Growth and the Location of Innovation and Industry in a Model of Occupational Choice

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Abstract

This paper investigates the relationship between growth and spatial patterns of research and manufacturing activity in a two-region model of occupational choice that features incomplete knowledge spillovers and trade costs. The allocation of heterogeneous workers into high-skilled and low-skilled employment shapes patterns of comparative advantage, and in a world with free movement of investment capital, innovation and manufacturing occur in the locations that provide the lowest costs. Although manufacturing activity always occurs in both regions, innovation activity may concentrate in the region with the larger share of industry, or disperse across regions. Focusing on long-run equilibria with dispersed innovation, we find that although the aggregate growth rate is unaffected by shifts in production and research activity, the relatively wealthy region always has larger shares of the manufacturing and innovation industries. As such, spatial patterns of economic activity have important implications for the welfare of each region. Finally, we show that, in some cases, net offshoring flows from the wealth abundant region to the wealth scarce region in both production and innovation.

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1 Introduction

In an increasingly integrated world economy, geographical patterns of industry play a fundamental role in the determination of patterns of trade, investment and regional innovation activity. In particular, access to the technical knowledge of manufacturing firms, and a high-skilled labour force are key conditions for regions hoping to attract innovation firms that offer the high wages associated with employment in research and development. This paper develops a two-region model of endogenous growth in order to investigate patterns of trade, offshoring, production, and innovation activity.

In recent years a large body of literature has developed that examines the relationship between spatial patterns of industry and regional levels of innovation, and a general consensus has developed that suggests a positive relationship between industry concentration and the rate of economic growth. A key feature of this literature, however, is a tendency towards the catastrophic agglomeration of innovation activity in one region when spillovers of technical knowledge are imperfect across regions. The theoretical concentration of innovation is problematic for several reasons. First, casual observation of patterns of R&D activity make it clear that innovation is, at least to some extent, internationally dispersed with no sign of convergence towards concentration in a few particular regions. In fact, trends in the domestic R&D expenditures reported by the OECD Science, Technology, and Industry Scoreboard 2007 suggests a pattern towards greater dispersion of innovation activity. Second, many geography models suggest that despite the loss of manufacturing and innovation activity to larger regions, smaller regions will still benefit from the increased innovation activity of those larger regions. This may not be the case, however, when smaller regions continue to innovate. Third, models that feature the concentration of innovation activity are not capable of explaining recent trends of offshoring in innovation from developed to developing regions.

In this paper we develop a two-region model of endogenous growth and occupational choice, and explore the relationship between regional growth and spatial patterns of manu-
facturing and innovation activity. The allocation of heterogeneous workers into high-skilled and low-skilled employment shapes patterns of comparative advantage, and in a world with free movement of investment capital, innovation and manufacturing occur in the locations that provide the lowest costs. In particular, our model is capable of producing long-run equilibria for which innovation activity occurs in both regions, even in the presence of imperfect cross-regional knowledge spillovers.

The remainder of the paper proceeds as follows. In Section 2 we develop the basic structure for our model of occupational choice and endogenous growth. Section 3 provides a characterization of long-run equilibria for which innovation activity is regionally dispersed. Concluding remarks are provided in Section 5.

2 The Model

This section develops a basic two-region model of occupational choice and endogenous growth. There are three types of economic activity: a traditional sector produces a homogeneous good, a manufacturing sector produces horizontally differentiated varieties, and an innovation sector generates new product designs for firms entering the manufacturing sector. Labor is the only factor of production, but workers are differentiated with respect to skill level and are free to choose employment in the activity that provides the highest wage income: production or innovation. The two regions are referred to as the East and West, and are symmetric in all respects with the exception of their initial levels of non-labour wealth, $K$ and $K^*$, where an asterisk denotes variables associated with the West. Although there is no labor migration between regions, trade may occur in traditional goods, manufacturing products, and product designs. As the setups for each region are symmetric, we focus on the eastern region while introducing the basic structure of the model.
2.1 Households

The demand side of the economy is made up of dynastic households that solve a multistage utility maximization problem. The aggregated intertemporal utility of households residing in the East is

$$U = \int_0^\infty e^{-\rho t} \ln \left[ C_X(t)^\alpha C_Y(t)^{1-\alpha} \right] dt,$$

where $C_X(t)$ and $C_Y(t)$ are consumptions of a CES composite of manufactured varieties and the traditional good, $\alpha$ is the share of household expenditure on manufacturing goods, and $\rho$ is the subjective discount rate. Following Dixit and Stiglitz (1977), the specific form for the manufacturing composite is

$$C_X = \left[ \int_0^n c(i)^\beta di + \int_0^{n^*} c^*(j)^\beta dj \right]^\frac{1}{\beta}, \quad (1)$$

where $c(i)$ and $c^*(j)$ are the respective demands for varieties $i$ and $j$ of the $n$ and $n^*$ masses of product varieties produced in the East and in the West. In addition, the degree of product differentiation, $\beta = 1 - 1/\sigma$, is determined by the elasticity of substitution between any two varieties, $\sigma > 1$.

