R&D policies, trade and process innovation

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Abstract

In this paper we set up a simple trade model with two countries that host one firm each. The firms may invest in R&D to reduce their marginal production costs, and each government may grant R&D subsidies to its domestic firm. We show that it is optimal for a government to provide higher R&D subsidies the lower the level of trade costs. This is true even if the firms are independent monopolies. If the firms produce imperfect substitutes, the policy competition in R&D subsidies may imply that one of them suspends production. An equilibrium with production in both countries is sustainable if the countries harmonize their subsidy rates, but may result in excessive R&D investments.

Keywords: trade, R&D, subsidies, process innovation

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1 Introduction

Research and development (R&D) is of great importance both from firms' and governments' point of view. Industrial R&D may result in new goods, higher product quality or lower production costs and consumer prices. In many industries R&D is considered to be of vital importance to survive in the market; in particular, this seems to be true for industries with strong international competition. From the governments' point of view, there are at least two reasons why industrial R&D activities should be supported. First, public-goods aspects of R&D imply that the market on its own typically supplies less than the optimum R&D. Second, R&D policies capture important strategic trade policy effects, and may hence be applied to improve the position of domestic firms in international markets. However, in an international setting it is also clear that national R&D policies do not necessarily lead to a global optimum. There is, on the one hand, an obvious danger of harmful policy competition between countries, implying too high subsidies. On the other hand, with cross-border public goods, governments pursuing national interest only, will typicality supply too little subsidies from a global point of view. Hence, while there are good reasons to recommend active use of public policies to stimulate R&D in firms, it is not always obvious what the optimal policies should look like.

In this paper we study optimal national and international R&D policies in a number of different settings. We look at the importance of international trade for R&D investments and R&D policies; we analyze possible effects of policy competition between countries, and we study the optimal design of global or regional R&D policies in cases where countries cooperate. For this purpose we construct a simple model of two countries with one firm in each. The firms produce horizontally differentiated goods, and there is intra-industry trade between the countries. The firms can invest in process-improving R&D to reduce (marginal) production costs; and they do so as long as they find such investments profitable. Governments can influence the R&D decision through subsidizing R&D. We specify a two-stage game, where governments at the first stage simultaneously set subsidies, and the firms thereafter choose R&D, production levels and sales in the two markets.

The degree of horizontal product differentiation plays a key role in the model. If the two goods are poor substitutes, the consumers value highly the availability of different products, and competition between the firms is not particularly strong; hence, there is room for both goods in the market. If, on the other hand, the goods are close substitutes, the firms compete fiercely, while consumers do not care much whether they choose one variant or the other; in such an industry there may be equilibria where only one of the firms survives. In both cases R&D and R&D policies matter, but perhaps for different reasons and with different results. When goods are poor substitutes, each firm's choice of R&D level follows from a cost-minimizing trade-off between lower (marginal) production costs and higher (R&D) investment costs, without much focus on what the other firm chooses. For the government, the motive for active policies is to ensure that not only profits but also consumer surplus effects are taken into account when R&D levels are determined. When goods are close substitutes, the consumers will be relatively price sensitive. Each government will therefore take into account the fact that its domestic firm can gain a large share of the market even if it has only a small cost advantage relative to its competitor. All else equal, the governments will thus grant higher R&D subsidies the closer substitutes the goods are. Hence, there is a "profit shifting" or "business stealing" motive for R&D subsidies in addition to the public-good motive discussed above.

A number of interesting results come out of our analysis. For instance, we show that the level of trade costs is important for optimal R&D investments and R&D policies. The reason for this is that freer trade increases the size of the market, and makes it more profitable even for a monopoly to invest in cost-reducing R&D. Thereby, freer trade actually leads not only to more exports but also to more domestic sales. The government - focusing on consumer surplus as well as profits (net of subsidies) - realizes this, and finds that the bigger the market, the higher is the R&D effect of a given subsidy, and the stronger are the incentives to subsidize R&D. Hence, trade liberalization leads to more R&D, higher R&D subsidies and more sales both in domestic and foreign markets. In this case, the motive for subsidies is not to promote exports *per se*; the size of the export market is only important because it matters for the choice of R&D investments and hence for domestic consumer surplus[2]

[2] The effect is similar to what Krugman (1984) labelled "import protection as export promotion", in that it focusses on the links between the size of the market and the marginal costs of production. However, while Krugman's focus was on how to promote exports, in our case export is a means to ensure

From the strategic trade policy literature, it is well known that R&D subsidies may be a second-best option if export subsidies are not available (see e.g. Spencer and Brander (1983) and Leahy and Neary (2000)), and in some respects such subsidies can also be a more robust policy recommendation than export policies (see e.g. Bagwell and Staiger (1994) and Brander (1995)). This strand of the literature nonetheless argues that policy competition tends to result in excessive R&D from the subsidizing countries' point of view. This is due to the business stealing effect. However, all the studies mentioned above make the simplifying assumption that all production is exported to a third country. Haaland and Kind (2004) depart from this simplification, and focus directly on domestic consumer surplus effects of R&D subsidies and on the need for international policy coordination to avoid harmful strategic trade policy[3]. The analysis shows that policy competition gives "wrong" subsidies, but not necessarily too high subsidies, compared to a coordinated solution. If goods are close substitutes, policy competition implies too high subsidies; if on the other hand, goods are fairly differentiated, a coordinated solution would give higher subsidies than the non-cooperative outcome of the policy competition.

In the present paper we modify and extend this analysis. In particular, while the above analysis only looked at symmetric outcomes, we now study carefully all possible outcomes. When goods are close substitutes, policy competition may be so fierce that it is impossible for both firms to survive in the market. We thus find multiple equilibria where one of the firms is not active in the market. Depending on the degree of product differentiation in the industry, we may have a stable symmetric equilibrium, an unstable symmetric equilibrium, or no symmetric equilibria at all. In the latter two cases there may be stable asymmetric equilibria, even if the countries at the outset are completely symmetric.

Furthermore - and perhaps more surprising - we find that with coordinated policies between the two countries, a harmonization of the R&D subsidies is not necessarily welfare maximizing. When the two goods are sufficiently close substitutes, it will not be optimal

lower costs and higher domestic sales.

[3] Leahy and Neary (2001) analyse policy competition and coordination between countries in a similar setting.

from the society's point of view to invest in process innovation in both industries. It would be better to concentrate R&D efforts to one of the industries, to save investment costs. Hence, the optimal common R&D policy for the two countries could be to subsidize R&D in one of the countries, but not in the other. Indeed, it may even be optimal to tax R&D in the other country. The intuition for this is related to the public-goods aspect of R&D and the fact that with close substitutes the added value for the consumers of having process development in both firms is not very high. So to avoid duplication of the investment costs, the common first-best policy could be to stimulate R&D in one firm and reduce the R&D incentives in the other.

2 The model

Demand side

We employ a model with two intrinsically symmetric countries and two firms. Firm 1 is located in and owned by residents of Country 1, while Firm 2 is located in and owned by residents of Country 2. The population size in each country is equal to 1, and the utility function of a representative consumer is given by

$$U_i = \alpha q_{ii} + \alpha q_{ji} - \left(\frac{q_{ii}^2}{2} + \frac{q_{ji}^2}{2} + b q_{ii} q_{ji} \right), \tag{1}$$

where q_{ii} and q_{ji} are consumption of the goods produced by the domestic and the foreign firm, respectively. The first subscript thus indicates in which country the good is produced, and the second subscript in which country the good is consumed.

Equation (1) is a standard quadratic utility function where the parameter $b \in [0, 1)$ measures the degree of horizontal differentiation between the goods; the goods are completely independent if $b = 0$, while they are identical in the limit $b = 1$. More generally, the two goods are closer substitutes from the consumers' point of view the higher is b.

