is weakened compared with when there is no second-stage bargaining. But in their presence, more investment by each country lowers the own relative valuation, which strengthens the investment incentive. The change in the second-stage bargaining is described by Figure 4.

6.2. **Example 2.** Consider instead the following model.

$$f^i(z_i, a_i) = a_i\sqrt{z_i},$$
$$D(z_T) = \gamma z_T.$$

As in the previous subsection, we obtain

$$\frac{\partial S(\mathbf{q},\mathbf{a})}{\partial q_i} = \frac{q_i a_i^2}{2\gamma \sum_k q_k} - \frac{\sum q_k^2 a_k^2}{4\gamma\left(\sum_k q_k\right)^2} - \frac{a_i^2}{2\gamma} + \frac{\sum_k a_k^2}{4\gamma},$$

so that at the symmetric equilibrium we obtain

$$\frac{\partial^2 S(\mathbf{1},\mathbf{a})}{\partial a_i \partial q_i} = -\frac{a_i}{8\gamma} < 0,$$
$$\frac{\partial^2 S(\mathbf{1},\mathbf{a})}{\partial a_j \partial q_i} = \frac{3a_i}{8\gamma} > 0,$$
$$\frac{\partial S(\mathbf{1},\mathbf{a})}{\partial a_i} = \frac{a}{4\gamma} > 0.$$

The results get reversed compared with Example 1, and the situation is described by Figure 5.

7. Concluding Remarks

In this paper, we analyzed a two-period two-country model with or without anticipating a future international agreement on environmental control. Several remarks may be in order before we conclude the paper.

In order to simplify our analysis and in order to enable us to track down the likely outcomes, we employed several crucial assumptions. One of the most important is that an international agreement being binding. Any foreseeable international agreement on global environment

