Market Size and Firm Location in a Non-tradable Service Industry*

Hiroshi Kurata\textsuperscript{a,†}, Takao Ohkawa\textsuperscript{b}, Makoto Okamura\textsuperscript{c}

\textsuperscript{a} Faculty of Economics, Tohoku Gakuin University, Japan
\textsuperscript{b} Department of Economics, Ritsumeikan University, Japan
\textsuperscript{c} Economics Department, Hiroshima University, Japan

Abstract

This paper investigates the welfare effects of firm location in a non-tradable service industry. When firms determine their own locations in either of two regions with difference in market size, firm location is insufficient in the region with large market for consumers while excessive for producers and the economy as a whole. Although an expansion of difference in market size may or may not increase firms’ output, it is unambiguously unfavorable for producers. On the other hand, the expansion is favorable for consumers and the economy as a whole.

Keywords: Firm Location; Non-tradable good; Market Size; Inefficiency; Cournot Competition

\textit{JEL classifications:} F21; L11; L13

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\textsuperscript{†}Corresponding Author; Address: 1-3-1 Tsuchitoi, Aoba-ku, Sendai, 980-8511, Japan; Tel/Fax: +81-22-721-8977; E-mail: hkurata@tscc.tohoku-gakuin.ac.jp
1 Introduction

Recently, service industry, or tertiary (i.e., third-stage) industry has played an important role in the world economy. For example, the Ministry of Economics, Trade and Industry (METI) Japan (2007) reports that the share of service industry in real GDP is 67.9% in Japan, and 70.9% in the United States in 2002. The tendencies of large shares of service industry are also observed in other countries.

In the service industry, in particular, this paper focuses on non-tradable services, such as hotels, rental car services, and convenience stores. In such non-tradable service industries, firms can provide their services only in the market where they locate. That is, services are provided in several segmented markets.

Taking this property into our considerations, firm location in the non-tradable service industry has crucially affected by two factors. The first factor is market size. The segmented markets generally have differences in size. It seems that markets with larger population are more attractive for firms than those with smaller population, because the former has more demand. At the same time, however, large markets bring severe competition to firms. For example, urban areas have more hotels than rural areas, and there are more hotels offering cheaper rate in the urban areas than in the rural areas.

The second factor is strategic interactions of location choice. Since firms normally face some financial or managerial resource constraints, they cannot locate in all segmented markets. Then, when each firm chooses its own location, it must take the rival firms' location into its consideration. This implies that firms' location choices have strategic interactions. Thus, considering firm location in the non-tradable good

\footnote{For the detail, see the report “Towards Innovation and Productivity Improvement in Service Industries” on the METI webpage: http://www.meti.go.jp/english/report/data/0707SPRING.html.}

\footnote{Kurata, Ohkawa and Okamura (2008) categorize service as tradable service and non-tradable service. The former includes, for instance, computer programming, while the latter includes, for instance, hotels and rental car services.}
industries, it is critical to include both the difference in market size and the strategic interactions of firms’ location choices.

The purpose of this paper is to clarify the welfare effects of firm location in a non-tradable service industry. In order to describe characteristics of the non-tradable service, we construct the following simple model: Firms determine to locate in either of two regions with difference in market size.\(^3\) The difference in market size is described by the number of consumers (Haufler and Wooton, 1999). After determining their location, firms compete in each market in the Cournot sense.

On the welfare effect of firm location in the non-tradable service industry, we address two questions. The first question is how the firm location, which is determined by each firm, is inefficient from the welfare viewpoint. Under imperfect competition, insofar as each firm behaves non-cooperatively, one can see that firm location is inefficient from the welfare point of view. But it is not so straightforward to see whether firms insufficiently or excessively locate. The second question is whether a change in the difference in market size, \(i.e.,\) concentration or dispersion of population, is favorable or not.

We obtain the following results: For the first question, firms insufficiently locate in the larger market for consumers, while excessively locate for producers and for the economy as a whole. For the second question, an expansion of difference in market size unambiguously decreases firms’ profits. In particular, one may think that the expansion of difference in market size sounds favorable for firms locating in the larger region, but actually it is not favorable, because of severe competition. On the other hand, the expansion of the difference in market size is favorable for consumers and

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\(^3\)Our model is similar to that in “subsidy-game” literature \((e.g.,\) Barros and Cabral, 2000). In their model, governments in two countries perform subsidy-game in the setting where a foreign firm locates into either of two countries. Since their focus is on the subsidy game, the number of foreign firm is one. On the other hand, our model considers oligopolistic firms because of including strategic behaviors between firms.
for the economy as a whole. These results are explained by the property of the non-
tradable service industries.

