Abstract

Using labor union’s bargaining power as an indication of government policy on labor standards, this paper analyzes the competition between a domestic (North) firm and a foreign (South) firm, and their relationship with labor standards. First, we show that an increase in labor standards raises the domestic firm’s profits and reduces that of the foreign firm, if the labor union is sufficiently employment-oriented. On the contrary, if the union is sufficiently wage-oriented, ‘a race to the bottom’ of labor standards may occur and it becomes more intensified under free trade than under a protective trade system. Second, Northern labor standards are higher than Southern ones on condition that the Northern union is more employment oriented than the Southern union. Third, the North’s imposing a tariff to force the Southern government to raise its labor standards is effective only if the Southern union is wage-oriented. Thus, in order to raise Southern labor standards, both countries may need some form of economic integration, if the North does not want to abandon its free trade system.

Key Words: Labor-Management Bargaining, Labor Standards, Labor Union, Duopoly, Tariff, Economic Integration

JEL Classification: F10, F16, J50, J80, L13
1. Introduction

As tariff barriers are decreasing worldwide, non-tariff barriers such as environmental standards, health standards and labor standards are on the rise. This paper examines the endogenous choice of labor standards (LS) in a model of international duopoly where a domestic firm competes against a foreign exporting firm. Some argue that LS issues appear to be about humanitarianism, but in fact they are about competitiveness. Labor unions in the developed countries on the one hand have championed LS in the name of human rights and social justice in the developing countries. On the other hand, they perceive increased competition from poor countries as unfair because LS there is low, hence urging for trade sanctions against countries that do not enforce a set of agreed standards in the workplace. They argue that weak standards and inadequate enforcement of standards are means for generating artificially low wages and augmenting the natural comparative advantage that low-wage countries have in labor-intensive goods.

In this model, we analyze LS in the context of labor-management negotiations, and treat the labor union’s bargaining power as an indication of how importantly governments view LS issues. In other words, we assume that the level of LS in each country is endogenously determined by its government. This postulate is based on several stylized facts and studies. First, one of the four Core Labor Standards presented by the International Labor Organization (ILO) is ‘freedom of association and the effective recognition of the rights to collective bargaining’,¹ which basically represents how strong the union is v.s. the firm. Second, according to studies by Moene and Wallerstein (2003), Sweden and Norway have experienced almost full employment after World War II. They attribute this extraordinary phenomenon to the union’s strong bargaining power, which is a symbol of high LS in Scandinavia. Third, some governments in the developed countries want to include LS issues in WTO (the World Trade Organization) negotiations, hoping to raise the labor costs/wages in the developing countries and in turn reduce their competitiveness. For instance, the U.S. and France campaigned for LS and a “social clause” at WTO meetings in Singapore in 1996 and Seattle in 1999; The European Union also brought such issues to the WTO’s Doha conference in 2001.

We allow the labor union to have a biased preference towards either wages or employment. It is argued that unions tend to be employment-oriented during recession, when securing jobs is a priority. In contrast, during business boom, they tend to be wage-oriented and have stronger

¹ The other three are (i) the elimination of all forms of forced or compulsory labor; (ii) the effective abolition of child
demands for wage hikes. In an interesting survey of British trade unions, Clark and Oswald (1993) find that union preferences are more heavily weighted toward employment than would be implied by the so-called rent-maximization behavior (i.e., maximizing the sum of union members’ rents), even though union leaders care more about wages than employment. In addition, in a rich country, unions might be concerned more about increasing the size of union membership than wages, while union workers in poor countries might be more interested in the wage level, given that many non-union workers are earning near existence-level wages. Thus, we assume the union and the firm negotiate over two issues, wages and employment, and the negotiated equilibrium would depend on the union’s preference. This consideration allows us to examine different firm performances arising out of the union’s preference, and how this changes the government’s optimal choice of LS.

Our main results can be summarized as follows. Firstly, an increase in labor standards can raise the domestic firm’s profit and reduce that of the foreign firm if labor unions are sufficiently employment-oriented. This arises because employment-oriented unions are willing to sacrifice some wage demands for higher employment. The opposite is true, if labor unions are sufficiently wage-oriented. Furthermore, with wage-oriented labor unions, the so-called ‘a race to the bottom’ of labor standards may arise and it becomes more intensified under a free trade system than under a protective trade system.

Secondly, Northern labor standards can be optimally set at a higher level than Southern ones, if the Northern union is more employment-oriented than the Southern union. The intuition is that compared with wage-oriented ones, employment-oriented unions demand relatively higher employment, which raises national welfare and firm profits under certain conditions. Thus the government is more willing to grant higher LS/bargaining power to more employment-oriented unions.

Lastly, the North’s imposing a tariff to force the Southern government to raise its labor standards is effective only if the Southern union is wage-oriented. Alternatively, in order to raise Southern labor standards, both North and South may need a further economic integration (i.e., some form of joint welfare maximization) if the North does not want to give up its free trade system.

These results shed light on the current international debate on the enforcement issue of high labor standards in poor countries. The debates basically involve the following two fundamental questions: First, can labor standards in poor countries be raised for better working and wage conditions? Second, can employment in rich countries be secured with the raised labor standards in

labor; and (iii) the elimination of discrimination in respect of employment occupation. See ILO (1999).
poor countries? Stern and Terrell (2003) surveyed several empirical studies and found that there is little compelling evidence to give a positive reply to those questions: Firstly, a higher labor standard does not necessarily improve wages and working conditions in poor countries. Secondly, a low labor standard in poor countries does not necessarily destruct jobs in rich countries. Our theoretical results provide plausible explanations to why those empirical findings are observed: the effects of labor standards depend on how labor-management relations are resolved, and whether unions are more interested in wages or employment.

In the existing literature, LS has been treated as a source of externality in a general equilibrium framework (for instance, see Bagwell and Staiger (2001), Brown, Deardorff and Stern (1996), Rodrik (1996) and Srinivasan (1995)), which is assumed to directly increase consumer utility or national welfare. In these analyses, neither workers nor firms are directly positively affected by an improvement in LS. Recently, Chau and Kanbur (2006) show that whether a race to the bottom (of environmental or labor standards) is possible or not depends on the Northern demand curve, the size of big exporters relative to each other, and the relative size of the competitive fringe of small exporters. Zhao (2006) models LS as a firm’s choice variable such as working conditions in a partial equilibrium framework. Also, the implication of child-labor practices in developing countries for international trade has drawn quite some attention (for instance, see Brown, Deardorff and Stern (2003), Basu and Chau (2004) and Neumayer and Soysa (2005)). In contrast, we treat the bargaining power of labor unions as a government policy variable on labor standards, emphasizing the role of governments in LS issues.

The rest of the paper is organized as follows. Section 2 sets up the basic model of North and South trade with labor-management negotiations incorporated. Section 3 investigates the equilibrium properties with regard to the economic effects of labor standards and import tariffs. Section 4 examines optimal labor standards and tariff policies of governments. Section 5 extends the model to a case of asymmetric labor unions and also examines the implications of North-South economic integration on the choice of labor standards. And finally, section 6 concludes.

