Domestic Subsidies as Disguised Protection and Trade Agreements

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Abstract

In this paper, we investigate how domestic subsidies are treated in international agreements when the use of domestic subsidy is necessary to address a market imperfection that leads to under-production in the import-competing sector. We consider an incomplete-information model in which a government, having incentive to use its domestic subsidy as a means of import protection, can disguise its protective use of subsidy as a legitimate use of stimulating domestic production. We find that any optimal agreement must contain both flexible and rigid treatments of domestic subsidy: it needs a flexible treatment of subsidy to internalize the production externality and needs a rigid treatment of subsidy to raise the world price and trade volume in their agreement.

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1 Introduction

Subsidies have long been regarded as a subject of disputes in the international trading system. In a sense, international disputes over subsidies are not surprising, since a proper treatment of subsidies is not obvious in an international agreement: while the use of subsidy may be a necessary and thus legitimate instrument that addresses market imperfections, it may also act as protective measure that erodes the efficiency of an international agreement enhanced by tariff reductions. In practice, the WTO legal system has changed in the direction of tightening the use of subsidy: the WTO has introduced a new restriction on subsidies that is represented by the Agreement on Subsidies and Countervailing Measures (the SCM agreement).\textsuperscript{1} In this regard, Sykes (2005, 2009) reports that, though the WTO legal system is successful in ensuring that unanticipated subsidy programs do not frustrate the reasonable expectations associated with negotiated trade commitments, it is far less successful in addressing domestic subsidies. In particular, Sykes argues that the legal system fails to distinguish domestic subsidies that are socially necessary ("good" subsidies) from those used as protective measure ("bad" subsidies).\textsuperscript{2} This feature of the WTO legal system then raises the concern that, while having no capacity to distinguish good subsidies from bad subsidies, the system imposes a strict regulation on the use of domestic subsidy.

In this paper, we investigate how domestic subsidies are treated in an optimal international agreement. In particular, we develop a model of trade in which the use of domestic subsidy is necessary to address a market imperfection that leads to under-production in the import-competing sector. In this environment, a government may have two purposes when selecting its domestic subsidy. According to the celebrated targeting principle, the best government intervention is to use domestic subsidy and internalize the affected margin directly.\textsuperscript{3} Thus, on the one hand, the government may use subsidies as a necessary and "legitimate" intervention that addresses the market imperfection. On the other hand, the government may use subsidies as a means of import protection and disguise its protective use of subsidy as a legitimate use; by doing so, the government can lower the world price of the foreign export good and enjoy a terms-of-trade gain.

We consider a 2-country 2-good partial-equilibrium model in which trade occurs in two

\textsuperscript{1}The WTO has introduced a substantial restriction on the use of domestic subsidies that was not present in the GATT, and is moving toward further restrictions on domestic policies in general. See Sykes (2005) and Bagwell and Staiger (2006).

\textsuperscript{2}Sykes (2009) maintains that the WTO rules that purport to distinguish permissible from impermissible government activity are often incoherent and do not capture the full effects of government activity on business enterprise. He argues that it is arguably impossible to develop general principles that distinguish two types of subsidies.

\textsuperscript{3}See Bhagwati and Ramaswami (1963) and Johnson (1965).
symmetric countries where markets are perfectly competitive. This simple model is augmented in two key respects. First, a domestic production of import good by the home country generates a positive externality; thus, the use of subsidy in the import-competing sector is a legitimate intervention with the presence of a production externality. We assume that the production externality does not cross borders. Second, the home government privately observes the production externality and thus privately values the use of its subsidy to the domestic production. In particular, we develop an incomplete-information model with a continuum of possible externality types, where externality types are iid across countries. We assume that the home government intervenes in its import sector only, but uses two policy instruments: a domestic production subsidy and an import tariff.

The starting point of our analysis is to present standard features that are similarly observed in the literature. In the first-best policies, the home government selects its subsidy at the marginal externality and achieves zero tariff. In the (non-cooperative) Nash policies, while the home government selects its subsidy at the marginal externality, it unilaterally raises its import tariff to capture the terms-of-trade gain. Note that the first-best policies cannot be achieved. A central incentive problem is that, subsequent to a tariff-reduction negotiation, the home government has incentive to raise its subsidy for the protective purpose, in order to lower the world price of the foreign export good and thus enjoy a terms-of-trade gain. This problem causes the concern that the use of domestic subsidy may offset the benefit of negotiated tariff commitments. In fact, the problem has long been a justification of the continuing attempts by the WTO to regulate the use of subsidy. The concern would be greater when the home government with private information can disguise its protective use as a legitimate use of subsidy and circumvent the negotiated tariff commitments.4

In this paper, an international agreement acts to specify the policy set from which governments can select their policy pairs of subsidy and tariff. Following Horn, Maggi and Staiger (2010), we assume that international agreements are perfectly enforceable once governments agree on the policy set; hence, a government must select its policies only from the policy set that is specified by the agreement. Though this assumption simplifies our analysis, we allow for a different class of incentive compatibility constraint: the policy set is specified such that the home government with one externality type must not gain from selecting the policy pair that is prescribed for this government when it has a different externality type. We say

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4Indeed, proposal by the European Communities (WTO, 2002, pp. 2-3) describes the concern: “Significant amounts of financial support are increasingly granted by governments for ostensibly general activities which in fact directly benefit the production of certain products. These disguised subsidies can have equally severe trade-distorting effects and they are potentially much more harmful than more direct subsidies since they confer benefits in a largely non-transparent manner.” The EC then proposes to increase and clarify the scope for actions against disguised subsidies.
that an agreement is *optimal* when the associated policy set maximizes the expected global welfare and satisfies the incentive compatibility constraint. We explore various scenarios of international agreements, adopting the stage game: (i) two governments write an agreement that specifies the set of two policy instruments from which governments can select their policies, (ii) the home government privately observes its own externality types and (iii) the home government selects its policies from the policy set specified by the agreement.

We first consider a policy set in which the world price (of the foreign export good) is constant. This policy set can be represented by a decreasing function on the space of subsidy and tariff, since the home government, having two protective instruments, can lower the world price by raising either tariff or subsidy. Along this function, the home country’s import is constant and the home government has no incentive to manipulate its terms of trade; hence, it selects the Pigouvian subsidy that internalizes the production externality. This policy set thus acts as a sorting (separating) scheme that elicits a truthful revelation of externality types. To achieve a higher expected global welfare, among many policy sets that entail sorting, the policy set in which the world price is higher is preferred to any policy set in which the world price is lower. This result is readily established; given the Pigouvian subsidy for each type under any sorting scheme, a decrease in tariff raises the world price and the global welfare.

As a potential candidate of agreements, we may consider the agreement that induces the highest world price among the agreements that entail sorting for *all* externality types. Under this agreement, the home government is granted *flexibility* when choosing its subsidy: it is allowed to select any subsidy as long as its policy choice preserves the specified world price. Such a sufficient flexibility ensures that the home government truthfully reveals its externality type and promotes the domestic productive efficiency in the import-competing sector. In regard to the treatment of subsidy, this agreement exercises the targeting principle in an optimal way: the home government uses a first-best instrument (the Pigouvian subsidy) to internalize the production externality while lowering its import tariffs to reduce the negative terms-of-trade externality on the foreign welfare.

Our first main finding is that this separating agreement is not optimal: it may be improved on by an alternative agreement that entails pooling (rigidity) at the top (the interval of types adjoining the highest type). A strength of the agreement is that it uses the first-best subsidy. A weakness, however, is that it requires high tariffs along the policy set in which the world price is constant. Governments may look for some way to keep the subsidy-efficiency advantage while reducing tariffs by developing another policy set that has a flatter slope than before. This new set, however, induces lower-externality types to raise their subsidies and mimic higher-externality types. Hence, the (global) welfare gain associated
with the subsidy-efficiency advantage can be enjoyed only if the welfare loss associated with the “informational cost” in the form of high import tariffs is also experienced. The finding has some policy implications. First, an optimal agreement does not adhere strictly to the targeting principle in its treatment of subsidy. If an agreement uses a first-best instrument to remedy the market failure that leads to under-production, then it requires the use of high import tariffs that additionally stimulates domestic production and thus affords excessive protection to the import-competing sector. Second, any optimal agreement uses a partially rigid subsidy in order to reduce the informational cost in the form of import tariffs and increase trade volume.

We next explore a pooling agreement in which the use of subsidy is fully rigid or banned. In this agreement, while governments regulate the use of subsidy in a rigid way and sacrifice the domestic productive efficiency, they achieve zero tariff for all externality types. Our second finding is that this agreement is not optimal: any agreement in which the use of subsidy is banned or fully rigid can be improved on by an alternative agreement in which, for types at the bottom (the interval of types adjoining the lowest type), the world price is constant and the use of subsidy is flexible. Intuitively, the alternative agreement can grant the home government a flexible use of subsidy as long as its subsidy choice preserves the original world price. This agreement then entails a sorting interval at the bottom without lowering the world price for higher types (without imposing a negative externality on the foreign welfare). This finding implies that the SCM agreement appears to create an overly strict legal environment in its treatment of subsidy. As Bagwell and Staiger (2006) detail, the WTO has introduced a new restriction on subsidies: in contrast with the preceding GATT rules, the SCM agreement allows that the use of domestic subsidy may be “actionable” regardless of whether it nullifies or impairs the market access expectations associated with prior tariff commitments.

We extend the second finding to investigate the possibility that governments tailor the degree to which the use of subsidy is regulated, together with commitments to zero tariff. This possibility corresponds to circumstances under which governments subsequently strengthen the regulation on the use of subsidy in order to improve on a prior tariff-liberalization agreement. Our third finding is that, regardless of the degree to which the use of subsidy is regulated, an agreement in which import tariffs are bound to zero (or any constant level) is not optimal: it can be improved on by an alternative agreement in which, for types at the bottom, the world price is constant and the use of subsidy is flexible. In regard to the continuing attempts by the WTO to regulate the use of subsidy, this finding has an important implication: the subsequent attempts to improve on any tariff-liberalization agreement by strengthening the regulation on subsidy choices may be on the wrong track if governments
are not granted some flexibility to address market imperfections in the import-competing sector.