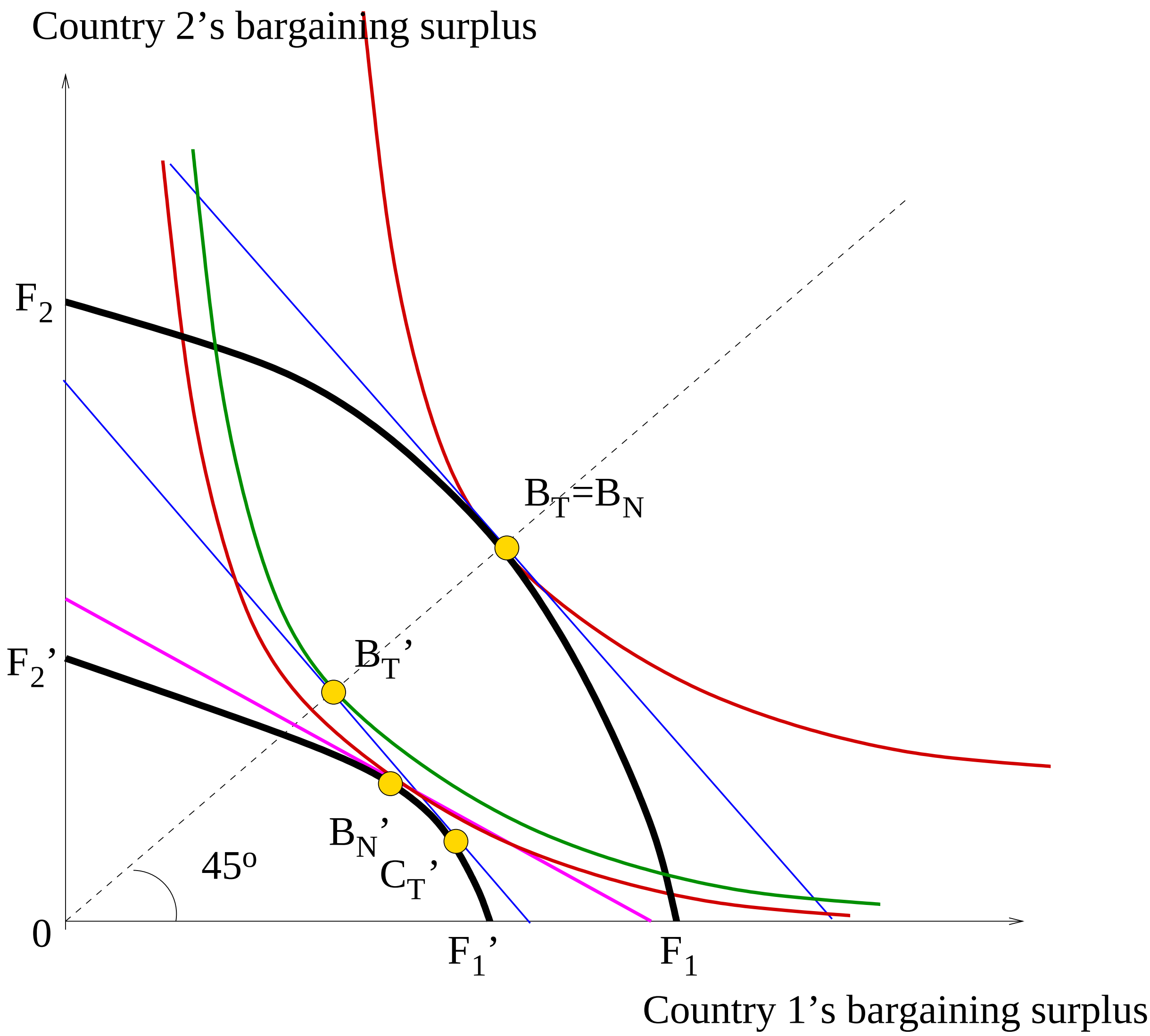

Figure 4. Example 1

will lack enforcing power other than self-enforcing property, because there is no world government that can enforce the agreement. Ideally, we should analyze a two period model with second period agreement being designed only to satisfy the self-enforcing property, or even better the renegotiation-proof self-enforcing property. Unfortunately, little is known about renegotiation-proof equilibrium of this nature and restricting feasible outcomes to be either self-enforcing or renegotiation-proof would make our analysis more complicated than necessary. Therefore, instead of using these equilibrium concepts to be satisfied for all feasible negotiation outcomes, we simply assumed that a binding contract is possible and any feasible outcome is potentially agreeable.