2. The Basic Model Setup

Consider two countries, the North (N) and the South (S), each having one firm, i.e., respectively N and S. Both firms produce an identical product which is sold in country N only,\(^2\) under the

\(^2\) Since LS issues arise out of North’s claims that low Southern LS helps to improve South’s competitiveness in the North,
following inverse demand function, \( p = p(q_N + q_S) \), with \( p' < 0 \), where \( q_i \) denotes the output of firm \( i \), for \( i = N, S \). The Northern government imposes a tariff \( t \) on imports from the South.

**Firms:** Labor is the only input required to produce the outputs in a one-to-one ratio by a proper choice of units. Given a wage rate of \( w_i \), firm \( i \)'s profits can be written respectively as:

\[
\pi_N = (p - w_N)q_N, \\
\pi_S = (p - w_S)q_S - tq_S.
\]

(1)

**Labor markets:** In both countries, workers are organized into unions. The union utility in each country can be represented by this simple function:

\[ u_i = w_i^\beta q_i, \]

(2)

where \( \beta > 0 \) is a parameter for union bias toward wages (See Pemberton (1988), Mezzetti and Dinopoulos (1991), and López and Nalyor (2004) for a similar definition). That is, if \( \beta > 1 \), then the union is said to be wage-oriented (more interested in wages than employment); if \( \beta < 1 \), then the union is said to be employment-oriented (less interested in wages than employment); and finally if \( \beta = 1 \), then the union is said to be neutral.

Wages and employment are negotiated between the union and the firm in each country. We adopt Nash bargaining to determine the negotiation equilibrium:

\[ G_i(w_i, q_i) = (u_i)^\theta_i (\pi_i)^{1-\theta_i}, \]

(3)

where \( \theta_i \) is the bargaining power of the union in country \( i \). For reasons discussed in the introduction, we use \( \theta_i \) to represent the LS in country \( i \) and assume that it is determined endogenously by country \( i \)'s government.

**Governments:** The Northern and Southern governments respectively care about the following welfare functions:

\[
\Phi_N = \pi_N + u_N(w_N, q_N) + v(q_N + q_S) - (q_N + q_S)p + tq_S, \\
\Phi_S = \pi_S + u_S(w_S, q_S),
\]

(4a)

(4b)

where in the North, it is the sum of firm profits, union utility, consumer surplus and tariff revenue. The term \( v(q_N + q_S) \) is the utility of consuming \( q_N + q_S \), with \( v'(\cdot) = p \). In the South, since there is no consumption, national welfare is the firm profits plus union utility.

we ignore what might be going on in the S market, though it should be straightforward to introduce a segmented S market.
**Game structure:** The game has two stages. In the first stage, each government determines its LS $\theta_i$ simultaneously, and the Northern government also determines the import tariff imposed on Southern imports; and in the second stage, the labor union and the firm negotiate to determine wages and employment in each country simultaneously. To ensure consistency, the game is solved by backward induction.

**Equilibrium Solutions:** The FOCs (first order conditions) in the second stage are as follows, for $i = N, S$, $i \neq j$,

\[
\frac{1}{G_i} \frac{\partial G_i(w_i, q_i)}{\partial w_i} = \frac{\beta \theta_i}{w_i} - \left(1 - \frac{\theta_i}{\pi_i}\right) \left(q_i\right) = 0, \quad (5a)
\]

\[
\frac{1}{G_i} \frac{\partial G_i(w_i, q_i)}{\partial q_i} = \frac{\theta_i}{q_i} + \left(1 - \frac{\theta_i}{\pi_i}\right) \left(\frac{\partial \pi_i}{\partial q_i}\right) = 0, \quad (5b)
\]

where $\frac{\partial \pi_i}{\partial q_i} = p'q_i + p - w_i - t < 0$ with $t = 0$ if $i = N$ and $t > 0$ otherwise. Using (5a), one sees that (5b) implies $w_N / \beta + p'q_N + p - w_N = 0$ for the North and $w_S / \beta + p'q_S + p - w_S - t = 0$ for the South. Without the union, the firm could have maximized its profits by setting $\frac{\partial \pi_i}{\partial q_i} = 0$. However, in (5b) we have $\frac{\partial \pi_i}{\partial q_i} < 0$, implying that in the bargaining equilibrium each firm produces more than the level that would maximize its profits. This arises because with positive bargaining power, the union can bargain for more employment as well as higher wage.

The four FOCs (two for each country) can be further simplified as, for $i = N, S$,

\[
\frac{\partial \tilde{G}_i}{\partial w_i} = \beta \theta_i \pi_i - (1 - \theta_i)w_i q_i = 0 \quad (6a)
\]

\[
\frac{\partial \tilde{G}_i}{\partial q_i} = \frac{w_i \theta_i}{\beta} + \frac{\partial \pi_i}{\partial q_i} = 0 \quad (6b)
\]

where $\frac{\partial \tilde{G}_i}{\partial w_i} = \frac{w_i \pi_i}{G_i} \frac{\partial G_i}{\partial w_i}$ and $\frac{\partial \tilde{G}_i}{\partial q_i} = \frac{\pi_i}{(1 - \theta_i)G_i} \frac{\partial G_i}{\partial q_i}$. We can straightforwardly solve the FOCs to obtain the four endogenous variables, $(w_N, q_N, w_S, q_S)$ as functions of the three policy variables, $(\theta_N, \theta_S, t)$, which are given in stage 2. Then we can endogenize the optimal choice of the policies in stage 1.
3. Comparative Static Analysis

Now we investigate the impact of the three policies imposed by the Northern and Southern governments on firms and unions. Detailed derivations are relegated to Appendix 1.

3.1 Effects on Outputs and Wages

The impacts of an increase in LS are, for \( i, j = N, S; i \neq j \),

\[
\frac{dw_i}{d\theta} > 0, \tag{7a}
\]

\[
\frac{dq_i}{d\theta} \begin{cases} > 0 & \text{if } \beta < 1, \\ < 0 & \text{if } \beta > 1, \end{cases} \tag{7b}
\]

\[
\frac{dq_j}{d\theta} \begin{cases} < 0 & \text{if } \beta < 1, \\ > 0 & \text{if } \beta > 1, \end{cases} \tag{7c}
\]

\[
\frac{dw_j}{d\theta} \begin{cases} < 0 & \text{if } \beta < 1, \\ > 0 & \text{if } \beta > 1. \end{cases} \tag{7d}
\]

Expression (7a) says that an increase in LS raises the negotiated wage, as expected, since the increase in LS raises the union’s bargaining power. Expression (7b) states that an increase in country \( i \)'s LS raises (reduces) firm \( i \)'s output if the union is employment (wage)-oriented. The reason is, an employment (wage)-oriented union demands a higher level of employment (wage) at the expense of a lower wage (less employment). This effect is strengthened if LS rises.

Expressions (7c) and (7d) follow expression (7b), reflecting the effects of an increase in country \( i \)'s LS on country \( j \)'s output and wages. Specifically, the sign of (7c) is the exact opposite of (7b), because outputs \( q_j \) and \( q_i \) are substitutes. These effects further lead to corresponding changes of the negotiated wage in the other country, resulting in (7d).