In order to analyze the firm location, we apply the way in the “excess entry”
literature (e.g., Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987). The
“excess entry” literature focuses on firm entry into a market and determines the number
of firms in the market endogenously. On the other hand, our analysis gives the total
number of firms exogenously, but determines the number of firms locating in each
region endogenously, in order to focus on the firm location.4

In the last two decades, firm location have been studied mainly in the economic
geography (e.g. Krugman, 1991; Fujita, Krugman and Venables, 1999; Baldwin, et al.,
2003). However, firm location in non-tradable service industry has not been analyzed
in the literature. The main focus of the economic geography is on whether firms
agglomerate or disperse as a result of the movement of labor. Although our focus
is similar to that in the economic geography, it is quite different in that we consider
the situation where firms locate only in the both two regions and that firm location is
explained as a result of firms’ strategic interactions. In this sense, our analysis provides
a new insight into the studies on firm location.

The rest of this paper is organized as follows. Section 2 provides the basic model.
Section 3 derives the equilibrium location of firms and clarifies characteristics of the
equilibrium. Section 4 considers the first question: how the equilibrium location is inef-
ficient, i.e., insufficient or excessive. Section 5 examines the second question: whether
the expansion of difference in market size is favorable or not. Finally, section 6 provides
brief concluding remarks.

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4Elberfeld (2003) derives a partly similar result in a different context; on firms’ technology choice.
2 The Model

There are two regions: regions $L$ and $S$, and each region has $m_i$ identical consumers ($i = L, S$). Throughout this paper, we assume that $m_L > m_S$; that is, region $L$ has larger population than region $S$.

We focus on a non-tradable service industry in these regions. Let $x_i$ and $X_i$ be consumption by each consumer and total consumption in region $i$, respectively ($i = L, S$). The total consumption in region $i$ is expressed by

$$X_i = m_i x_i.$$  \hspace{1cm} (1)

We assume a consumer in region $i$ has a demand function as a following form: $x_i = a - p_i$, where $p_i$ is the price of the non-tradable service, and $a$ is a positive constant. Then, from equation (1), we derive the inverse demand function in region $i$ as

$$p_i = a - \frac{1}{m_i} X_i, \quad i = L, S.$$ \hspace{1cm} (2)

In this industry, $N$ symmetric firms provide services. The total number of firms $N$ is exogenously given and fixed throughout this paper. Each firm can provide the service only in the market where it locates because its service is non-tradable. Because of some managerial resource constraints, these firms cannot locate in the both regions.\footnote{This assumption is adopted in Barros and Cabral (2000) and Fumagalli (2003).}

We denote $n_i$ as the number of firm locating in region $i$ ($i = L, S$).

Each firm must incur constant marginal cost $c$ for provision of service and entry cost $f$ to enter the market. Cost function of firm $j$ locating in region $i$ is given by $c_{ij} = c q_{ij} + f$, where $q_{ij}$ is an output of the firm. Note that the subscript $ij$ expresses the value related to firm $j$ in region $i$ ($i = L, S; j = 1, \ldots, N$).
We consider a following two-stage game. In the first stage, each firm simultaneously determines its own location. In this setting, firms must incur entry cost $f$. In the second stage, given firms’ location decision, firms compete in each market in the Cournot sense.

3 Equilibrium Location

We derive the subgame perfect equilibrium of the game. In the second stage, given the number of firms $n_i$, firm $ij$’s gross profit, $\pi_{ij}$, is then written by

$$\pi_{ij} = (p_i(X_i) - c)q_{ij}. \quad (3)$$

Firm $ij$’s net profit is thus expressed by $\pi_{ij} - f$. Note that we have the relationship with $X_i = n_i \sum_j q_{ij}$. From equation (3), the profit-maximizing condition for each firm is

$$p'_i(X_i)q_{ij} + p_i(X_i) - c = 0. \quad (4)$$

We focus on the symmetric equilibrium, and thus $q_{ij} = q_i$ and $\pi_{ij} = \pi_i$ and $X_i = n_i q_i$.