In light of the previous analyses, we next show that any optimal agreement contains both rigid and flexible treatments of subsidy. Our fourth finding is that any optimal agreement entails a sorting interval at the bottom in which the world price is constant and the use of subsidy is flexible and it also entails a pooling interval at the top in which the use of subsidy is rigid with zero tariff. A flexible treatment of subsidy at the bottom increases the home welfare for types in the interval without lowering the world price for higher types. A rigid treatment of subsidy at the top has a positive externality on the global welfare for lower types: it increases the world price in particular for lower types in the sorting interval. Thus, any optimal agreement contains two contrasting aspects of treating domestic subsidies found in the literature and reality: a flexible treatment may be associated with the message of the targeting principle and a rigid treatment may be associated with the attempts by the WTO to strengthen the regulation on subsidy choices. These two aspects are conditional on the externality scale: any optimal agreement treats subsidies in a flexible (rigid) manner when the production externality is sufficiently small (large).

We finally present a more comprehensive characterization an optimal agreement. Our fifth finding begins with a monotonicity of the world price and the subsidy choice: in any optimal agreement, (i) the world price is nonincreasing in types and (ii) the subsidy choice is nondecreasing in types. It next highlights the costs that governments would incur when the agreement includes a policy subset in which the world price is not constant: the presence of such a policy subset entails an interval of types in which the world price falls and the subsidy choice is higher than the marginal externality. We then present an example in which an optimal agreement takes a very simple form: in the associated policy set, any policy mix has one constant world price.

Though the treatment of subsidies has long been a central issue in the international trading system, it has received little attention from the theoretical literature. Lee (2007) firstly presents an incomplete-information model in which governments with private information cooperate over two policies when they are tempted to disguise the use of domestic policy and circumvent the negotiated tariff commitments. Whereas Lee (2007) assumes two externality types and linear functions, our model allows for a continuum of possible externality types and for a larger family of demand and supply functions. Our paper also relates to a recent literature of trade agreements among governments with private information. This literature includes Bagwell (2009), Bagwell and Staiger (2005), Feenstra and Lewis (1991), Martin and Vergote (2008) and Park (2006). All of these papers, however, focus on an agreement on one policy instrument among privately informed governments.
Bagwell and Staiger (2006) consider a model in which the use of domestic subsidy is necessary to address market imperfections that lead to under-production. They show that the non-violation complaint rules of GATT represent a proper treatment of domestic subsidy: following a tariff negotiation, a government has flexibility when choosing its subsidy provided that its choice does not impose a negative terms-of-trade externality on its trading partner. In our model, by contrast, any optimal agreement contains both rigid and flexible treatments of subsidy. Bagwell and Staiger (2006) also show that the SCM agreement may be criticized as causing a chilling effect on the incentive of governments to negotiate tariff liberalization: when domestic subsides are treated severely under the SCM agreement, governments may hesitate to undertake tariff negotiations, since tariffs then may be the best remaining means of assisting the import-competing sector. As Sykes (2009) argues, the chilling effect would not occur if governments were able to distinguish good subsidies from bad subsidies and then exclude the use of good subsidies from the object of restriction. In our paper, the policy implications presented above are explicitly based on an incomplete-information model.

The paper is organized as follows. Section 2 describes the basic model and shows that the model inherits the standard features observed in the literature. In Section 3, we explore various scenarios of international agreements and derive some features observed in any optimal agreement. Section 4 concludes. In the Appendix, we offer additional expositions not contained in the main text and provide proofs.

2 The Model

We consider a 2-country 2-good partial-equilibrium model of trade augmented to allow for the presence of a (positive) production externality. According to the celebrated targeting principle, the best government intervention is to use domestic subsidy and internalize the affected margin directly. The model describes an environment in which a government may have two purposes when it uses its domestic subsidy. On the one hand, the government has a legitimate purpose and uses its domestic subsidy to address market imperfections that result in too little production. On the other hand, the government uses its domestic subsidy as a means of import protection and yet disguises its protective use of domestic subsidy as a legitimate use; by doing so, the government can lower the world price of the foreign export good and enjoy a terms-of-trade gain.

2.1 The Basic Trade Model

We assume that trade occurs in two countries where markets are perfectly competitive. The
home country exports good $y$ to the foreign country in exchange for imports of good $x$. 
For good $x$, the home country has a downward-sloping demand function $D(p^d)$ for the consumer (local) price $p^d$ and an upward-sloping supply function $Q(p^s)$ for the supplier (local) price $p^s$. The two functions are positive and twice-continuously differentiable. Letting superscript asterisks denote foreign variables, for good $x$, we define the corresponding demand and supply functions of the foreign country as $D^*(p^{d*})$ and $Q^*(p^{s*})$. Our model allows for two possibilities together: (i) the domestic production of good $x$ by the home country generates a positive production externality; (ii) the home government privately observes the marginal production externality and thus privately values the use of its subsidy to the domestic production. In particular, we consider an incomplete-information model with a continuum of possible externality types, where externality types are iid across countries. Externality types are represented by the (marginal) production externality, denoted as $\theta$. Externality type $\theta$ is drawn from the support $[0, \theta]$ according to the twice-continuously differentiable distribution function, $F(\theta)$, where $\theta > 0$. We define the density as $f(\theta) \equiv F'(\theta)$ where $f(\theta) > 0$ for all $\theta \in [0, \theta]$. Producers ignore the external effects of their production on the aggregate production, and thus their supply functions are not directly affected by $\theta$. We also assume that the production externality does not cross borders. We may then represent the aggregate value of the production externality as $\theta Q(p^s)$ for the home country with externality type $\theta$.5

To deliver our main points simply, we introduce two non-negative policy instruments into the home import sector only: the home government intervenes in its import sector and uses a domestic production subsidy $s$ and an import tariff $\tau$.6 We assume that all policy instruments are non-prohibitive and expressed in specific terms. In the absence of policies by the foreign government, the foreign consumer and supplier prices are the same and denoted as the world (offshore) price: $p^w = p^{s*} = p^{d*}$. The markets in two countries are integrated, so that a foreign supplier receives the same price for sales in the foreign country that it receives for sales in the home country after paying tariffs: $p^{d*} = p^d - \tau$. The wedge between the home supplier price and the home consumer price is domestic subsidies: $p^s = p^d + s$. These pricing relationships similarly hold in the import sector of the foreign country. We may rewrite these pricing equations in a useful form:

\begin{equation}
    p^d = p^w + \tau \quad \text{and} \quad p^s = p^w + \tau + s.
\end{equation}

5The aggregate value of externality is similarly represented in Ederington (2002), Lee (2007) and Horn, Maggi and Staiger (2010).

6We can readily extend the model to a symmetric setting in which the foreign government also intervenes in its import sector and uses a domestic production subsidy $s^*$ and an import tariff $\tau^*$. 

Equilibrium prices are then determined by the market-clearing condition:

\[ D(p^d) + D^*(p^{sd}) = Q(p^s) + Q^*(p^{ss}). \] (2)

Plugging the consumer and supplier prices into the market-clearing condition, we can find that the equilibrium world prices, denoted by \( \hat{p} \), are functions of policy pairs: \( \hat{p}(s, \tau) \). The equilibrium consumer and supplier prices may then be denoted by \( \hat{p}^d(s, \tau) = \hat{p}(s, \tau) + \tau \) and \( \hat{p}(s, \tau) = \hat{p}^u(s, \tau) + \tau + s \). It is also immediate from the market-clearing condition (2) that, if the home government raises \( s \) or \( \tau \), then it can lower the world price of the foreign export good:

\[ \frac{\partial \hat{p}^u}{\partial s} = \frac{Q'}{D' - Q' - (Q^{u'} - D')} < 0 \] (3)

\[ \frac{\partial \hat{p}^u}{\partial \tau} = -\frac{D' - Q'}{D' - Q' - (Q^{u'} - D')} < 0. \] (4)

In addition, an increase in \( s \) or \( \tau \) by the home government promotes the domestic production of the foreign export good, \( Q(\hat{p}^s) \), and reduces the home import, \( D(\hat{p}^d) - Q(\hat{p}^s) \).

We assume that the welfare function of each country is separable across sectors, \( x \) and \( y \). We can thus focus on the welfare functions in the home import sector \( x \) which is the foreign export sector. The home welfare includes consumer surplus, profits, revenue from the import tariff, expenditures on the production subsidies and the aggregate value of the production externality. We may write the home welfare for externality type \( \theta \) as:

\[ W(s, \tau; \theta) \equiv CS(\hat{p}^d) + \Pi(\hat{p}^s) + \tau \cdot M(s, \tau) - s \cdot Q(\hat{p}^s) + \theta \cdot Q(\hat{p}^s), \] (5)

where \( M(s, \tau) \equiv D(\hat{p}^d) - Q(\hat{p}^s) \). As noted above, the equilibrium local prices in (5) are \( \hat{p}^d = \hat{p}^v(s, \tau) + \tau \) and \( \hat{p}^s = \hat{p}^u(s, \tau) + \tau + s \). A policy pair \( (s, \tau) \) also affects the world price and thus the foreign welfare. The foreign welfare is the sum of consumer surplus and profits:

\[ W^*(s, \tau) \equiv CS^*(\hat{p}^{sd}) + \Pi^*(\hat{p}^{ss}), \] (6)

where \( \hat{p}^{sd} = \hat{p}^{su} = \hat{p}^u(s, \tau) \). Observe that an increase in \( s \) or \( \tau \) by the home government imposes a negative terms-of-trade externality on the foreign exporters. An increase in \( s \) or \( \tau \) causes a decline in the world price \( \hat{p}^u \) which equals the foreign local prices. Such a policy change by the home government is harmful to the foreign exporters and the foreign welfare.\(^8\)

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\(^7\)This result is derived in the Appendix A. As in Bagwell and Staiger (2002), when an increase in \( s \) or \( \tau \) shifts in the home import demand curve, the consequent “price effect” (the terms-of-trade loss for foreign country) is always accompanied by the “quantity effect” (the market-access loss for foreign exporters).

