International Trade with a Public Intermediate Good and the Gains from Trade

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Abstract

We develop a one-primary factor, two-consumer good and two-country model of international trade where a country-specific public intermediate good is supplied efficiently through the Lindahl pricing rule in each country. Then it is shown that the country with larger factor endowment exports the good whose productivity is more sensitive to the public intermediate good. As for the gains from trade, we show that an incompletely specializing country necessarily loses from trade. We also presents the necessary and sufficient condition for a completely specializing country to gain from trade.

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1. Introduction

In recent decades, many trade theorists have studied the influences of public intermediate goods on the fundamental theorems in the traditional trade theory. (See Manning and McMillan (1979), Kahn (1980), Tawada and Okamoto (1983), Tawada and Abe (1984), Okamoto (1985), Altenburg (1987) and Ishizawa (1988).) But all of these studies paid attention to a small open economy and did not give any explicit analysis in a two-country framework. Another lack in these studies is the analysis of a welfare aspect. Therefore, whether a country can gain from trade still remains as an open question in an economy with public intermediate goods. An exception is the study made by Manning and McMillan (1979) who considered an economy with one primary factor, two consumer goods and one pure public intermediate good and showed by a simple Ricardian type of argument that a country necessarily gains from trade in a small open economy. But their assertion is not so robust because, as we will show in our analysis, a phenomenon that a country will lose from trade is not peculiar but sufficiently plausible in a two-country economy. Since Manning and McMillan concentrated on a small open economy, they did not argue how to determine the production pattern and the terms of trade both of which are crucial to the argument of the gains from trade.

In this paper, in order to deal with these subjects, we consider an economy where two tradable goods, one primary factor, one pure public intermediate good and two countries. And we assume that the public intermediate good is supplied efficiently by means of the Lindahl pricing rule. Moreover, we introduce a simple Marshallian adjustment process into the model in order to determine the patterns of trade. Then we discuss the gains from trade, according to the pattern of trade realized by this dynamic process.

Three main conclusions emerge from the analysis. The first is that the country with larger factor endowment exports the good whose productivity is more sensitive to the public intermediate good. The second is that an incompletely specializing country necessarily becomes worse off by free trade. Finally, concerning a completely specializing country, we propose the necessary and sufficient condition for the country to have a gain from trade.

This paper is organized as follows. Section 2 describes the model and exhibits some basic results regarding the production side. Section 3 examines the autarkic economy and characterizes the adjustment process. Then Section 4 investigates the patterns of trade attained at the trading steady-state equilibrium and Section 5 deals with the question of the gains from trade. Concluding remarks are presented in the last section.

2. The Model

We consider an economy with two consumer goods called good 1 and good 2, one primary factor called labor, and one pubic intermediate good which is collectively used in the production of all consumer goods.

2.1 production technologies and the labor endowment

We suppose that the production technologies of two consumer goods and public intermediate good are described as,

(1) $$Q_i = A_i(R)L_i, \quad A_i'(\cdot) > 0, \; A_i''(\cdot) < 0, \quad i = 1,2,$$

(2) $$R = f(L_R), \quad f'(\cdot) > 0, \; f''(\cdot) > 0,$$

respectively, where Q_i and R are the outputs of good i and the public intermediate good, respectively, L_i and L_R are the amounts of labor used in the production of good i and the public intermediate good, respectively, $f(\cdot)$ is the production function of the public intermediate good, and $A_i(R)$ is the marginal productivity of labor in the ith industry for a given R.

It is assumed that the elasticity of marginal labor productivity with respect to the provision of public input, $\eta_i(R) \equiv A_i'(R)R / A(R)$, is always greater in the first industry than the other, namely,

(3) $$\eta_1(R) > \eta_2(R) \quad \text{for all } R > 0.$$

This suggests that the productivity of the first industry is more sensitive to the public input than that of the second industry.

The labor endowment is assumed to be given and constant. The full employment condition of labor is

(4) $$L_1 + L_2 + L_R = L,$$

where L is the labor endowment.

2.2 the efficient supply of the public intermediate good and the Lindahl pricing rule

The production possibility frontier is the locus of (Q_1, Q_2) satisfying $Q_2 = \Gamma(Q_1, L)$ for a given L, where $\Gamma(Q_1, L)$ represents the maximum output of good 2 when Q_1 and L are fixed; that is,

$$\Gamma(Q_1, L) \equiv \max_{L_1, L_2, L_R, R} A_2(R) L_2$$

$$\text{subject to } A_1(R)L_1 = Q_1, \ R = f(L_R) \text{ and (4).}$$

The optimal conditions of the above problem are derived as

(5) $$A_1'(\cdot)Q_1 / A_1(\cdot)^2 + A_2'(\cdot)Q_2 / A_2(\cdot)^2 = 1/f'(\cdot),$$

(6) $$Q_1 / A_1(\cdot) + Q_2 / A_2(\cdot) + f^{-1}(R) = L.$$

Let $\Lambda(Q_1, L)$ be the optimal supply of the public intermediate good for given Q_1 and L. Then $\Gamma(\cdot)$ and $\Lambda(\cdot)$ constitute the solution to the simultaneous equations given by (5) and (6). Hence, by using (5) and (6) we can obtain the partial derivatives of $\Gamma(\cdot)$ and $\Lambda(\cdot)$ with respect to Q_1 and L as follows.

(7) $$\partial\Lambda / \partial Q_1 = -(\eta_1 - \eta_2) / RA_1\Delta > 0,$$

(8) $$\partial\Gamma / \partial Q_1 = -A_2 / A_1 < 0,$$

(9) $$\partial\Lambda / \partial L = -\eta_2 / R\Delta > 0,$$

(10) $$\partial\Gamma / \partial L = A_2 > 0,$$

where $\Delta = \Sigma_{i=1}^{2}[A_i'' \cdot A_i - 2(A_i')^2]Q_i / A_i^3 + f''/(f')^3 < 0$. Partial differentiation of (8) with respect to Q_1 and L, together with the use of (7) and (9), yields

(11) $$\frac{\partial^2\Gamma}{\partial Q_1^2} = \frac{A_2(\eta_1 - \eta_2)}{A_1 R} \cdot \frac{\partial\Lambda}{\partial Q_1} = -\frac{A_2(\eta_1 - \eta_2)^2}{(A_1 R)^2 \Delta} > 0,$$

(12) $$\frac{\partial^2\Gamma}{\partial L \partial Q_1} = \frac{A_2(\eta_1 - \eta_2)}{A_1 R} \cdot \frac{\partial\Lambda}{\partial L} = -\frac{A_2(\eta_1 - \eta_2)\eta_2}{A_1 R^2 \Delta} > 0.$$

It is obvious from (11) that the production possibility frontier is strictly convex to the origin, as shown in Figure 1.

(Figure 1)

Suppose that the government sets the price of the public intermediate good by the

Lindahl pricing rule. Then the output of the public intermediate good is determined by profit maximization. The profit maximizing conditions are expressed as,

$$p_R = p_1 A_1'(\cdot) L_1 + p_2 A_2'(\cdot) L_2,$$

$$p_R f'(\cdot) = w,$$

where p_i and p_R are the prices of good i and the public intermediate good, respectively, and w is the wage rate. We assume that the production of the public intermediate good is financed through the lump sum tax on total income (total wage bill plus profit). All firms within each industry are assumed to take the quantity of the pubic intermediate good as given and act as if they operate under constant returns to scale. Perfect competition and profit maximization in all private industries, together with the optimal conditions of public good supply, bring forth of

(13) $$p_1 / p_2 = A_2 / A_1 = -\partial\Gamma(Q_1, L) / \partial Q_1.$$

Thus, if both consumer goods are produced in the market equilibrium, production takes place at the point of the production possibility frontier where the budget line is tangent. It is shown in Figure 1.