Households maximize lifetime utility by choosing an expenditure-saving path that follows the Ramsey saving rule described by the following Euler equation:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho, \quad (2)$$

where $r(t)$ is the common world interest rate, and a dot over a variable denotes differentiation with respect to time. We assume that there is a perfect international market for investment funds, and thus the interest rate and the evolution of expenditure are the same in both regions: $r = r^*$ and $\dot{E}/E = \dot{E}^*/E^*$. For the remainder of the paper we suppress time notation when doing so does not create ambiguity.
With per-period expenditure determined by (2), the optimal allocation of eastern household expenditure across traditional and manufacturing goods is

\[ P_X C_X = \alpha E, \quad P_Y C_Y = (1 - \alpha)E, \]  

(3)

where \( P_X \) and \( P_Y \) are, respectively, a price index for manufacturing varieties, and the price of traditional goods. In particular, the price index is given by

\[ P_X = \left[ \int_0^n p(i)^{1-\sigma} \, di + \int_0^{n^*} \tau^{1-\sigma} p^*(j)^{1-\sigma} \, dj \right]^{1/\sigma}, \]  

(4)

where \( p(i) \) is the price of an eastern produced variety \( i \), and \( p^*(j) \) is the price of a western produced variety \( j \). Households residing in the East incur an iceberg trade cost \( \tau > 1 \) when purchasing goods that have been produced in the West.

Regarding the price index as the unit expenditure function for households’ purchases of manufactured varieties, Shephard’s Lemma and aggregation across eastern households yields the following demands for varieties produced locally \( c(i) \) and varieties produced in the West \( c^*(j) \):

\[ c(i) = p(i)^{-\sigma} P_X^{\sigma-1} \alpha E, \quad c^*(j) = \tau^{-\sigma} p^*(j)^{-\sigma} P_X^{\sigma-1} \alpha E. \]  

(5)

Western households have similar demands over manufacturing varieties.

2.2 Occupational choice

The populations of the eastern and western regions have masses of \( M \), and as discussed above, workers are heterogenous with respect to skill levels. Individual skill levels are determined in each region according to a continuous uniform skill distribution functions with support \([0, 1]\), that is, \( F(s) = s \) and \( F^*(s^*) = s^* \), where \( s \) and \( s^* \) represent the skill levels of individuals residing in the East and West, respectively.
We adopt the occupational choice framework introduced in Roy (1951) and adapted by Saint-Paul (2004), whereby workers are free to choose between employment in production or innovation. Production workers are classified as low-skilled and innovation workers are classified as high-skilled. In each period although all workers are capable of providing one unit of effective low-skilled labour per period, a worker with skill level $s$ can only provide $s$ units of effective high-skilled labour. Workers are free to move between sectors with no cost of switching employment.

The labor markets are competitive, and adjustments in the allocation of labor between production and innovation ensure that all firms pay the same effective low-skilled wage ($w_L$) and effective high-skilled wage ($w_H$). As workers are free to change employment at any time, each worker will choose the activity that provides the highest wage income. For example, a worker with skill level $s$ will choose between $w_L$ and $sw_H$, the incomes that can be earned in production and innovation, respectively.

If both production and innovation are active in the region concerned, there will exist a marginal worker who earns equal incomes in production and innovation and is indifferent between employment type. We denote the skill level of this marginal worker by $z$. This threshold skill level satisfies $w_L = zw_H$, and provides a convenient means of capturing the relative effective wage between low-skilled and high-skilled workers, that is $z = w_L/w_H$.

We can also use the threshold skill level $z$ to describe the allocation of labor between production and innovation as all workers with skill levels less than $z$ will choose employment in production and all workers with skill levels greater than $z$ will choose employment in innovation. It then follows that the eastern region’s effective supplies of low-skilled and high-skilled labor are

$$L(z) = M \int_0^z dF(s) = zM, \quad H(z) = M \int_z^1 sdF(s) = \frac{(1 - z^2)M}{2}. \quad (6)$$

Taking the partial derivatives of these effective labour supplies with respect to $z$ we can
show that the threshold skill-level also captures the opportunity cost associated with a change in the allocation of labour, that is $z = -H'(z)/L'(z)$. The western region has a corresponding threshold skill-level ($z^*$), relative effective wage ($w^*_L/w^*_H$), and effective supplies low-skilled and high-skilled labor ($L^*$ and $H^*$).

### 2.3 The traditional sector

Firms in the traditional sector produce a homogenous good for supply to a perfectly competitive world market that is characterized by free trade. Production employs a constant returns to scale technology whereby one unit of effective low-skilled labor produces one unit of output. Under these assumptions the price of traditional goods is same in the East and West, and equals the common effective low-skilled wage. We set the traditional good as the model numeraire, and consequently $w_L = w^*_L = 1$.