Letting p_{ii} and p_{ji} denote the end-user prices of the two goods in country i, we may express consumer surplus as $CS_i = U_i - p_{ii}q_{ii} - p_{ji}q_{ji}$. Provided that trade takes place, optimal consumer behavior implies that $\partial CS_i / \partial q_{ii} = \partial CS_i / \partial q_{ji} = 0$. From this we find that the inverse demand curves are given by

$$p_{ii} = \alpha - (q_{ii} + bq_{ji}) \text{ and } p_{ji} = \alpha - (q_{ji} + bq_{ii}). \tag{2}$$

Supply side

The firm located in country i incurs trade costs $\tau \geq 0$ per unit it exports to country j. In absence of R&D investments the marginal production cost of firm i is equal to c. In this case the profit margins on domestic sales and exports are given by $(p_{ii} - c)$ and $(p_{ij} - c - \tau)$, respectively. However, each firm may invest in R&D in order to reduce its marginal costs. More specifically, firm i reduces its marginal production costs to $(c - x_i)$ by investing $C(x_i) = x_i^2 + f$ in process innovation, where the parameter f represents the fixed costs of setting up an R&D project. We may thus write the profit function of firm i as

$$\pi_i = (p_{ii} - (c - x_i))q_{ii} + (p_{ij} - (c - x_i) - \tau)\, q_{ij} - x_i^2 + s_i x_i - f, \tag{3}$$

where $s_i \geq 0$ is the R&D subsidy level the firm receives from its domestic government.

Clearly, the firms may find it optimal to invest in R&D until marginal costs equal zero if $(\alpha - c)$ is sufficiently large, particularly if they receive R&D subsidies. We shall assume that $(\alpha - c)$ is not so high that this happens.[4]

Welfare in each country is given by the sum of domestic consumer surplus and profit minus R&D subsidies:

$$W_i = CS_i + \pi_i - s_i x_i. \tag{4}$$

Note that consumer surplus may be written as

$$CS_i = \frac{1}{2}\left(q_{ii}^2 + q_{ji}^2\right) + bq_{ii}q_{ji}. \tag{5}$$

In the following we consider a two-stage game, where the governments set R&D subsidies at stage 1 and the firms set quantities and decide R&D levels at stage 2.[5]

[4] A sufficient condition for $(c - x_i) > 0$ to be true is that $c/\alpha \geq 4/5$. See the section "Optimal cooperative R&D subsidies" in the Appendix.

[5] The literature often assumes that the firms set R&D levels before they choose output. This makes sense if the firms both are willing and able to make commitment with respect to their R&D investments. Otherwise, it seems more natural to assume that R&D efforts and output are determined simultaneously, as we do in this paper. See further discussion in the final section.

2.1 Benchmark: Optimal R&D subsidies to a monopoly

As a benchmark we assume that the firms are monopolies in their own market segments, which amounts to setting $b = 0$. This means that there are no strategic interactions between the firms, so that they choose R&D investments and output independent of each other.

Holding R&D investments fixed, profit maximizing output for firm i is found by setting $\partial\pi_i/\partial q_{ii} = \partial\pi_i/\partial q_{ij} = 0$ if there is trade. This yields monopoly outputs

$$q_{ii} = \frac{\alpha - (c - x_i)}{2} \text{ and } q_{ij} = \frac{\alpha - \tau - (c - x_i)}{2}. \tag{6}$$

Suppose f is sufficiently small that the firm chooses to invest in R&D. The cost of increasing R&D investment by one unit is equal to $(2x_i - s_i)$, while the benefit - in terms of reduced marginal production costs - equals $(q_{ii} + q_{ij})$. The benefit is thus increasing in total output. Profit maximizing behavior implies that $(2x_i - s_i) = (q_{ii} + q_{ij})$, or

$$x_i = \frac{q_{ii} + q_{ij} + s_i}{2}. \tag{7}$$

Combining (6) and (7) we find that output equals

$$q_{ii} = \frac{4(\alpha - c) - \tau + 2s_i}{4} \text{ and } q_{ij} = \frac{4(\alpha - c) - 3\tau + 2s_i}{4} \tag{8}$$

while R&D investment is

$$x_i = \frac{2(\alpha - c) - \tau + 2s_i}{2}. \tag{9}$$

Not surprisingly, we see that export is decreasing in the level of trade costs. More interesting, the same is true also for domestic sales and R&D investments. The reason for the latter is that higher trade costs reduce export and thus the firm's willingness to invest in cost reductions. This leads to higher marginal production costs $(c - x_i)$ and therefore lower output also domestically.

It is well known from, e.g., Spencer and Brander (1983) that a government may have incentives to grant R&D subsidies to domestic firms in order to improve their competitive position. This has been labelled the "business stealing effect" in the literature. However, there are no strategic interactions between the firms if $b = 0$, and therefore no business stealing effect. Consequently, the government in country i cannot use R&D subsidies to increase profit net of R&D subsidies for its domestic firm;

$$\frac{\partial\left(\pi_i - s_i x_i\right)}{\partial s_i} = -s_i < 0 \text{ for } s_i > 0. \tag{10}$$

It should further be noted that the government and the firm in country i have coinciding interests in utilizing the monopoly power abroad. All else equal, the quantity q_{ij} given by equation (6) is therefore optimal both from the government's and firm's point of view. *Given output,* it is moreover straight forward to show that the government would prefer R&D investments such that $x_i^* = x_i = \left(q_{ii} + q_{ij}\right)/2$.[6] Thus, also in this respect the government and the firm have coinciding interests (c.f., equation (7) with $s_i = 0$). However, the monopolist's output at home is too low from the government's point of view. The fact that q_{ii} is increasing in s_i therefore suggests that it is welfare improving for the government to subsidize cost-reducing R&D, since it will be optimal to make the firm increase domestic supply and therefore investment in R&D. Thereby the government is able to reduce the domestic consumer price and increase consumer surplus (also foreign consumer surplus increases, but this is irrelevant for the government in country i):[7]

$$\frac{\partial p_{ii}}{\partial s_i} = -\frac{1}{2} < 0 \text{ and } \frac{\partial CS_i}{\partial s_i} = \frac{1}{2} q_{ii} > 0. \tag{11}$$

The consumers gain more from a given price reduction the more they consume of the good. We therefore see that $\partial CS_i/\partial s_i$ is increasing in q_{ii}. This in turn indicates that the government should optimally increase the subsidy level if trade costs fall, since output is higher the lower the level of trade costs. Formally, setting $\partial W_i/\partial s_i = \partial\left(\pi_i - s_i x_i\right)/\partial s_i + \partial CS_i/\partial s_i = 0$ we have (with superscript M for monopoly):

$$s_i^M = \frac{4\left(\alpha - c\right) - \tau}{6}; \ \frac{\partial s_i}{\partial \tau} < 0. \tag{12}$$

We can now state:

Proposition 1: *Suppose that the firms are monopolists in their own market segments. The governments will then subsidize domestic R&D. Trade liberalization ($d\tau < 0$) makes it optimal to increase the subsidy level.*

[6] Provided that this does not imply negative marginal costs, $\left(c - x_i^*\right)$.

[7] Using (2) and (8) with $b = 0$ we find $p_{ii} = c + \tau/4 - s_i/2$ and $p_{ij} = c + 3\tau/4 - s_i/2$

As noted above, there are no strategic interactions between the firms (or the governments) if $b = 0$. The mechanisms through which trade makes it optimal for governments to subsidize R&D is therefore qualitatively different from those that have been analyzed in strategic trade policy papers. Indeed, the only reason why the government increases R&D subsidies when we open up for trade in the present context, is that this makes the domestic economy more efficient. The output of the R&D project - here more cost efficient production technologies - is a non-rival good that should be provided in a greater quantity the larger the activity level of the firm. Other things equal, trade liberalization increases total output and therefore makes it optimal to invest more in R&D both from a private and social point of view.

3 R&D policies with intra-industry trade

In the rest of the paper we assume that $b \in (0, 1)$, which means that the two goods are imperfect substitutes. It should be noted that the quadratic utility function described by equation (1) has the realistic feature that total market demand is decreasing in b, all else equal.[8] This reflects the common assumption that consumers have convex preferences, so that the size of the market tends to be smaller the less differentiated the goods.

3.1 Market equilibrium

At the last stage the firms simultaneously choose quantities and R&D investments. An equilibrium with intra-industry trade is thus given by $\partial\pi_i/\partial x_i = \partial\pi_i/\partial q_{ii} = \partial\pi_i/\partial q_{ij} = 0$. Holding quantities fixed, we find that $\partial\pi_i/\partial x_i = 0$ implies

$$x_i = \frac{q_{ii} + q_{ij} + s_i}{2}, \tag{13}$$

which is the same expression as we had for the monopoly. The incentives to invest in cost reduction is consequently also in this case increasing in total output and the subsidy level.

[8] This is most easily seen by assuming that the goods are sold at a fixed price $\bar{p}$. We then find that consumer demand is given by $q_{ii} = q_{ji} = (\alpha - \bar{p}) / (1 + b)$.