In addition, the effects of the Northern tariff can be obtained as follows: \( dq_N / dt > 0 \), \( dw_N / dt > 0 \), \( dq_S / dt < 0 \), and \( dw_S / dt < 0 \), as expected.

The above results are important to warrant a lemma.

**Lemma 1:** (i) An increase in country \( i \)'s LS raises the negotiated wage in the country, but it raises the output only if the labor union is employment-oriented, and lowers it if the union is
wage-oriented. (ii) An increase in country i’s LS reduces the output of the competing country if the union is employment-oriented, and raises it if the union is wage-oriented.

We can draw some interesting implications from Lemma 1. There exists a hypothesis that a higher LS might provide incentives for workers to work harder and thus increase output.\(^3\) Our results suggest that the increase in LS does provide incentives (in the form of a higher negotiated wage), but it leads to higher output only if the union is employment-oriented. If the union is wage-oriented, then an improvement in LS would lower outputs instead, because the union might sacrifice employment/output for a higher wage.

In addition, humanitarian groups, labor unions and politicians in some Northern countries claim that a lower LS in the South enables it to be more competitive and sell more in Northern markets. Thus, if Southern LS were forced up, Northern firms could sell more and Northern workers gain more. Our results in (7c) and (7d) show that this is only true if unions are employment oriented.

3.2 Effects on Firm Profits

How about firm’s profitability? First, let us check whether a rise in LS in a country raises firm’s production costs in both countries. Differentiation gives:

\[
\frac{dC_i}{d\theta_i} = \frac{dw_i}{d\theta_i} q_i + \frac{dq_i}{d\theta_i} w_i, \quad i = N, S
\]

where \( C_i = w_i q_i \). While the first term on the RHS (right hand side) is positive since \( \frac{dw_i}{d\theta_i} > 0 \) as in (7a), the second term is positive if \( \beta < 1 \) and negative if \( \beta > 1 \) as in (7b). One can verify that the net effect is positive for a large domain of \( \beta \), which leads to the following lemma.

Lemma 2: \( \frac{dC_i}{d\theta_i} > 0 \) for a large domain of \( \beta \in (0, \bar{\beta}) \) with \( \bar{\beta} \in (1 + \sqrt{3}, \infty) \).

Proof: We only prove the case for the North. That for the South can be done analogously. Detailed calculations yield:

\(^3\) For instance, in Zhao (2006), an increase in LS improves working conditions and induces higher work efforts, resulting in higher outputs.
\[
dC_N/d\theta_N = \left[ (2\theta_3(\beta-1)+3) - (\beta-1)\theta_N(\theta_3(\beta-1)+2) \right] q_Nq_S (p')^2 (\beta\pi_N + q_Nw_N) / \Delta.
\]

The sign of the expression in square brackets depends on the value of \( \beta \). First, if \( \beta < 1 \), it is positive. Second, if \( \beta > 1 \), we set the expression in brackets equal to zero and solve for \( \beta \), which gives:

\[
\bar{\beta} = (-\theta_N + \theta_S + \theta_N\theta_S + \sqrt{\theta_N^2 + 2\theta_N\theta_S + \theta_S^2})/(\theta_N\theta_S).
\]

Here \( \bar{\beta} > 0 \) for \( \theta_i \in (0,1) \). In particular, \( \bar{\beta} \in (1+\sqrt{3}, \infty) \), and \( \bar{\beta} = 1 + \sqrt{3} \) when \( \theta_i = 1 \), while \( \bar{\beta} = \infty \) when \( \theta_i = 0 \). Therefore, Lemma 2 holds for a large domain of \( \beta \in (0, \beta) \) with \( \beta \in (1+\sqrt{3}, \infty) \). QED

The above result implies that the bargaining power granted the union works as a costly factor to firm’s production activities, which is in line with the conventional wisdom that an increase in LS would raise production costs and lower profits.

Now we turn to the effects of LS on firm profits, and show that the conventional wisdom is only partially correct in the present model. Appendix 2 proves the following results:

\[
\frac{d\pi_N}{d\theta_N} > 0 \text{ if } \beta < \beta^*, \quad \text{where } \beta^* < 1,
\]

(8a)

\[
\frac{d\pi_S}{d\theta_i} < 0 \text{ if } \beta > \beta^*, \quad \text{where } \beta^* < 1,
\]

(8b)

\[
\frac{d\pi_i}{d\theta_i} \begin{cases} < 0 & \text{if } \beta < 1, \\ > 0 & \text{if } \beta > 1, \end{cases} \quad \text{where } i, j = N, S \text{ and } i \neq j ,
\]

(8c)

\[
\frac{d\pi_N}{dt} > 0, \quad \frac{d\pi_S}{dt} < 0.
\]

(8d)

First, (8a) and (8b) say that an increase in LS raises firm profits if the labor union is sufficiently employment-oriented, but reduces them otherwise. The first part is against conventional wisdom. Suppose that unions are employment-oriented. The increase in Northern LS raises the union wage and employment by (7a) and (7b), but it also lowers the Southern firm’s employment and thus output by (7c). The former two effects work negatively to the Northern firm’s profits through costs, while the last effect intensifies competition in the market. If the unions are sufficiently employment-oriented (\( \beta < \beta^* \) for the North and \( \beta < \beta^* \) for the South), the competition effect may outweigh the cost effect and as a result the Northern firm’s profit increases.

Next, (8c) shows that an increase in LS in a country reduces firm profits in the other country if
labor unions are employment-oriented, but it raises them otherwise. Suppose that unions are employment-oriented. An increase in Southern LS reduces the Northern employment by (7c), which deteriorates the Northern firm’s competitiveness in the market. This effect may dominate the beneficial effect on costs (as through (7c) and (7d)), and thus the Northern firm’s profit decreases.

Finally, (8d) is as expected, saying that the tariff increases the profit of the Northern firm but reduces that of the Southern one. We summarize these results as:

**Proposition 1:** An increase in country i’s LS, (i) lowers the profit of this country, unless the union is sufficiently employment oriented, in which case it may raise the profit; (ii) reduces (raises) the profit of the competing country if the union is employment (wage) oriented.

This proposition implies that regulations to raise the Southern LS may hurt the Northern firm if the union is employment oriented, contrary to the original intentions of Northern labor activists and other Northern interest groups who lobby to force up Southern LS. However, such regulations are effective if unions are wage oriented.