2.3 preferences

Suppose that social preferences are described by the homothetic utility function

$$u = U(C_1, C_2) = \phi[v(C_1, C_2)], \qquad \phi'(\cdot) > 0,$$

where C_i is the social consumption for good i and $v(\cdot)$ is monotonely increasing, strictly quasi-concave and linearly homogeneous in C_1 and C_2.

Normalizing the price of good 2 to unity and denoting the price of good 1 relative to good 2 by p, the expenditure function with the level of utility, u, is expressed as

$$E(p,u) = e(p) \cdot \phi^{-1}(u), \qquad e'(\cdot) > 0, \ e''(\cdot) < 0.$$

Thus the indirect utility level, V, and the consumption ratio of good 1 to good 2, C_1 / C_2, are described, respectively, as

(14) $$V = \phi[I / e(p)] \equiv V(p, I),$$

(15) $$C_1 / C_2 = \gamma / (1-\gamma) p \equiv Z(p),$$

where $I \equiv pQ_1 + Q_2$ is national income and $\gamma \equiv e'(p) p / e(p)$ is the share of income spent on good 1. Then the consumption ratio of good 1 to good 2 has a negative relation

to its relative price since

(16) $$dZ / dp = -\varepsilon \cdot Z /(1-\gamma) p < 0,$$

where $\varepsilon \equiv -e''(\cdot) p / e'(\cdot)$ is the price elasticity of compensated demand for good 1.

3. Autarky

Now we adopt the following Marshallian adjustment process: The output of good 1 rises if the corresponding demand price, p_D, exceeds the corresponding supply price, p_S and vice versa. So the precise description of the adjustment process is

$$\dot{Q}_1 = p_D - p_S,$$

where dot denotes a time derivative.

The demand price is defined as the relative price of good 1 that clears the commodity markets for a given output level of good 1. From (15) and the market-clearing condition, say $C_1 / C_2 = Q_1 / Q_2$, the autarkic demand price, p_D, is thus represented as the solution to

(17) $$Z(p_D) = Q_1 / \Gamma(Q_1, L).$$

Total differentiation of (17) with respect to p_D and Q_1 and the use of (8) and (16), we obtain

(18) $$\chi_{p_D Q_1} = -(1-\gamma)(A_2 Q_1 / A_1 Q_2 + 1) / \varepsilon < 0,$$

where $\chi_{XY} \equiv (dX / dY)/(Y / X)$ denotes the elasticity of X with respect to Y. The above equation suggests that the autarkic demand price is decreasing in the domestic output level of good 1.

The supply price is defined as the relative average production cost of good 1. By using (13) we can express it as $p_S = -\partial\Gamma(Q_1, L) / \partial Q_1$. So, from (8) and (11) the elasticity of p_S with respect to Q_1 is given by

(19) $$\chi_{p_S Q_1} = -(\eta_1 - \eta_2)\chi_{\Lambda Q_1} < 0.$$

In view of (7) and (19), we find that the supply price is also decreasing in the domestic output level of good 1.

Let us denote the autarkic demand price and the supply price by $p_D = D(Q_1, L)$ and $p_S = S(Q_1, L)$, respectively. Then the autarkic demand curve, showing the locus of the output-price pair (Q_1, p_D) such that $p_D = D(Q_1, L)$ for a given L, is displayed

by the curve DD in Figure 2, and the supply curve, showing the locus of the output-price pair (Q_1, p_S) such that $p_S = S(Q_1, L)$ for a given L, is as depicted by the curve SS. If the demand curve is steeper than the supply curve and there is at least one intersection, there exists a globally stable and unique autarkic steady-state equilibrium E. It is shown in Figure 2. Evaluating (18) at the autarkic steady-state equilibrium we have $\chi_{p_D Q_1} = -1/\varepsilon$, which, together with (19), suggests that the stability condition is expressed as

(20) $$-\chi_{p_S Q_1} < 1/\varepsilon, \quad \text{for } Q_1 \text{ such that } D(Q_1, L) = S(Q_1, L)$$

Hereafter we assume that this stability condition holds.

(Figure 2)

4. Trade in a Two-Country World

We now consider free trade between two countries, say the home and foreign countries. The two countries are assumed to be identical in the production technologies and preferences but not necessarily in the labor endowment. The public intermediate good is supposed to be country-specific and have no international spillovers. In what follows we attach a superscript * to each variable pertaining to the foreign country.

4.1 trading equilibrium

To see what the equilibrium will be after opening trade, we will begin with the description of the Marshallian dynamic process under free trade. It is easily shown that, under free trade, the world consumption ratio of good 1 to good 2, $(C_1 + C_1^*)/(C_2 + C_2^*)$, is also expressed as $Z(p) = \gamma/(1-\gamma)p$ for a given world relative price, p. From this fact and the world market-clearing condition expressed by

$$(C_1 + C_1^*)/(C_2 + C_2^*) = (Q_1 + Q_1^*)/(Q_2 + Q_2^*),$$

the world demand price, p_D, should be the solution to

(21) $$Z(p_D) = (Q_1 + Q_1^*)/[\Gamma(Q_1, L) + \Gamma^*(Q_1^*, L^*)].$$

Let us denote the world demand price by $p_D = \tilde{D}(Q_1, Q_1^*, L, L^*)$. Then the Marshallian adjustment process under free trade is described as follows.

(22) $$\dot{Q}_1 = F(Q_1, Q_1^*, L, L^*) \equiv \tilde{D}(Q_1, Q_1^*, L, L^*) - S(Q_1, L),$$

(23) $$\dot{Q}_1^* = F^*(Q_1^*, Q_1, L^*, L) \equiv \tilde{D}(Q_1, Q_1^*, L, L^*) - S^*(Q_1^*, L^*),$$

where $S(\cdot)$ and $S^*(\cdot)$ is the country-specific supply prices.

Differentiating (21) totally with respect to p_D, Q_1 and Q_1^*, and using (8) and (16), the world demand price is shown to be decreasing in Q_1 and Q_1^*, namely,

(24) $$\partial\tilde{D} / \partial Q_1 < 0, \qquad \partial\tilde{D} / \partial Q_1^* < 0.$$

This immediately leads to the fact that $F(\cdot)$ and $F^*(\cdot)$ are decreasing in Q_1^* and Q_1, respectively, that is,

(25) $$\partial F / \partial Q_1^* < 0, \qquad \partial F^* / \partial Q_1 < 0.$$

As for the country-specific supply prices, $S(\cdot)$ and $S^*(\cdot)$, it is assured that

(26) $$\partial S / \partial Q_1 < 0, \qquad \partial S^* / \partial Q_1^* < 0.$$

For the time being, in order to simplify the discussion we assume that the two countries are the same in regard to their labor endowments. Then, since two countries are the same in any respect, the existence of the public intermediate good itself plays a pivotal role in trade creation if trade occurs between countries.

Figure 3 represents the phase diagram of the dynamic system expressed by (22) and (23). Following Ethier (1979, 1982), we call the locus of output pair (Q_1, Q_1^*) satisfying $\dot{Q}_1 = F(\cdot) = 0$ the home *allocation curve*. Similarly we call the locus of output pair (Q_1, Q_1^*) satisfying $\dot{Q}_1^* = F^*(\cdot) = 0$ the foreign allocation curve. Under the assumption that the two countries are identical in labor endowment, the home and foreign allocation curves can be drawn as the curves AEB and CED, respectively. The two curves are symmetric about the 45° line, on which they intersect at a point E, and the intersection corresponds to the autarkic steady-state output level of good 1.