Solving $\partial\pi_i/\partial q_{ii} = \partial\pi_i/\partial q_{ij} = 0$ when we hold R&D investments fixed we further have

$$\begin{aligned} q_{ii} &= \frac{1}{2+b}(\alpha - c) + \frac{b}{4-b^2}\tau + \frac{2x_i - bx_j}{4-b^2} \\ q_{ij} &= \frac{1}{2+b}(\alpha - c) - \frac{2}{4-b^2}\tau + \frac{2x_i - bx_j}{4-b^2}. \end{aligned} \tag{14}$$

Higher trade costs make the home market more protected from foreign competition. For any given R&D investment, we therefore find a positive relationship between domestic sales and trade costs $(\partial q_{ii}/\partial\tau > 0)$. However, the direct effect of higher trade costs is to reduce export $(\partial q_{ij}/\partial\tau < 0)$, and it is easily verified that total sales for each firm is decreasing in τ $(\partial(q_{ii} + q_{ij})/\partial\tau < 0)$. Equation (13) therefore tells us that higher trade costs imply less cost-reducing R&D investments, which in turn reduce both domestic sales and exports $(\partial q_{ii}/\partial x_i = \partial q_{ij}/\partial x_i > 0)$. The latter effect suggests that higher trade costs may imply that sales at home fall. Indeed, from the analysis above we know that this is true in the monopoly case $(b = 0)$. With $b \in (0, 1)$ we can combine (13) and (14) to find that output equals[9]

$$\begin{aligned} q_{ii} &= \frac{1}{1+b}(\alpha - c) - \frac{1-2b}{2(2-b)(1+b)}\tau + \frac{s_i - bs_j}{2(1-b^2)} \\ q_{ij} &= \frac{1}{1+b}(\alpha - c) - \frac{3}{2(2-b)(1+b)}\tau + \frac{s_i - bs_j}{2(1-b^2)}. \end{aligned} \tag{15}$$

while

$$x_i = \frac{1}{1+b}(\alpha - c) - \frac{1}{2(1+b)}\tau + \frac{2-b^2}{2(1-b^2)}s_i - \frac{b}{2(1-b^2)}s_j. \tag{16}$$

From (15) and (16) we have the following:

Proposition 2: *Holding subsidies fixed, trade liberalization* $(d\tau < 0)$ *leads to higher domestic output if* $b < 1/2$ *and to higher export and more R&D investments for all* $b \in [0, 1)$.

The reason why trade liberalization reduces domestic sales for $b > 1/2$, is that the goods are then so close substitutes that the firm loses sales due to higher import competition even though it invests more in cost-reducing R&D. Absent of trade costs and subsidies

[9] We have here implictly assumed that $c - x_i \geq 0$; see discussion below.

($\tau = s_1 = s_2 = 0$), we further see that quantities and R&D investments are decreasing in b. This reflects the fact that the total market size is smaller the less differentiated the goods are, as noted above.

To see the effectiveness of granting R&D subsidies when there is competition between the firms, we can use equations (15) and (16) to find

$$\frac{\partial^2 q_{ii}}{\partial s_i \partial b} = \frac{\partial^2 q_{ij}}{\partial s_i \partial b} = \frac{\partial^2 x_i}{\partial s_i \partial b} = \frac{b}{(1-b^2)^2} > 0.$$

If firm i receives higher subsidies, it will thus respond by increasing output and R&D investment more the higher is b. This may seem a bit surprising, since the size of the market is decreasing in b. The intuition for this result is that the firms compete more fiercly the less differentiated their goods are - a firm that has only a small cost advantage can capture a relatively large share of the market if the competitor produces a close substitute. Hence, a given increase in the subsidy level gives rise to a larger R&D investment and output expansion the higher is b. For the same reason, we also find that $sign\,(\partial q_{ii}/\partial s_j) = sign\,(\partial q_{ij}/\partial s_j) = sign\,(\partial x_i/\partial s_j) < 0$ with

$$\frac{\partial^2 q_{ii}}{\partial s_j \partial b} = \frac{\partial^2 q_{ij}}{\partial s_j \partial b} = \frac{\partial^2 x_i}{\partial s_j \partial b} = -\frac{1+b^2}{2\,(1-b^2)^2} < 0.$$

3.2 R&D policy competition

First-order conditions with R&D policy competition

In the first stage of the game the governments non-cooperatively choose subsidy levels to maximize domestic welfare. Solving $\partial W_i/\partial s_i = 0$ simultaneously for the two countries we find a symmetric outcome given by $s_1 = s_2 \equiv s^{PC}$ (superscript PC for policy competition):

$$s^{PC} = \frac{2\,(1+b^2)}{3+4b-3b^2-2b^3}(\alpha - c) - \frac{1-2b+3b^2}{(2-b)\,(3+4b-3b^2-2b^3)}\tau. \tag{17}$$

Inserting for (17) into (16) we further find

$$x_i^{PC} = \frac{5-b^2}{3+4b-3b^2-2b^3}\,(\alpha - c) - \frac{4-3b+b^3}{(2-b)\,(3+4b-3b^2-2b^3)}\tau. \tag{18}$$

The subsidy level and R&D investments are thus decreasing in τ, which is what we should expect from the monopoly case. Since the size of the market is smaller the less differentiated the goods are, one might further expect that s^{PC} and x_i^{PC} to be monotonically decreasing in b. However, this is not true - differentiating equations (17) and (18) with respect to b, we find that both $s(b)$ and $x(b)$ are U-shaped.

To see the intuition for this result, it is useful to note that we can write the profit level of firm i as

$$\pi_i = q_{ii}^2 + q_{ij}^2 - x_i^2 + s_i x_i.$$

Differentiating profits net of R&D subsidies we have

$$\frac{\partial \left(\pi_i - s_i x_i\right)}{\partial s_i} = \frac{q_{ii} + q_{ij}}{1 - b^2} - \frac{\left(2 - b^2\right) x_i}{1 - b^2},$$

where the first term shows the increase in operating profits for firm i subsequent to a marginal increase in s_i and the second term shows the resulting higher R&D costs. To isolate the business stealing effect, suppose for the moment that both countries set subsidies so as to maximize domestic profit net of subsidies (i.e., they do not take consumer surplus into account when they set the subsidy levels). Solving $\partial \left(\pi_i - s_i x_i\right) / \partial s_i = 0$ for $i = 1, 2$ we then find that the countries would end up with the common subsidy level

$$s^{\pi} = \frac{2b^2}{2 + 2b - 2b^2 - b^3} \left(\alpha - c - \tau/2\right); \ \frac{\partial s^{\pi}}{\partial b} > 0; \ \frac{\partial^2 s^{\pi}}{\partial b^2} > 0.$$

This would be the equilibrium subsidy level if all output were exported to a third country. Each country would then subsidize its domestic firm's R&D in order to give it a competitive advantage over the other firm. This is a pure business-stealing effect, which is present for all $b > 0$ ($s^{\pi} = 0$ at $b = 0$). Since the business-stealing effect becomes increasingly stronger the closer substitutes the goods are, the subsidy level is an increasing and convex function of b. This explains why $s^{PC}(b)$ is upward-sloping for high values of b; the business stealing effect is then so strong that it dominates over the fact that the size of the market is decreasing in b.

We can now state:

Lemma 1: *Assume that the countries set R&D subsidy levels non-cooperatively at stage 1. In this case the first-order conditions give rise to a U-shaped relationship between* *the subsidy level and b in each country, while each firm's marginal cost forms an inverted U-shaped curve of b.*

The U-shaped relationship between s^{PC} and b is shown in the left-hand side panel of Figure 1. The Figure also illustrates that trade liberalization (trade costs reduced from $\tau = 1/4$ to $\tau = 0$) gives rise to a positive vertical shift in the curve s^{PC}.