From equations (2), (3) and (4), in equilibrium, output of each firm, $q_i$, profit of each firm, $\pi_i$, and total output, $X_i$, can be written as functions of the number of firms $n_i$; i.e., $q_i = q_i(n_i)$, $\pi_i = \pi_i(n_i)$, and $X_i = X_i(n_i)$. In the following, we normalize $a - c = 1$.

Under the setup, we have the following solutions ($i = L, S$):

$$q_i = \frac{m_i}{n_i + 1}, \quad \pi_i = \frac{m_i}{(n_i + 1)^2}, \quad \text{and} \quad X_i = n_i q_i = \frac{m_i n_i}{n_i + 1}. \quad (5)$$

Here, let us introduce the firm-number elasticity of firm’s output in market $i$ by

$$\theta_i(n_i) \equiv -\frac{n_i}{q_i} \frac{\partial q_i}{\partial n_i} \quad (i = L, S).$$

The elasticity $\theta_i(n_i)$ has the following properties.$^6$

$^6$Proofs of upcoming Lemmas and Propositions will appear in Appendix.
Lemma 1

(i) \( \theta_i(n_i) \) is in the interval \((0,1)\);
(ii) \( \theta_i(n_i) \) is independent of the population \(m_i\);
(iii) \( \theta_i(n_i) \) is increasing and strictly concave in \(n_i\).

We now turn into the first stage. In the first stage, if the resulting net profits are different between markets, any firms locating in the region with less net profit have incentive to move into the other region. Therefore, the equilibrium location must guarantee that the both regions have equal net profits.\(^7\) We thus define the equilibrium location as follows.

**Definition: Equilibrium Location**

The equilibrium location \((n^e_L, n^e_S)\) is a pair of \((n_L, n_S)\) such that

(i) \( n_L + n_S = N \)
(ii) \( \pi_L(n_L) - f = \pi_S(n_S) - f \), and
(iii) For given \(n_i\), equation (4) holds \((i = L, S)\).

Let us define \( \gamma \equiv \sqrt{\frac{m_S}{m_L}} \) as a parameter expressing the difference in market size. Notice that \(0 < \gamma < 1\) because \(m_L > m_S\). From conditions (i) and (ii) of the equilibrium location, we obtain

\[
(n^e_L, n^e_S) = \left( \frac{N + 1 - \gamma}{1 + \gamma}, \frac{\gamma(N + 1) - 1}{1 + \gamma} \right).
\]

From equation (6), it is straightforward to see that \(n^e_L > n^e_S\) because \(\gamma < 1\). In this paper, we focus on the case \(n^e_i \geq 1\) for all \(i = L, S\).\(^8\) We thus assume that \(\frac{2}{N} \equiv \frac{\gamma}{N} \leq \gamma\) in the following analysis. We use superscript \(e\) to express the value at the subgame.

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\(^7\)We ignore the “integer problem” of the game.
\(^8\)In other words, we do not focus on the case of perfect agglomeration, i.e., the case where all firms locate in a region, in this paper.
perfect equilibrium. Substituting equation (6) into equations (5), we obtain

\[
q^e_L = \frac{1 + \gamma}{N + 2} m_L, \quad \pi^e_L = \left( \frac{1 + \gamma}{N + 2} \right)^2 m_L, \quad \text{and} \quad X^e_L = \frac{N + 1 - \gamma}{N + 2} m_L;
\]

\[
q^e_S = \frac{1 + \gamma}{\gamma(N + 2)} m_S, \quad \pi^e_S = \left\{ \frac{1 + \gamma}{\gamma(N + 2)} \right\}^2 m_S, \quad \text{and} \quad X^e_S = \frac{\gamma(N + 1) - 1}{\gamma(N + 2)} m_S.
\]

From equations (7), we find a property of the equilibrium location.

**Proposition 1**

The equilibrium price in market \( L \) is always lower than that in market \( S \).

Since market \( L \) has larger population, one may expect the equilibrium price in market \( L \) is higher. Proposition 1 shows that, however, large population attracts many firms, and the price rather becomes lower than that in market \( S \).