(Figure 3)

We further proceed to investigate the two allocation curves in more details. Because of (26) and the assumption that the two countries are completely identical,

(27) $$p_S = S(Q_1, L) \ > \text{(resp. <)} \ S^*(Q_1^*, L^*) = p_S^*, \text{ as } Q_1^* \ > \text{(resp. <)} \ Q_1.$$

From (22), (23) and (27), it follows that, in the upper area of the 45° line, $\dot{Q}_1 = F(\cdot) < 0$ for any output pair (Q_1, Q_1^*) satisfying $\dot{Q}_1^* = F^*(\cdot) = 0$. Taking this fact into consideration and using the decreasingness property of $F(\cdot)$ with respect Q_1^*, we find that the home allocation curve lies below the foreign allocation curve in the upper

area of the 45° line, as shown in Figure 3. In a similar way, it is exhibited that the foreign allocation curve lies below the home allocation curve in the lower area of the 45 ° line.

Moreover, The slope of the home (resp. foreign) allocation curve is negative at the intercept on the horizontal (resp. vertical) axis such as in Figure 3, due to the decreasigness property of $F(\cdot)$ (resp. $F^*(\cdot)$) with respect to Q_1^* (resp. Q_1).

Therefore, one typical relationship between two allocation curves is possibly shown as in Figure 3.

The arrows in Figure 3 indicate the qualitative laws of motion of Q_1 and Q_1^*. From (25) we find that $\dot{Q}_1 = F(\cdot)$ is positive (resp. negative) in the lower (resp. upper) area of the home allocation curve. Thus Q_1 rises (resp. falls) over time in the lower (resp. upper) area of the curve AEB, as depicted by the horizontal arrows in the figure. Similarly we can denote the dynamic behavior of Q_1^* by the vertical arrows. $\overline{Q}_1$ is defined as the maximum output level of good 1 in home when $Q_2 = 0$. So is $\overline{Q}_1^*$ in foreign[1]. Then the whole of each allocation curve is accommodated into the box OGJI in Figure 3. Therefore, if the point representing the country-output combination of good 1 slightly moves from the point E to somewhere in the upper (resp. lower) region of the 45° line, the point D (resp. B) will eventually become the trading steady-state equilibrium. In this case, the country exporting good 1 is diversified and the other country is specialized in good 2 and exports it.

4.2 patterns of production and preferences

We now consider the relationship between the patterns of production and preferences. In view of (15), it is plausible to think that the greater the taste for good 1, the larger shares of income spent on good 1, for a given p. Thus we suppose

[1] Therefore, $\overline{Q}_1$ ($\overline{Q}_1^*$) is the output level of good 1 in the home (foreign) production possibility frontier when Q_2 (Q_2^*) is equal to zero.

preferences to satisfy this property[2].

Suppose that the economy is in equilibrium initially. Then, the world excess demand for good 1 appears if the taste for good 1 becomes greater. In this case, the relative price of good 1 has to rise in order to restore the market equilibrium. This implies that the greater the taste for good 1, the higher the world demand price, $p_D = \tilde{D}(Q_1, Q_1^*, L, L^*)$, for given (Q_1, Q_1^*, L, L^*). Hence $\dot{Q}_1$ (resp. $\dot{Q}_1^*$) becomes positive in sign for any output pair (Q_1, Q_1^*) that assures $\dot{Q}_1$ (resp. $\dot{Q}_1^*$) to be zero initially. This, together with (25), implies that in Figure 3 the home (resp. foreign) allocation curve shifts up (resp. rightward) according to an intensification in the taste for good 1.

Reminding the assumption that $L = L^*$, we consider the case where the taste for good 1 is sufficiently strong, so that the segment AE (resp. CE) of the curve AED (resp. CED) intersects the line GJ (resp. IJ). Then, according to the qualitative laws of motion of Q_1 and Q_1^* indicated by arrows in Figure 3, the good 1 exporting country will specialize in its exporting good and the other country will diversify at the trading steady-state equilibrium. Conversely, as for the case where the taste for good 1 is not so strong, the segment AE (resp. CE) of the curve AEB (resp. CED) is within the box OGJI but the segment BE (resp. DE) intersects it. Then each country is expected to specialize in its exporting good at the trading equilibrium. If the taste for good 1 is sufficiently weak, the whole allocation curves are located within the box OGJI just as Figure 3 shows. Then, as stated earlier, the country exporting good 1 is diversified and the other country is specialized in good 2 and exports it.

4.3 comparative advantage and country size

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[2] Suppose that preferences are described by the following CES function:

$$U(C_1, C_2) = [\alpha C_1^{\rho} + (1-\alpha) C_2^{\rho}]^{1/\rho}, \qquad \rho < 1, \ \rho \neq 0, \ 0 < \alpha < 1,$$

where α represents the extent to which good 1 is preferred, compared to good 2. Then we can see that, holding prices of consumer good constant, an increase in α raises the share of income spent on good 1.

Allowing the home and foreign countries to have different labor endowments, we investigate what pattern of trade will be attained after opening trade. Hence we first consider the autarkic economy again and suppose an increase in the labor endowment of one country. Then it is seen from (10) that the output of good 2 increases for a given output level of good 1 in that country. So the right-hand side of (17) becomes less than its left-hand side. By (16), however, the market equilibrium can be restored with a rise in the autarkic demand price while the output level of good 1 is constant. Thus the autarkic demand curve shifts up corresponding to an increase in the labor endowment, which is shown by the upward arrow in Figure 2. By contrast, from (8), (12) and (13) it is clear that the home supply curve shifts downward in response to an increase in the labor endowment. This is shown by the downward arrow in the figure. By the use of these facts we can obtain the following lemma.

Lemma 1

Under the assumption that $\eta_1(R) > \eta_2(R)$ *for all* $R > 0$*, a country with larger labor endowment has a lower relative autarkic price of good 1 and a higher autarkic output ratio of good 1 to good 2.*

Proof. Suppose that the relative autarkic price of good 1 is p_A in Figure 2. Then an increase in the labor endowment shifts up the demand curve from DD to D'D' and it shifts down the supply curve from SS to S'S', so that the relative autarkic price declines from p_A to p'_A. This implies that the country with larger labor endowment faces a lower relative autarkic price of good 1. Furthermore, from (16) and (17), it is easily verified that a lower autarkic relative price leads to a higher autarkic output ratio of good 1 to good 2. Thus we can conclude that the larger country has a higher autarkic output ratio of good 1 to good 2. **Q.E.D.**

Let us turn our attention to the trading equilibrium. Suppose that the home and foreign allocation curves are, respectively, represented by the curves AEB and CED in Figure 3 initially. Let the foreign labor endowment, L^*, increase. As already

mentioned, an increase in labor endowment lowers the supply price of good 1. So an increase in L^* leads to a fall in the foreign supply price, $p_S^* = S^*(Q_1^*, L^*)$. From (10), (16) and (21), we find that the world demand price, $p_D = \tilde{D}(Q_1, Q_1^*, L, L^*)$, rises in response to an increase in L^*. Therefore, as a result of an increase in L^*, the sign of $\dot{Q}_1$ (resp. $\dot{Q}_1^*$) becomes positive for any output pair (Q_1, Q_1^*) that lies on the curve AEB (resp. CED). This, together with (25), asserts that the home and foreign allocation curves shift outward in response to an increase in L^*[3]. Bearing these shifts of the allocation curves in mind and using Lemma 1, we can obtain the following proposition with respect to the patterns of trade.

Proposition 1

Suppose that production technologies and preferences are the same between two countries. Then, under the assumption that $\eta_1(R) > \eta_2(R)$ for all R, a country with larger labor endowment exports good 1 and the other country exports good 2 at the trading equilibrium.