The curve labelled $MC(s = 0)$ in the right-hand side panel of Figure 1 shows that marginal costs ($MC = c - x_i$) are increasing in b if the firms do not receive R&D subsidies. This reflects the negative relationship between b and the size of the market. With subsidies, on the other hand, we have an inverted U-shaped curve; indeed, with subsidies the firms actually have the highest cost-reducing investments in the smallest market (i.e. in industries where b is close to 1).[10]

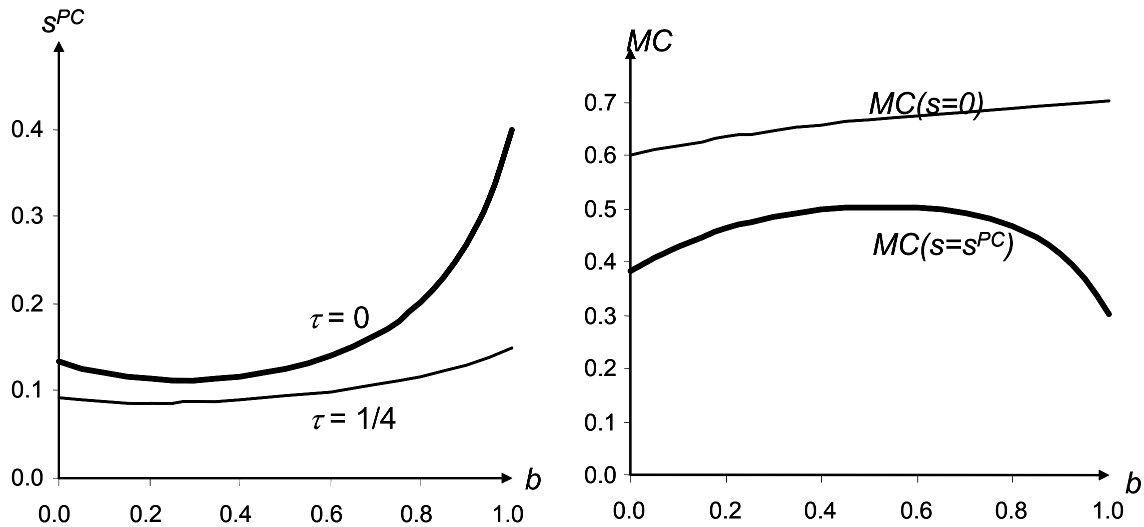

Figure 1: *FOCs for subsidy levels and marginal costs with policy competition.*

Equilibrium with R&D policy competition

The above analysis shows the first-order conditions for subsidies set non-cooperatively by the governments. We will now analyze whether these first-order conditions characterize a (unique) equilibrium. To this end we have to check the second-order conditions and the stability of the system. In order to simplify the algebra, we will in the following assume

[10] In all the figures we assume that $\alpha = 1$ and $c = 0.8$.

that $\tau = 0$.

The second-order conditions for the firms' choice of quantities and R&D investments at stage 2 are satisfied. However, when the countries compete in subsidies at stage 1 we find that

$$\frac{\partial^2 W_i}{\partial s_i^2} = -\frac{(3-b^2)(1-2b^2)}{4(1-b^2)^2},$$

which means that the second-order conditions hold if $b < (1/2)\sqrt{2} \approx 0.707$. We further have

$$\frac{\partial W_i}{\partial s_i} = \frac{(1+b^2)}{2(1+b)^2(1-b)}(\alpha - c) - \frac{(3-b^2)(1-2b^2)}{4(1-b^2)^2}s_i - \frac{b(1+b^2)}{4(1-b^2)^2}s_j, \tag{19}$$

where the term before s_i is negative if and only if the second-order conditions are satisfied. Since the first term is positive, we therefore find (at least a local) optimum at $W_i/\partial s_i = 0$.

Solving $W_i/\partial s_i = 0$ for the range of b where the second-order conditions hold, we find the reaction function

$$s_i(s_j) = \frac{2(1-b)(1+b^2)}{(3-b^2)(1-2b^2)}(\alpha - c) - \frac{(1+b^2)b}{(3-b^2)(1-2b^2)}s_j. \tag{20}$$

The system is stable if $\left|\frac{\partial s_i(s_j)}{\partial s_j}\right| < 1$. From equation (20) we find that this is satisfied for $b < 0.591$, but not for larger values of b.

The reaction curves $s_1(s_2)$ and $s_2(s_1)$ are illustrated in Figure 2. The left-hand side panel of Figure 2 shows the reaction curves for $b = 0.5$, in which case the stability conditions are satisfied. If the countries initially have different subsidy levels - $s_1 > s_2$, say - then each country's best response to the other country's subsidy level leads to a convergence where the countries eventually end up with the same subsidies.[11] The stability conditions are, however, not satisfied in the right-hand side panel of Figure 2, where $b = 0.65$. Here the figure indicates that we will eventually end up with a positive subsidy level in Country 1 and zero subsidies in Country 2 if initially $s_1 > s_2$. The reason for this is that the consumers consider the goods to be so close substitutes if $b > 0.591$ that one of the countries may find it optimal to set subsidy levels which are so high that its domestic firm captures the whole market. Note that it is sufficient for country 1 to set

[11] Here we follow the conventions in the literature and use the terms "reaction" and "response" even though the countries set the subsidy levels simultaneously.

$s_1 = s_1^A$ in order to ensure that Country 2 sets $s_2 = 0$ (where $s_1^A > s^{PC}$).

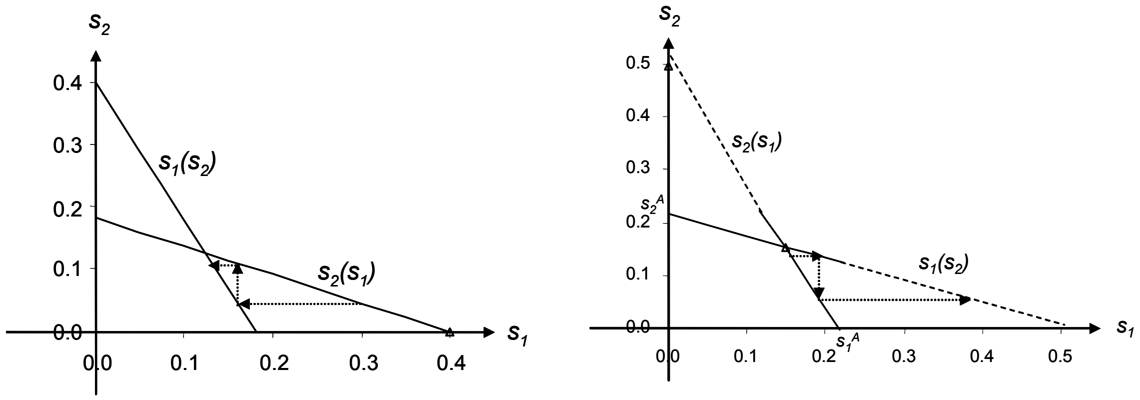

Figure 2: *Stability in R&D competition.*

The right-hand side panel of Figure 2 thus suggests that we have a stable equilibrium where only one of the countries grants R&D subsidies if the symmetric equilibrium is unstable. To verify this, assume that $b \in (0.591, 0.707)$, i.e., in the range where the system is unstable but the second-order conditions hold. Suppose Country 1 believes that Country 2 sets $s_2 = 0$. Maximizing welfare in Country 1 with respect to s_1 under the restriction that output and R&D investments in Firm 2 are non-negative, we have (with superscript A for asymmetry)

$$s_1^A = \frac{2(1-b)}{b}(\alpha - c) \tag{21}$$

Inserting for s_1^A and $s_2 = 0$ into equations (15) and (16) we find that Firm 2 will be inactive ($q_{22} = q_{21} = x_2 = 0$). Given that $s_2 = 0$, it is thus optimal for Country 1 to grant so high subsidies that Firm 1 becomes a monopolist. However, comparing with the monopoly subsidy level s_1^M (see equation (12)), we find that $s_1^A - s_1^M = 2(3-4b)(\alpha - c)/(3b) > 0$ in the relevant area of b. Country 1 must therefore use a subsidy level which is higher than its first-best choice.[12]

[12] Country 1 is aware of the fact that the foreign firm in stage 2 invests in R&D and supplies a positive output even if $s_2 = 0$ unless $s_1 \geq s_1^A$. As this would have a negative welfare effect in country 1, it is optimal for country 1 to set $s_1 = s_1^A > s_1^M$.

Next, suppose that Country 2 believes $s_1 = s_1^A$. Using equation (19) we then find

$$\frac{\partial W_2}{\partial s_2} = -\frac{(3-b^2)(1-2b^2)}{4(1-b^2)^2} s_2 < 0 \text{ for } s_2 > 0 \text{ and } b < 0.707,$$

from which it follows that Country 2's best response to $s_1 = s_1^A$ is $s_2^A = 0$.