\(^8\)The policy change that lowers the world price is harmful to the foreign exporters but beneficial to the foreign consumers. The foreign consumer gain, however, amounts to a transfer from the foreign producers to the foreign consumers. Thus, when the world price falls, the net foreign welfare decreases.
The home government with externality type \( \theta \) cares about the negative externality on the foreign exporters, when it selects \((s, \tau)\) that maximizes the global welfare:

\[
W^G(s, \tau; \theta) \equiv W(s, \tau; \theta) + W^*(s, \tau).
\]  

Note also that the policy choice, \( s \) or \( \tau \), is a function that maps from the set of externality types \([0, \theta]\) to the set of possible subsidy levels \([0, \infty)\). The typical policy choices made by the home government with externality type \( \theta \) can be denoted by \( s(\theta) \) and \( \tau(\theta) \). The associated expected home welfare and expected global welfare are given by \( E_\theta W(s(\theta), \tau(\theta); \theta) \) and \( E_\theta W^G(s(\theta), \tau(\theta); \theta) \), repetitively.

### 2.2 First-Best and Nash Policies

We first characterize the first-best policies. The globally efficient policies for the home government with externality type \( \theta \), denoted by \( s^E(\theta) \) and \( \tau^E(\theta) \), are the policies that maximize the global welfare \( W^G(s, \tau; \theta) \):\(^9\)

\[
s^E(\theta) = \theta \text{ and } \tau^E(\theta) = 0 \text{ for all } \theta.
\]  

In the first-best policies, the home government selects its subsidy at the marginal externality and achieves zero tariff. We next characterize the (non-cooperative) Nash policies, assuming that \( W(s, \tau; \theta) \) is strictly concave in \( s \) and \( \tau \) for all \( \theta \).\(^{10}\) The Nash policies for the home government with externality type \( \theta \), denoted by \( s^N(\theta) \) and \( \tau^N(\theta) \), are the policies that maximize the home welfare \( W(s, \tau; \theta) \). Taking derivatives with respect to \( s \) and \( \tau \), we can find that the Nash policies satisfy

\[
s^N(\theta) = \theta \text{ and } \tau^N(\theta) = \frac{E^*(\tilde{p}^w)}{E^*(\tilde{p}^w)} \text{ for all } \theta,
\]  

where \( \tilde{p}^w = \tilde{p}^w(s = s^N(\theta), \tau = \tau^N(\theta)) \) and \( E^*(\tilde{p}^w) = Q^*(\tilde{p}^w) - D^*(\tilde{p}^w) \). In the Nash policies, while the home government selects its subsidy to internalize the marginal externality, it uses import tariffs to capture the terms-of-trade gain based on the inverse of the foreign export elasticity. In fact, the assumption that government intervention is non-prohibitive requires that the highest externality type \( \theta \) should be below a certain level so that import volume is positive. For the analyses below, we now explicitly make the following assumption:

**Assumption 1.** (i) \( W(s, \tau; \theta) \) and \( W^*(s, \tau) \) are strictly concave in \( s \) and \( \tau \); (ii) \( M(s = \theta, \tau = 0) > 0 \).

\(^9\)In the Appendix, we find the first-best and Nash policies.

\(^{10}\)The assumption is satisfied for a large family of demand and supply functions, including linear functions.
Under the assumption (i), the global objective $W^G(s, \tau; \theta)$ is also concave in $s$ and $\tau$. Under the assumption (ii), our subsequent analyses will focus on international agreements under which government intervention is non-prohibitive.

### 2.3 Standard Features

We next emphasize that our trade model inherits standard features observed in the literature. To highlight a central incentive problem apparent in the model, we assume that the home government must select its policies from the policy set in which the world price is constant:

$$\{(s, \tau) : \tilde{p}^{sw}(s, \tau) = \tilde{p}^{sw}(s = \theta, \tau = 0)\}. \quad (10)$$

The home government with externality type $\theta$ then selects the policy pair that maximizes $W(s, \tau; \theta)$ subject to the policy set (10). For any $(s, \tau)$ in the set, the world price is constant at $\tilde{p}^{sw}(s = \theta, \tau = 0)$ and thus the foreign welfare $W^*(s, \tau)$ is constant. Since an increase in $s$ or $\tau$ lowers the world price, the policy set (10) can be uniquely represented by the iso-world price function, $\tau = \tau_{sep}(s)$:

$$\tau_{sep}(s) = \frac{Q'}{D' - Q'}[s - \theta]. \quad (11)$$

This iso-world price function is strictly decreasing and crosses the point $(\theta, 0)$; the slope, $\frac{d\tau_{sep}}{ds} = \frac{Q'}{D' - Q'} < 0$, is given by (3) and (4) and is measured at the policies on the function. Along this function, the home country’s import is constant and the home government has no incentive to use its subsidy and manipulate its terms of trade; hence, the home government with externality type $\theta$ addresses the production externality with the efficient subsidy: $s(\theta) = \theta$. We formalize this finding.

**Lemma 1.** If the home government with externality type $\theta$ must select its policies from the policy set (8), then its subsidy choice equals the marginal production externality: $s(\theta) = \theta$ for all $\theta$.

The proof is in the Appendix. Given the subsidy choice $s(\theta) = \theta$, the home government with externality type $\theta$ is induced to choose $\tau_{sep}(\theta)$ from the iso-world price function $\tau = \tau_{sep}(s)$. Thus, the policy set (10) acts as a sorting (separating) scheme and elicits a truthful revelation of all externality types.

We extend this result and develop some additional points. Suppose first that the home government must select its policies from an alternative policy set in which import tariffs are sufficiently low and *constant* for all $\theta$. This policy set then poses an incentive problem that typically occurs subsequent to a tariff-reduction negotiation: the home government has incentive to raise its subsidy above the marginal externality in order to lower the world price.
and thus enjoy a terms-of-trade gain.\footnote{A similar incentive problem is also observed in models by Bagwell and Staiger (2001) and Lee (2007) among many others. A different feature here is that we use the private-information setting with continuous types.} This incentive problem is evident in the first-best policies in (8). To compare the set (10) to any other set that keeps the same world price in (10), suppose next that the home government must select its policies from the set has only two policy pairs:

\[ \{(s, \tau) : (s_1, \tau_1), (s_2, \tau_2)\} \]  

where \( \hat{p}^w(s_1, \tau_1) = \hat{p}^w(s_2, \tau_2) = \hat{p}^w(s = \bar{\theta}, \tau = 0) \).  

(12)

The difference from (10) is that this policy set entails pooling: the home government with some different types pools at one policy pair. We can then infer from Lemma 1 that the home welfare is strictly higher under (10) than under (12), except for the case \( \theta \in \{s_1, s_2\} \) in which the home government has the same welfare under both policy sets. We may develop a general point from this result.

**Lemma 2.** For all \( \theta \), the home welfare is at least as high under (10) as under any other policy set that preserves the world price at \( \hat{p}^w(s = \bar{\theta}, \tau = 0) \). For some \( \theta \), the home welfare is strictly higher under (10) than under any other policy set that preserves the world price at \( \hat{p}^w(s = \bar{\theta}, \tau = 0) \).

We next establish the previous lemmas at a general level. To this end, we next assume that the home government must select its policies from the set:

\[ \{(s, \tau) : \hat{p}^w(s, \tau) = \hat{p}^w(s = \bar{\theta}, \tau = \hat{\tau})\} \]  

(13)

where \( \hat{\tau} \geq 0 \) is constant. This can also be represented by a strictly decreasing function \( \tau = \hat{\tau}^{sep}(s) \). We can then show that Lemma 1 and 2 hold: (i) along the iso-world price function, the home government with externality type \( \theta \) selects the efficient production subsidy: \( s(\theta) = \theta \) and (ii) the home government prefers to select its policies from (13) than from any other policy set that preserves the world price at \( \hat{p}^w(s = \bar{\theta}, \tau = \hat{\tau}) \).

We also note that, if \( \hat{\tau} \) rises from zero to a positive level, then the iso-world price function shifts up, so that the home government with externality type \( \theta \) selects a higher tariff; as a result, for each type \( \theta \), the world price falls and thus the foreign welfare decreases. In regard to the effect on the home welfare, we assume that, if the world price falls as \( \hat{\tau} \) rises from zero to a positive level, then the home government with externality type \( \theta \) becomes better off as it selects a higher tariff and the same subsidy at \( \theta \) along the new iso-world price function. In particular, we make the following assumption:

**Assumption 2.** For all \( \theta \), the home welfare under the policy set (13) increases in \( \hat{\tau} \) when \( \hat{\tau} = 0 \).
Under Assumption 2, our subsequent analyses will focus on circumstances under which the home government with any externality type has incentive to increase its tariff in order to enjoy a terms-of-trade gain.\footnote{Indeed, this assumption is satisfied if and only if import tariffs are lower under the set \((10)\) than under the Nash policies: \(\tau^{sep}(\theta) < \tau^N(\theta)\) for all \(\theta\). This inequality holds for a large family of demand and supply functions (including linear functions), if \(\theta\) is not too large and the term \(\frac{E^*(p^g)}{E^*(p^w)}\) in (9) is not too small. A sufficiently small \(\frac{E^*(p^g)}{E^*(p^w)}\) is observed when the home country is very small and thus has little incentive to manipulate terms of trade.}

3 International Agreements

In this section, we explore various scenarios of international agreements that treat domestic subsidies differently. We first investigate an agreement that grants the home government flexibility in its use of subsidy in order to achieve the efficient (Pigouvian) production subsidy. We next analyze an agreement that imposes a rigidity on the use of subsidy in order to achieve the efficient (zero) import tariff.

We consider the following stage game: (i) two governments write an agreement that specifies the policy set of policy pairs from which governments can select their policies, (ii) the home government privately observes its own externality types and (iii) the home government selects its policies from the policy set specified by the agreement. We assume that the home government privately observes its externality type while it publicly observes policy choices. The foreign government observes policies selected by the home government, but it does not observe the foreign externality type. Thus, the foreign government cannot verify whether the use of subsidy by the home government equals or is more than the level that is necessary to address the production externality. We may refer to the international agreement on \textit{two} policy instruments as a trade agreement, in that our model describes circumstances under which governments negotiate over import tariffs subject to their incentives to use domestic subsidies and manipulate terms of trade.