Proof. Without loss of generality we can assume that $L^* > L$. Then the home and foreign allocation curves can be depicted by the curves A'E'B' and C'E'D' in Figure 3. By the assumption that $L^* > L$, Lemma 1 implies that

$$\frac{Q_{1A}}{\Gamma(Q_{1A}, L)} < \frac{Q_{1A} + Q_{1A}^*}{\Gamma(Q_{1A}, L) + \Gamma^*(Q_{1A}^*, L^*)} < \frac{Q_{1A}^*}{\Gamma^*(Q_{1A}^*, L^*)}, \tag{28}$$

where Q_{1A} and Q_{1A}^* are, respectively, the home and foreign autarkic output levels of good 1. From (16), (17), (21) and (28), it follows that

$$D^*(Q_{1A}^*, L^*) < \tilde{D}(Q_{1A}, Q_{1A}^*, L, L^*) < D(Q_{1A}, L), \tag{29}$$

where $D(Q_1, L)$ and $D^*(Q_1^*, L^*)$ are the country-specific autarkic demand prices. Noticing that the autarkic demand price becomes equivalent to the supply price at the

[3] An increase in L^* moves the intersection of the two allocation curves into the lower area of the 45° line. We understand the reason by noticing that a fall in the foreign supply price due to an increase in L^* enlarges the value of Q_1 that equalizes the home and foreign supply prices for any specific value of Q_1^*.

autarkic equilibrium, and using (22), (23) and (29), we can obtain

(30) $$\dot{Q}_1 = F(Q_{1A}, Q_{1A}^*, L, L^*) < 0, \qquad \dot{Q}_1^* = F^*(Q_{1A}, Q_{1A}^*, L, L^*) > 0.$$

From (30) it is evident that an autarkic output pair (Q_{1A}, Q_{1A}^*) locates somewhere in the region D'E'A' in Figure 3. Let F be such a point. Then, according the adjustment process, the trading equilibrium attained at G' in Figure 3, where the foreign country exports good 1. Thus we can conclude that a larger country with larger labor endowment exports good 1 and the other country exports good 2 at the trading equilibrium. **Q.E.D.**

An intuitive explanation for Proposition 1 is as follows. The larger country can take advantage of an abundant supply of labor to produce the public intermediate good, so that it faces a lower relative autarkic price of good 1. This is because good 1 is more sensitive to the provision of pubic input than good 2. Hence, if the two countries commerce free trade, the larger country exports good 1 and the smaller country exports good 2.

5. Gains from Trade

Now we are in a position to investigate the welfare effects of trade. In this section we first focus our analysis on the relation ship between the production patterns and the gains from trade in a country, and then we explore the welfare effects of trasde to each country in a two-country framework.

To start with, we shall derive some properties of the home national income, $I = pQ_1 + Q_2$. Consider home country producing both consumer goods and let us denote its national income by

$$g(p, L) \equiv pQ_1(p, L) + Q_2(p, L),$$

where $Q_i(p, L)$ is the amount of good i produced by competitive firms within the ith industry when the relative price of good 1 and the labor endowment are given by p and L, respectively. Because for given p and L the competitive output level of good 2 is expressed as $Q_2(p, L) = \Gamma[Q_1(p, L), L]$, we can rewrite the national income

as

(31) $$g(p,L) = pQ_1(p,L) + \Gamma[Q_1(p,L),L].$$

Partial differentiation of (31) with respect to p and use of (13) yield

(32) $$\partial g / \partial p = Q_1(p,L).$$

We can make the analysis of foreign national income in a similar manner. Thus we can establish the following lemma.

Lemma 2

Consider home country and let $\overline{Q}_i$ be the maximum output level of good i ($i = 1,2$) when the other consumer good is not produced. Then, the home national income in the case where both consumer goods are produced, $g(p,L)$, satisfies the following:

$$g(p,L) < p\overline{Q}_1 \text{ (resp. } \overline{Q}_2\text{)}, \quad \text{for all } p > p_l \text{ (resp. } p < p_u\text{)},$$

where $p_l \equiv -\partial\Gamma(\overline{Q}_1,L)/\partial Q_1$ and $p_u \equiv -\partial\Gamma(0,L)/\partial Q_1$ are, respectively, the lower and upper limits of p such that both consumer goods can be produced.

The foreign national income also satisfies this property.

Proof. We treat the home country case only, since the foreign country case can be shown in the same way. We should recall that the production possibility frontier is strictly convex to the origin and that the budget line is tangent to the production possibility frontier when both consumer goods are produced, as depicted in Figure 1. From the figure we can see that $Q_1(p,L)$ is decreasing in p, say $\partial Q_1 / \partial p < 0$. This fact, together with (32), says that $\partial^2 g / \partial p^2$ is negative for all $p \in [p_l, p_u]$. Thus it follows that for any $p \neq p'$

$$\begin{aligned} g(p,L) &< g(p',L) + [\partial g(p',L)/\partial p](p-p') \\ &= p'Q_1(p',L) + Q_2(p',L) + Q_1(p',L)(p-p') \\ &= pQ_1(p',L) + Q_2(p',L). \end{aligned}$$

It is clear from Figure 1 that p_l (resp. p_u) is the value of p such that $Q_1(p,L) = \overline{Q}_1$ (resp. $Q_2(p,L) = \overline{Q}_2$). Hence, replacing p' with p_l (or p_u) in the above equation, we can obtain Lemma 2.

Q.E.D.

Let us turn to the analysis of the gains from trade. For the time being we consider the case where both consumer goods are produced and we denote the indirect trade utility function as $V(p,L)$. Then, from (14) it follows that

(33) $$V(p,L)=\phi[g(p,L)/e(p)].$$

Concerning the properties of $V(p,L)$, we can show the following lemma.

Lemma 3

Consider home country and let p_A be a relative home autarkic price of good 1. Then the home indirect trade utility in the case where both consumer goods are produced, $V(p,L)$, has the following properties:

(i) $V(p,L)$ is strictly decreasing (resp. increasing) in p if p is higher (resp. lower) than p_A. Hence $V(p,L)$ is maximized at p_A.

(ii) $V(p,L)$ is smaller than $\phi[p\overline{Q}_1/e(p)]$ (resp. $\phi[\overline{Q}_2/e(p)]$) for all $p>p_l$ (resp. $p<p_u$).

The foreign indirect trade utility also satisfies the above property.

Proof. Consider the case of home countryPartially differentiating (33) with respect to p and using (32) yield

(34) $$\partial V/\partial p=[\phi'(\cdot)/e(\cdot)][Q_1(p)-\gamma g(p,L)/p].$$

The last term in (34), $\gamma g(p,L)/p$, represents the consumption level of good 1 when the relative price and the labor endowment are given by p and L, respectively. Therefore $\partial V/\partial p=0$ if and only if $p=p_A$. Therefore, to show that $\partial^2 V/\partial p^2$ is negative at p_A suffices to prove Lemma 3 (i). Deriving the second order derivative of (33) with respect to p and evaluating it at p_A, we have

(35) $$\frac{\partial^2 V(p_A,L)}{\partial p^2}=\frac{\phi'(\cdot)Q_1(p_A,L)}{e(p_A,L)p_A}\cdot\left[\varepsilon+\frac{\partial Q_1(p_A,L)}{\partial p}\cdot\frac{p_A}{Q_1(p_A,L)}\right],$$

from (32).