Combining the eastern and western demands for traditional goods (3), the world supply of effective low-skilled labor to the production of traditional goods is

$$L_Y + L_Y^* = (1 - \alpha)(E + E^*), \quad (7)$$

where we have set the the common world price for traditional goods to unity.

### 2.4 The manufacturing sector

The manufacturing sector features incumbent firms that produce horizontally differentiated varieties and compete according to monopolistic competition (Dixit and Stiglitz, 1977). These firms are free to locate in either region with negligible costs of relocation and profit repatriation. Accordingly, all firms base production in the region that supports the highest level of operating profit.

Each firm employs a constant returns to scale production technology that is symmetric across regions with one unit of effective low-skilled labor required for each unit of output
produced. Then, given the instantaneous demands (5), profit maximization leads to the well known constant markup over marginal cost pricing rule \( p = 1/\beta \), and it follows that the optimal operating profit of a firm with its production based in the eastern region, for instance, is

\[
\pi = px - x = \frac{x}{\sigma - 1},
\]  

(8)

where \( x \) is firm level output and describes the scale of production.

Setting supply equal to the sum of demands (5) yields the respective equilibrium output levels for eastern and western based firms as

\[
x = \alpha\beta \left[ \frac{E}{n + \varphi n^*} + \frac{\varphi E^*}{\varphi n + n^*} \right], \quad x^* = \alpha\beta \left[ \frac{\varphi E}{n + \varphi n^*} + \frac{E^*}{\varphi n + n^*} \right],
\]  

(9)

where \( \varphi \equiv \tau^{1-\sigma} \) describes the freeness of trade with a value of \( \varphi = 0 \) indicating prohibitively large trade costs and a value of \( \varphi = 1 \) indicating free trade.

As firms choose the location that offers the greatest operating profit, the spatial pattern of manufacturing activity across regions continues to adjust until operating profits are the same for firms located in either region, \( \pi = \pi^* \). Consequently, referring back to (8), we can see that the firm level scale of production is also equalized across regions, \( x = x^* \). Taking eastern and western expenditures as constant for the moment, substitution of (9) into this condition yields a condition for the eastern share of manufacturing activity that ensures firm scale and operating profit are the same in both regions:

\[
\gamma \equiv \frac{n}{N} = \frac{E - \varphi E^*}{(1 - \varphi)(E + E^*)},
\]  

(10)

where \( N = n + n^* \) is the total number of incumbent firms. In particular, this condition shows that the pattern of manufacturing activity depends on relative market size and the freeness of trade. An increase in the size of the eastern market (\( E \)), or a decrease in size of
the western market \((E^*)\), will increase the eastern share of manufacturing industry. On the other hand, an increase in the freeness of trade \((\phi)\) increases the share of industry locating in the larger market.

Using (10) with (9) and (8), the optimal operating profit of a firm located in either region is now obtained as

\[
\pi = \pi^* = A(E + E^*) \left(\frac{1}{1 + A}\right)N^* ,
\]

(11)

where \(A = \alpha/(\sigma - \alpha) > 0\) is used to consolidate demand parameters.

With a unit-coefficient production technology, the total demand for labour from manufacturing activity is determined by the total supply of manufacturing goods, \(nX + n^*x^*\):

\[
L_X + L^*_X = \alpha\beta (E + E^*) ,
\]

(12)

where we have used (9) and (10).

### 2.5 The innovation sector

New product designs are created by high-skilled researchers employed at competitive innovation firms. Each new product design enables the entry of a new firm into the manufacturing sector. In addition, technical knowledge is generated as a by-product of the product design process. This accumulation of knowledge generates the intertemporal externality introduced by Romer (1990) and Grossman and Helpman (1991) whereby the success of current innovation activity increases the productivity of future research. As a result of this externality, the innovation sector becomes the engine of economic growth.

Following Martin and Ottaviano (1999), we assume that the stock of technical knowledge is preserved within the production processes of incumbent manufacturing firms. This technical knowledge is studied by researchers and applied in product development resulting in what are referred to as Jacob-type knowledge spillovers. These spillovers are imperfect,
however, as access to technical knowledge requires a sufficient proximity to manufacturing activity. To simplify our analysis, we suppose that although spillovers are perfect within a region, they are imperfect across regions.

A new product design can be created in the East using $1/(n + \lambda n^*)$ units of effective high-skilled labour, where $n$ and $n^*$ respectively proxy for knowledge spillovers from eastern and western based manufacturing firms. The parameter $0 \leq \lambda \leq 1$ indicates the degree of knowledge spillovers from production based in a different region (Baldwin and Forslid, 2000).\footnote{In contrast to Martin and Ottaviano (1999), Baldwin and Forslid (2000) assume that knowledge is contained in the stock of product designs that have been introduced to date.} It then follows that new product creation occurs in the East and West according to

$$
\dot{m} = (n + \lambda n^*)H,
\dot{m}^* = (n^* + \lambda n)H^*.
$$

(13)

where $\dot{m}$ and $\dot{m}^*$ are the numbers of new products created in the East and West at each moment in time.