We follow Horn, Maggi and Staiger (2010) and assume that international agreements are perfectly enforceable once governments agree on the policy set; hence, a government must select its policies only from the policy set that is specified by the agreement. Though this assumption simplifies our analysis, we allow for a different class of incentive compatibility constraint: the policy set is specified such that the home government with one externality type must not gain from selecting the policy pair that is prescribed for this government when it has a different externality type. This incentive constraint is analogous to the standard truth-telling constraint encountered in mechanism-design problems. We also assume that the objective of international agreement is to find the incentive compatible policy set in which
the expected global welfare is maximized. We say that an agreement is optimal when
the associated policy set maximizes the expected global welfare and satisfies the incentive
compatibility constraint: formally, an agreement is optimal when its policy set \{(s, \tau)\} satisfies
\[
E_\theta W^G(s(\theta), \tau(\theta); \theta) \geq E_\theta W^G(\widehat{s}(\theta), \widehat{\tau}(\theta); \theta)
\]
for any alternative set \{(\widehat{s}, \widehat{\tau})\}
and
\[
W(s(\theta), \tau(\theta); \theta) \geq W(s(\theta), \tau(\theta); \theta) \text{ for all } \theta \text{ and } \widehat{\theta} \neq \theta.
\]
(\text{IC}(\theta))
Equivalently, an agreement is not optimal if there exists an alternative policy set in which
the expected global welfare is higher than in the original set and the incentive compatibility
constraint holds.

3.1 Flexible Treatment of Subsidy
In this subsection, we consider a (full) separating agreement in which the home government
internalizes all externality types with the Pigouvian subsidies. The policy set specified by the
agreement must satisfy the incentive compatibility constraint: the home government with
externality type \(\theta\) must make the subsidy choice, \(s(\theta) = \theta\); \(\theta = \arg \max_s W(s, \tau; \theta)\) for all \((s, \tau)\) in the policy set. Together with this constraint, governments would tailor the import
tariff for type \(\theta\), \(\tau(\theta)\), in order to maximize the expected global welfare \(E_\theta W^G(s(\theta), \tau(\theta); \theta)\).

Two findings can be established when governments maximize the expected global welfare.
First, among the policy sets in which the world price is constant at \(\hat{p}^w(s = \overline{s}, \tau = \overline{\tau})\), the policy set that entails full sorting is preferred to any policy set that entails a partial or
full pooling. This result is immediate from our previous argument that follows Lemma 2.
Second, among the policy sets that entail full sorting, the policy set in which the world price
is higher is preferred to any policy set in which the world price is lower. This result is readily
established by referring to the policy set (13): if the iso-world price function shifts up as \(\widehat{\tau}\)
rises from zero, then the global welfare \(W^G(s(\theta), \tau(\theta); \theta)\) decreases for all \(\theta\). We can thus
present the following lemma.

13 As noted above, we can readily develop a modified symmetric model in which the home (foreign) government
with externality type \(\theta\) selects policy schedules, \(s(\theta)\) and \(\tau(\theta)\) (\(s^*(\theta)\) and \(\tau^*(\theta)\)). With this modification, we may
write the home welfare as \(W_x(s(\theta), \tau(\theta); \theta) + E_\theta W_y(s^*(\theta), \tau^*(\theta))\). The first (second) term is the welfare of the
import (export) sector. The second term takes an expected value because of non-observability of the foreign externality
types. Likewise, we may write the foreign welfare as \(W^*_x(s^*(\theta), \tau^*(\theta); \theta) + E_\theta W^*_y(s(\theta), \tau(\theta))\). We then define the
expected global welfare as the objective function: \(E\theta W^G_x(s(\theta), \tau(\theta); \theta) \equiv E_\theta W_x(s(\theta), \tau(\theta); \theta) + W^*_y(s(\theta), \tau(\theta))\) for the
sector \(x\) and similarly \(E_\theta W^G_y(s^*(\theta), \tau^*(\theta); \theta)\) for the sector \(y\). We can then find that the two symmetric sets of policies
that increase these two objectives also increase the expected home welfare, \(E_\theta[W_x(s(\theta), \tau(\theta); \theta) + W_y(s^*(\theta), \tau^*(\theta))]\),
and the expected foreign welfare, \(E_\theta[W^*_x(s^*(\theta), \tau^*(\theta); \theta) + W^*_y(s(\theta), \tau(\theta))]\). Our model thus corresponds to the
model that focuses on the sector \(x\) and searches for the policy schedules that maximizes \(E_\theta W^G_x(s(\theta), \tau(\theta); \theta)\).
Lemma 3. For all $\theta$, the expected global welfare is strictly higher under the policy set (10) than under any other policy set that is a subset of $\{(s, \tau) : \tilde{p}^w(s, \tau) \leq \tilde{p}^w(s = \overline{\theta}, \tau = 0)\}$.

The proof is in the Appendix. We next observe that any (full) separating agreement involves a fixed world price only within the region $\{(s, \tau) : \tilde{p}^w(s, \tau) \leq \tilde{p}^w(s = \overline{\theta}, \tau = 0)\}$: if an agreement ever includes a policy mix $(\tilde{s}, \tilde{\tau})$ at which $\tilde{p}^w(\tilde{s}, \tilde{\tau}) > \tilde{p}^w(s = \overline{\theta}, \tau = 0)$, then it must entail at least a partial pooling with $s(\theta) \neq \theta$ for some $\theta$. Hence, the best separating agreement has the policy set (10). Intuitively, the iso-world price function, $\tau = \tau^\text{sep}(s)$, offers a lower import tariff for a given subsidy than does any other iso-world price function that acts as a sorting scheme.\footnote{It is straightforward to show that the best separating agreement strictly improves on the (non-cooperative) Nash policies, given that $\tau^\text{sep}(\theta) < \tau^N(\theta)$ for all $\theta$ under Assumption 2.}

This best separating agreement has distinct features in its treatment of subsidy. As noted above, the agreement has the policy set (10) that acts as a revelation mechanism. Under the agreement, the home government is granted flexibility when choosing its subsidy: it is allowed to select any subsidy below $\overline{\theta}$ as long as its policy choice preserves the world price at $\tilde{p}^w(s = \overline{\theta}, \tau = 0)$. Such a sufficient flexibility in the use of subsidy is necessary to ensure that the home government truthfully reveals its externality type and promotes the domestic productive efficiency in the import-competing sector. In regard to the treatment of domestic subsidy, the agreement exercises the targeting principle in an optimal way: the home government uses a first-best instrument (the Pigouvian subsidy) to internalize the production externality while lowering its import tariffs to reduce the negative terms-of-trade externality on the foreign welfare.

We next argue that the best separating agreement has a weakness: though the agreement achieves the efficient production subsidy, it entitles the home government to choose high import tariffs for a wide range of externality types. We can show that an alternative agreement exists that improves on the best separating agreement. Suppose that, for a critical type $\theta^c \in (0, \overline{\theta})$, the alternative agreement has the policy set:

$$\{(s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(s = \theta^c, \tau = 0)\}.$$  \hfill (14)

In comparison with the set (10), this policy set involves a higher world price and instead restricts the subsidy choice to the level below $\theta^c < \overline{\theta}$: the home government can select any subsidy below $\theta^c$ as long as its policy choice preserves the world price at $\tilde{p}^w(s = \theta^c, \tau = 0)$. For all $\theta < \theta^c$, it entails sorting: $s(\theta) = \theta$ and $\tau(\theta) < \tau^\text{sep}(\theta)$. For all $\theta \geq \theta^c$, however, it entails pooling at $(\theta^c, 0)$: $s(\theta) = \theta^c$ and $\tau(\theta) = 0$. Observing that, if $\theta^c \to \overline{\theta}$, then the alternative agreement approaches the best separating agreement, we differentiate the expected global
welfare under the alternative agreement with respect to $\theta^C$. We then establish that the best separating agreement is not optimal: it can be improved on by the alternative agreement in which the use of subsidy is rigid for types at the top (the interval of $\theta$ adjoining $\bar{\theta}$, $[\theta^C, \bar{\theta}]$).

**Proposition 1.** An agreement in which the use of subsidy satisfies $s(\theta) = \theta$ for all $\theta$ is not optimal: it can be improved on by an alternative agreement in which the world price is raised by a rigid treatment of subsidy for some $\theta$ at the top.

The proof is in the Appendix. This finding is quite general, in that it holds for any distribution function $F$. Intuitively, governments face a trade-off when contemplating the best separating agreement. An advantage of the agreement is that it uses the first-best subsidy. A disadvantage, however, is that the global welfare is reduced by high import tariffs. Governments may look for some way to keep the subsidy-efficiency advantage while reducing tariffs. They might consider an alternative policy set, represented by a function $\tau = \tau^{alt}(s)$, that is strictly decreasing and is flatter than the function $\tau = \tau^{sep}(s)$. Such an alternative set, however, will induce lower-externality types to raise their subsidies and mimic higher-externality types. Hence, the (global) welfare gain associated with the domestic productive efficiency can be enjoyed only if the welfare loss associated with the “informational cost” in the form of high import tariffs is also experienced.

Our finding has some policy implications. First, it shows that an optimal agreement does not adhere strictly to the targeting principle in its treatment of subsidy. If an agreement uses a first-best instrument to remedy the market failure that leads to under-production, then it requires the use of high import tariffs that additionally stimulates domestic production and thus affords excessive protection to the import-competing sector. Second, our finding implies that any optimal agreement entails at least partial pooling: it can reduce the informational cost of high import tariffs by sacrificing the domestic productive efficiency for some $\theta$. In later analysis, we will confirm that any optimal agreement uses a partially rigid subsidy in order to reduce import tariffs and increase trade volume. A different rationale for trade agreements to adopt a rigid treatment of subsidy is suggested by Horn, Maggi and Staiger (2010): using a model in which the WTO/GATT regulation is regarded as an incomplete contract, they show that, if trade volume is large, then an optimal agreement may be made partially or fully rigid in order to save contracting costs. Whereas a partial rigidity leads to an increase in trade volume in our paper, large trade volume leads to a partial or full rigidity in their model.