### 3.3 Effects on Union Utility

Next, we look into the effects of an increase in LS on union utility. For \( i = N, S \),

\[
\frac{du_i}{d\theta_i} = \beta \frac{u_L}{w_i} \frac{dw_i}{d\theta_i} + \frac{u_L}{w_i} \frac{dq_i}{d\theta_i}.
\]

(9)

While the first term on the RHS is positive since \( \frac{dw_i}{d\theta_i} > 0 \) as in (7a), the second term is positive if \( \beta < 1 \) and negative if \( \beta > 1 \) as in (7b), which leads to:

**Lemma 3:** \( \frac{du_i}{d\theta_i} > 0 \) for \( \beta \in (0, \infty) \).

**Proof:** Again we only prove the case for the North. After some calculations, using \( \frac{dw_N}{d\theta_N}, \frac{dq_N}{d\theta_N} \) and the equality of \( \beta \theta_N p^* q_N = -w_N \) in Appendix 1, we can rearrange (9) as follows.

\[
\frac{du_N}{d\theta_N} = \frac{1}{\Delta} (p^*)^2 (q_N)^2 q_S u_N \beta w_N (\beta \pi_N + q_N w_N) \left[ (2\theta_S (\beta - 1) + 3) - \frac{\beta - 1}{\beta} \theta_N (\theta_S (\beta - 1) + 2) \right],
\]
which is positive for $\beta \in (0, \infty)$, since the expression in square brackets is positive:

$((\beta - 1)/\beta) \theta_N \in (-\infty, 1)$ and $\left(2\theta_S (\beta - 1) + 3\right) > \left(\theta_S (\beta - 1) + 2\right) > 0$.  \textbf{QED}

This lemma shows that a higher LS increases the union’s utility regardless of its preference towards wage versus employment.

And the Northern tariff has the following effects:

$$\frac{du_N}{dt} = \beta \frac{u_N}{w_N} \frac{dw_N}{dt} + \frac{u_N}{q_N} \frac{dq_N}{dt} > 0,$$

$$\frac{du_S}{dt} = \beta \frac{u_S}{w_S} \frac{dw_S}{dt} + \frac{u_S}{q_S} \frac{dq_S}{dt} < 0,$$

because $dw_N / dt$ and $dq_N / dt$ are positive and $dw_S / dt$ and $dq_S / dt$ are negative (see section 3.1 and Appendix 1). As expected, a higher tariff protection of the North against the South would effectively increase the Northern labor union’s wage, employment and thus utility, regardless of the union’s preference towards wage and employment. And exactly the opposite applies to the Southern union.

3.4 Effects on Consumer Surplus

Since output is consumed in the North only, its consumer surplus can be expressed as

$$\varphi(q_N + q_S) \equiv \nu(q_N + q_S) - (q_N + q_S) p.$$ Differentiation yields;

$$\frac{d\varphi}{d\theta_N} = -p'\frac{(q_N + q_S)}{d\theta_N} \frac{d(q_N + q_S)}{d\theta_N}$$

where $\nu' = p$ and $d(q_N + q_S)/d\theta_N = (p'q_s / \Delta) (\beta \pi_N + q_N w_N) (\theta_s (\beta - 1) + 1)/((\beta - 1)/\beta)$, which is positive if $\beta < 1$ and negative if $\beta > 1$. Thus, if the unions are employment oriented, an increase in LS raises the total quantities provided by both firms, lowering the market price. As a result, consumers benefit. However, if the unions are biased toward wages, a higher LS reduces their negotiated employments and the total quantities provided in the market as well, increasing the market price and lowering consumer surplus.

4. Optimal LS and Tariff
In this section we solve for optimal policies in terms of LS and tariffs, by maximizing national welfare consisting of consumer surplus, firm profits, labor union utility and the tariff revenue, wherever applicable.

4.1 The Northern Government

By substitution, the North’s welfare function in (4a) can be rewritten as:

\[
\Phi_N = v(q_N + q_S) - pq_S - w_N q_N + u_N(w_N, q_N) + t q_S.
\]

Differentiation yields:

\[
\frac{\partial \Phi_N}{\partial \theta_N} = \left[ (p - p'q_s) \frac{d(q_N + q_S)}{d\theta_N} - p \frac{dq_S}{d\theta_N} \right] + \left[ t \frac{dq_S}{d\theta_N} + (\beta w_N^\theta - w_N) \frac{dq_N}{d\theta_N} \right] + \left[ (w_N^\theta - w_N) \frac{dq_N}{d\theta_N} \right]
\]

where \( v' = p, \ \frac{d(q_N + q_S)}{d\theta_N} \) is positive if \( \beta < 1 \) and negative if \( \beta > 1 \) (see Section 3.4).

If \( \beta < 1 \), the first square bracket on the RHS is positive and the second one is negative. If \( \beta > 1 \), then the signs are reversed. And regardless of \( \beta \), the last bracket is negative. To find out the optimal Northern LS, we may solve \( \frac{\partial \Phi_N}{\partial \theta_N} = 0 \) for \( \theta_N \). Let us denote it by \( \theta_N^* \).

On the other hand, the welfare-maximizing optimal tariff is given by:

\[
\frac{\partial \Phi_N}{\partial t} = \left[ (p - p'q_s) \frac{d(q_N + q_S)}{dt} - p \frac{dq_S}{dt} \right] + \left[ t \frac{dq_S}{dt} + q_S - p \frac{dq_S}{dt} \right] + \left[ (\beta w_N^\theta - w_N) \frac{dq_N}{dt} \right] + \left[ (w_N^\theta - w_N) \frac{dq_N}{dt} \right]
\]

where \( \frac{d(q_N + q_S)}{dt} = p' q_N q_S \left( \theta_S (\beta - 1) + 1 \right) < 0 \).

The first bracket is negative. The second one is positive, provided that the tariff is small. And the third one depends on \( \beta \): It is negative if \( \beta < 1 \) and positive if \( \beta > 1 \). However, we can show that the optimal tariff \( t^* \) must be positive. The proof is straightforward: Suppose \( \frac{\partial \Phi_N}{\partial t} \big|_{t=0} = 0 \). Substituting \( t = 0 \) into the long expression above for \( \frac{\partial \Phi_N}{\partial t} \), we obtain \( \frac{\partial \Phi_N}{\partial t} \big|_{t=0} > 0 \) because of \( t \left( \frac{dq_S}{dt} \right) < 0 \), resulting in a contradiction.

4.2 The Southern Government

Maximizing (4b) yields:

\[
\frac{\partial \Phi_S}{\partial \theta_S} = \left[ p' \frac{d(q_N + q_S)}{d\theta_S} q_s + (\beta w_S^\theta - w_S) \frac{dq_N}{d\theta_S} \right] + \left[ (p - t) \frac{dq_S}{d\theta_S} \right] + \left[ (w_S^\theta - w_S) \frac{dq_S}{d\theta_S} \right]
\]

where \( \frac{d(q_N + q_S)}{d\theta_S} = (p' q_N / \Delta) (\beta \pi_S + q_S w_S) \left( \theta_N (\beta - 1) + 1 \right) ((\beta - 1) / \beta) \) is positive if
\( \beta < 1 \) and negative if \( \beta > 1 \). If \( \beta < 1 \), the first bracket on the RHS is negative and the second is positive; if \( \beta > 1 \), the first bracket becomes positive and the second becomes negative. Regardless of \( \beta \), the last bracket is negative. To find out optimal Southern LS, we may solve \( \frac{\partial \Phi_S}{\partial \theta_S} = 0 \) for \( \theta_S^* \). Let us denote it by \( \theta^*_S \).