### 4 Inefficiencies of Firm Location

In this section, we consider the first question for the firm location. Under given population \( m_i \) (\( i = L, S \)), we examine how the equilibrium location is inefficient from the viewpoints of producers, consumers, and the economy as a whole. Here we consider the marginal firm relocation from region \( S \) to region \( L \), introducing a new policy parameter \( n \) such that \( \frac{dn_L}{dn} = 1 \) and \( \frac{dn_S}{dn} = -1 \) because the total number of firms is fixed.

First, we focus on the producers’ viewpoint. Let \( PS_i \) and \( PS \) be producer surplus in market \( i \) (\( i = L, S \)) and total producer surplus, respectively. \( PS_i \) is given by

\[
PS_i(n_i) = n_i(\pi_i(n_i) - f),
\]

and the total producer surplus is organized as \( PS = PS_L(n_L) + PS_S(n_S) \). Partially
differentiating equation (8), we have

\[
\frac{\partial PS_i}{\partial n_i} = (\pi_i - f) + n_i p_i' x_i \frac{dX_i(n_i)}{dn_i} + n_i (p_i - c_i) \frac{dx_i(n_i)}{dn_i}
\]

\[
= (\pi_i - f) - n_i \pi_i (1 - \theta_i) - \pi_i \theta_i. \tag{9}
\]

The first term of equation (9) represents the direct effect of the relocation, which expresses a relocating firm’s profit. The second term is the price effect of the relocation, which illustrates effect of the relocation on the market price. And the third term shows the strategic effect of the relocation, which corresponds to changes in the incumbents’ profits through strategic interactions among firms. From equation (9), we obtain

\[
\frac{dPS}{dn} = \{(\pi_L - f) - (\pi_S - f)\}
\]

\[
- \{n_L \pi_L (1 - \theta_L) - n_S \pi_S (1 - \theta_S)\} - (\pi_L \theta_L - \pi_S \theta_S). \tag{10}
\]

Evaluating equation (10) at the equilibrium location, the first term vanishes from the definition of the equilibrium location. We then have the following result.

**Proposition 2**

*From producers’ viewpoint, firms excessively locate in region L.*

Proposition 2 means that a slight relocation of firms from region S to region L at the equilibrium location reduces joint profits. Therefore, the equilibrium location is too concentrated for producers in region L.

We next examine the consumer’s viewpoint. Let \( CS_i \) and \( CS \) be consumer surplus in market \( i \) (\( i = L, S \)) and total consumer surplus, respectively. \( CS_i \) is given by

\[
CS_i(n_i) = \int_0^{X_i(n_i)} p_i(z)dz - p_i(X_i(n_i))X_i(n_i), \tag{11}
\]
and the total consumer surplus is organized as $CS = CS_L + CS_S$. Partially differentiating equation (11) with respect to $n_i$, we have

$$\frac{\partial CS_i}{\partial n_i} = -p_i'(X_i)X_i \frac{dX_i}{dn_i} = n_i \pi_i (1 - \theta_i) > 0. \tag{12}$$

This is the reverse of the price effect appeared in equation (9), because consumers’ expenditure exactly equals to the total revenue of firms in market $i$. Considering equation (12) in each market yields

$$\frac{dCS}{dn} = n_L \pi_L (1 - \theta_L) - n_S \pi_S (1 - \theta_S). \tag{13}$$

Evaluating equation (13) at the equilibrium location, we find the following result.

**Proposition 3**

*From consumers’ viewpoint, firms insufficiently locate in region $L$.*

Proposition 3 states that a marginal relocation of firms from region $S$ to region $L$ raises consumer surplus.

Finally, we investigate the viewpoint of the whole economy. Let $W$ be the social welfare where $W = PS + CS$. From equations (10) and (13), we have

$$\frac{dW}{dn} = \{(\pi_L - f) - (\pi_S - f)\} - (\pi_L \theta_L - \pi_S \theta_S). \tag{14}$$

Let $\pi^e$ be the equilibrium gross profit. Notice that $\pi_L(n_L^e) = \pi_S(n_S^e) = \pi^e$ holds at the equilibrium location. Evaluating equation (14) at the equilibrium yields

$$\frac{dW}{dn} \bigg|_{(n_L^e, n_S^e)} = -\pi^e(\theta_L - \theta_S) < 0. \tag{15}$$

The sign condition of equation (15) is derived from Lemma 1 (iii).
Proposition 4

From the viewpoint of the economy as a whole, firms excessively locate in region $L$.