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15 Allow for any alternative agreement under which the policy set is represented by a decreasing function $\tau = \tau^{alt}(s)$ that is flatter than $\tau = \tau^{sep}(s)$ for any $s$. We can show that this alternative agreement is not optimal. A limiting case of this alternative agreement is an agreement with zero tariffs we discuss below.
3.2 Rigid Treatment of Subsidy

In this subsection, we explore the possibility that governments regulate the use of subsidy in a rigid way and sacrifice the domestic productive efficiency, in order to reduce import tariffs. In particular, we consider an agreement in which the use of subsidy is restricted to be fully rigid for all $\theta$. A special case of the agreements is that the use of subsidy is banned (restricted to zero).

Our objective here is to present that governments would not reach an agreement in which the use of subsidy is banned or fully rigid. Among all possible agreements in which the use of subsidy is fully rigid, we characterize the best pooling agreement. Suppose that the home government selects a policy pair: $s(\theta) \equiv s^p$ and $\tau(\theta) \equiv \tau^p$ where $s^p$ and $\tau^p$ are constant. The policy set is a singleton, $\{(s^p, \tau^p)\}$, and thus is apparently incentive compatible; no externality type has incentive to mimic other types. The best pooling agreement maximizes the expected global welfare, $E_\theta[W^G(s^p, \tau^p; \theta)]$. Since all prices are constant for $\theta$ in this agreement, referring to the global welfare in (5), we can find that the expected global welfare $E_\theta[W^G(s^p, \tau^p; \theta)]$ equals $W^G(s^p, \tau^p; \theta)$, except that the last term in (3) now becomes the expected production externality, $E[\theta] \cdot Q(\widehat{p}^*).$ The best pooling agreement is then characterized by $s^p = E[\theta]$ and $\tau^p = 0$. This agreement thus indicates that governments can save the informational cost of high import tariffs by imposing a strong restriction on the use of subsidy.

We now show, however, that, even when the support of possible externality types is small, this agreement is not optimal: the welfare gain associated with tariff liberalization is dominated by the welfare loss associated with the domestic productive inefficiency. An immediate finding is that the use of subsidy is not banned in any optimal agreement: the agreement in which $s(\theta) \equiv 0$ can be improved on by the best pooling agreement. Further, the best pooling agreement can be improved on by an alternative agreement in which the policy set is

$$\{(s, \tau) : \widehat{p}^w(s, \tau) = \widehat{p}^w(s = E[\theta], \tau = 0)\}. \tag{15}$$

A new feature is that the home government is granted flexibility to select any subsidy below $E[\theta]$ as long as its policy choice preserves the original world price $\widehat{p}^w(s = E[\theta], \tau = 0).$ In effect, the inclusion of such a flexible treatment of subsidy acts to expand the policy set along an iso-world price function for $\theta$ at the bottom (the interval adjoining type 0). The new policy set entails sorting for $\theta \leq E[\theta]$ and pooling at the point $(E[\theta], 0)$ for $\theta > E[\theta]$. With the new policy set, the home government becomes strictly better off for $\theta < E[\theta]$ and remains indifferent for $\theta \geq E[\theta]$. The home-welfare improvement does not impose the

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16 Note that trade volume is not maximized in an optimal agreement, since it is maximized when $s(\theta) \equiv 0$ and $\tau(\theta) \equiv 0.$
negative terms-of-trade effect on the foreign producers and thus increases the expected global welfare. We summarize our findings.

**Proposition 2.** The use of subsidy is neither banned nor fully rigid in any optimal agreement: any agreement in which the use of subsidy is banned or fully rigid can be improved on by an agreement in which, for \( \theta \) at the bottom, the world price is constant and the use of subsidy is flexible.

We now discuss some policy implications. Under GATT rules, domestic subsidies were treated in a tolerant manner. A government could make a non-violation complaint against another subsidizing government whose a new subsidy program nullified or impaired the market access expectations associated with prior tariff commitments. The subsidizing government would then be expected to make a policy adjustment that returned market access to its original level (under no obligation to remove the subsidy). The WTO has introduced a new restriction on subsidies that is represented by the SCM agreement. In contrast with the preceding GATT rules, the SCM agreement allows that the use of domestic subsidy may be “actionable” regardless of whether it nullifies or impairs the market access expectations associated with prior tariff commitments. The use of domestic subsidy may now be actionable even if the relevant product is not subject to any tariff commitment or the subsidy already exists at the time of any tariff commitment.\(^{17}\)

Bagwell and Staiger (2006) consider a model in which the use of domestic subsidy is necessary to address market imperfections that lead to under-production. They show that the SCM agreement may be criticized as causing a chilling effect on the incentive of governments to negotiate tariff liberalization: when domestic subsidies are treated severely under the SCM agreement, governments may hesitate to undertake tariff negotiations, since tariffs then may be the best remaining means of assisting the import-competing sector. Sykes (2009) argues that the chilling effect would not occur if governments were able to distinguish socially necessary subsidies (“good” subsidies) from those used as protective measure (“bad” subsidies) and then exclude the use of good subsidies from the object of subsidy restriction. Sykes maintains, however, that, due to the complexity of modern economy and the wide panoply of government activity that affects business activities, the WTO rules lack the capacity to distinguish good subsidies from bad ones.

In this paper, we explicitly develop an incomplete-information model in which the foreign government cannot directly verify whether the home government uses its subsidy as protective measure beyond what is necessary to address the production externality. Our

\(^{17}\)We here follow Bagwell (2008) to discuss the key difference between GATT and WTO rules on subsidies.
finding indicates that governments would not reach an agreement in which the use of subsidy is banned or fully rigid; the SCM agreement thus appears to create an overly strict legal environment in its treatment of subsidy. Our model also suggests a dilemma faced by a strict regulation on the use of subsidy. As is hinted by Sykes (2009), a strict regulation on subsidy choices may be supported by some degree of separability between good and bad subsidies. Our model shows, however, that externality types are truthfully revealed only in the range where subsidies are treated in a flexible manner; good and bad subsidies are grouped together in the range where subsidies are treated in a rigid way.

3.3 Tariff Commitment and Subsequent Subsidy Regulation

In this subsection, we consider the possibility that governments tailor the degree to which the use of subsidy is regulated, together with commitments to zero or any constant tariff for all $\theta$. This possibility corresponds to circumstances under which governments subsequently strengthen the regulation on the use of subsidy in order to improve on a prior tariff-liberalization agreement in which tariffs are bound to zero or any constant level. We show that, whether the regulation is lax or strict, any agreement with zero or any constant tariffs can be improved on by an alternative agreement in which, for some $\theta$, a flexible use of subsidy is allowed as along as it preserves a fixed world price.

Since the policy set is not a singleton in any optimal agreement, we restrict attention to the agreements in which multiple subsidy choices are allowed. We begin with an agreement in which the policy set consists of two policy pairs: $\{(s, \tau): (s_1, 0), (s_2, 0)\}$ where $s_1$ and $s_2$ are constant with $s_2 > s_1$. Given that higher-externality types are more willing to increase subsidies ($\frac{\partial^2 W(s, \tau; \theta)}{\partial \theta \partial s} > 0$), subsidy choices can be represented by a step function that has a jump at $\theta_c \in [0, \theta)$: $s(\theta) = s_1$ for all $\theta \leq \theta_c$ and $s(\theta) = s_2$ for all $\theta > \theta_c$.\(^{18}\) The level of jump is made such that $(s_1, 0)$ and $(s_2, 0)$ are indifferent for type $\theta_c$. We now show that this agreement is not optimal. Consider two possible cases: $s_1 > 0$ and $s_1 = 0$. For the first case ($s_1 > 0$), we can construct an alternative agreement in which the policy set consists of two parts:

$$\{(s, \tau) : \bar{p}^w(s, \tau) = \bar{p}^w(s = s_1, \tau = 0) \text{ and } (s_2, 0)\}.$$  \hspace{1cm} (16)

The first part allows a more flexible use of subsidy than before, whereas the second part remains the same. By offering such a flexible use of subsidy, the alternative set expands the choice set for the home government while preserving the world price $\bar{p}^w(s = s_1, \tau = 0)$. Observing that the original agreement involves a higher world price at $(s_1, 0)$ than at $(s_2, 0)$,\(^{18}\) Note that, if the support of possible externality types is sufficiently small ($\theta$ is sufficiently small), then only one policy pair $(s_2, 0)$ is selected for all $\theta$. The policy set is then equivalent to a singleton in which case the agreement is not optimal as shown in Proposition 2 and thus can be ignored.

\hspace{1cm} 18
we find that the alternative agreement expands the choice set while preserving the higher world price. This finding means that the alternative agreement does not lower the world price for any $\theta$. We thus conclude that the alternative agreement increases the expected home welfare without imposing a negative externality on the foreign welfare: the original agreement is not optimal. As in the Appendix, we can also show that the original agreement with the second case ($s_1 = 0$) is not optimal. Further, we may extend our argument to an agreement in which its policy set reduces the degree of subsidy restriction and offers more options: $\{(s, \tau) : (s_1, 0), (s_2, 0), \ldots, (s_K, 0)\}$.

In line with such extensions, we next consider the agreement in the limiting case where import tariffs are bound to zero for all $\theta$ and subsidies are left to the discretion of the home government. In this agreement with zero tariffs, the home government would select its subsidy above a certain level, $\hat{s} > 0$. Letting $\hat{s}' \equiv \min\{\hat{s}, \overline{\theta}\}$, we construct an alternative agreement in which the policy set has two parts:

$$\{(s, \tau) : \hat{p}_w(s, \tau) = \hat{p}_w(s = \hat{s}', \tau = 0) \text{ and } [(\hat{s}', 0), (\overline{\theta}, 0)]\}.$$  \hspace{1cm} (17)

The first part provides the home government with flexibility to select any policies that preserve the world price $\hat{p}_w(s = \hat{s}', \tau = 0)$. The second part is a line segment and represents a discretionary choice for any subsidy $s \in [\hat{s}', \overline{\theta}]$ under zero tariffs. If $\hat{s} > \overline{\theta}$ ($\hat{s}' = \overline{\theta}$), then the alternative agreement is the best separating agreement. Observe that the original agreement involves the world price that is lower than $\hat{p}_w(s = \overline{\theta}, \tau = 0)$. It is immediate from Lemma 3 that the expected global welfare is higher under the alternative agreement than under the original agreement. If $\hat{s} \leq \overline{\theta}$ ($\hat{s}' = s$), then there exist some types $\theta < \hat{s}$ for which the home welfare is higher under the alternative agreement than under the original agreement. Intuitively, by offering a flexible use of subsidy for types at the bottom (an interval of types adjoining the type 0), the alternative agreement expands the original policy set while it preserves the highest possible world price $\hat{p}_w(s = \hat{s}, \tau = 0)$ in the original set; hence, it does not lower the world price for any $\theta$. Therefore, the agreement in the limiting case is not optimal.