The creation of a new product enables access to a infinite stream of operating profits through entry into the manufacturing sector. Accordingly, the value of a product design, which we denote by $v$, equals the present value of the stream of operating profits. Free entry into the innovation sector ensures that when there is active innovation, the value of a product design will equal the cost of product development:

$$
v \leq \frac{w_H}{n + \lambda n^*}, \quad v^* \leq \frac{w_H^*}{n^* + \lambda n}.
$$

(14)

These free entry conditions highlight the mechanism that determines comparative advantage in innovation activity.

Free entry into the manufacturing sector reduces the rate of return to investment in a new product design to equal the risk-free interest rate, $r$. Taking the time derivative of (14) with respect to time, we derive the no-arbitrage condition introduced by Grossman and
Helpman (1991):

\[ r \leq \frac{\pi}{v} + \frac{\dot{v}}{v}, \]  

(15)

where the first term on the right hand side is the dividend rate, and the second term is the rate of capital gains.

With free trade in product designs, a new firm entering the manufacturing sector will have its product developed in the region with the lowest product development cost. Accordingly, when there is active research in both regions, product development costs equalize and \( z\theta = z^*\theta^* \), where we have used \( w_H = 1/z \) and \( w_H^* = 1/z^* \). This condition can be solved for the eastern share of manufacturing activity that equates product development costs in the East and the West:

\[ \delta \equiv n = \frac{z^* - \lambda z}{(1 - \lambda)(z + z^*)}, \]

(16)

where we have used the notation \( \delta \) to differentiate between (16) and (10). This condition indicates how the attractiveness of each region as a location for innovation activity is affected by changes in the labor allocations of each region and the degree of interregional knowledge spillovers. For example, an increase in eastern labor employed in innovation (a decrease in \( z \)) raises the share of industry required in order for the East to receive knowledge spillovers sufficient for equal product development costs across regions.

### 2.6 Long-run equilibrium

We examine long-run equilibria for which the allocation of labour is constant across production and innovation. Consequently, the skill thresholds and the share of manufacturing activity located in each region are constant in the steady state, that is \( \dot{z} = 0, \dot{z}^* = 0, \) and \( \dot{\gamma} = 0 \). Given the symmetric setup of the model, for specific levels of trade costs and knowledge spillovers, cross-region differences in labour allocations and the associated shares of
production and innovation activity are determined solely by shares of asset wealth. This section closes the model by deriving conditions for regional expenditures and the aggregate growth rate which clear the labour market of each region.

Beginning with the demand for low-skilled labour in production, aggregate expenditure can be obtained as a function of the skill thresholds by summing across (7) and (12) for both regions:

\[ E + E^* = (1 + A) [L + L^*]. \]  \hspace{1cm} (17)

This condition ties the supply of low-skilled labour to the product markets.

Next, employing the innovation functions (13), the long-run aggregate rate of innovation can be obtained as

\[ g \equiv \frac{\dot{N}}{N} = \theta H + \theta^* H^*, \]  \hspace{1cm} (18)

where \( \theta = \gamma + \lambda \gamma^* \) and \( \theta^* = \gamma^* + \lambda \gamma \) describe the contribution of knowledge spillovers from industry to innovation in the East and the West, respectively. The existence of a competitive international market for investment funds ensures that eastern and western households have equal access to investment opportunities, and asset wealth therefore grows at the same rate \((g)\) in each region. Furthermore, constant regional shares of industry require the number of firms locating in each region to grow at the same rate. Thus, in long-run equilibrium \( K, K^*, n, n^*, \) and \( N \) will all grow at the same pace regardless of the levels of production and innovation activity taking place in each region.

We can now derive conditions for regional expenditures. The expenditure of a specific region equals the sum of household wage and investment income minus the cost of investing in new asset wealth. For example, eastern expenditure is \( E = L + w_H H + K \pi - \dot{K} v. \)

Accordingly, making use of \( w_H = 1/z, w^* = 1/z^*, \dot{z} = 0, \) and \( \dot{z}^* = 0, \) with (11), (14),
(17), and (18), regional expenditures can be written as functions of the skill thresholds:

\[
E = L + kA(L + L^*) + \frac{(1 - k)H}{z} - \frac{kH^*}{z^*},
\]

\[
E^* = L^* + (1 - k)A(L + L^*) + \frac{kH^*}{z^*} - \frac{(1 - k)H}{z},
\]

where from (14) we have invoked \(z\theta = z^*\theta^*\) as active innovation in both regions requires equal product development costs, \(v = v^*\). These conditions show that an increase in a region’s share of asset wealth has two opposing effects on that region’s expenditure. The first is the positive effect of an increase in investment income (the second term), and the second is the negative effect of an increase in the region’s share of the aggregate cost of innovation (the fourth and fifth terms).