Proposition 4 implies that a marginal firm relocation to region $L$ reduces economic welfare.

Intuitions behind the above results are explained as follows. When each firm makes its own location decision, it considers only the resulting net profit in each market, i.e., the direct effect. On the other hand, no firms take the price effect and strategic effect in equation (9) into accounts. Since the latter two effects are larger in market $L$ than in market $S$ at the equilibrium location, the relocation reduces producer surplus. We have seen that the relocation changes consumer surplus exactly by the negative of the price effect (equation(12)). The relocation, thus, raises consumer surplus. At the equilibrium location, a change in social welfare only consists of the strategic effect, because profits are equalized across markets and the price effect disappears. Since $n_L^e > n_S^e$ holds, the strategic effect in market $L$ dominates that in market $S$ (equation (15)). Economic welfare is, therefore, reduced by the relocation.

5 Effect of Market Size

In this section, we consider the second question; i.e., how the difference in market size affects producers, consumers, and economy as a whole. Similar to the analysis in Section 4, we consider a marginal change in population from region $S$ to region $L$, i.e., an expansion of difference in market size between markets, assuming the total population $M$ is fixed; i.e. $m_L + m_S = M$. Then, the marginal change in population is represented by a new parameter $m$ such that $\frac{dm_L}{dm} = 1$ and $\frac{dm_S}{dm} = -1$.

Notice that the change in population affects not only on equilibrium outputs, profits and total outputs, but also on the equilibrium numbers of firms locating in the both
regions. So, at first, let us clarify the effect of the expansion of difference in market size on the firm location. Partially differentiating the equilibrium location (equation (6)) with respect to $m$ derives the following relationship.

**Lemma 2**

The expansion of the difference in market size increases $n^e_L$ and decreases $n^e_S$.

We now consider the effect of the expansion of the difference in market size on producers. Differentiating equations (7) with respect to $m$ yields the following result.

**Proposition 5**

Define the threshold number of firms as $\bar{N} \equiv 2 + 2\sqrt{2}$.

(i) If $N < \bar{N}$, the expansion of difference in market size unambiguously increases the equilibrium firm’s output in market $L$. If $N \geq \bar{N}$, the expansion of difference in market size may or may not increase the equilibrium firm’s output in market $L$, depending on difference in market size. That is,

$$\frac{\partial q^e_L}{\partial m} > 0 \quad \text{if} \quad \tilde{\gamma} \leq \gamma \leq 1,$$

and

$$\frac{\partial q^e_L}{\partial m} < 0 \quad \text{if} \quad \gamma \leq \gamma < \tilde{\gamma},$$

where $\tilde{\gamma} \equiv \sqrt{2} - 1$ is the threshold difference in market size such that $\frac{\partial q^e_L}{\partial m} = 0$. In contrast, it unambiguously decreases the equilibrium firm’s output in market $S$.

(ii) The expansion of difference in market size unambiguously decreases the equilibrium profits in the both markets.

Proposition 5 (i) tells that the total number of firms and the original difference in market size are critical whether the change in population increases firms’ output. The reason is explained as follows. Differentiating equation (5), the effect on $q^e_L$ is decom-
posed by

\[ \frac{\partial q_L}{\partial m} = \frac{1}{n_L + 1} - \frac{m_L}{(n_L + 1)^2} \frac{\partial n_L^e}{\partial m}. \quad (16) \]

We call the first term of equation (16) as \textit{market-size effect} on output; \textit{i.e.}, an increase in output because of the change in population, which is positive, and the second term as \textit{competitive effect} on output, \textit{i.e.}, an increase in firms by the market expansion, which is negative. Define \( f(\gamma) = \frac{1}{n_L^e} \) and \( g(\gamma) = \frac{m_L}{(n_L^e + 1)^2} \frac{\partial n_L^e}{\partial m} \), which express the absolute values of the market-size effect and the competitive effect, respectively. For a relatively small \( N \), the positive market-size effect always dominates the competitive effect, because the absorbing demand per firm is enough large (see Figure 1 (i)). However, for a relatively large \( N \), since the absorbing demand per firm is small, the positive market-size effect is dominated by the negative competitive effect if the difference in market size is not so large (see Figure 1 (ii)). On the same line of reason, the effect on \( q_S \) consists of

\[ \frac{\partial q_S}{\partial m} = -\frac{1}{n_S^e + 1} - \frac{m_L}{(n_S^e + 1)^2} \frac{\partial n_S^e}{\partial m}. \quad (17) \]

The first term of equation (17) expresses the market-size effect on output; \textit{i.e.}, a decrease in output because of the change in population, which is negative. The second term illustrates the competitive effect on output; \textit{i.e.}, decreasing firms locating in the region, which is positive. In market \( S \), irrespective of the total number of firms, the negative market-size effect always dominates the competitive effect, because the number of firms are sufficiently small and the losing demand per firm is pretty large. Firms’ output in market \( S \) is thus decreased by the expansion of difference in market size.