To summarize, a zero-tariff agreement is not optimal regardless of whether the regulation on subsidy choices is lax or stringent. In fact, this finding can be directly extended to any agreement in which tariffs are bound to a constant level for all $\theta$.

**Proposition 3.** An agreement in which import tariffs are bound to zero for all $\theta$ is not optimal regardless of the degree to which the use of subsidy is regulated: it can be improved

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\footnote{We can also extend our argument to show that an agreement is not optimal when its policy set includes line segments with zero tariffs such as $\{(s, \tau) : ([\hat{s}_1, \overline{s}_1], 0), \ldots, ([\hat{s}_K, \overline{s}_K], 0)\}$.}
on by an alternative agreement in which, for some \( \theta \) at the bottom, the world price is constant and the use of subsidy is flexible.

Our analysis builds on a central trade-off faced by governments: a flexible use of subsidy can promote the domestic productive efficiency and yet it incurs the informational cost of high import tariffs. We find that any optimal agreement contains both rigid and flexible treatments of subsidy. Proposition 1 shows that an optimal agreement does not adhere strictly to the targeting principle in its treatment of subsidy: a partially rigid treatment of subsidy is necessary to lower import tariffs and increase trade volume. Proposition 2 and 3 show that an optimal agreement allows positive import tariffs and affords a flexible use of subsidy: a flexible treatment of subsidy is necessary to expand the policy choice set and promote the domestic productive efficiency for some \( \theta \). In regard to the continuing attempts by the WTO to regulate the use of subsidy, our findings have an important implication: the subsequent attempts to improve on any tariff-liberalization agreement by strengthening the regulation on subsidy choices may be on the wrong track if governments are not granted some flexibility to address market imperfections in the import-competing sector.

3.4 Flexible and Rigid Treatment of Subsidy

In light of the previous analyses, we find that, for any distribution function \( F \), any optimal agreement contains both rigid and flexible treatments of subsidy. In this subsection, we show that any optimal agreement exhibits these two conflicting aspects in two different ranges of externality types: there exist forces in favor of flexibility for types at the bottom (sorting interval adjoining the lowest-externality type 0) and in favor of rigidity for types at the top (pooling interval adjoining the highest-externality type \( \theta \)).

We now present two important features that are found in any optimal agreement. First, any optimal agreement exhibits sorting for types at the bottom, \([0, \theta_c]\). Intuitively, having a sorting interval at the bottom, an agreement can grant the home government a flexible subsidy choice for types at the bottom without lowering the world price for higher types (without imposing a negative externality on the foreign welfare). Second, any optimal agreement entails a pooling interval for types at the top, \([\theta^e, \bar{\theta}]\) where \( \theta^e \geq \theta_c \). Intuitively, a rigid treatment of subsidy for types at the top has a positive externality on the global welfare for lower types: it serves to increase the world price in particular for lower types in the sorting interval where a flexible use of subsidy is allowed as long as it preserves the associated world price. These two findings are now summarized.

**Proposition 4.** In any optimal agreement, (i) there exists \( \theta_c \in (0, \bar{\theta}) \) such that the policy
set entails a sorting interval for \( \theta \in [0, \theta_c] \) in which the world price is constant and the use of subsidy is flexible and (ii) there exists \( \theta^* \in [\theta_c, \bar{\theta}] \) such that the policy set entails a pooling interval for \( \theta \in [\theta^*, \bar{\theta}] \) in which the use of subsidy is rigid with zero tariff.

The proof is in the Appendix. In summary, we first find that any optimal agreement contains two contrasting aspects of treating domestic subsidies: a flexible treatment may be associated with the message of the targeting principle and a rigid treatment may be associated with the attempts by the WTO to strengthen the regulation on subsidy choices. We next find that these two aspects are conditional on the externality scale: any optimal agreement treats subsidies in a flexible (rigid) manner when the production externality is sufficiently small (large). An immediate finding also follows from the sorting interval at the bottom. For any continuous distribution \( F \), there is no possibility of selecting zero subsidy in any optimal agreement: a strictly positive subsidy is surely observed in any optimal agreement.

We now present a more comprehensive characterization an optimal agreement. We first show that any optimal agreement maintains the maximal world price in the sorting interval at the bottom: the world price for \( \theta \in [0, \theta_c] \) is at least as high as the world price for \( \theta > \theta_c \). The sorting interval at the bottom may be denoted by

\[
\{(s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(s(\theta_c), \tau(\theta_c)) \text{ for all } s \leq s(\theta_c) = \theta_c\}. \tag{18}
\]

Suppose that, in an optimal agreement, there exists a type \( \theta_0 > \theta_c \) such that \( \tilde{p}^w(s(\theta_0), \tau(\theta_0)) > \tilde{p}^w(s(\theta_c), \tau(\theta_c)) \). For this agreement to be incentive compatible, \( s(\theta_0) > s(\theta_c) \) and the policy set must be located in the region:

\[
\{(s, \tau) : W(s, \tau; \theta_c) \leq W(s(\theta_c), \tau(\theta_c); \theta_c) \text{ and } W(s, \tau; \theta_0) \leq W(s(\theta_0), \tau(\theta_0); \theta_0)\}. \tag{19}
\]

We next find the policy point that maximizes \( \tilde{p}^w(s, \tau) \) in the set:

\[
\{(s, \tau) : W(s, \tau; \theta_0) = W(s(\theta_0), \tau(\theta_0); \theta_0)\}. \tag{20}
\]

This set is the iso-welfare function for type \( \theta_0 \) that crosses the point \( (s(\theta_0), \tau(\theta_0)) \). The world price increases as the iso-world price function, \( \{(s, \tau) : \tilde{p}^w(s, \tau) = k \text{ for a constant } k > 0\} \), shifts down along the set (20). The world price is maximized at the point where the iso-world price function is tangential to the set (20) if the tangent point is above zero tariff or at zero tariff. Otherwise, the world price is maximized at the point where the iso-world price function crosses the set (20) at zero tariff. Let the point be denoted by \( (s_0, \tau_0) \). It then follows that

\[20\text{If } s(\theta_0) \leq s(\theta_c), \text{ then the policy mix } (s(\theta_0), \tau(\theta_0)) \text{ must be located below the iso-world price function that corresponds to the sorting interval at the bottom. The policy mix will not be chosen by any type.} \]
If the policy set includes a jump, then any subsidy increase associated with the jump lowers levels because of incentive compatibility. If the jump occurs above zero tariff then an alternative agreement exists that has a result is thus confirmed.

\{ (s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(s_0, \tau_0) \text{ for all } s \leq s_0 \}.

The policy set for \( s > s_0 \) remains the same. Observe that, for all \( \theta \leq s_0 \), the policy choice is made along this sorting interval: \( s(\theta) = \theta \) for all \( \theta \leq s_0 \). Since the sorting interval at the bottom is lengthened at a higher world price, the expected global welfare increases. We can extend this argument to show that, in any optimal agreement, the world price is nonincreasing in \( \theta \): \( \tilde{p}^w(\theta_2, \tau(\theta_2)) \leq \tilde{p}^w(\theta_1, \tau(\theta_1)) \) for any \( \theta_2 > \theta_1 \).

We next show that, in any optimal agreement, the subsidy choice, \( s(\theta) \), is nondecreasing in \( \theta \). This result directly follows from the monotonicity of the world price as shown above. Suppose that an optimal agreement contains policy points, \( (s(\theta_1), \tau(\theta_1)) \) and \( (s(\theta_2), \tau(\theta_2)) \), such that \( s(\theta_2) \leq s(\theta_1) \) for some \( \theta_2 > \theta_1 \). The monotonicity of the world price and incentive compatibility of type \( \theta_1 \) imply that \( (s(\theta_2), \tau(\theta_2)) \) must be located in the region:

\[ \{ (s, \tau) : \tilde{p}^w(s, \tau) \leq \tilde{p}^w(s(\theta_1), \tau(\theta_1)) \text{ and } W(s, \tau; \theta_1) \leq W(s(\theta_1), \tau(\theta_1); \theta_1) \}. \quad (21) \]

A contradiction is then generated: \( (s(\theta_1), \tau(\theta_1)) \) is preferred to \( (s(\theta_2), \tau(\theta_2)) \) for type \( \theta_2 \). Hence, \( s(\theta) \) is nondecreasing in \( \theta \) in any optimal agreement.

A difficulty with analyzing an optimal agreement arises in particular when the policy set is not continuous and includes jumps. To describe how a jump behaves, we first find that, if the policy set includes a jump, then any subsidy increase associated with the jump lowers the world price: if an optimal agreement includes a jump from a policy point \( (s(\theta_1), \tau(\theta_1)) \) to \( (s(\theta_2), \tau(\theta_2)) \) where \( s(\theta_2) > s(\theta_1) \), then \( \tilde{p}^w(s(\theta_1), \tau(\theta_1)) > \tilde{p}^w(s(\theta_2), \tau(\theta_2)) \). Observe that the inequality \( s(\theta_2) > s(\theta_1) \) implies \( \theta_2 \geq \theta_1 \) and that incentive compatibility implies \( \theta_2 \neq \theta_1 \). It follows that \( \theta_2 > \theta_1 \) and so \( \tilde{p}^w(s(\theta_1), \tau(\theta_1)) \geq \tilde{p}^w(s(\theta_2), \tau(\theta_2)) \). If the jump occurs at zero tariff \( (\tau(\theta_1) = 0) \), then the strict inequality, \( \tilde{p}^w(s(\theta_1), \tau(\theta_1)) > \tilde{p}^w(s(\theta_2), \tau(\theta_2)) \), immediately holds. If the jump occurs above zero tariff \( (\tau(\theta_1) > 0) \) and \( \tilde{p}^w(s(\theta_1), \tau(\theta_1)) = \tilde{p}^w(s(\theta_2), \tau(\theta_2)) \), then an alternative agreement exists that has a continuous policy set for \( s \in [s(\theta_1), s(\theta_2)] \):

\[ \{ (s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(s(\theta_1), \tau(\theta_1)) \text{ for } s \in [s(\theta_1), s(\theta_2)] \}. \quad (22) \]

The policy set for \( s \notin [s(\theta_1), s(\theta_2)] \) remains the same. The change in the policy set increases the home welfare for some \( \theta \in (\theta_1, \theta_2) \) without affecting the foreign welfare for any \( \theta \). The result is thus confirmed.