With conditions for regional expenditures in hand, we can close the model by deriving conditions for the equilibrium labour allocations of each region using the allocation curve technique.\(^2\) Substituting the time derivatives of (14), along with (10), (13), and (14), into the no-arbitrage conditions (15) yields

\[
\rho = z\theta A(L + L^*) - \theta H - \theta^* H^*,
\]

\[
\rho = z^*\theta^* A(L + L^*) - \theta H - \theta^* H^*,
\]

where \(\theta\) and \(\theta^*\) determine the weights of industry shares in knowledge spillovers to innovation for each region.

The allocation curves determine the long-run labour allocations the ensure equilibrium in the investment market. Although the model we have developed does not feature the catastrophic agglomeration of manufacturing industry, innovation activity may concentrate fully in one region. We refer to the case where all innovation occurs in one region as a concentrated equilibrium, and the case where innovation occurs in both regions as a dispersed equilibrium. In the latter case both allocation curves hold, while in the former case only

\(^2\)The allocation curve technique is developed in Ethier (1982).
the allocation curve of the innovating region binds. The next section provides a characterization of dispersed equilibria.

3 Dispersed equilibrium

This section examines the key features of dispersed equilibria. While the allocation curves (21) and (22) do determine the unique labour allocations associated with a dispersed steady state, their non-linear nature complicates the characterization of long-run equilibria. To resolve this issue we introduce two alternative equilibrium conditions.

The first condition is an investment locus that equates the return to product development across regions. As discussed above, a constant share of industry in each region requires a common growth rate for \(n\) and \(n^*\). As both the operating profits of manufacturing firms and product development costs must be equalized across regions in a dispersed equilibrium, we have \(\gamma = \delta\), and it follows that \(\theta = (1 + \lambda)z^*/(z + z^*)\) and \(\theta^* = (1 + \lambda)z/(z + z^*)\). Substituting these conditions with (6) into either of the allocation curves, (21) or (22), yields

\[
\rho = \frac{(1 + \lambda)[(1 + 2A)z^* - 1]}{2} M. \tag{23}
\]

We refer to this condition as the investment locus, and it traces out all combinations of the eastern and western skill thresholds that ensure a common rate of return to product development across regions. The investment locus has a strictly positive slope in \(z-z^*\) space as depicted by the curve \(aa\) shown in Figure 1.

With the equilibrium level of investment determined by (23), all we require is a condition to pin down the threshold skill combinations that equate (10) and (16). Thus, our second long-run equilibrium condition is simply \(\gamma = \delta\), which using (6) can be written as

\[
z [B + kC] = z^* [B + (1 - k)C], \tag{24}
\]
where \( B = (1 + \lambda + 2\rho) [1 - \varphi + (\lambda - \varphi)(1 + 2A)] \), and \( C = 2(1 + 2A)(1 - \lambda)(1 + \varphi)\rho \).

We refer to this condition as the share locus, and it extends linearly from the origin in \( z-z^* \) space, as illustrated by the line 0b in Figure 1. Moreover, for \( B > kC \) and \( B > (1 - k)C \), the slope of the share locus is strictly positive.

If the share locus has a positive slope, we can show that a unique long-run equilibrium with active innovation in both regions exists if the allocation of asset wealth lies within a specific range. In particular, an examination of (16) indicates that the equalization of product development costs across regions requires \( 1/\lambda > z^*/z > \lambda \). This constraint is illustrated by the dash lines in Figure 1. If the share locus lies between these border conditions, both regions will have active innovation sectors. Thus, comparing the slope of the share locus with the slopes of the boundary conditions, we can derive the range of asset wealth allocation for which a unique dispersed equilibrium exists.

**Lemma 1** (Dispersed innovation activity): The existence of a unique dispersed equilibrium requires (i) \( \underline{k} \geq k \geq \bar{k} \), where

\[
\underline{k} = \frac{\lambda C - (1 - \lambda)B}{(1 + \lambda)C}, \\
\bar{k} = \frac{C + (1 - \lambda)B}{(1 + \lambda)C},
\]
and (ii) \(1 - B/C \geq k \geq B/C\).

While the first condition ensures equal product development cost in both regions, the second condition is required for a stable equilibrium. If Lemma 1 is satisfied, the investment and share loci can be used to solve for the long-run labour allocation of each region:

\[
z = \left( \frac{1 + \lambda + 2\rho}{(1 + \lambda)(1 + 2A)M} \right)^{1/2} \left( \frac{B + (1 - k)C}{B + kC} \right)^{1/2},
\]

\[
z^* = \left( \frac{1 + \lambda + 2\rho}{(1 + \lambda)(1 + 2A)M} \right)^{1/2} \left( \frac{B + kC}{B + (1 - k)C} \right)^{1/2}.
\]

A casual inspection of these equilibrium threshold skill levels indicates a clear pattern between a region’s share of asset wealth and its long-run allocation of labour between production and innovation.