4.3 The Analysis

We are in a position to state:

**Proposition 2:** Regardless of the tariff system, the optimal LS in each country is weaker if \( \beta > 1 \) than if \( \beta < 1 \). That is, \( \theta^*_{i} \big|_{\beta > 1} < \theta^*_{i} \big|_{\beta < 1} \).

**Proof:** We only prove the case for the North. That for the South can be done analogously. Suppose \( \beta^- = 1 - \varepsilon \) and \( \beta^+ = 1 + \varepsilon \), where \( \varepsilon > 0 \) and small. Denote the optimal LS \( \theta^-_N \) that satisfies \( \frac{\partial \Phi_N}{\partial \theta_N} = 0 \) when \( \beta^- = 1 - \varepsilon \), and \( \theta^+_N \) that satisfies \( \frac{\partial \Phi_N}{\partial \theta_N} = 0 \) when \( \beta^+ = 1 + \varepsilon \).

After some calculation, we obtain the following FOC:

\[
\frac{\partial \Phi_N}{\partial \theta_N} \big|_{\beta^-} = (p'q_s / \Delta)((1 - \varepsilon)\pi_N + q_N w_N)A = 0,
\]

where \( A = \begin{bmatrix}
\left(-\varepsilon \over 1 - \varepsilon\right) & \left( (p - p'q_s)(1 - \varepsilon \theta_s) + (p - t) + (w_{N}^{1+\varepsilon} - w_N)(2 - \varepsilon \theta_s) \right)
\end{bmatrix}.
\]

Now, changing \( \beta^- = 1 - \varepsilon \) to \( \beta^+ = 1 + \varepsilon \) in the above FOC yields:

\[
\frac{\partial \Phi_N}{\partial \theta_N} \big|_{\beta^+} = (p'q_s / \Delta)((1 + \varepsilon)\pi_N + q_N w_N)B,
\]

where \( B = \begin{bmatrix}
\left( \varepsilon \over 1 + \varepsilon\right) & \left( (p - p'q_s)(1 + \varepsilon \theta_s) + (p - t) + (w_{N}^{1+\varepsilon} - w_N)(2 + \varepsilon \theta_s) \right)
\end{bmatrix}.
\]

Then, \( \frac{\partial \Phi_N}{\partial \theta_N} \big|_{\beta^+} \) is not zero any more since \( \theta^-_N \) is the optimal LS for the case of
\( \beta^- = 1 - \varepsilon \). To verify the sign, subtract the former from the latter. Then we have:

\[
\frac{\partial \Phi_N}{\partial \theta_N} |_{\beta^-, \theta^-} - \frac{\partial \Phi_N}{\partial \theta_N} |_{\beta^-, \theta^-} < 0.
\] (11)

To see this negative sign, note first that \( B > A > 0 \). And the term in front of \( B \) in \( \frac{\partial \Phi_N}{\partial \theta_N} |_{\beta^-, \theta^-} \) is negative and has a larger absolute value than a similar term in front of \( A \) in \( \frac{\partial \Phi_N}{\partial \theta_N} |_{\beta^-, \theta^-} \). Thus, (11) is negatively signed. Also, since \( \frac{\partial \Phi_N}{\partial \theta_N} |_{\beta^-, \theta^-} \) is zero, we must have \( \frac{\partial \Phi_N}{\partial \theta_N} |_{\beta^-, \theta^-} < 0 \). This implies that \( \theta_N^+ |_{\beta^-, \theta^-} < \theta_N^- |_{\beta^-, \theta^-} \). So in general, we have \( \theta_N^+ |_{\beta^+, \theta^+} < \theta_N^- |_{\beta^-} \), which is true under free trade as well. Refer to Figure 1. QED

![Figure 1: LS - wage vs. employment oriented unions](image)

Proposition 2 implies that each government may tend to choose weaker LS when unions are wage-oriented. Wage-oriented unions pursue higher wages at the expense of a lower negotiated employment, raising the firm’s costs and lowering its profitability. As a consequence, the government may try to reduce the union’s bargaining power by choosing a lower labor standard. This finding is associated with the argument of ‘a race to the bottom’, which we will further discuss later in Proposition 4. Next, we investigate how the tariff affects the Northern LS compared with free trade.
**Proposition 3:** Each country’s LS is lower under the optimal tariff than under free trade if $\beta < 1$, and the opposite is true if $\beta > 1$. Formally,

(i) If $\beta < 1$, then $\theta_i^* |_{t > 0} < \theta_i^* |_{t = 0}$;  
(ii) If $\beta > 1$, then $\theta_i^* |_{t > 0} > \theta_i^* |_{t = 0}$.

**Proof:** Again it suffices to prove the case for the North only. Under the optimal tariff, $t^*$, the optimal LS $\theta_N^*$ satisfies,

\[
\frac{\partial \Phi_N}{\partial \theta_N} = \left( p^q \Delta \right) \left( \beta \pi_N + q_N w_N \right) \left[ \frac{(\beta - 1)}{\beta} \left( p - p' q_N \right) \left( \theta_s (\beta - 1) + 1 \right) + \left( p - t^* \right) \right] - p \frac{q_N}{w_N} \left( \beta w_N - w_N \right) \left( 2 \theta_s (\beta - 1) + 3 \right) = 0 .
\]

Now consider a free trade system with $t=0$. Then from the above expression one can verify that $\frac{\partial \Phi_N}{\partial \theta_N} |_{t=0} > 0$ if $\beta < 1$ and $\frac{\partial \Phi_N}{\partial \theta_N} |_{t=0} < 0$ if $\beta > 1$, which implies the proposition. These are illustrated in Figure 2.

QED

![Figure 2: LS - optimal tariff vs. free trade](image)

Proposition 3 implies that imposing a tariff to force the South to raise its LS is only effective if the union is wage-oriented. If the union is employment oriented, the South would choose a lower...
LS in response to Northern pressure. Thus, it further implies that trade liberalization in the North may raise Southern LS in the latter case, which is in line with the argument that the best way to raise Southern LS is to keep Northern markets open. In addition, this case confirms the empirical findings of Neumayer and Soysa (2006) that countries that are more open to trade have fewer rights violations than more closed ones. In the last part of this section, we look into the issue of “a race to the bottom” of LS. We can establish:

**Proposition 4:** A race to the bottom of LS arises only under two conditions: each government does not care about union utility, or the union is sufficiently wage-oriented. In other cases, it does not arise.

**Proof:** (i) Let us first prove the case for the South, whose welfare consists of firm profits and union utility, since consumption occurs in the North only. Figure 3 shows that an increase in LS raises firm profits and union utility if the union is sufficiently employment-oriented (i.e. if \( \beta < \beta' \) in Figure 3). If the union is wage-oriented (\( \beta > 1 \)), then an increase in LS reduces firm profits, but it still raises union utility. Therefore, the optimal LS can be still positive. It becomes zero only if the union is sufficiently wage oriented, at a point \( \beta \geq \beta'' \), where \( \beta'' \):

\[
\frac{\partial \Phi_s}{\partial \theta_s} = \frac{\partial \pi_s}{\partial \theta_s} + \frac{\partial u_s}{\partial \theta_s} = 0.
\]

Also, in the special case that the government does not care about union utility, the union utility does not enter the government’s objective function and thus the government chooses zero LS if \( \beta > \beta' \) (i.e., \( \frac{\partial \Phi_s}{\partial \theta_s} = \frac{\partial \pi_s}{\partial \theta_s} = 0 \) if \( \beta > \beta' \)).