In contrast, Proposition 5 (ii) says that firms’ profits in the both regions decrease irrespective of the total number of firms and the original difference in market size.
From equation (5), we obtain the effect of the expansion of difference in market size on the firm profits in market \( L \) and \( S \), respectively, as

\[
\frac{\partial \pi^e_L}{\partial m} = \frac{1}{(n^e_L + 1)^2} - \frac{2m_L}{(n^e_L + 1)^3} \frac{\partial n^e_L}{\partial m}, \quad \text{and} \quad (18)
\]

\[
\frac{\partial \pi^e_S}{\partial m} = -\frac{1}{(n^e_S + 1)^2} - \frac{2m_S}{(n^e_S + 1)^3} \frac{\partial n^e_S}{\partial m}. \quad (19)
\]

The first term in equation (18) (resp. equation (19)) expresses the market-size effect on profit; i.e., an increase (resp. a decrease) in profit because of the change in population, while the second term illustrates the competitive effect on profit; i.e., the effect because of an increase (resp. a decrease) in the number of locating firms, which is negative (resp. positive). Since more firms locates in the market \( L \), the market-size effect on profit is relatively small in market \( L \). Thus, the positive market-size effect is dominated by the negative competitive effect in market \( L \), and vice versa in market \( S \).

Next, we take a look at the effect on consumers. Considering equations (7) and (10), we have the following result.

**Proposition 6**

The expansion of difference in market size increases (resp. decreases) the equilibrium total output and consumer surplus in market \( L \) (resp. market \( S \)), and increases the total consumer surplus.

Without the change in population, there are more population in region \( L \) than in region \( S \). Because of the change in population, more consumers goes into region \( L \), which attracts more firms, and, as a result, increases the total output in market \( L \). Note that the total output increases even if the output per firm decreases (see Proposition 5 (i)). In contrast, the total output in market \( S \) is decreased by the change in population at the same time. The increase in market \( L \) outweighs the decrease in market \( S \) because region \( L \) has more population. Therefore, the total consumer surplus is increased by
the expansion of difference in market size.

Now let us consider the effect of expansion of the difference in market size on the social welfare. Proposition 5 states that the expansion of the difference in market size decreases the total producer surplus, while Proposition 6 tells that it increases the total consumer surplus. The overall effect is thus clarified by comparing the effects on producer surplus and consumer surplus.

Proposition 7

The expansion of difference in market size increases the social welfare.

Proposition 7 shows that the effect of the expansion of the difference in market size on consumer surplus dominates that on producer surplus, irrespective of difference in market size and the total number of firm.

Note that for any change in population, the results in Section 4 are valid; i.e., the firm location is either excessive or insufficient as long as \( m_L > m_S \) is satisfied.

6 Concluding Remarks

In non-tradable service industries, regions possible for firms to locate are limited in the most cases. Since the markets in the regions normally have difference in sizes, each firm determines its own location considering difference in market size and its rivals’ location choices.

In this paper, we construct the simple model where firms locate in either of two regions with difference in market size to answer two questions; the first is how firm location is inefficient, i.e. insufficient or excessive, from the welfare point of view, and the second is how an expansion of difference in market size between regions affects producers, consumers, and the economy as a whole.

For the first question, we show that firm location is insufficient for consumers,
while excessive for producers and the overall economy. This result implies that firms tend to concentrate in the larger market. For the second question, we find that the expansion of difference in market size does not necessarily increases output of each firm in the larger region, and unambiguously decreases firms’ profit in the both regions. In contrast, the expansion of difference in market size is always favorable for consumer and the overall economy. This result explains that proceeding of firms’ concentration is not beneficial for producers, while beneficial for consumers and the overall economy, because competition becomes severer.