As all regional asymmetries stem from the gap in asset wealth, we formally summarize the relationship between \(k\) and the skill thresholds in the following proposition:

**Proposition 1 (Asset wealth and labor allocation):** An increase in the eastern share of wealth lowers the skill threshold in the East and raises the skill threshold in the West.

**Proof:** Using (25) and (26) we obtain

\[
\frac{dz}{dk} = -z^*(2B + C)C < 0, \quad \frac{dz^*}{dk} = \frac{z(2B + C)C}{2(B + (1 - k)C)^2} > 0.
\]

Thus, if we measure economic activity in terms of labour employment, when populations are symmetric with respect to size and the distribution of skill, the region with the greatest share of asset wealth has the largest share of innovation activity and the smallest share of overall production activity. It is also interesting to note, however, that although the relatively poor region has a smaller level of innovation activity, the average skill level of researchers is higher. Returning to Figure 1, the investment locus is unaffected by changes in the allocation of asset wealth, and an increase in \(k\) causes the share locus to rotate counterclockwise about the origin. For \(k > 1/2\), we have \(z > z^*, L < L^*, H > H^*\), and the
eastern region has the greatest share of innovation activity, as shown in Figure 1.

We are also interested in the spatial pattern of manufacturing activity. Examining the effects of a change in $k$ on (16) we obtain the following corollary:

**Corollary 1 (Asset wealth and industry share):** The relatively wealthy region has a larger concentration of manufacturing industry.

*Proof:* Noting that $\gamma = \delta$ in equilibrium, taking the total derivative of (16) with respect to $k$ yields

$$\frac{d\delta}{dk} = \frac{(1 + \lambda)}{(1 - \lambda)(z + z^*)^2} \left[ z \frac{dz^*}{dk} - z^* \frac{dz}{dk} \right] > 0,$$

where we have used Proposition 1.

The relatively wealthy region, therefore, has the largest shares of both innovation and manufacturing activity in a dispersed equilibrium, and becomes a net exporter of manufacturing goods and a net importer of traditional goods. Corollary 1 can also be used to rank regional expenditures. Referring back to (10) we can see that if the wealthy region has the larger share of industry it must also have a larger level of regional expenditure. Therefore, for $k > 1/2$, as the eastern region has a greater share of industry, eastern expenditure must be greater than western expenditure, $E > E^\ast$. Thus, the dispersed equilibrium features a standard home market effect whereby the greatest share of firms locate production in close proximity to the largest market when there are positive trade costs.

Turning next to the relationship between trade costs and labour allocations we have the following proposition:

**Proposition 2 (Trade costs and labor allocations):** A decrease in trade costs decreases the skill threshold of the relatively wealthy region and raises the skill threshold of the relatively poor region: $dz/d\varphi \leq 0$ and $dz^*/d\varphi \geq 0$ for $k \geq 1/2$, and $dz/d\varphi > 0$ and $dz^*/d\varphi < 0$ for $k < 1/2$. 

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Proof: Using (25) and (26) we obtain

\[
\frac{dz}{d\varphi} = \frac{z^*(1 - 2k) [BC'(\varphi) - CB'(\varphi)]}{2(B + kC)^2}, \quad \frac{dz^*}{d\varphi} = -\frac{z(1 - 2k) [BC'(\varphi) - CB'(\varphi)]}{2(B + (1 - k)C)^2},
\]

where \(B'(\varphi) = -(1 + \lambda + 2A) < 0\), and \(C''(\varphi) = 2(1 + 2A)(1 - \lambda)\rho > 0\).

These results can be used directly to obtain the following corollary for the impact of a change in trade costs on the spatial pattern of the manufacturing industry.

**Corollary 2** (Trade costs and industry shares): A decrease in trade costs raises the share of industry located in the wealthy region: \(d\gamma/d\varphi \geq 0\) for \(k \geq 1/2\), and \(d\gamma/d\varphi < 0\) for \(k < 1/2\).

Proof: Taking the derivative of (16) with respect to \(\varphi\) obtains

\[
\frac{d\delta}{d\varphi} = \frac{(1 + \lambda)}{(1 - \lambda)(z + z^*)^2} \left[ z \frac{dz^*}{d\varphi} - z^* \frac{dz}{d\varphi} \right],
\]

which can be signed using the results summarized in Proposition 2.