(ii) The proof for the North is more complicated, since Northern welfare includes consumer surplus and tariff revenue. Let us look at the case of near free trade, i.e., \( t \approx 0 \), then the effect on the tariff revenue disappears. Since an increase in LS raises consumer surplus \( (d \varphi / d \theta_N > 0) \) if \( \beta < 1 \), it moves the point for \( \partial \Phi_N / \partial \theta_N = 0 \) to the right of \( \beta'' \), say a point such as \( \beta''' \). That is, only if \( \beta \geq \beta''' \), then the Northern government would choose a zero level of LS. QED
5. Some Extensions

In this section we introduce two extensions of the basic model. One is to incorporate asymmetric labor unions in terms of preferences across countries and the other is to look into the effects of economic integration. We analyze how these affect LS choices in the two countries.

5.1 Asymmetric Labor Unions

So far, we have treated both labor unions as having identical preferences towards wage versus employment. What if this is not the case? Here we can consider four asymmetric cases: Firstly, the Northern labor union is wage-oriented while the Southern union is employment-oriented, i.e., \( \beta_N > 1 \) and \( \beta_S < 1 \). Secondly, the Northern union is employment oriented while Southern union is wage oriented, i.e., \( \beta_N < 1 \) and \( \beta_S > 1 \). Thirdly, the union is more wage oriented in country \( i \) than that in country \( j \), i.e., \( \beta_i > \beta_j > 1 \). Lastly, the union is more employment-oriented in country \( i \) than that in \( j \), i.e., \( \beta_i < \beta_j < 1 \). We also examine the first two cases with and without free trade. The results are summarized as follows.

**Proposition 5:** When the labor unions in the two countries have asymmetric preferences over wages and employments, the governments set their optimal LS as follows:

(i) \( \theta^*_N |_{\beta_N > 1, r > 0} < \theta^*_N |_{\beta_N > 1, r > 0} < \theta^*_S |_{\beta_S < 1, r > 0} < \theta^*_S |_{\beta_S < 1, r > 0} \).
(ii) \( \theta^*_S |_{\beta_S > 1, t = 0} < \theta^*_S |_{\beta_S > 1, t > 0} < \theta^*_N |_{\beta_N > 1, t = 0} < \theta^*_N |_{\beta_N < 1, t = 0} \),

(iii) \( \theta^*_t |_{\beta_t > \beta_t > 1} < \theta^*_t |_{\beta_t > 1} \) for a given tariff \( t \), and

(iv) \( \theta^*_t |_{\beta_t = 1} < \theta^*_t |_{\beta_t < 1} \) for a given tariff \( t \).

\textbf{Proof}: Suppose \( \beta_N \neq \beta_S \). Then, from Proposition 3, we have \( \theta^*_N |_{\beta_N > 1, t = 0} < \theta^*_N |_{\beta_N > 1, t > 0} \) and \( \theta^*_N |_{\beta_N < 1, t = 0} < \theta^*_N |_{\beta_N < 1, t > 0} \); and \( \theta^*_S |_{\beta_S > 1, t = 0} < \theta^*_S |_{\beta_S < 1, t = 0} \) and \( \theta^*_S |_{\beta_S > 1, t = 0} < \theta^*_S |_{\beta_S > 1, t > 0} \). In addition, from Proposition 2, we obtain \( \theta^*_N |_{\beta_N > 1} < \theta^*_S |_{\beta_S < 1} \) and \( \theta^*_N |_{\beta_N < 1} < \theta^*_S |_{\beta_S > 1} \) given a level of the tariff. Using all these rankings, we derive (i) and (ii) in Proposition 5.

For (iii) and (iv), it suffices to prove the case for the North only. First, for (iii), suppose that \( \beta_N = \beta_S > 1 \) and the North chooses a \( \theta^*_N \) that satisfies the following FOC condition:

\[
\frac{\partial \Phi_N}{\partial \theta_N} = \left( \frac{p' q_S}{\Delta} \right) \left( \beta_N \pi_N + q_N w_N \right) \left[ \left( \frac{\beta_N - 1}{\beta_N} \right) \left( \frac{(p - p') (\theta_S (\beta_S - 1) + 1) + (p - t')'}{2 \theta_S (\beta_S - 1) + 3} \right) \right] = 0.
\]

Given \( \beta_S \), if \( \beta_N \) further increases slightly, then \( \frac{\partial \Phi_N}{\partial \theta_N} |_{\theta_N, \beta_N} < 0 \) from the above FOC. The new optimal level of LS for the North becomes lower. As for (iv), suppose that \( \beta_N = \beta_S < 1 \) and the North chooses a \( \theta^*_N \) that satisfies the above FOC. Now, if \( \beta_N \) further decreases slightly, we can similarly verify \( \frac{\partial \Phi_N}{\partial \theta_N} |_{\theta_N, \beta_N} > 0 \). And the new optimal level of LS for the North becomes higher. \( \text{QED} \)

It is quite common to observe that developed countries sustain relatively higher labor standards than developing countries. The above Proposition shows that this may be a reflection of different preferences of their labor unions: The Northern union may be more interested in employment than the Southern one. And in the extreme case that the Northern union is employment oriented while the Southern one is wage oriented, the LS differential between the two countries is the highest under free trade.
5.2 Economic Integration and Southern LS

Does regional economic integration increase LS? From Proposition 3, we learned that the Southern government may choose a higher LS under the optimal tariff system than under free trade system if labor unions are wage-oriented. Put another way, this implies that the Northern tariff on Southern imports is an effective way to raise Southern labor standards if labor unions are wage-oriented.

In this section, we further investigate the issue of economic integration. In particular, what we have in mind is the effect of some Southern countries’ accessions to the World Trade Organization or to the European Union, where member countries lose their discretionary choice of import tariffs (i.e., free trade is mandatory). And they may as well cooperate over non-tariff issues such as labor standards. Our question is whether such a deeper economic integration increases their LS or not.

To see this formally, we change the game structure as follows. In the first stage, both governments determine their LS cooperatively, and the Northern government abides by the agreed-upon zero tariff; and in the second stage, the labor union and the firm negotiate to determine wages and employment in each country simultaneously.

The second stage of the game can be solved as in earlier sections. Now to find out the optimal LS, both governments maximize their joint welfare choosing the two LS, given a zero-tariff system. They yield the following FOCs:

\[ \frac{\partial \Phi_N}{\partial \theta_N} + \frac{\partial \Phi_S}{\partial \theta_S} = 0 \quad \text{and} \quad \frac{\partial \Phi_N}{\partial \theta_N} + \frac{\partial \Phi_S}{\partial \theta_S} = 0. \]

Let us denote the Northern optimal LS as \( \theta^F_N \) and the Southern one as \( \theta^F_S \), where the superscript \( E \) stands for economic integration. We are now in a position to compare whether \( \theta^E_i \) is greater or smaller than the optimal LS, \( \theta^*_i \), under the full discretionary regime over tariffs and LS in previous sections.