Our analysis is so simple that it is easily applied in some realistic situations. Especially, it is good for explaining foreign direct investment in the non-tradable service industries.

In our analysis, we do not consider labor employment explicitly. In addition, we do not examine the relationship between the inefficiencies of firm location and the change in population. Including these topics may bring several interesting results. We have left these topics for our future research.

Appendix

A. Proof of Lemma 1

From equation (5), we have

$$\theta(n_i) = \frac{n_i}{q_i} \frac{\partial q_i}{\partial n_i} = - \frac{n_i}{m_i} \left( - \frac{m_i}{(n_i + 1)^2} \right) = \frac{n_i}{n_i + 1}. \quad (A1)$$
From (A1), the statements (i) and (ii) are straightforward. Differentiating (A1) with respect to \( n_i \),

\[
\frac{d\theta(n_i)}{dn_i} = \frac{1}{(n_i + 1)^2} > 0 \quad \text{and} \quad \frac{d^2\theta(n_i)}{dn_i^2} = -\frac{2}{(n_i + 1)^3} < 0.
\] (A2)

Equations (A2) means that \( \theta(n_i) \) is increasing and strictly concave in \( n_i \). The statement (iii) is thus proved.

\[ \square \]

B. Proof of Proposition 1

From equations (6) and (7), we see that \( n^e_L > n^e_S \) and \( q^e_L > q^e_S \). Since \( X_i = n_i q_i \), we find that \( X^e_L = n^e_L q^e_L > X^e_S = n^e_S q^e_S \). Substituting this relationship into equation (2), therefore, we obtain \( p_L = a - \frac{1}{m_L} X_L < p_S = a - \frac{1}{m_S} X_S \). \[ \square \]

C. Proof of Proposition 2

Let \( \pi^e \) be the equilibrium gross profit. Noting that \( \pi_L(n^e_L) = \pi_H(n^e_H) = \pi^e \) holds at the equilibrium, equation (10) is

\[
\frac{\partial PS}{\partial n} \bigg|_{(n^e_L, n^e_S)} = -\pi^e \left\{ n^e_L (1 - \theta_L) - n^e_S (1 - \theta_S) \right\} - \pi^e (\theta_L - \theta_S)
\]

\[
= -\pi^e \left\{ \frac{n^e_L}{n^e_L + 1} - \frac{n^e_S}{n^e_S + 1} \right\} - \pi^e (\theta_L - \theta_S),
\]

using (4) in the second equation. Since \( n^e_L > n^e_S \) holds at the equilibrium and Lemma 1(iii), the sign of RHS of the above equation is negative. \[ \square \]
D. Proof of Proposition 3

Using (4) and (7), at the equilibrium location, we have

\[
\frac{\partial C_S}{\partial n} \bigg|_{(n_L^e, n_S^e)} = \pi^e \left\{ \frac{n_L^e}{n_L^e + 1} - \frac{n_S^e}{n_S^e + 1} \right\} > 0,
\]

because \( n_L^e > n_S^e \) holds at the equilibrium. ■

E. Proofs of Proposition 5

Differentiating equations (7) with respect to \( m \), we have,

\[
\frac{\partial q_L}{\partial m} = \frac{(\gamma + 1)^2 - 2}{2\gamma(N + 2)}, \quad \frac{\partial q_S}{\partial m} = \frac{(\gamma - 1)^2 - 2}{2\gamma(N + 2)} \quad \text{and} \quad \frac{\partial \pi_L}{\partial m} = \frac{\partial \pi_S}{\partial m} = -\frac{(1 + \gamma)(1 - \gamma)}{2\gamma(N + 2)^2}. \tag{A3} \tag{A4}
\]

From equation (A3), the sign of \( \frac{\partial q_L}{\partial m} \) depends on its denominator. Define \( \tilde{\gamma} = \sqrt{2} - 1 \) such that \((\gamma + 1)^2 - 2 = 0\). Supposing that \( N < \bar{N}, \tilde{\gamma} < \gamma \) holds. Then, the denominator is always positive, and thus \( \frac{\partial q_L}{\partial m} \geq 0 \). But, supposing that \( N \geq \bar{N} \), if \( \tilde{\gamma} \leq \gamma < 1 \) (resp. \( \gamma \leq \tilde{\gamma} < \gamma \)), the denominator is positive (resp. negative), and thus \( \frac{\partial q_L}{\partial m} \geq 0 \) (resp. \( \frac{\partial q_L}{\partial m} < 0 \)).