We now have:

Proposition 3: *The symmetric equilibrium is stable for $b \in [0, 0.591)$ and unstable for $b \in (0.591, 0.707)$. For $b \in (0.591, 0.707)$ there exists a stable equilibrium with $s_i^A = \frac{2(1-b)}{b}(\alpha - c)$ and $s_j^A = 0$. The subsidy level s_i^A is decreasing in b. Production is equal to zero in the firm that does not receive subsidies.*

The reason why s_i^A is decreasing in b is that the cost advantage that Country 1 will have to give its domestic firm in order to foreclose Firm 2 is smaller the less differentiated the consumers perceive the goods to be.

We have now characterized the equilibrium for $b \in [0, 0.707)$. For higher values of b there does not exist any equilibrium in pure strategies if the fixed costs f of setting up a research project equal zero. This is due to the fact that the business-stealing effect is then so strong that each country has an incentive to overbid the other in subsidy levels. Indeed, as shown by equation (15), the firms become infinitely sensitive to differences in subsidy levels in the limit $b \to 1$. However, with a fixed cost of setting up research projects, it takes more than a marginal increase in profits to choose positive R&D investments, and in the Appendix we show that there exists a stable asymmetric equilibrium in pure strategies if f is sufficiently high. This equilibrium has the following properties:

Proposition 4: *Assume that $b > \frac{1}{2}\sqrt{2}$. There does not exist any equilibrium in pure strategies if $f < (7/9)(\alpha - c)^2$. If $f > (7/9)(\alpha - c)^2$ there exists a stable asymmetric equilibrium where one country does not provide R&D subsidies $(s_j = 0)$ and the other country sets $s_i = s_i^A = \frac{2(1-b)}{b}(\alpha - c)$ for $b \leq 3/4$ and $s_i = s_i^M = 2(\alpha - c)/3$ for $b \in [3/4, 1)$. Production is equal to zero in the firm that does not receive subsidies.*

Figure 3 illustrates the equilibrium subsidy levels for the case where f is sufficiently high to ensure the existence of equilibria in pure strategies for all $b \in [0, 1)$. Note in particular that the subsidy level used by Country 1 is the same if the firms produce independent goods ($b = 0$) as if $b \in [3/4, 1)$. Even if Country 1 could foreclose Firm 2 from the market by setting $s_1 = s_1^A$ in the latter area of b, that would yield a subsidy level that is lower than the welfare-maximizing one for Country 1.

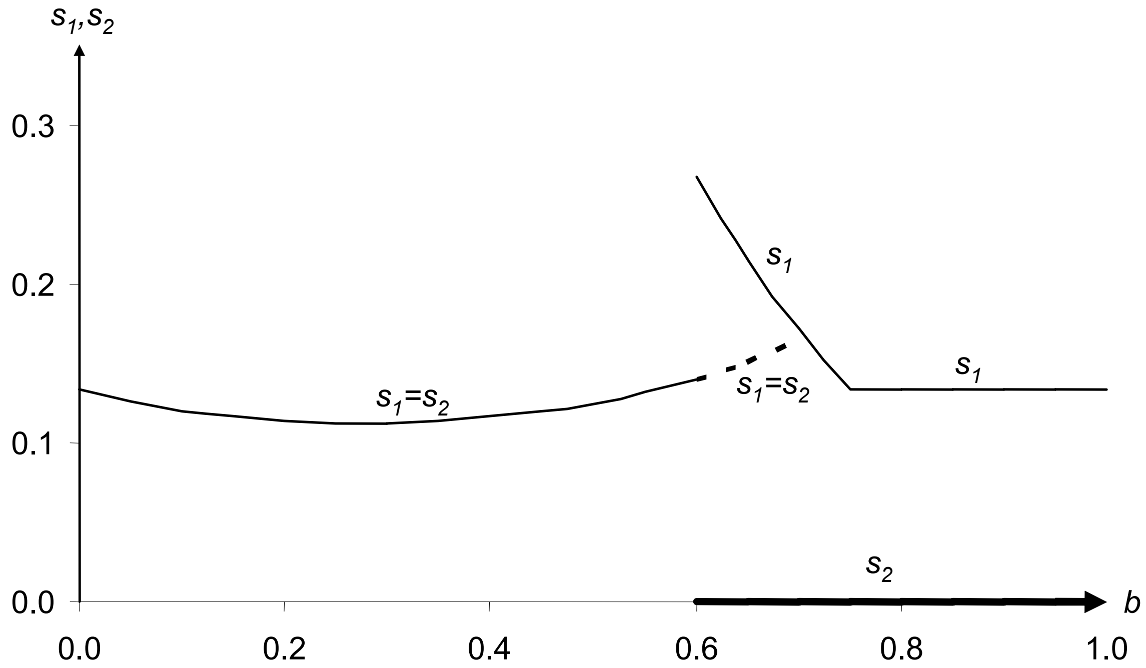

Figure 3: *Equilibrium subsidy levels with policy competition.*

3.3 Policy coordination

The above analysis shows that there is a rationale for national governments to subsidize R&D; however, there are at least two reasons why the national subsidies are not necessarily optimal from a global point of view. First, national governments do not take costs and benefits for foreign consumers or firms into consideration; second, the business-stealing motive and the accompanying policy competition cannot be optimal in a global sense. Hence, there is a need for international policy coordination; however, it is not obvious what type of coordination one should have. A natural approach, motivated by the literature on tax competition, would be to say that coordination should imply harmonization of policies across countries. If R&D subsidies are bound to be at the same level in the two countries, there will be no policy game, and the subsidies could be used to correct for the public-goods aspects of R&D. In section 3.3.1 harmonized R&D policies are studied, and the implications of such policies are discussed. Harmonization of R&D subsidies implies a symmetric outcome in the two countries, with the same R&D levels and identical quantities produced and sold. While such a symmetric outcome may seem reasonable given that the countries are symmetric, it is, in fact, not always optimal from a global point of view. In section 3.3.2 we study optimal coordinated policies, and show that depending on the degree of product differentiation, the optimal global solution could either be one with the same subsidies to both firms or one in which only one of the firms are subsidized.

3.3.1 Harmonized R&D subsidies

Assume that the countries agree on a common subsidy level $s_1 = s_2 = s$ that maximizes aggregate welfare in the two countries. The solution to this problem is straight forward. Solving $\partial (W_1 + W_2) / \partial s = 0$ we find (with superscript H for harmonized subsidies):[13]

$$s^H = \frac{2}{1 + 3b + b^2} (\alpha - c) \, ; \, \frac{\partial s}{\partial b} < 0. \tag{22}$$

The subsidy level is thus monotonically decreasing in b. This is true for two reasons. First, because the size of the market is decreasing in b. Second, because there is stronger competition between the firms the less differentiated goods they produce. All else equal, higher competition implies that output for each firm increases, and thus their incentives to invest in cost reduction. This in turn means that the need to provide R&D subsidies is lower the higher is b.

Inserting for s into equation (15) we have

$$x_i^H = \frac{3 + b}{1 + 3b + b^2} (\alpha - c) \, ; \, \frac{\partial x_i^H}{\partial b} < 0.$$

When the countries harmonize their subsidies, we thus see that the larger is b, the lower are subsidy levels and R&D investments. The latter implies that marginal costs are

[13] The second-order condition equals $\frac{\partial^2 W}{\partial s^2} = -\frac{b^2+3b+1}{(1+b)^2}$, and is thus negative for all $b \in [0.1]$.

increasing in b, as illustrated in Figure 4. [14]