Making use of (13), we find that $Q_1(p,L)$ is the solution to

$p = S(Q_1, L) \equiv -\partial\Gamma(Q_1, L)/\partial Q_1$ in regard to Q_1. Thus $[\partial Q_1(p,L)/\partial p][p/Q_1(p,L)]$ is rewritten as

$$\text{(36)} \qquad \frac{\partial Q_1(p,L)}{\partial p} \cdot \frac{p}{Q_1(p,L)} = \left[\frac{Q_1(p,L)}{p} \Big/ \frac{\partial Q_1(p,L)}{\partial p} \right]^{-1}$$

$$= \left[\frac{\partial S(Q_1,L)}{\partial Q_1} \cdot \frac{Q_1}{S(Q_1,L)} \right]^{-1}$$

$$= 1/\chi_{p_S Q_1} .$$

By the use of (20), (35) and (36), we can show that $\partial^2 V/\partial p^2$ is negative at p_A. The property (ii) is easily derived from Lemma 2.

The case of foreign country can be shown in the same way.

Q.E.D

In Figure 4, the curve ABC represents the locus of the price-utility pair (p,V) such that $V = V(p,L)$ for a given L. Since a country expands the production of its exporting good by opening trade and $Q_1(p,L)$ is decreasing in p, we can confirm that the segment AB (resp. CB) corresponds to the locus of the price-utility pair (p,V) for which a country is incompletely specialized and exports good 1 (resp. good 2). On the other hand, the curves AE and CD represent the loci of the price-utility pair (p,V) satisfying $V = \phi[p\overline{Q}_1/e(p)]$ and $V = \phi[\overline{Q}_2/e(p)]$, respectively. That is, the curve AE(resp. CD) indicates the utility level of a country specializing completely in good 1 (resp. good 2). Thus we can establish the following proposition as to the relationship between the production patterns and the gains from trade.

(Figure 4)

Thus we can establish the following proposition as to the gains from trade.

Proposition 2

Suppose that $\eta_1(R) > \eta_2(R)$ for all $R > 0$. Then, as for the relationship between the production patterns and the gains from trade, the following hold.

(i) The country that is incompletely specialized after trade becomes worse off.

(ii) The country specializing in good 1 gains from trade if and only if international price of good 1 exceeds the level $\underline{p}$ such that $\phi[\underline{p}\overline{Q}_1/e(\underline{p})] = V_A$, where V_A represents

the autarkic utility level.

(iii) The country specializing in good 2 gains from trade if and only if international price of good 1 falls short of $\overline{p}$ *such that* $\phi[\overline{Q}_2 / e(\underline{p})] = V_A$.

Proof: Proposition 2 (i) immediately follows from Lemma 3 (i). Next let us prove Proposition 2 (ii) by the use of Figure 4. In the figure, the utility level of the good 1 exporting country is indicated by the curve BAFE and that of the good 2 exporting country is denoted by the curve BCFD. Moreover, the price levels $\underline{p}$ and $\overline{p}$ in the figure represent the values of p satisfying $V_A = \phi[p\overline{Q}_1 / e(p)]$ and $V_A = \phi[\overline{Q}_2 / e(p)]$, respectively. Thus the country specializing in good 1 (resp. good 2) loses from trade if and only if the relative price of good 1 falls short of $\underline{p}$ (resp. exceeds $\overline{p}$). **Q.E.D.**

Let us give an intuitive explanation of Proposition 2. First of all, we should remember that the autarkic demand curve is assumed to be steeper than the supply curve at the intersection. This assumption, together with homothetic preferences, says that on the production possibility frontier the marginal rate of substitution exceeds (resp. falls short of) the marginal rate of transformation if Q_1 is smaller (resp. larger) than the autarkic output level Q_{1A} [4]. An indifference curve corresponding to the autarkic utility level can be represented by the curve UU' and the curve FF' indicates the production possibility frontier in Figure 5. In this case, the budget line of the incompletely specializing country is as depicted by the line II' that is tangent to the production possibility frontier and lies below the autarkic indifference curve. Therefore, the country that incompletely specializes after trade becomes worse off. Concerning the interpretation of (ii) and (iii) of Proposition 2, let the lower (resp. upper) threshold price $\underline{p}$ (resp. $\overline{p}$) be given by the slope of the line that starts from the point F' (resp. F) and is tangent to the autarkic indifference curve UU' in Figure 6. Thus, we can see that the

[4] Note that the autarkic demand price p_D is equal to the marginal rate of substitution at the consumption point and $p_S = -\partial\Gamma(Q_1, L) / \partial Q_1$.

country specializing completely in good 1 (resp. good 2) becomes better off by trade if and only if international price p_e exceeds $\underline{p}$ (falls short of $\overline{p}$).

(Figures 5 and 6)

Finally, we consider an economy that consists of the two countries, say the home and foreign countries. Suppose that they are identical in all respects initially and that the foreign labor endowment has increased. Then, by (10) and (12), the foreign production possibility frontier shifts upward and becomes flatter. So the foreign country sees its national income rise for given p and it faces the lower p_l, which is the lower limit of p such that both consumer goods can be produced. Moreover, it follows from Lemma 1 that $p_A > p_A{}^*$. These facts, together with Proposition 1, imply that if the foreign labor endowment exceeds the home labor endowment, the home and foreign utility levels under free trade are, respectively, indicated by the curves BCFD and B'A'FE' in Figure 7. Proposition 2 shows that trade can be welfare-decreasing for one country, but from Figure 7 we find that it is not possible for both countries to lose from trade. For example, suppose that an international price of good 1 is given by $p' \in (\overline{p}, p_u)$ in the figure. Then, the foreign country completely specializes in good 1 and enjoys a rise in its welfare level although the home welfare level falls by trade for any production patterns.

(Figure 7)

6. Concluding Remarks

We have considered trade between two countries in a simple general equilibrium model with a pubic intermediate good. By the introduction of Marshallian dynamic adjustment process, we have developed a rigorous argument on the gains from trade as well as the patterns of trade in the two-country model. Three significant results have derived from our analysis.

The first is that under free trade the larger country exports the good whose productivity is more sensitive to the pubic intermediate good. We should notice that the result bears a close resemblance to the result shown in the similar framework but dealing with the case of increasing returns to scale. For example, Ethier (1982) and

Tawada (1989) dealt with a one-factor, two-good and two-country model in which one industry is subject to increasing returns to scale and verified that under free trade the larger country exports the good of the industry subject to such externalities.

The second is that, in spite of the efficient supplies of the public intermediate good, the country that produces both consumer goods under free trade necessarily becomes worse off. This result seems to be important when compared with that of the case of external economies in production. In the case of external economies, there exists a market distortion that the price line cuts the production possibility frontier at an equilibrium, which gives a negative effect to the gains from trade theorem. Nevertheless, it is possible for an incompletely specializing country to have a gain from trade in this case. In the case of the public intermediate good, however, the source of the result that trade is harmful to a country is only the shape of the production possibility frontier, which is convex to the origin, under the assumption of efficient supply of the public intermediate good. Therefore, it is inevitable for an incompletely specializing country to have a loss from trade. In this reason, we can easily infer that the traditional gains from trade must hold in the case of semi-public intermediate goods since the production possibility frontier becomes concave to the origin in this case. (See Tawada(1980).) Thirdly, we proposed a necessary and sufficient condition for an completely specializing country to have a gain from trade. According to this condition, if the international equilibrium price is far away from the autarkic equilibrium prices, one of two countries must lose from trade. According to Figure 7, if the international equilibrium price falls in between two autarkic equilibrium prices, both countries specialize and have a benefit from trade.

Finally, we can consider two extensions of our analysis. In this paper we have assumed that the public intermediate good has no international spillovers. But there are many public intermediate goods that serve in production internationally. The infrastructure of information networks, satellite communication systems, international research and developments etc. are the typical examples. Therefore our present analysis needs to be extended to the case of international public intermediate goods. Another important extension is to suppose public intermediate goods to be durable,

since the public intermediate goods play a role of infrastructure for private production. Then the public intermediate should be characterized as a capital stock and the analysis should be dynamic.