In contrast, speaking of effects on the equilibrium output in market \( S \) and equilibrium profits in the both firms, \( \frac{\partial q_S}{\partial m} < 0 \) and \( \frac{\partial \pi_i}{\partial m} < 0 \) \((i = L, S)\), from equations (A3) and (A4) and \( 0 < \gamma < 1 \). ■
F. Proof of Proposition 6

Differentiating equations (7) with respect to $m$ yields

$$\frac{\partial X^e_L}{\partial m} = \frac{1 + 2\gamma(N + 1) - \gamma^2}{2\gamma(N + 2)} \quad \text{and} \quad \frac{\partial X^e_S}{\partial m} = \frac{1 - 2\gamma(N + 1) - \gamma^2}{2\gamma(N + 2)}. \quad (A5)$$

From equations (2) and (10), $CS_i$ is calculated by $CS_i = \frac{X^2_i}{2m_i}$. Using (A5), we have

$$\frac{\partial CS_L}{\partial m}_{(n^e_L,n^e_S)} = \frac{X^e_L m_L}{2} \left( \frac{\partial X^e_L}{\partial m} - \frac{X^e_L}{m_L} \right) = \frac{(N + 1 + \gamma)(1 + \gamma(N + 1))}{2\gamma(N + 2)^2} > 0, \quad \text{and} \quad (A6)$$

$$\frac{\partial CS_S}{\partial m}_{(n^e_L,n^e_S)} = \frac{X^e_S m_S}{2} \left( \frac{\partial X^e_L}{\partial m} + \frac{X^e_S}{m_S} \right) = \frac{(N + 1 + \gamma)(\gamma(N + 1) - 1)}{2\gamma(N + 2)^2} < 0. \quad (A7)$$

Thus, the migration from region $S$ to $L$ increases (resp. decreases) the equilibrium total output and consumer surplus in market $L$ (resp. market $S$).

Finally, summing equations (A6) and (A7) yields that

$$\frac{\partial CS}{\partial m}_{(n^e_L,n^e_S)} = \frac{\partial CS_L}{\partial m}_{(n^e_L,n^e_S)} + \frac{\partial CS_S}{\partial m}_{(n^e_L,n^e_S)} = \frac{(N + 1)(1 + \gamma)(1 - \gamma)}{\gamma(N + 2)^2} > 0. \quad (A8)$$

Therefore, the migration increases the total consumer surplus. \qed

G. Proof of Proposition 7

As we see in Section 4, $PS = PS_L + PS_S = \sum_{i=L,S} n_i (\pi_i - f)$. Thus, differentiating $PS$ with respect to $m$ yields

$$\frac{dPS}{dm} = \frac{\partial n_L}{\partial m} (\pi_L - f) + n_L \frac{\partial \pi_L}{\partial m} + \frac{\partial n_S}{\partial m} (\pi_S - f) + n_S \frac{\partial \pi_S}{\partial m}. \quad (A9)$$

The first and third terms express the effect of change in the number of locating firms, and the second and forth terms show the effect of change in firms’ profit. However,
at the equilibrium location, the first and third term vanishes. Therefore, evaluating equation (A9) at the equilibrium location, we have

\[
\left. \frac{dPS}{dm} \right|_{(n_L, n_S)} = n_L \frac{\partial \pi^*_L}{\partial m} + n_S \frac{\partial \pi^*_S}{\partial m} = -\frac{N(1 + \gamma)(1 - \gamma)}{\gamma(N + 2)^2} < 0. \tag{A10}
\]

From equations (A10) and (A8),

\[
\left. \frac{\partial W}{\partial m} \right|_{(n_L, n_S)} = \left. \frac{\partial PS}{\partial m} \right|_{(n_L, n_S)} + \left. \frac{\partial CS}{\partial m} \right|_{(n_L, n_S)} = \frac{(1 + \gamma)(1 - \gamma)}{\gamma(N + 2)^2} > 0.
\]

Therefore, the migration from region S to L increases the social welfare. ■
References


Figure 1 (i): Difference in market sizes and the effect of migration on output for $N < \bar{N}$

Figure 1 (ii): Difference in market sizes and the effect of migration on output for $\bar{N} \leq N$