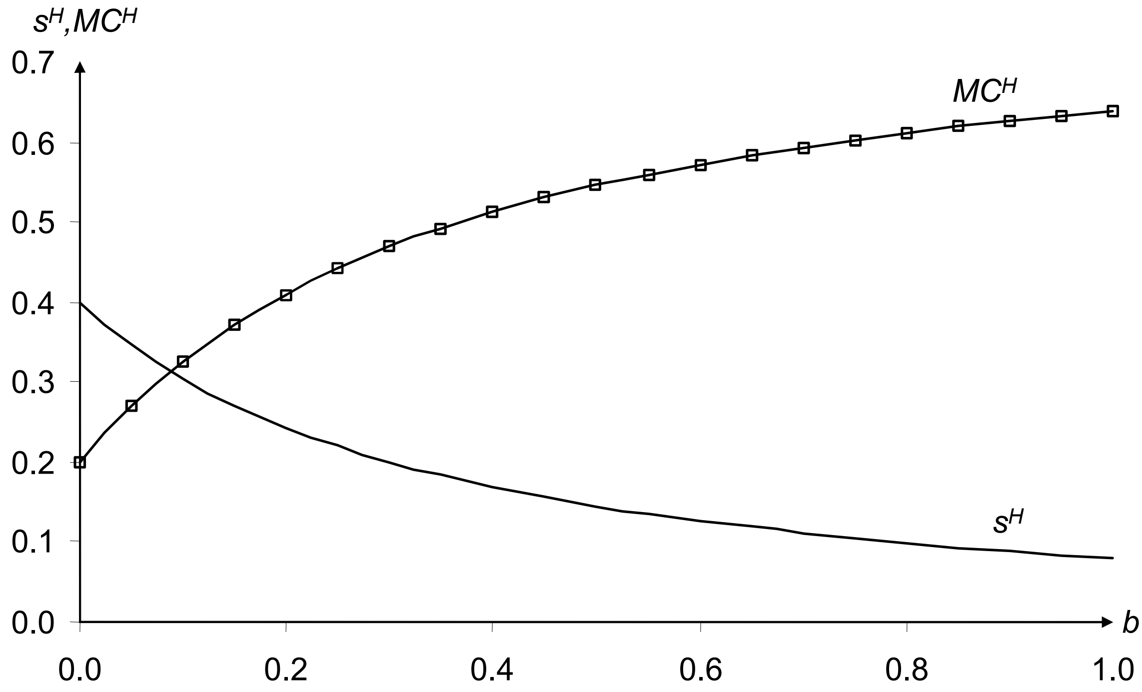

Figure 4: *Subsidy levels and marginal costs with harmonized subsidies.*

We can now state:

Proposition 5: *Suppose that the countries choose a common subsidy level that maximizes aggregate welfare. Subsidy levels are then lower, and marginal production costs higher, the closer substitutes the consumers perceive the goods to be.*

3.3.2 Optimal coordinated R&D subsidies

The countries have only one policy instrument if they harmonize their R&D policies; the common subsidy level s. Though the harmonization policy internalizes the business

[14] With harmonized subsidies we have $c - x_i = \frac{2c(b+2)^2 - 2\alpha(b+3)}{2(b^2+3b+1)}$, which implies that we depending on the parameter values we may get $c - x_i = 0$ for low values of b. In particular, if $c \leq (3/4)\,\alpha$ it will be optimal for the countries to provide subsidies which ensures that $MC = 0$ in the neighborhood of $b = 0$.The parameter values we use ensures that this does not happen.

stealing effect, it is not necessarily optimal to subsidize both firms. To see this, we now allow the subsidies s_1 and s_2 to differ.

Solving $\partial W/\partial s_i = 0$ $(i = 1, 2)$ we find that the first-order conditions imply $s_i = s^H$, i.e., the same subsidy level as in the case with policy harmonization (equation (22)). In particular, this implies that

$$s_i = s_i^* = 2(\alpha - c) \tag{23}$$

in absence of competition (which is true for $b = 0$). Recall from equation (12) that $s_i = s_i^M = 2(\alpha - c)/3$ if only one good is produced in a non-cooperative equilibrium. The intuition for why $s_i^* > s_i^M$ is that the cooperative equilibrium maximizes aggregate welfare (i.e., takes into account the positive consumer surplus effects in both countries of subsidizing R&D).

The second-order conditions for optimum are satisfied only if $b < b^{SOC} \equiv \frac{3}{2} - \frac{1}{2}\sqrt{5} \approx 0.38$. For higher values of b the optimal policy is to subsidize only one of the firms (see Appendix). Setting $s_2 = 0$ in this case and solving $\partial W/\partial s_1 = 0$ we find

$$s_1 = \frac{2(1-b)^2}{b^4 - 4b^2 + 1}(\alpha - c)\,; \; \frac{\partial s_1}{\partial b} > 0 \tag{24}$$

and

$$x_2 = \frac{b^3 - b^2 - 2b + 1}{1 - 4b^2 + b^4}(\alpha - c)\,, \; \frac{\partial x_2}{\partial b} < 0. \tag{25}$$

Since $x_2 > 0$ for $b < 0.44$, we see that the subsidy level given to Firm 1 is too small to completely foreclose the competitor. This is in sharp contrast to the case with policy competition, where the firm that receives zero subsidies ceases to produce (and makes no R&D investments). The reason why policy competition may lead to complete foreclosure is the fact that each of the countries have incentives to grant so high subsidies that its domestic firm monopolizes the market (for sufficiently high values of b). This business stealing effect is not present with policy cooperation. In the Appendix we nonetheless prove the following:

Proposition 6: *Assume that $b > b^{SOC}$ and that the countries are able to tax R&D. In this case it is optimal to subsidize one firm ($s_i > 0$) and tax the other firm ($s_j \leq 0$) such that the latter is inactive.*

By taxing R&D in one of the firms, the countries could prevent unnecessary duplication of R&D expenses. However, below we require $s_i \geq 0$ as R&D taxation does not seem a realistic policy option. Note that in principle the countries could set s_1 so high that the cost-reducing R&D investments in Firm 1 are large enough to keep Firm 2 out of the market. Equation (25) shows, however, that for $b < 0.44$ this is not optimal; the R&D expenses of doing so would be too high. Additionally, there is also a gain for the consumers of having access to both varieties. This advantage is smaller, though, the less differentiated the goods are. Therefore s_1 is increasing in b.

Firm 2 ceases to produce at $b = 0.44$. Technically, the analysis of the optimal subsidy policy for higher values of b is analogous to the one with policy competition. Given that only Firm 1 is active in the market, there exists a first-best subsidy level. According to equation (23) this subsidy level is equal to $s_1 = s_1^* \equiv 2(\alpha - c)$ with policy cooperation. However, from equations (15) and (16) we find that s_1^* is too low to prevent Firm 2 from entering the market at stage 2 if $b < 1/2$. Similar to the case with policy competition, the countries must therefore set $s_1^A = \frac{2(1-b)}{b}(\alpha - c)$ in order to completely foreclose Firm 2 (c.f., equation (21)) when $s_1^* < s_1^A$. Formally, in the Appendix we prove the following:

Proposition 7: *Assume that the countries set non-negative subsidy levels cooperatively to maximize aggregate welfare. There exists a symmetric equilibrium with $s_1 = s_2 = s^H$ for $b \in (0, 0.38)$. For higher values of b the equilibrium is asymmetric. Suppose that*

i) $b \in (0.38, 0.44)$. Then $(s_i, s_j) = \left(\frac{2(1-b)^2}{b^4 - 4b^2 + 1}(\alpha - c), 0\right)$. Both firms produce and invest in R&D.

ii) $b \in (0.44, 0.50)$. Then $(s_i, s_j) = \left(s_1^A, 0\right)$. The non-subsidized firm is completely foreclosed from the market.

iii) $b \in (0.50, 1.00)$. Then $(s_i, s_j) = (2(\alpha - c), 0)$. Only the subsidized firm is active in the market. The subsidy level is equal to first-best.

Figure 5 illustrates the relationship between the subsidy levels and b graphically. Both with policy competition (see Figure 3) and with policy cooperation we have that the equilibrium is symmetric for sufficiently low values of b, while only one firm receives subsidies for higher values of b. In this sense there are clear similarities between the outcome with competition and cooperation, even though the reasons for the asymmetry are fundamentally different. Moreover, with cooperation there is no gain per se from monopolizing the market, in which case the non-subsidized firm is not necessarily completely foreclosed from the market (though foreclosure through taxation would have been optimal). Note also that the symmetric equilibrium breaks down for lower values of b with policy coordination than with policy competition.