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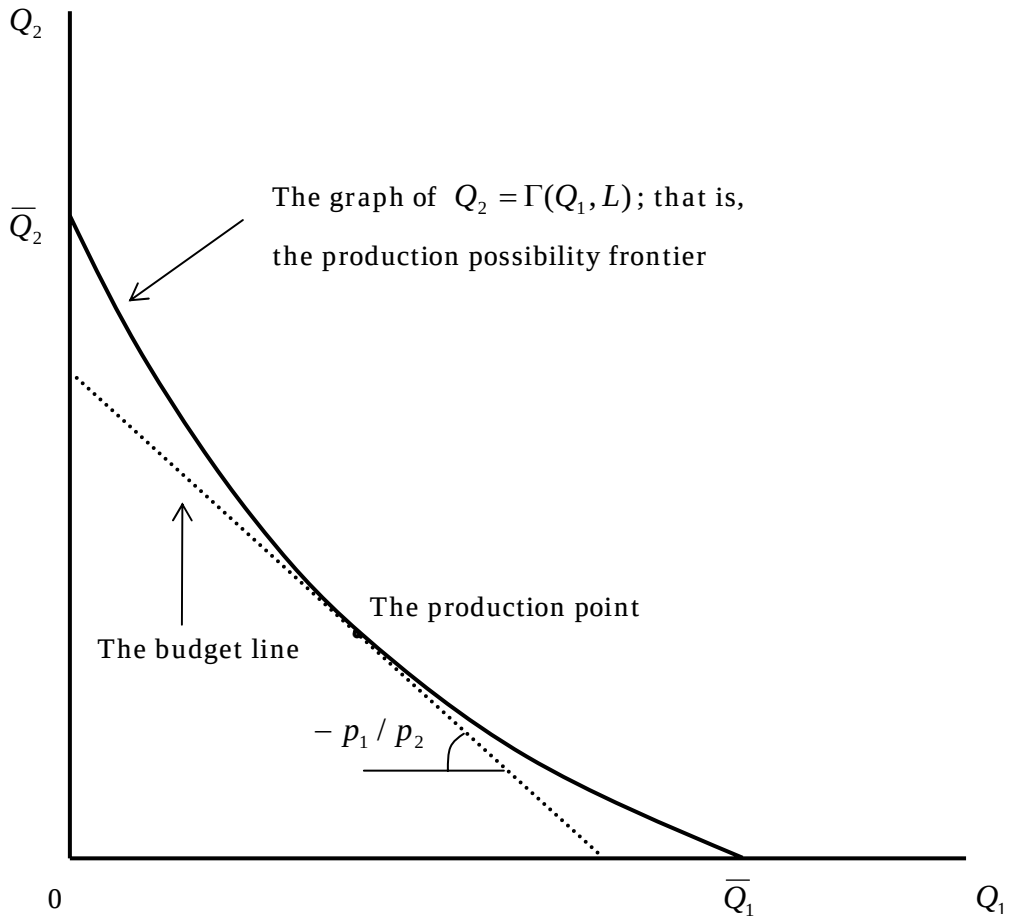
Q_2
$\overline{Q}_2$
The graph of $Q_2 = \Gamma(Q_1, L)$; that is,
the production possibility frontier
The budget line
The production point
$-p_1 / p_2$
0
$\overline{Q}_1$
Q_1