\(^{21}\)The proof of this part is in the Appendix.

\(^{22}\)If an optimal agreement has a jump from one point to another, then the two points have different subsidy levels because of incentive compatibility.
As noted above, to achieve a higher global welfare in an interval of $\theta$, governments prefer to employ a sorting interval ($s(\theta) = \theta$ in that interval) at a higher (constant) world price. We now show that the presence of a jump in a policy set may act to reduce the global welfare in an interval of $\theta$: a jump entails a pooling interval and the subsidy increase associated with the jump lowers the world price. Suppose first that an optimal agreement involves a jump from the endpoint of the sorting interval at the bottom to another policy point $(s_1, \tau_1)$ where $s_1 > s(\theta_c)$. We then obtain the result: if the jump occurs above zero tariff ($\tau(\theta_c) > 0$), then the jump is made such that the policy point $(s(\theta_c), \tau(\theta_c))$ is indifferent to $(s_1, \tau_1)$ for type $\theta_c$: $W(s(\theta_c), \tau(\theta_c); \theta_c) = W(s_1, \tau_1; \theta_c)$. To see this, observe that $(s_1, \tau_1)$ should be selected from the region:

$$\{(s, \tau) : \bar{p}^w(s, \tau) < \bar{p}^w(s(\theta_c), \tau(\theta_c)) \text{ and } W(s, \tau; \theta_c) \leq W(s(\theta_c), \tau(\theta_c); \theta_c)\}. \quad (23)$$

Thus, the result does not hold only if $W(s(\theta_c), \tau(\theta_c); \theta_c) > W(s_1, \tau_1; \theta_c)$. This strict inequality in turn implies that there exists a type $\tilde{\theta} > \theta_c$ such that $(s(\theta_c), \tau(\theta_c))$ is preferred to $(s_1, \tau_1)$ for all $\theta \in [\theta_c, \tilde{\theta}]$. Then an alternative agreement exists that lengthens the sorting interval at the bottom along the iso-world price function $\{(s, \tau) : \bar{p}^w(s, \tau) = \bar{p}^w(s(\theta_c), \tau(\theta_c))\}$. The sorting interval can be lengthened up to the point $(\tilde{\theta}, \tau(\tilde{\theta}))$ if $\tau(\tilde{\theta}) \geq 0$. Otherwise, the sorting interval can be lengthened up to the point in which the iso-world price function crosses zero tariff. Letting this crossing point be $(s_0, 0)$, we define $s_c \equiv \min\{s_0, \tilde{\theta}\}$. The alternative agreement can then have the sorting interval at the bottom:

$$\{(s, \tau) : \bar{p}^w(s, \tau) = \bar{p}^w(s(\theta_c), \tau(\theta_c)) \text{ for all } s \leq s_c\}. \quad (24)$$

Since the expected global welfare increases with this change, the result is confirmed.23 This result can be readily extended to a jump that is made from any sorting interval above zero tariff.

Suppose next that an optimal agreement involves a jump from an endpoint of a sorting interval, $(s(\theta_1), \tau(\theta_1))$, to a starting point of another sorting interval, $(s_2, \tau_2)$, where $s_2 > s(\theta_1)$. We may then develop two additional points. First, since the two policy points, $(s(\theta_1), \tau(\theta_1))$ and $(s_2, \tau_2)$, are indifferent for type $\theta_1$, the agreement entails a pooling interval, $(\theta_1, s_2]$: pooling at $(s_2, \tau_2)$ for all $\theta \in (\theta_1, s_2)$. Second, the agreement entails an interval in which domestic distortions take the form of “over-subsidy”: $s(\theta) > \theta$ for all $\theta \in (\theta_1, s_2)$. Indeed, a jump from an endpoint of any sorting interval entails an analogous pooling interval with over-subsidy, as long as the jump is not followed by another jump that entails pooling. A similar

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23 The result is based on the assumption that the jump occurs above zero tariff ($\tau(\theta_c) > 0$); the sorting interval at the bottom cannot be lengthened when the jump is made at zero tariff. Note that any jump at zero tariff ($\tau(\theta_c) = 0$) entails pooling for a range of $\theta$, whether it is made from a sorting interval or a pooling point.
pooling interval with over-subsidy can also be observed when an optimal agreement involves two consecutive jumps: (i) jump from an endpoint of any sorting interval, \((s(\theta_1), \tau(\theta_1))\), to \((s_2, \tau_2)\) where \(s_2 > s(\theta_1)\) and (ii) jump from \((s_2, \tau_2)\) to \((s_3, \tau_3)\), where \(s_3 > s_2\). If the last point \((s_3, \tau_3)\) is not followed by another jump, then the agreement entails two consecutive pooling intervals with over-subsidy over the range \([\theta_1, s_3]\): there exists \(\tilde{\theta} \in (\theta_1, s_2]\) such that the agreement entails pooling at \((s_2, \tau_2)\) for all \(\theta \in (\theta_1, \tilde{\theta}]\) and pooling at \((s_3, \tau_3)\) for all \(\theta \in (\tilde{\theta}, s_3]\). We may extend these points and conclude that any jump entails an pooling interval with over-subsidy and lowers the world price.

Another scenario is the limiting case in which the number of jumps is infinitely large: an optimal agreement includes an interval in which subsidy choice is continuous and yet distorted \((s(\theta) \neq \theta)\). This case occurs when an agreement includes a continuous policy subset whose slope is flatter than the policy subset that entails a sorting interval in which \(s(\theta) = \theta\). Suppose, for example, that an agreement includes a continuous policy subset between two policy points, \((s(\theta_1), \tau(\theta_1))\) and \((s(\theta_2), \tau(\theta_2))\), whose slope is sufficiently flat. This agreement similarly entails an interval with over-subsidy: for all \(\theta \in [\theta_1, \theta_2]\), domestic distortions take the form of over-subsidy \((s(\theta) > \theta)\) and the world price falls along the interval.

We now summarize the findings.

**Proposition 5.** Allow for any optimal agreement. (i) The world price is nonincreasing in \(\theta\): thus, the world price is at least as high in the sorting interval at the bottom as in any other interval. (ii) The subsidy choice is nondecreasing in \(\theta\). (iii) If the agreement includes any policy subset in which the world price is not constant, then it entails an interval of types such that the world price is lower for types in that interval than for types below that interval and (b) \(s(\theta) > \theta\) for interior types of that interval.

In an optimal agreement, the policy subset that entails a sorting interval of \(\theta\) is the one in which the world price is constant: in the policy subset in which the world price is constant, the use of subsidy is flexible and externality types are truthfully revealed. Proposition 5 (iii) highlights the costs that governments would incur when their agreement includes a policy subset in which the world price is not constant: the presence of such a policy subset reduces the global welfare in an interval of \(\theta\). It, however, has a positive affect on the global welfare for \(\theta\) other than in that interval. For example, the presence of a jump in an interval serves to reduce import tariffs for types below that interval and also offer the possibility of employing

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24We ignore the case in which the policy set is steeper than the sorting interval in which \(s(\theta) = \theta\), since the world price is nonincreasing in \(\theta\) in an optimal agreement.
a sorting interval for types above that interval. Therefore, the existence and magnitude of “non-sorting” intervals are subtle and determined by the expected positive and negative effects that are conditional on the distribution function $F$.

We lastly present an example of an optimal agreement. Assume that demand and supply functions are linear in the home import-competing sector: $D(p^d) = \alpha - \beta p^d$ and $Q(p^s) = \gamma p^s$ for the home country and $D^*(p^{d*}) = \alpha - \beta p^{d*}$ and $Q^*(p^{s*}) = \gamma^* p^{s*}$ for the foreign country, where $(\alpha, \beta, \gamma, \gamma^*) > 0$ and $\gamma^* > \gamma$. Assume also that the marginal production externality, $\theta$, is uniformly distributed over the support $[0, \overline{\theta}]$. The key respect that governments would consider to increase the expected global welfare is how to lengthen the sorting intervals at higher world prices. A difficulty is that governments face two forces in conflict between lengthening a sorting interval and raising the world price in that interval. If governments intends to prolong the sorting interval at the bottom, for example, then they must lower the world price for that interval.

We therefore proceed with a simple agreement in which the sorting interval at the bottom crosses the pooling interval at the top: $\theta_c = \theta^c$ in Proposition 4. In this agreement, governments would find the optimal level of world price when they face two competing forces: a flexible use of subsidy for types below $\theta_c$ and a rigid use of subsidy for types above $\theta_c$. Following the proof of Proposition 1 in the Appendix, we can find the optimal level of $\theta_c$ (denoted by $\theta_c^{opt}$) that maximizes the expected global welfare. Specifically, if $\beta = 1$, $\gamma = \frac{1}{2}$ and $\gamma^* = 2$, then the optimal choice is unique at $\theta_c^{opt} = \frac{2}{3} \overline{\theta}$ for all $\alpha$ and all $\overline{\theta}$.

This specific example indicates that a flexible use of subsidy may be necessary to address market imperfections in a wide range of parameter.