**Proposition 6:** After economic integration, if countries cooperate over LS, (i) then they tend to choose higher LS than in the absence of the economic integration if labor unions are wage-oriented; (ii) If labor unions are employment-oriented, the effect of economic integration on LS is ambiguous.

**Proof:** When plugging the individual optimal LS into the FOCs, the following equalities must hold,
These equations are not necessarily zero. To see their signs, note first,

\[
\frac{\partial \Phi_s}{\partial \theta_s} |_{\theta_s^*} = \frac{\partial \Phi_N}{\partial \theta_N} |_{\theta_N^*} + \frac{\partial \Phi_S}{\partial \theta_N} |_{\theta_N^*} = 0 + \frac{\partial \Phi_S}{\partial \theta_N} |_{\theta_N^*} + 0.
\]

The implications of the above proposition are interesting. Suppose that labor unions are wage-oriented. Then the free trade system of the North results in a lower LS in the South as compared to under the optimal tariff, as shown in Proposition 3. And in order for the North to raise Southern LS, it must abandon the free trade regime and impose a positive tariff against Southern imports. However, Proposition 6 says that if both countries cooperate over the LS to maximize their

D is positive if \( \beta > 1 \) and negative if \( \beta < 1 \), while E is always positive. Therefore, if \( \beta > 1 \), we have \( \frac{\partial (\Phi_N + \Phi_S)}{\partial \theta_N} |_{\theta_N^*} = \frac{\partial \Phi_S}{\partial \theta_N} |_{\theta_N^*} > 0 \); that is, the optimal LS under economic integration is greater than without integration. If \( \beta < 1 \), \( \frac{\partial (\Phi_N + \Phi_S)}{\partial \theta_N} |_{\theta_N^*} = \frac{\partial \Phi_S}{\partial \theta_N} |_{\theta_N^*} \) is either positive or negative, depending on the relative size of \( D \) and \( E \).

Second, \( \frac{\partial \Phi_N}{\partial \theta_s} |_{\theta_s^*} = \frac{G + K}{\Delta} \), where \( G \) and \( K \) can be written similarly as \( D \) and \( E \), with the subscripts \( N \) and \( S \) switched. Since the Northern government cares about consumer surplus, \( G \) shows that an increase in the Southern LS brings ambiguous effects. However, if we assume \( \beta \) is not extremely high or small, the bracket in \( G \) becomes positive. Then, if \( \beta > 1 \), \( \frac{\partial (\Phi_N + \Phi_S)}{\partial \theta_s} |_{\theta_s^*} = \frac{\partial \Phi_N}{\partial \theta_s} |_{\theta_s^*} > 0 \), implying that the optimal LS with economic integration is greater than without integration; If \( \beta < 1 \), \( \frac{\partial (\Phi_N + \Phi_S)}{\partial \theta_s} |_{\theta_s^*} = \frac{\partial \Phi_N}{\partial \theta_s} |_{\theta_s^*} \) is either positive or negative, depending on the relative size of \( G \) and \( K \). QED
joint welfare under the free trade system, then it is possible for the North to induce the Southern
government to choose a higher LS, without giving up the free trade regime.

6. Concluding Remarks
In this paper, with a setting of a Northern firm competing against a Southern exporter in the
Northern market, we investigated how governments set labor standards when labor unions have a
biased preference towards either wages or employment. The following results are noteworthy. First,
given a level of labor standards and tariff, an increase in a country’s labor standards raises the profit
of the country and reduces that of the competing country, if labor unions are sufficiently
employment-oriented. Otherwise (i.e., with wage-oriented unions), the opposite is true. Second,
given any tariff level, governments choose higher labor standards when labor unions are
employment-oriented than when they are wage-oriented. Third, the Southern government chooses
higher labor standards when the tariff is at the Northern government’s disposal than when it is not
(i.e., under free trade), if labor unions are wage-oriented. Otherwise, the opposite is true. Lastly, ‘a
race to the bottom’ of labor standards may arise if either (i) each government does not care about
union’s utility or (ii) the union is sufficiently wage-oriented. A race to the bottom of labor standards
is more likely to arise under free trade than under an optimal tariff.

We extended the analysis to two more interesting cases. First, we considered asymmetric
preferences of labor unions. We found that the North sets higher LS than the South, when the
Northern union is more employment-oriented than the Southern union. Second, we consider the
effect of economic integration between the North and the South on their LS decisions. We showed
that both countries cooperatively choose higher LS in order to maximize their join welfare even if
their labor unions are wage-oriented.

Another interesting extension would be to introduce multinational corporations and foreign
direct investment, where the Northern firm has a Southern branch and it bargains with the Southern
labor union. The Northern multinational firm can use this situation as a threat against the Northern
labor union as well as the Southern one, in case either of the negotiations breaks down. The threat
of going multinational would reduce the union wage premium regardless of union preferences.
However, the Southern labor union has a better position than the Northern one since it deals with
two firms, the Southern firm and the Southern branch of the Northern firm. This would positively
affect the negotiated wage and employment. The final effect might be ambiguous. We leave it for
future studies.
Appendix 1

Totally differentiating the FOCs (6a–6b) in the second stage yields (for $i, j = N, S; i \neq j$):

\[
\begin{bmatrix}
\frac{\partial^2 \bar{G}_N}{\partial w_N^2} & \frac{\partial^2 \bar{G}_S}{\partial w_N^2} & 0 & \beta \theta_N \frac{\partial \pi_N}{\partial q_S} \\
\frac{\partial^2 \bar{G}_N}{\partial w_N \partial q_N} & \frac{\partial^2 \bar{G}_S}{\partial w_N \partial q_N} & 0 & \beta \theta_N \frac{\partial \pi_N}{\partial q_S} \\
1 - \beta & \beta & \frac{\partial^2 \pi_N}{\partial q_N^2} & \frac{\partial^2 \pi_N}{\partial q_N^2} \\
0 & \beta \theta_S \frac{\partial \pi_S}{\partial q_S} & \frac{\partial^2 \bar{G}_S}{\partial w_S^2} & \frac{\partial^2 \bar{G}_S}{\partial w_S \partial q_S} \\
0 & \frac{\partial^2 \pi_S}{\partial q_S^2} & 1 - \beta & \frac{\partial^2 \pi_S}{\partial q_S^2}
\end{bmatrix}
\begin{bmatrix}
dw_N \\
dq_N \\
dw_S \\
dq_S
\end{bmatrix}
= - \begin{bmatrix}
\frac{\partial^2 \bar{G}_N}{\partial w_N \partial \theta_N} \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
d\theta_N \\
d\theta_S \\
d\theta_S \\
d\theta_S
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
dt
\]

where \( \frac{\partial^2 \bar{G}_i}{\partial w_i^2} = \beta \theta_i \frac{\partial \pi_i}{\partial w_i} - (1 - \theta_i) q_i = \left[ (1 - \beta) \theta_i - 1 \right] q_i < 0, \)