Figure 1

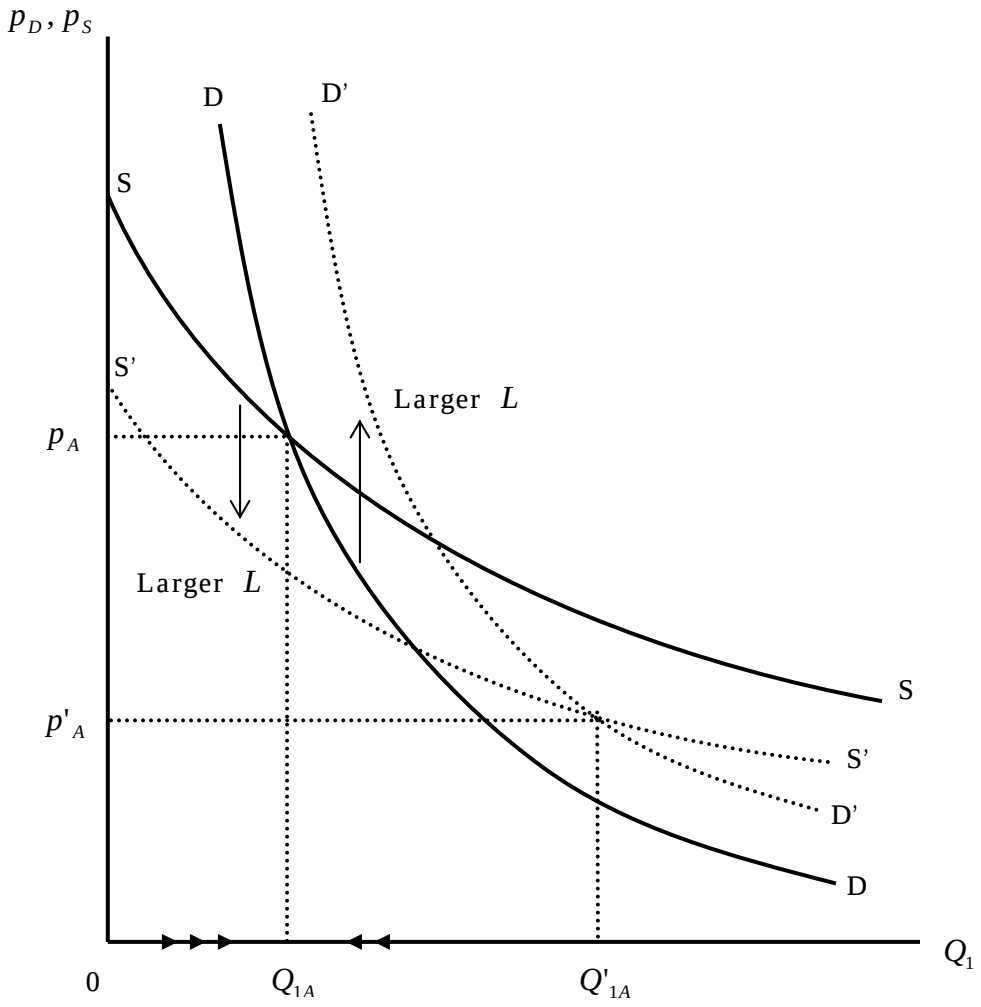

Figure 2

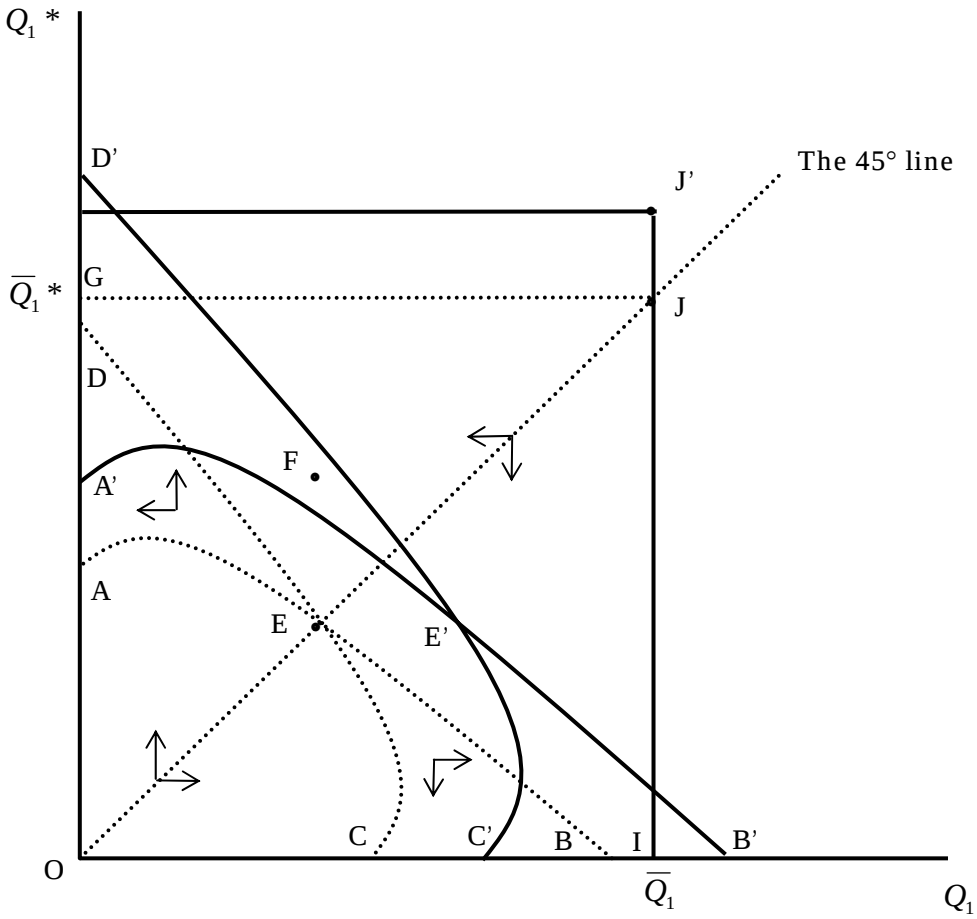

Figure 3

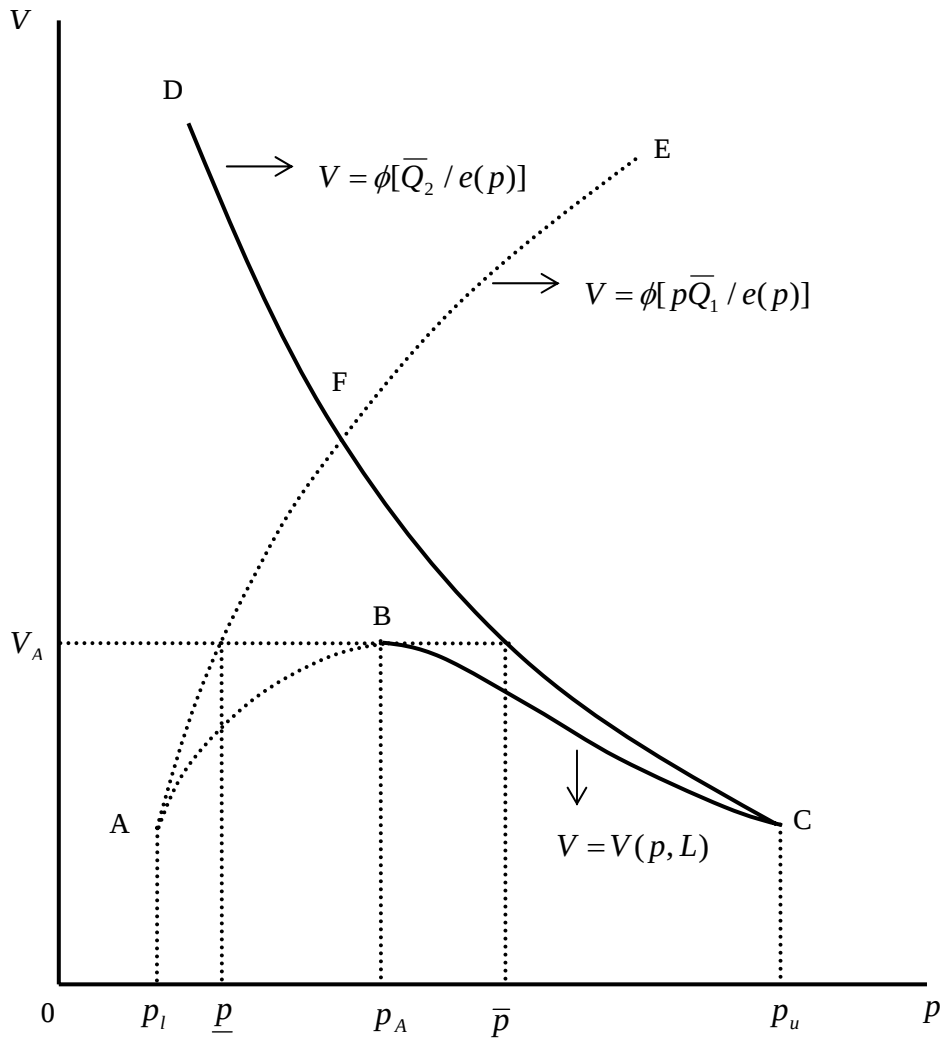
V
D
E
$V = \phi[\overline{Q}_2 / e(p)]$
$V = \phi[p\overline{Q}_1 / e(p)]$
F
B
V_A
A
C
$V = V(p, L)$
0
p_l
$\underline{p}$
p_A
$\overline{p}$
p_u
p