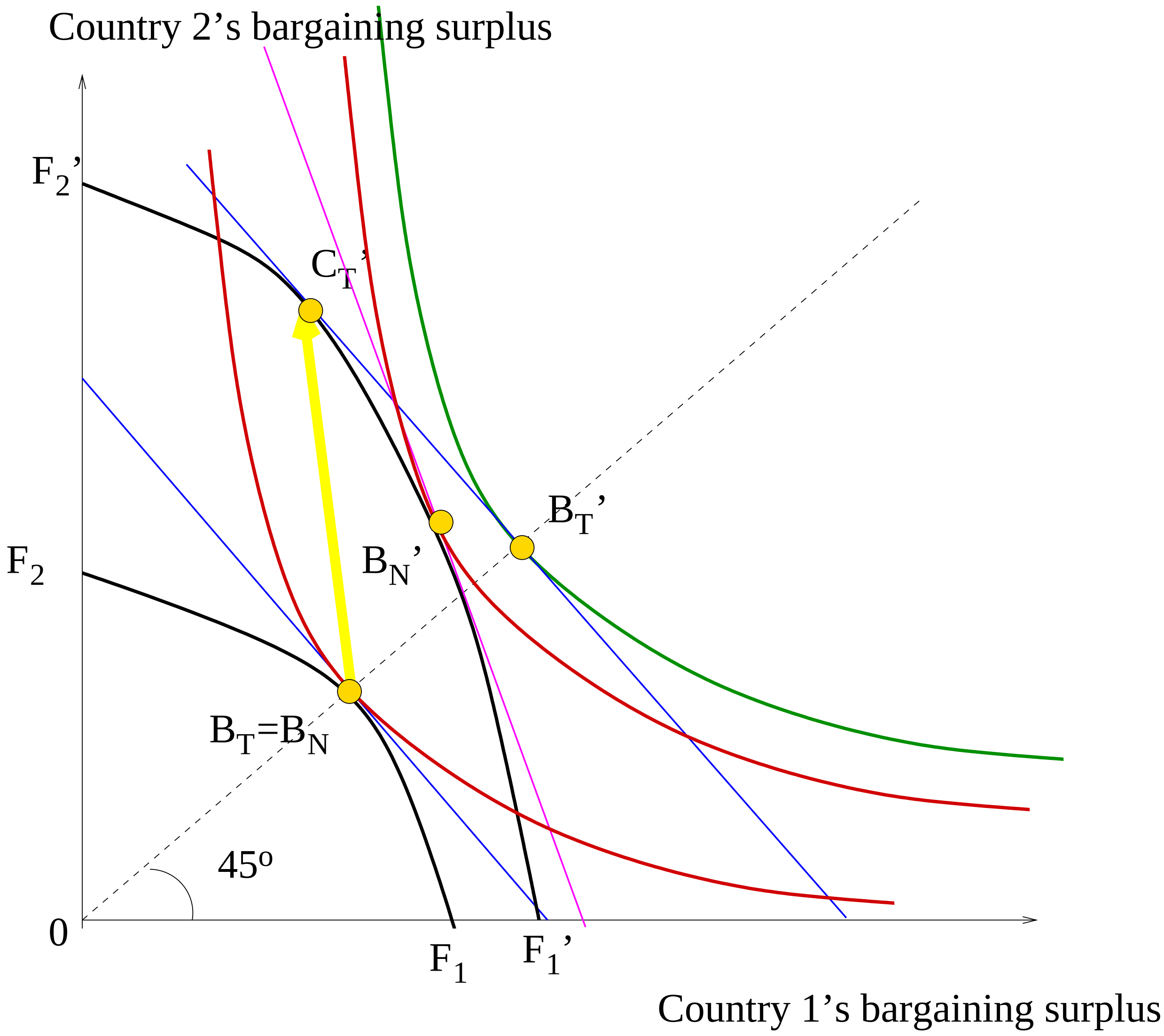

Figure 5. Example 2

The specific mathematical formulation which we employed for national welfare (3) may also be restrictive. In particular, most of our results that strategic interactions hinge on absence of income effects possibly arising from environmental damage in more general formulations. Nonetheless, most important message of the paper is that the incentives for ex ante investment ciritcally depends on whether an international negotiation is anticipated with or without transfers. Since creation of international market for tradable emission permits requires a certain allocation of initial permits among the countries, the emissions trading involves international transfers of income. Our analysis reveals that feasibility of such trading system as global environment management strategy affects the ex-ante abatement investment by each country.

Lastly, in the model, we treated the two periods, ex ante and ex post, without paying any attention to their lengths. However, the ex ante investment is a flow variable, and its impact on the ex post becomes larger as the length of the first period (ex ante period) becomes longer. An obvious implication of this observation is that, once the possibility of a future international agreement becomes non-negligible, the sooner an agreement get struck, the less impact this negative incentive affects the ex ante investment for improving emission efficiency.

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Appendix A. Abatement Investment as Technical Progress\re

When talking of abatement investment, the previous literature has been somewhat ambiguous in its characterization. Energy conservation, often referred to as the most important strategy in the global warming problem, is a kind of technical progress which raises the total factor productivity given the available resources for production other than R&D resources. However there is also another type of abatement investment, which induces the private sector to save or decrease the carbon-gas emissions chosen voluntarily. The latter is a certain type of costly investment to install emission-saving devices, unlikely to be undertaken by the private incentive, but is thus enforced by the government.