\( \frac{\partial^2 \bar{G}_i}{\partial w_i \partial q_i} = \beta \theta_i \frac{\partial \pi_i}{\partial q_i} - (1 - \theta_i) w_i = -w_i < 0, \)

\( \frac{\partial^2 \bar{G}_i}{\partial q_i^2} = \beta \theta_i p' q_i < 0, \)

\( \frac{\partial^2 \bar{G}_i}{\partial w_i \partial \theta_i} = \frac{1 - \beta}{\beta}, \)

\( \frac{\partial^2 \pi_i}{\partial q_i^2} = \frac{p'' q_i + 2 p'}{0}, \)

\( \frac{\partial^2 \pi_i}{\partial q_i \partial q_i} = p'' q_i + p' < 0, \)

\( \frac{\partial^2 \bar{G}_i}{\partial q_i \partial \theta_i} = -\beta \theta_S q_i < 0, \)

\( \frac{\partial^2 \bar{G}_i}{\partial q_i \partial q_i} = -1 < 0. \)

The determinant of the 4 by 4 matrix on the left hand side can be expanded as:

\[
\Delta = \left[ \frac{\partial^2 \bar{G}_N}{\partial w_N^2} \frac{\partial^2 \bar{G}_S}{\partial w_S^2} - \frac{\partial^2 \pi_N}{\partial q_N^2} \frac{\partial^2 \pi_S}{\partial q_S^2} - \frac{\partial^2 \pi_N}{\partial q_N \partial q_S} \frac{\partial^2 \pi_S}{\partial q_S \partial q_N} \right] + \frac{(1 - \beta)^2}{\beta} \left[ \beta \theta_N \frac{\partial \pi_N}{\partial q_S} - \beta \theta_S \frac{\partial \pi_S}{\partial q_S} - \frac{\partial^2 \bar{G}_N}{\partial w_N \partial \theta_N} \frac{\partial \pi_N}{\partial q_S} - \frac{\partial^2 \bar{G}_S}{\partial w_S \partial \theta_S} \frac{\partial \pi_S}{\partial q_S} \right]
\]

We have \( \beta \theta_i \pi_i = (1 - \theta_i) q_i w_i \) for \( i = N, S \) from (6a), and \( w_N / \beta + p' q_S + p - w_N = 0 \) and \( w_S / \beta + p' q_S + p - t - w_S = 0 \) from (6b). Multiplying \( \beta \theta_i \) to the second equations in (6b) and using \( \beta \theta_i \pi_i = (1 - \theta_i) q_i w_i \), we obtain \( \beta \theta_i p' q_i + w_i = 0 \). This implies \( \beta \theta_i \frac{\partial \pi_i}{\partial q_i} = \frac{\partial^2 \bar{G}_i}{\partial w_i \partial q_i}, \)

making the 2nd bracket on the RHS of \( \Delta \) zero. Hence, \( \Delta > 0 \) provided that the own effects (the 1st term in 1st bracket) dominate the cross effects (the rest of terms in 1st bracket).

For simplicity we evaluate the comparative static results at \( p'' = 0 \). (Our results hold as long as the marginal revenue is decreasing in output, i.e., as long as \( p'' \) is not extremely positive.) Then we obtain the results shown in section 3.1 as follows. For \( i, j = N, S \) and \( i \neq j; \)

\[
\Delta \frac{dw_i}{d\theta_j} = \left( 2 \theta_j (\beta - 1) + 3 \right) (p')^2 q_j (\beta \pi_i + q_i w_i) > 0.
\]
\[ \Delta \frac{dq_i}{d\theta_i} = \left( \beta \frac{\beta - 1}{\beta} \right) \left( \theta_j (\beta - 1) + 2 \right) p' q_j (\beta \pi_i + q_i) \begin{cases} > 0 & \text{if } \beta < 1 \\ < 0 & \text{if } \beta > 1 \end{cases} \]
\[ \Delta \frac{dw_j}{d\theta_j} = (\beta - 1) \theta_i (p')^2 q_i (\beta \pi_j + q_j) \begin{cases} < 0 & \text{if } \beta < 1 \\ > 0 & \text{if } \beta > 1 \end{cases} \]
\[ \Delta \frac{dq_i}{d\theta_j} = -\left( \beta \frac{\beta - 1}{\beta} \right) p' q_j (\beta \pi_j + q_j) \begin{cases} < 0 & \text{if } \beta < 1 \\ > 0 & \text{if } \beta > 1 \end{cases} \]
\[ \Delta \frac{dw_j}{dt} = \theta_i \beta (p') q_j q_s > 0, \quad \Delta \frac{dq_i}{dt} = -p' q_i q_s > 0, \quad \Delta \frac{dw_s}{dt} = -\Delta p' \theta_s \frac{dq_s}{dt} < 0. \quad \text{QED} \]

Appendix 2

Proof for (8a) and (8b):
\[ \frac{d\pi_N}{d\theta_N} = \left( (\beta \pi_N + q_N w_N) p' q_s / \Delta \beta \right) \left[ (\beta - 1)(\theta_j (\beta - 1) + 2)(p - w_N) - p' q_N (\beta (2 + \theta_j) + (1 + \theta_j)) \right]. \]
After some calculation, we can verify that \( d\pi_N / d\theta_N \) is negative if \( \beta > \beta^* \) and positive if \( \beta < \beta^* \), where \( \beta^* \in (0,1) \) is a critical value of \( \beta \) that gives \( d\pi_N / d\theta_N = 0 \). The proof of existence of the critical value is as follows. Suppose \( d\pi_N / d\theta_N = 0 \), implying:
\[ (\beta^* - 1)(\theta_j (\beta^* - 1) + 2)(p - w_N) = p' q_N [(\beta^* \theta_j + 2) + (1 - \theta_j)] \]
It is clear that since the RHS is negative, \( \beta^* \) in the LHS must be less than 1 (by definition it is greater than 0). And (9b) can be proved analogously, by replacing \( \beta^* \) with \( \beta^{**} \), where \( \beta^{**} \) is a critical value of \( \beta \) that yields \( d\pi_s / d\theta_s = 0 \). \quad \text{QED} \]

Proof for (8c):
\[ \frac{d\pi_N}{d\theta_S} = \beta - q_N (\beta \pi_S + q_s w_S) p' [p' q_N (1 - \theta_N) - (p - w_N)]. \]
Here, \( d\pi_N / d\theta_S < 0 \) if \( \beta < 1 \) and \( d\pi_N / d\theta_S > 0 \) if \( \beta > 1 \). We can straightforwardly prove for the case of \( d\pi_N / d\theta_S \) in a similar way. \quad \text{QED} \]

Proof for (8d):
\[ \frac{d\pi_N}{dt} = (p' q_N q_s / \Delta) [- (p - w_N) - p' q_N (\theta_N - 1)] > 0. \]
The proof for \( d\pi_s / dt < 0 \) can be done in a similar way straightforwardly. \quad \text{QED} \]
References