An improvement in the freeness of trade (\(\varphi\)) makes the relatively wealthy region more attractive as a location for production through the standard home market effect and hence increases the concentration of manufacturing industry in that region. As a result, technical knowledge spillovers from industry to innovation increase in the relatively wealthy region and decrease in the relatively poor region making product development cheaper in the wealthy region. The labour allocations of each region then adjust with high-skilled contracting in the wealthy region and expanding in the poor region. For the example provided in Figure 1, an increase in \(\varphi\) will rotate the share locus (24) to the left while raising \(z^*\) and lowering \(z\).

Finally, we discuss how changes in the level of interregional knowledge spillovers influence patterns of innovation and manufacturing activity.
Proposition 3 (Knowledge spillovers and labor allocations): While an increase in interregional knowledge spillovers decreases the skill threshold of the relatively poor region, the skill threshold of the relatively wealthy region may rise or fall.

Proof: Derivatives of (25) and (26) yield

\[ \frac{dz}{d\lambda} = \frac{z^*(1 - 2k)(BC'(\lambda) - CB'(\lambda))}{2(B + kC)^2} - \frac{\rho}{z^*} \frac{dz^*}{d\lambda} = \frac{z(1 - 2k)(CB'(\lambda) - BC'(\lambda))}{2(B + (1 - k)C)^2} - \frac{\rho}{z}. \]

where \( B'(\lambda) = 1 - \varphi \lambda + (\lambda - \varphi)(1 + 2A) + (1 + \lambda + 2\rho)(1 - \varphi + 2A) > 0 \), and \( C'(\lambda) = -2(1 + 2A)(1 + \varphi)\rho < 0 \).

Thus, an improvement in interregional knowledge spillovers always raises high-skilled employment in the relatively poor region. In particular, the increase in \( \lambda \) raises the productivity of innovation and allows for a lower average productivity of researchers thereby expanding high-skilled labour employment. This will also be the case for the wealthy region when the gap in asset wealth is not very large. In contrast, if the gap in asset wealth is large, the relatively wealthy region will also have a large share of manufacturing industry, and consequently the improvement in research productivity will lead to a contraction in high-skilled labour employment.

Corollary 3 (Knowledge spillovers and industry shares): The effect of an increase in knowledge spillovers an shares of manufacturing activity is ambiguous.

Proof: Taking the derivative of (16) with respect to \( \lambda \) gives

\[ \frac{d\delta}{d\lambda} = \frac{z^* - z}{(1 - \lambda)^2(z + z^*)} + \frac{(1 + \lambda)}{(1 - \lambda)(z + z^*)^2} \left[ z \frac{dz^*}{d\lambda} - z^* \frac{dz}{d\lambda} \right]. \]

From Propositions 1 and 2, the first term is positive and the second term is negative for \( k > 1/2 \).

Although we cannot derive concrete analytical results for the relationship between the level of interregional knowledge spillovers and labour allocations, numerical examples show that
an increase in $\lambda$ will raise the skill threshold of the relatively poor region and the lower that of the relatively wealthy region given our assumption of uniform skill distributions.

With an understanding of patterns of production and trade, we can now discuss the effect of a change in the distribution of asset wealth on the pace of innovation. Substituting $\theta = (1 + \lambda)z^*/(z + z^*)$ and $\theta^* = (1 + \lambda)z/(z + z^*)$ with (6), (25), and (26) into (18), the rate of innovation for the overall economy is

$$g = \frac{(1 + \lambda)M}{2} - \frac{1 + \lambda + 2\rho}{2(1 + 2A)}.$$ 

The aggregate rate of innovation is, therefore, not affected by changes in the pattern of manufacturing and innovation activity. As expected, the rate of innovation is positively related to population size and negatively related to the discount rate (Grossman and Helpman, 1991). In addition, an increase in the level of interregional knowledge spillovers has a positive effect on the innovation rate. While the aggregate innovation rate remains unchanged after shifts in manufacturing and research activity, however, regional innovation rates do not. In particular, referring back to (13) and applying the results of Proposition 1, the relatively wealthy region has both a larger share of manufacturing activity ($\gamma > 1/2$) and a higher level of employment in research ($z < z^*$). Thus, the relatively wealthy region will produce more product designs each period ($\dot{m} > \dot{m}^*$).

The patterns of production and research activity that we have derived above have interesting implications for the directions of net offshoring in the manufacturing and innovation sectors when combined with the distribution of asset wealth. Following Martin and Ottaviano (1999), we measure the extent of net offshoring in manufacturing using

$$\dot{n} - \dot{K} = (n - K)g = (\gamma - k)gN,$$

where we have used $\dot{n}/n = g$. A negative sign for this measure implies a net shift in manufacturing from the relatively wealthy region to the relatively poor region.
Proposition 4 (Offshoring in manufacturing): In the manufacturing sector net offshoring flows from the relatively wealthy region to the relatively poor region, for $\tilde{\varphi} < \varphi$, where

$$\tilde{\varphi} = \frac{(1 + \lambda)(1 + (1 + 2A)\lambda) + 2\rho}{(1 + \lambda)(1 + \lambda + 2A) + 2\rho[\lambda + (1 + \lambda)(1 + 2A)]},$$

(28)

Proof: See Appendix

This result is similar to the findings of Martin and Ottaviano (1999). Referring to Figure 1, where $k > 1/2$, the eastern region attracts the production of some western owned firms if trade costs are relatively high, as there is a lower level of competition. This competition effect must be balanced, however, against the home market effect that makes location in the western region attractive. In particular, for a sufficiently low level of trade costs the home market effect causes all western owned firms and some eastern owned firms to locate in the West, as eastern households can be supplied through exports at a relatively low cost.