Within the present framework, the latter type of abatement satisfies (i) $f_a(z, a) < 0$ and (ii) $f_{az}(z, a) < 0$.One may think that those properties do not hold for the first type of abatement investment. However, even when the abatement investment serves as the familiar technical progress, abatement may lead to $f_{az}(z, a) < 0$. We will demonstrate it below.

To make our analysis clear, particularly from the view-point of the literature in technical progress, let us reformulate it as below:

(A-1) $$y = G(a_n a_z z, a_n a_x x),$$

where a_z (or a_x) denotes the effective units of carbon gas emission (or labor) in production and a_n the efficiency level of both inputs in production.

As is often assumed in the literature of technical progress, we assume that the production function (A-1) is subject to constant returns to scale. In the language of the theory of technical progress, the following classification is possible with regard to the types of technical progress:

(Type-A): Emission-saving technical progress, expressed by an increase in a_z

(Type-B): Labor-saving technical progress, expressed by an increase in a_x

(Type-C): Hicks-neutral technical progress, expressed by an increase in a_n

One should note that in view of the above list of the types in technical progress and the production-function form in (A-1), the abatement efficiency parameter a_i shows the profile of the technology levels defined above, i.e., (a_z, a_x, a_n). The production function (1) is in fact represented as follows.

Let $e = \dfrac{a_z z}{a_x x}$ represent what we may call the *effective emission-labor ratio*, and $g(e)$ the output per effective unit of labor. Then the production function (A-1) can be expressed in terms of the effective unit of labor as below

$$y = a_n a_x x g(e) \left(= a_n a_x x g \left(\frac{a_z z}{a_x x} \right) \right). \tag{A-2}$$

Then the marginal product of carbon gas emission, which we denote by MP_z, is given by

$$MP_z = a_n a_z g' \left(\frac{a_z z}{a_x x} \right).$$

Thus the marginal product of carbon gas emission increases along with an increase in a_n (Hicks-neutral technical progress) and in a_x (labor-saving technical progress) as shown by

$$\begin{aligned}
\frac{\partial MP_z}{\partial a_n} &= a_z g'(e) > 0, \\
\frac{\partial MP_z}{\partial a_x} &= -a_n \left(\frac{a_z}{a_x} \right)^2 g"(e) \frac{z}{x} > 0, \\
\frac{\partial MP_z}{\partial a_z} &= a_n \left(g'(e) + e g"(e) \right) = a_n g'(e) \left(1 - \sigma s_x \right),
\end{aligned}$$

where $s_x := \frac{g(e) - e g'(e)}{g(e)}$ represents the factor-cost share of labor and $\sigma := \frac{-e g(e) g"(e)}{g'(e)(g(e) - e g'(e))}$ the elasticity of substitution between labor and carbon-gas emission.

Lemma 4. *When improvement in the abatement efficiency takes one of the following forms, then it enhances the marginal product of carbon gas emission and thus increases the individual country's optimal carbon gas emission.*

(i) Hicks-neutral technical progress.

(ii) Labor-saving technical progress.

(iii) Emission-saving technical progress given $\sigma < \frac{1}{s_x}$.

Hereafter we classify the types of improvement in abatement efficiency as follows:

- *Type H*: Higher emission productivity, giving rise to $f_{za}(\cdot) > 0$
- *Type L*: Lower emission productivity, giving rise to $f_{za}(\cdot) < 0$

Type H corresponds to the types of improvement in abatement efficiency listed in Lemma 4, and Type L the others leading to a decrease in the marginal product of carbon gas emission, particularly emission-saving technical progress with sufficiently great elasticity of substitution between labor and carbon-gas emission.