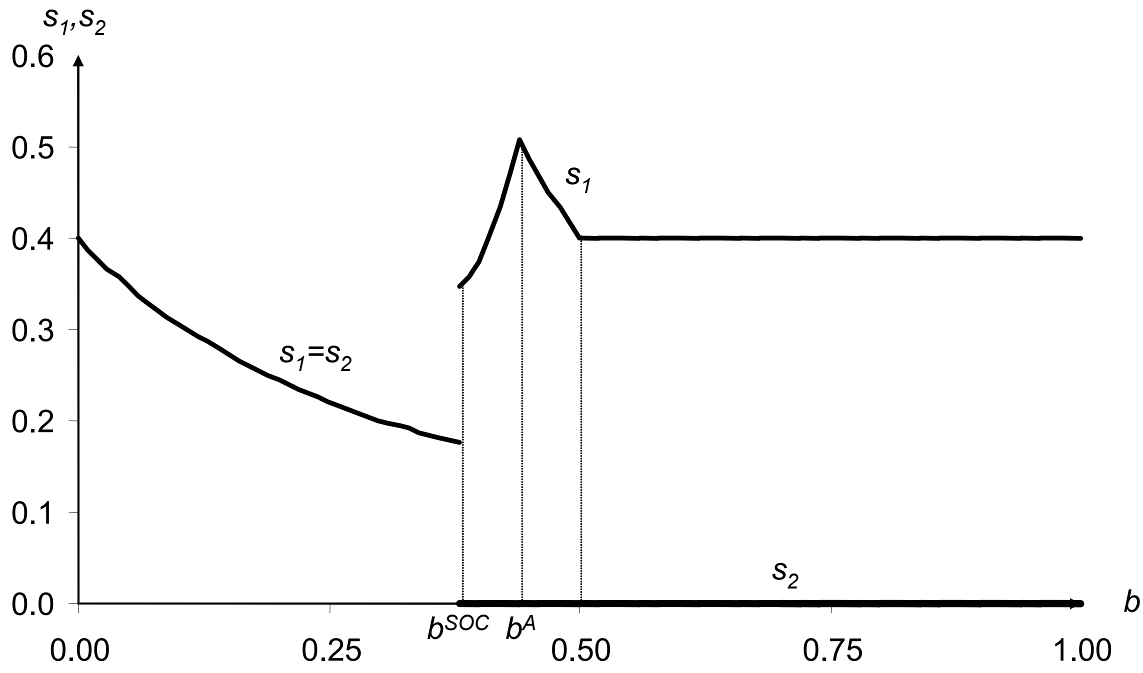

Figure 5: *Equilibrium subsidy levels with policy cooperation.*

4 Concluding remarks

In this paper we have studied optimal industrial R&D investments in an international setting. In a simple model with two countries and one firm in each, we have looked at the firms' R&D decisions and the governments' incentives to influence R&D levels through subsidies. Both non-cooperative policies and coordinated international policies are studied; for national (non-cooperative) policies there are both a public-goods motive and a business-stealing motive for R&D policies. With coordinated policies, the business-stealing motive disappears, while the public-goods motive is reinforced. A number of interesting conclusions come out of the analysis.

First, it is shown that international trade and trade costs are important for the firms' choice of R&D as well as for the governments' optimal policies towards R&D. Liberalization implies that the firms find it optimal to increase their cost-reducing R&D investments, since the market becomes bigger. And higher R&D implies lower marginal costs, lower prices and more sales in all markets. The government realizes that the R&D effect of a given subsidy will increase, and they thus find it optimal to increase the subsidies. Freer international trade thus implies more R&D, higher R&D subsidies and more sales, possibly also in the domestic market. The policy effects do not rely on any business-stealing motive; even for a monopoly it would be the case that optimal R&D subsidies and domestic sales increase when trade costs go down.

Second, we study in some detail policy competition between two governments pursuing national interest. Contrary to much of the literature we do not only focus on exports to third markets; we explicitly include the effects for domestic consumers in the analysis. We find that the effects of policy competition depend critically on the characteristics of the market. If the goods are poor substitutes, competition between the firms is not very strong, and for the governments the public-good motive for subsidies is more important than the business-stealing one. In such industries there will typically be a symmetric outcome, where both governments subsidize R&D in the domestic firm, and where both firms invest in R&D and sell their products in the two markets. When the goods are close substitutes, on the other hand, the business-stealing motive for subsidies dominates, and competition may become so tough that only one firm survives in the market. We analyze various regimes, and show that depending on the degree of product differentiation we may have a unique, stable symmetric equilibrium, an unstable symmetric equilibrium, or no symmetric equilibria at all. In the latter two cases, there will typically (but not always) be asymmetric equilibria in which only one of the firms survive in the market.

Third, we analyze policy coordination, and look at the optimal R&D policy from a global point of view. Given the potentially harmful effects of policy competition, it is not difficult to see why there is a need for policy coordination. However, contrary to what one might expect, policy coordination does not necessarily lead to a harmonization of the subsidies to the two firm. In fact, our analysis shows that when goods are fairly close substitutes, a coordinated policy may imply that only one of the firms receives R&D subsidies, while the other firm may or may not survive on its own. Hence, the surprising result is that both with policy competition and policy coordination we may end up with an asymmetric equilibrium where only one of the firms receives subsidies and is active in the market. However, the reason for such an outcome is different in the two cases. With policy competition it is the business-stealing effect. With coordinated policies, the asymmetric outcome appears to avoid unnecessary duplication of costly R&D investments.

In the model we have assumed a two-stage game where the firms at the second stage determine R&D and output simultaneously. Many of the contributions to the literature assume three stages, such that the firms at the second stage (i.e. after the subsidies are set) determine the R&D investments, and at the third stage produce and sell the goods. This assumption is not critical for our main results. With a three-stage game, there would be strategic motives for the firms' R&D decisions in addition to the cost-minimizing ones, but that would not change our results qualitatively. A second assumption to discuss, is the specific cost function for R&D. In the analysis it was shown that the fixed costs of an R&D project could be important for the existence of asymmetric equilibria. The convexity of the cost function may furthermore be of importance for the stability properties. Hence, the exact outcomes that we find may depend on the specific cost function. However, the main conclusions regarding the effects of trade liberalization and the possibilities of asymmetric as well as symmetric equilibria remain valid also with more general R&D functions.

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6 Appendix

Proof of Proposition 4:

Assume that $b > (1/2)\sqrt{2}$, in which case $\partial^2 W_i/\partial s_i^2 > 0$. If neither country grants subsidies we find that

$$W_i^{s=0} = \frac{2+b}{(1+b)^2}(\alpha - c)^2 - f.$$

This is an equilibrium if none of the countries have incentives to depart from zero subsidies. However, with $s_2 = 0$ we can use equation (19) to find

$$\frac{\partial W_1}{\partial s_1} = \frac{(1+b^2)}{2(1+b)^2(1-b)}(\alpha - c) + \frac{(3-b^2)(2b^2-1)}{4(1-b^2)^2}s_1 > 0. \tag{26}$$

Welfare in Country 1 is consequently monotonically increasing in s_1 as long as equation (26) holds (i.e., as long as there is intra-industry trade). This means that Country 1 will choose a subsidy level which is so high that Firm 2 is foreclosed from the market. The first-best subsidy level for Country 1 if Firm 2 is foreclosed, is the monopoly subsidy level

$s_1^M = 2(\alpha - c)/3$. However, Firm 2 will not be foreclosed as long as $s_1^A = \frac{2(1-b)}{b}(\alpha - c) > s_1^M$, which is true for $b < 3/4$.

We will now analyze the cases $b \in \left[(1/2)\sqrt{2}, 3/4\right]$ and $b \in [3/4, 1]$ separately.

Case A: $b \in \left[(1/2)\sqrt{2}, 3/4\right]$.

In order to ensure $q_{22} = q_{21} = 0$ at stage 2 for $b \in \left[(1/2)\sqrt{2}, 3/4\right]$, Country 1's best response to $s_2 = 0$ is $s_1^A = \frac{2(1-b)}{b}(\alpha - c)$ (the same as in the range $b \in \left(0.591, (1/2)\sqrt{2}\right)$. This subsidy level is higher than Country 1's first-best subsidy, but the lowest which forecloses the foreign firm at stage 2.

With $(s_1, s_2) = \left(s_1^A, 0\right)$we find that welfare in the two countries equal

$$W_1^A = \frac{8b - 2b^2 - 3}{2b^2}(\alpha - c)^2 - f \text{ and } W_2^A = \frac{(\alpha - c)^2}{2b^2}.$$

Since

$$W_1^A > W_1^{s=0},$$

it follows that Country 1's best response to $s_2 = 0$ is $s_1 = s_1^A$ also for $b \in \left[(1/2)\sqrt{2}, 3/4\right]$.