As dispersed equilibria feature active research in both regions, we can also examine the pattern of offshoring in the innovation sector. Applying (6), (25), and (26), net offshoring in innovation can be measured using

$$\dot{m} - \dot{K} = \frac{(1 + \lambda)(1 - 2k)(B + C)z^*(1 - zz^{**})N}{B + (1-k)C}.$$  

(29)

A negative sign for this condition implies that eastern households invest in a greater number of product designs than the eastern innovation sector produces, and that some innovation activity is therefore carried out offshore. This will be the case when $k > 1/2$, and we therefore have the following proposition:

Proposition 5 (Offshoring in innovation): In the innovation sector net offshoring flows from the relatively wealthy region to the relatively poor region.

While an increase in the share of asset wealth maintained by the relatively wealthy region raises its shares of manufacturing and innovation activity, the corresponding expansion in
high-skilled employment lowers the average productivity of researchers. As such, equal-
ization of product development cost requires that some western firms offshore their product
development to the innovation sector of the relatively poor region. This specific pattern of
offshoring in innovation is not obtainable in models where research activity is fully con-
centrated in the relatively wealthy region.

We conclude this section with a brief discussion of the welfare aspects of dispersed
equilibria. Combining the price and expenditure conditions, the respective indirect utilities
associated with eastern and western households are

$$V = \frac{1}{\rho} \ln D E^\theta^{s-1}, \quad V^* = \frac{1}{\rho} \ln D E^*\theta^*^{s-1},$$ (30)

where we have set the initial number of product varieties to unity, and $D = (\alpha \beta)^\alpha (1 -
\alpha)^{1-\alpha} e^{(\alpha \beta)\rho}$ is constant given that the aggregate rate of innovation is unaffected by changes
in patterns of production and innovation. Figure 2 provides an illustration of these indirect
utilities.

![Figure 2: Regional welfare comparison](image)

Figure 2 shows that an increase in a region’s share of asset wealth always leads to a
welfare improvement. There are two reasons. First, an increase in $k$ leads directly to an
increase in investment income. Second, the wealthy region has a higher share of high-
skilled employment. High-skilled workers not only earn a higher wage, their wages also increase with an expansion of high-skilled employment: \( w_H = 1/z > 1 \). Thus, parameter changes that increase the level of innovation activity in a region will lead to a welfare improvement.

4 Conclusion

In this paper we re-examine the relationship between industry concentration and economic growth using a two-region model of innovation-based endogenous growth that has been adapted to allow for occupational choice. The allocation of heterogeneous workers into high-skilled and low-skilled employment shapes patterns of comparative advantage, and in a world with free movement of investment capital, innovation and manufacturing occur in the locations that provide the lowest costs. Focusing on long-run equilibria with dispersed innovation, we find that although the aggregate growth rate is unaffected by shifts in production and research activity, the relatively wealthy region always has larger shares of the manufacturing and innovation industries. As such, spatial patterns of economic activity have important implications for the welfare of each region. Finally, we show that, in some cases, net offshoring flows from the wealth abundant region to the wealth scarce region in both production and innovation.

Appendix

First, we calculate the partial derivatives for the threshold asset shares derived in Lemma 1:

\[
\frac{\partial k}{\partial \varphi} = \frac{(1 - \lambda) [BC'(\varphi) - B'(\varphi)C]}{(1 + \lambda)C^2} > 0, \quad \frac{\partial \tilde{\varphi}}{\partial \varphi} = \frac{(1 - \lambda) [B'(\varphi)C - BC''(\varphi)]}{(1 + \lambda)C^2} < 0,
\]

where \( B'(\varphi) = -(1 + \lambda + 2A) < 0 \), and \( C'(\varphi) = 2(1 + 2A)(1 - \lambda)\rho > 0 \).

Second, we derive the the threshold \( \tilde{\varphi} \) discussed in Proposition 4. This threshold can be
obtained using (16), (25), and (26):

$$\gamma - k = \frac{(1 - 2k) [(1 - \lambda)B - \lambda C]}{(1 - \lambda)(2B + C)}.$$ 

The sign of this condition depends on the distribution of asset wealth and the term $(1 - \lambda)B - \lambda C$. Setting $(1 - \lambda)B = \lambda C$ and solving for $\varphi$ yields $\tilde{\varphi}$.

References