Figure 4

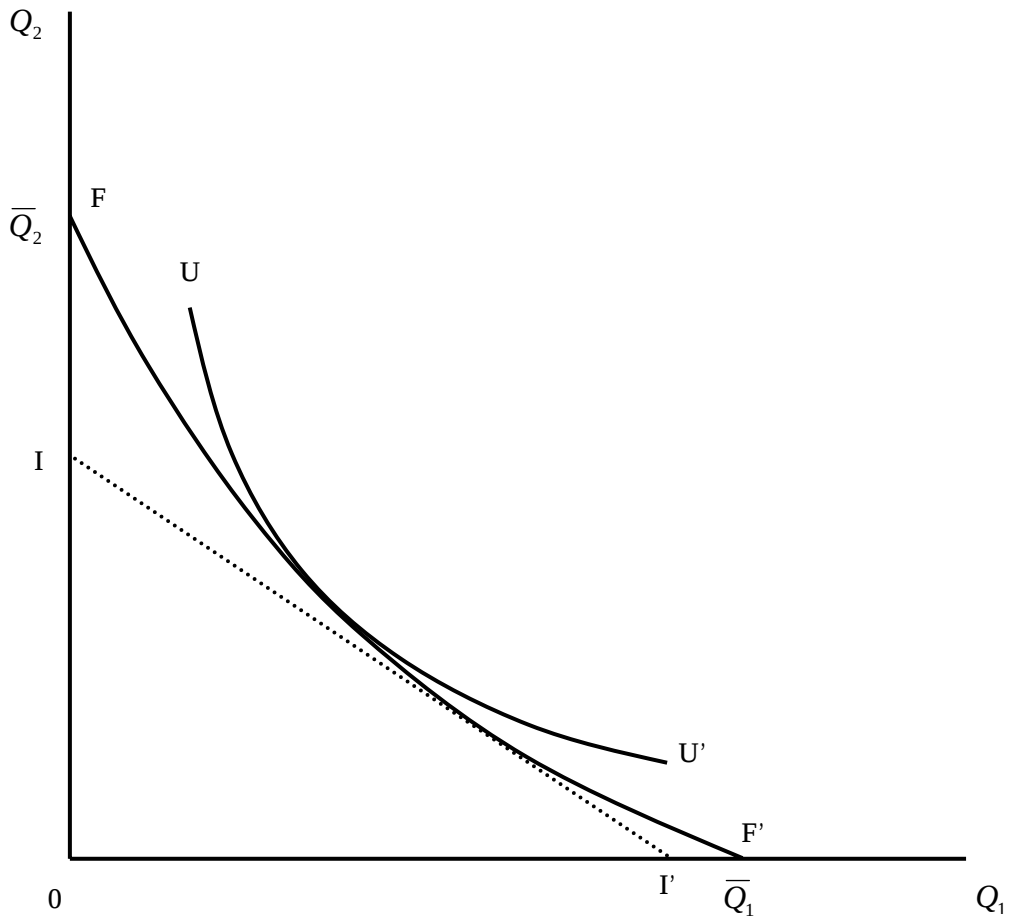

Figure 5

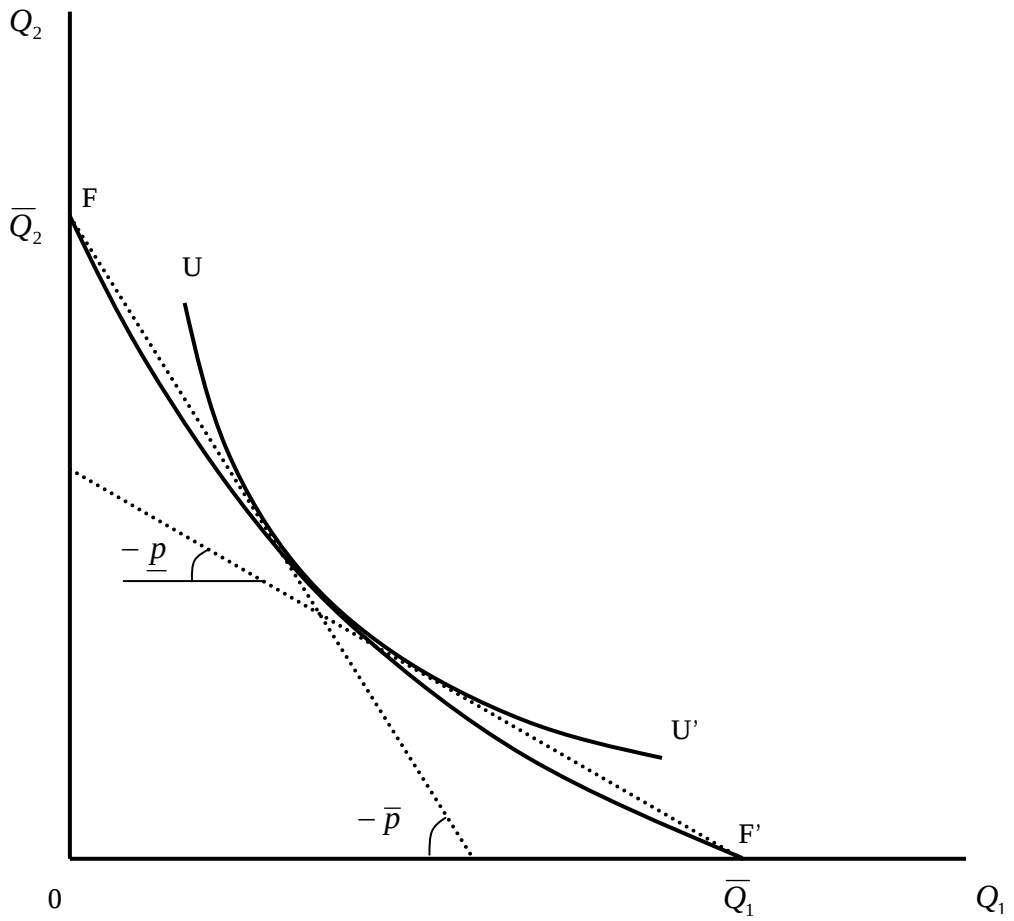

Figure 6

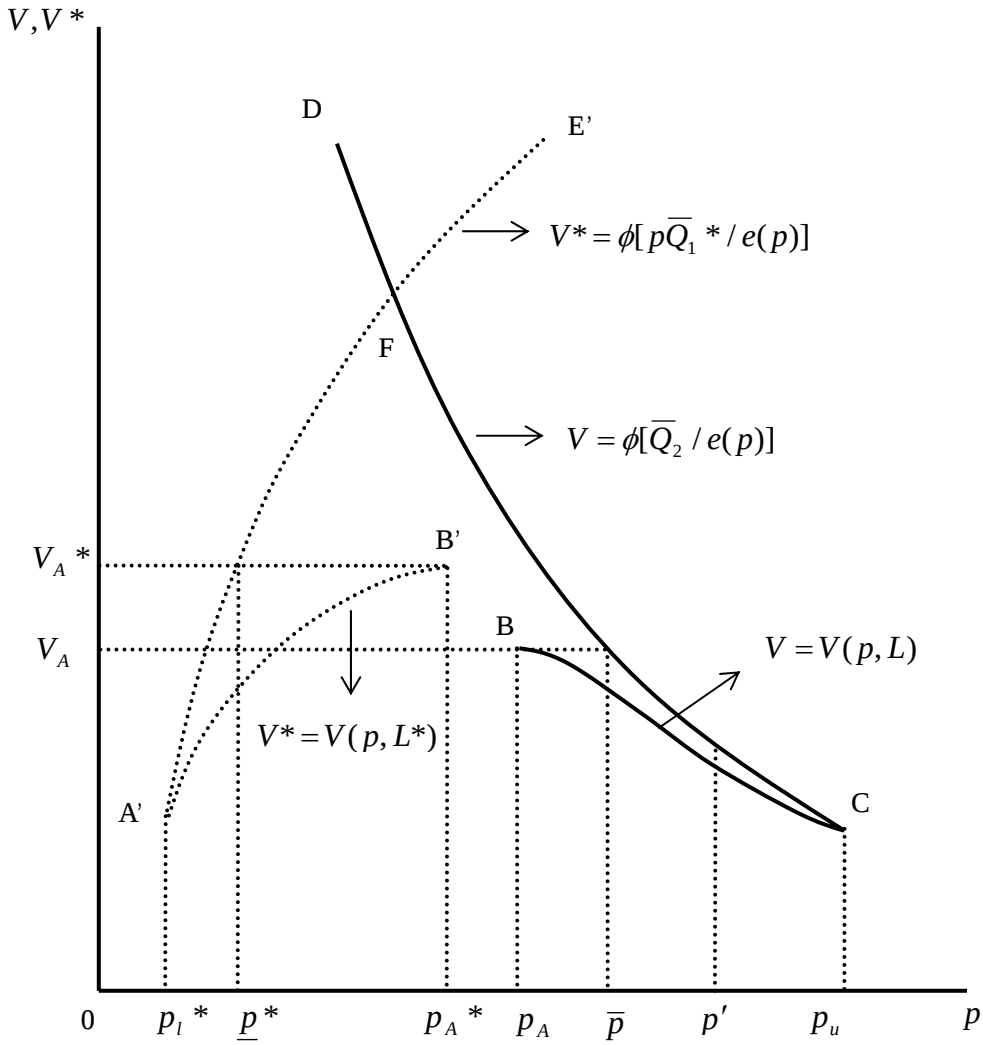
V, V^*
D
E'
$V^* = \phi[p\overline{Q}_1^* / e(p)]$
F
$V = \phi[\overline{Q}_2 / e(p)]$
V_A^*
B'
V_A
B
$V = V(p, L)$
$V^* = V(p, L^*)$
A'
C
0
p_l^*
$\underline{p}^*$
p_A^*
p_A
$\overline{p}$
p'
p_u
p

Figure 7