What is Country 2's best response to $s_1 = s_1^A$? Inserting for $s_1 = s_1^A$ into equation (19) we have

$$\frac{\partial W_2}{\partial s_2} = \frac{(3 - b^2)(2b^2 - 1)}{4(1 - b^2)^2} s_2 > 0 \text{ for } s_2 > 0.$$

This means that if Firm 2 performs R&D, then it will be optimal for Country 2 to grant subsidies which foreclose Firm 1 from the market (in which case Country 1's belief that $s_2 = 0$ and that Firm 2 is foreclosed from the market is wrong). Solving $q_{11} = q_{12} = 0$ with respect to s_2 for $s_1 = s_1^A$ we find $s_2^{'} = \frac{2\left(1-b^2\right)}{b^2}(\alpha - c)$ and

$$W_2^{'} = \frac{8b^2 - 2b^4 - 3}{2b^4}(\alpha - c)^2 - f.$$

Given that $s_1 = s_1^A$, it is not profitable for Country 2 to grant subsidies if $W_2^A > W_2^{'}$. This inequality holds if

$$f > f' \equiv \frac{(3 - b^2)(2b^2 - 1)}{2b^4}(\alpha - c)^2,$$

which reaches a maximum at $b = 3/4$, where $f' = (13/27)(\alpha - c)^2$. We can therefore conclude that there exists an asymmetric equilibrium $(s_i, s_j) = \left(\frac{2(1-b)}{b}(\alpha - c), 0\right)$ for $b \in \left[(1/2)\sqrt{2}, 3/4\right]$ if $f > (13/27)(\alpha - c)^2$.[15]

Case B: $b \in [3/4, 1)$

Given that $s_2 = 0$ and $b \in [3/4, 1)$, Country 1 will use its first-best subsidy level $s_1 = s_1^M$ to foreclose Firm 2 from the market. Welfare in the two countries is then equal to

$$\begin{aligned} W_1^B &= \frac{15}{9}(\alpha - c)^2 - f \text{ and} \\ W_2^B &= \frac{8}{9}(\alpha - c)^2. \end{aligned}$$

Using the same procedure as above, we find that Country 2's best response to $s_1 = s_1^M$ is $s_2 = \frac{2(4-3b)}{3b}(\alpha - c)$ or $s_2 = 0$, depending on the size of the fixed costs. With the subsidy levels $\left(s_1, s_2''\right) = \left(s_1^M, \frac{2(4-3b)}{3b}(\alpha - c)\right)$ we have

$$W_2'' = \frac{16b - 3b^2 - 8}{3b^2}(\alpha - c)^2 - f.$$

Subtracting $W_2^B - W_2''$ we find that $s_2 = 0$ is Country 2's best response to $s_1 = s_1^M$ for all $b \in [3/4, 1)$ if

$$f > \frac{7}{9}(\alpha - c)^2.$$

Q.E.D.

Optimal cooperative R&D subsidies

The second-order conditions for optimal subsidies when the subsidy levels may differ are

$$\begin{aligned} \frac{\partial^2 W}{\partial s_i^2} &= -\frac{1 - 4b^2 + b^4}{2(1-b^2)^2} < 0 \text{ for } b < \frac{1}{2}\sqrt{6} - \frac{1}{2}\sqrt{2} \approx 0.51764 \\ \left(\frac{\partial^2 W}{\partial s_i^2}\right)\left(\frac{\partial^2 W}{\partial s_j^2}\right) - \left(\frac{\partial^2 W}{\partial s_i \partial s_j}\right)^2 &= \frac{1 - 7b^2 + b^4}{4(1-b^2)^2} > 0 \text{ for } b < \frac{3}{2} - \frac{1}{2}\sqrt{5} = 0.38197, \end{aligned}$$

[15] Comparing welfare in the two countries we find $W_1^A - W_2^A > 0$ if $f < f^{crit} \equiv \frac{4b-b^2-2}{b^2}(\alpha - c)^2$.

which means that the SOCs are satisfied only if $b < \frac{3}{2} - \frac{1}{2}\sqrt{5} = 0.38$.

For $b > 0.38$ we have to look for corner solutions. It is straight forward to show that it is inoptimal to set $s_1 = s_2 = 0$. This leaves us with the the following candidates for optimum:

I) Set $s_2 = 0$ and choose a welfare maximizing level of s_1, possibly without foreclosing Firm 2 (alternatively, choose s_2 optimally, given that $s_1 = 0$)

II) Set $s_2 = 0$, and choose s_1 such that Firm 2 is completely foreclosed (alternatively, set $s_1 = 0$, and choose s_1 such that Firm 1 is completely foreclosed)

III) Set s_1 at the optimal level, given that only Firm 1 is present in the market (alternatively, set s_2 at the optimal level, given that only Firm 2 is present in the market).

IV) Set s_1 optimally, given that Firm 2 is foreclosed by setting $s_2 \leq 0$.

Case I:

Setting $s_2 = 0$ we have

$$\begin{aligned}\frac{\partial^2 W}{\partial s_1^2} &= -\frac{1}{2}\frac{b^4 - 4b^2 + 1}{(b-1)^2(1+b)^2} \\ &< 0 \text{ for } b < \frac{1}{2}\sqrt{6} - \frac{1}{2}\sqrt{2} = 0.517\,64.\end{aligned}$$

Provided all non-negativity constraints are satisfied, we can then solve $\partial W/\partial s_1 = 0$ to find (with superscript to signify Case I):

$$s_1^I = \frac{2(1-b)^2}{b^4 - 4b^2 + 1}(\alpha - c)\,; \ \frac{\partial s_1^I}{\partial b} > 0. \tag{27}$$

Inserting for s_1 and s_2 into equation (16) we find

$$x_1^I = \frac{3 - 5b + b^3}{1 - 4b^2 + b^4}(\alpha - c)$$

and[16]

$$x_2^I = \frac{b^3 - b^2 - 2b + 1}{1 - 4b^2 + b^4}(\alpha - c)\,.$$

[16] From this we find that a sufficient condition for $c - x_1 > 0$ is that $c/\alpha > 0.778$.

In the relevant area we find that $x_2^I > 0$ for $b \in (0.38, 0.44)$, in which case welfare is given by

$$W^I = \frac{2b^3 - 10b + 5}{1 - 4b^2 + b^4} (\alpha - c)^2 . \tag{28}$$

Case II:

Setting $s_2 = 0$ we find that $q_{22} = q_{21} = x_2 = 0$ if

$$s_1^{II} = s_1^A = 2\frac{1-b}{b} (\alpha - c) . \tag{29}$$

Using equations (16) and (29) we have

$$x_1^{II} = \frac{2-b}{b} (\alpha - c)$$

and

$$W^{II} = \frac{4b - 1 - b^2}{b^2} (\alpha - c)^2 . \tag{30}$$

Case III:

Given that Firm 2 does not produce, the optimal subsidy level to Firm 1 equals[17]

$$s_1^{III} = 2(\alpha - c), \tag{31}$$

from which it follows that

$$x_1^{III} = 3(\alpha - c).$$

This yields the welfare level

$$W^{III} = 3 (\alpha - c)^2 . \tag{32}$$

Case III is relevant only if $s_2 = 0$ and $s_1 = s_1^{III}$ does not lead Firm 2 to invest in R&D and produce at stage 2. Using equations (15) and (16) we find that this holds for $b \geq 1/2$.

Case IV:

Suppose the countries set $s_1 = s_1^{III}$. From equation (15) we find that $(q_{22} + q_{21}) = 0$ if

$$s_2^{IV} = -2 (1 - 2b) (\alpha - c) < 0 \text{ for } b < 1/2. \tag{33}$$

[17] This is most easily found by setting $b = 0$ in equation (22).

In this case Firm 2 will not produce or invest in R&D, and we therefore have

$$W^{IV} = 3(\alpha - c)^2 . \tag{34}$$

Proof of Proposition 6:

Comparing equations (28), (30) and (34) we find that welfare is highest in Case IV, where R&D taxes imply that Firm 2 is inactive. It can further be shown that welfare is higher by choosing $s_2 = s_2^{IV}$ than by setting s_2 such that $x_2 = 0$; in the latter case Firm 2 would with an optimal choice of s_1 have positive output for $b < 0.47$ (even though it does not invest in R&D). If we allow R&D taxes, we thus see that the countries would prefer to completely foreclose Firm 2 from the market for $b > b^{SOC}$.[18] Q.E.D.

Proof of Proposition 7:

Comparing equations (28) and (30) we find that welfare is highest if Firm 2 is not completely foreclosed from the market for $b < 0.445$. We further find from equations (30) and (32) that welfare is higher with $s_1 = s_1^{III}$ than with $s_1 = s_1^{II}$, which is feasible for $b > 1/2$.

[18] Welfare in the symmetric equilibrium equals $W = \frac{b+3}{b^2+3b+1}(\alpha - c)^2$. Since $W^{IV} > W$ for $b > 0.37$ it is thus optimal to tax and foreclose Firm 2 even for $b < b^{SOC}$.