Appendix B. Alternative Expression for the Externality Internalization Effect

In this section, we derive the alternative expression for the externality internalization effect in the text. For this purpose, we fist define

$$\begin{aligned} \xi(z,a) &:= -\frac{z f_{zz}(z,a)}{f_z(z,a)} > 0, \\ \delta(z_T) &:= \frac{z_T D"(z_T)}{D'(z_T)} > 0. \end{aligned}$$

$\xi(z,a)$ represents the own elasticity of the marginal productivity of emissions, and $\delta(z_T)$ the elasticity of the world marginal environmental damage with respect to the world total emissions. Using these elasticities, the slope of the individual country's quasi-reaction function at the symmetric equilibrium is shown to satisfy

$$\begin{aligned} r_z^i &= \frac{\theta D"(z_T)}{f_{zz}(z,a)} \text{ (where } z_T = 2z) \\ &= \frac{\theta D"(z_T)}{-\frac{f_z(z,a)}{z}\xi(z,a)} \\ &= -\frac{z D"(z_T)}{D'(z_T)\xi(z,a)} \ (\because (9)) \end{aligned}$$

$$\text{(B-3)} \qquad = -\frac{\delta(2z)}{2\xi(z,a)} < 0,$$

where use was made of $\xi(z,a) > 0$ and $\delta(z_T) > 0$. We also find that total differentiation of (**??**) gives rise to

$$\begin{aligned} z_t^\star(t) &= \frac{\theta D'(2z^\star(t))}{f_{zz}(z^\star(t),a) - 2t\theta D"(2z^\star(t))}, \\ &= \frac{z^\star(t)}{\frac{t f_{zz}(z^\star(t),a) z^\star(t)}{f_z(z^\star(t),a)} - \frac{2tD"(2z^\star(t))z^\star(t)}{D'(2z^\star(t))}}, \\ &= -\frac{z^\star(t)}{t\{\xi(z^\star(t),a) + \delta(2z^\star(t))\}}, \end{aligned}$$

where use was made of the definitions of $\xi(z,a)$ and $\delta(z_T)$. This result implies that the the emission-tax elasticity of the individual country's carbon gas emissions for the symmetric equilibrium, denoted by $\eta(t)$, is equal to $\frac{1}{\xi(z^\star(t),a)+\delta(2z^\star(t))}$, i.e., there holds

$$\eta(t) \quad := \quad -\frac{tz_t^\star(t)}{z^\star(t)} = \frac{1}{\xi\left(z^\star(t),a\right)+\delta\left(2z^\star(t)\right)}. \tag{B-4}$$

In view of (B-3) and (B-4), one can rewrite the externality internalization effect as below.

$$\begin{aligned}
\theta D'\left(2z^{iN}\left(\mathbf{a},\theta\right)\right)\frac{\partial z^{TN}\left(\mathbf{a},\theta\right)}{\partial a_i} &= \frac{1}{1-\sum_k r_z^k}\cdot\frac{z^{iN}f_{za}\left(z^{iN}\left(\mathbf{a},\theta\right),a\right)\theta D'\left(z^{TN}\left(\mathbf{a},\theta\right)\right)}{\xi\left(z^{iN}\left(\mathbf{a},\theta\right),a\right)f_z\left(z^{iN}\left(\mathbf{a},\theta\right),a\right)} \\
&= \frac{1}{1+\frac{\delta\left(2z^{iN}(\mathbf{a},\theta)\right)}{\xi\left(z^{iN}(\mathbf{a},\theta),a\right)}}\frac{z^{iN}\left(\mathbf{a},\theta\right)f_{za}\left(z^{iN}\left(\mathbf{a},\theta\right),a\right)}{\xi} \\
&= \frac{z^{iN}\left(\mathbf{a},\theta\right)f_{za}\left(z^{iN}\left(\mathbf{a},\theta\right),a\right)}{\xi\left(z^{iN}\left(\mathbf{a},\theta\right),a\right)+\delta\left(2z^{iN}\left(\mathbf{a},\theta\right)\right)} \\
&= \frac{z^\star(1)f_{za}\left(z^\star(1),a\right)}{\xi\left(z^\star(1),a\right)+\delta\left(2z^\star(1)\right)} \\
&= f_{za}\left(z^\star(1),a\right)z^\star(1)\eta(1),
\end{aligned}$$

which establishes (B).