A further possibility is that an agreement includes a policy subset in which the world price is not constant. We now argue that the optimal agreement may take a remarkable simple form: $\theta_c = \theta^c$. Fig. 1 illustrates an example in which the policy set includes a jump from $(s(\theta_c), \tau(\theta_c))$ to $(s(\theta_1), \tau(\theta_1))$ and entails pooling at $(s(\theta_1), \tau(\theta_1))$ for all $\theta \in (\theta_c, \theta_1]$. Since demand and supply functions are linear, the policy subset in which the world price is constant is linear. This policy set makes a parallel shift when the world price changes. The agreement with the jump offers a sorting interval up to higher types than an alternative agreement that has no such jump, though the jump lowers the world price for types above $\theta_1$. In contrast, the alternative agreement offers a unique sorting interval and incurs domestic distortions at zero tariff. We now find that, if $\overline{\theta}$ is sufficiently small, then this alternative agreement achieves a higher expected global welfare than the original agreement.

\[ W(s = \theta, \tau, \theta) \leq W(s = \theta + c, \tau; \theta). \]

Note that trade volume is always positive when $\alpha$ is sufficiently large.

Alternatively, we may assume that each type $\theta$ has a sufficiently large incentive to raise subsidy: for $c > 0$ sufficiently large, $W(s = \theta, \tau; \theta) \leq W(s = \theta + c, \tau; \theta)$. Intuitively, given...
that $\theta$ is uniformly distributed, the overall sorting interval that the alternative agreement offers at a higher expected world price is at least as long as the overall sorting interval that the original agreement offers. This finding can be extended to show that, to maximize the expected global welfare, governments would not gain from having any non-sorting interval under the assumptions.

4 Conclusions

In this paper, we investigate how domestic subsidies are treated when each government with private information has incentive to disguise its protective use of subsidies as a legitimate use of subsidies. We find that any optimal agreement entails a sorting interval at the bottom in which the world price is constant and the use of subsidy is flexible and also entails a pooling interval at the top in which the use of subsidy is rigid with zero tariff. A flexible treatment of subsidy at the bottom increases the home welfare for types in the interval without lowering the world price for higher types. A rigid treatment of subsidy at the top has a positive externality on the global welfare for lower types: it increases the world price in particular for lower types in the sorting interval. A flexible treatment may be associated with the message of the targeting principle and a rigid treatment may be associated with the attempts by the WTO to strengthen the regulation on subsidy choices.
5 Appendix A

We first show that the world price decreases in \( s \) and \( \tau \) in equilibrium. It is immediate from the market-clearing condition that

\[
\frac{\partial \hat{p}^w}{\partial s} = \frac{Q'}{D' - Q' - (Q'' - D''')} < 0
\]  
\( A1 \)

\[
\frac{\partial \hat{p}^w}{\partial \tau} = -\frac{D' - Q'}{D' - Q' - (Q'' - D'')} < 0.
\]  
\( A2 \)

We can also show that, in equilibrium, the domestic import decreases in \( s \) and \( \tau \): 

\[
\frac{\partial M}{\partial s} = \frac{\partial E^*}{\partial s} = (Q'' - D'') \frac{\partial \hat{p}^w}{\partial s} = \frac{(Q'' - D'')(Q')}{D' - Q' - (Q'' - D'')} < 0
\]  
\( A3 \)

\[
\frac{\partial M}{\partial \tau} = \frac{\partial E^*}{\partial \tau} = (Q'' - D'') \frac{\partial \hat{p}^w}{\partial \tau} = -\frac{(Q'' - D'')(D' - Q')}{D' - Q' - (Q'' - D'')} < 0.
\]  
\( A4 \)

We can finally show that the domestic production of import good increases in \( s \) and \( \tau \) in equilibrium:

\[
\frac{\partial Q}{\partial s} = Q \frac{\partial \hat{p}^s}{\partial s} = \frac{Q'(D' - (Q'' - D''))}{D' - Q' - (Q'' - D'')} > 0
\]  
\( A5 \)

\[
\frac{\partial Q}{\partial \tau} = Q \frac{\partial \hat{p}^s}{\partial \tau} = -\frac{Q'(Q'' - D'')}{D' - Q' - (Q'' - D'')} > 0.
\]  
\( A6 \)

From (A1)-(A4), we find that 

\[
-\frac{\partial \hat{p}^w/\partial s}{\partial \hat{p}^w/\partial \tau} = -\frac{\partial M/\partial s}{\partial M/\partial \tau} = \frac{Q'}{D' - Q'} < 0.
\]  
\( A7 \)

We then obtain two findings: (i) if the world price \( \hat{p}^w \) is constant in a set of \((s, \tau)\), then the equilibrium import volume \( M \) is also constant in that set and (ii) the slope \( \frac{\partial \tau}{\partial s} \) is strictly negative along the set.

**First-Best and Nash Policies.** Define \( \bar{p} \) by \( D(\bar{p}) = 0 \) and also define \( \underline{p} = \inf \{p : Q(p) > 0\} \). Consumer surplus and profit are respectively given by 

\[
CS(\hat{p}^d) = \int_{\hat{p}^d}^{\bar{p}} D(p)dp \text{ and } \Pi(\hat{p}^d) = \int_{\underline{p}}^{\bar{p}} Q(p)dp.
\]

We can directly find \( \frac{dCS(\hat{p}^d)}{dp} = -D(\hat{p}^d) \) and \( \frac{d\Pi(\hat{p}^d)}{dp} = Q(\hat{p}^d) \). We next write the pricing relationships: \( \hat{p}^d(s, \tau) = \hat{p}^w(s, \tau) + \tau \) and \( \hat{p}^s(s, \tau) = \hat{p}^w(s, \tau) + \tau + s \). Differentiating \( W^G(s, \tau; \theta) \) with respect to \( s \) and \( \tau \), we can find 

\[
\frac{\partial W^G(s, \tau; \theta)}{\partial s} = \tau \frac{\partial M}{\partial s} + [\theta - s] \frac{\partial Q}{\partial s} = 0
\]

\[
\frac{\partial W^G(s, \tau; \theta)}{\partial \tau} = \tau \frac{\partial M}{\partial \tau} + [\theta - s] \frac{\partial Q}{\partial \tau} = 0.
\]
We first show that \( s \leq \theta \). The first equation implies that, for a given \( \tau \geq 0 \), if \( s > \theta \), then \( \frac{\partial W_G(s, \tau; \theta)}{\partial \tau} < 0 \); hence, \( s \leq \theta \). Given \( s \leq \theta \), the second equation implies that, if \( \tau > 0 \), then \( \frac{\partial W_G(s, \tau; \theta)}{\partial \tau} < 0 \); hence, \( \tau = 0 \). It follows from the first equation that, if \( \tau = 0 \), then \( s = \theta \). Therefore, \( W_G(s, \tau; \theta) \) is maximized by \( \tau = 0 \) and \( s = \theta \). We next differentiate \( W(s, \tau; \theta) \) with respect to \( s \) and \( \tau \) under the assumption that \( W(s, \tau; \theta) \) is concave. We find

\[
\frac{\partial W(s, \tau; \theta)}{\partial s} = -\frac{\partial \overline{p}^w}{\partial s} M_x + \tau \frac{\partial M}{\partial s} + [\theta - s] \frac{\partial Q}{\partial s} = 0
\]

\[
\frac{\partial W(s, \tau; \theta)}{\partial \tau} = -\frac{\partial \overline{p}^w}{\partial \tau} M_x + \tau \frac{\partial M}{\partial \tau} + [\theta - s] \frac{\partial Q}{\partial \tau} = 0.
\]

These first-order conditions are satisfied when \( s = \theta \) and

\[
\tau = \frac{\frac{\partial \overline{p}^w}{\partial \tau}}{\frac{\partial M}{\partial \tau}} = \frac{\frac{\partial \overline{p}^w}{\partial s}}{\frac{\partial M}{\partial s}}.
\]

Using (A1)-(A4), we can rewrite these equalities as \( \tau = \frac{E^*(\overline{p}^w)}{E^*(\overline{p}^w)} \).

**6 Appendix B: Proofs**

**Proof of Lemma 1.** We show that the home government with externality type \( \theta \) selects \( s = \theta \) under the policy set in which \( \overline{p}^w \) is constant. The policy set can be represented by a decreasing function \( \tau = \tau(s) \) where \( \tau'(s) < 0 \). The home government with externality type \( \theta \) then finds the level of \( s \) that maximizes \( W(s, \tau(s); \theta) \). Using the pricing relationships, \( \overline{p}^d = \overline{p}^w(s, \tau(s)) + \tau(s) \) and \( \overline{p}^s = \overline{p}^w(s, \tau(s)) + \tau(s) + s \), and using \( \frac{dC_S(\overline{p}^d)}{d\overline{p}^d} = -D(\overline{p}^d) \) and \( \frac{dM(\overline{p}^s)}{d\overline{p}^s} = Q(\overline{p}^s) \), we find

\[
\frac{\partial W(s, \tau(s); \theta)}{\partial s} = [Q - D] \left( \frac{\partial \overline{p}^w}{\partial \tau} \frac{d\tau}{ds} + \frac{\partial \overline{p}^w}{\partial \tau} \right) + \tau \frac{\partial M}{\partial \tau} \frac{d\tau}{ds} + \tau \frac{\partial M}{\partial s} + [\theta - s] \left( \frac{\partial Q}{\partial \tau} \frac{d\tau}{ds} + \frac{\partial Q}{\partial s} \right) \quad (A8)
\]

From (A7), it follows that the slope of the iso-world price function is

\[
\frac{d\tau}{ds} = -\frac{\frac{\partial \overline{p}^w}{\partial \tau}}{\frac{\partial \overline{p}^w}{\partial \tau}} = -\frac{\partial M}{\partial s} \quad (A9)
\]

The RHS of (A8) is then reduced to the last term:

\[
\frac{\partial W(s, \tau(s); \theta)}{\partial s} = [\theta - s] \left( \frac{\partial Q}{\partial \tau} \frac{d\tau}{ds} + \frac{\partial Q}{\partial s} \right) = [\theta - s] \frac{Q'D'}{D' - Q'}.
\]

The second equality of (A9) is given by (A5)-(A7). If \( s > \theta \), then \( \frac{\partial W(s, \tau(s); \theta)}{\partial s} < 0 \), and if \( s < \theta \), then \( \frac{\partial W(s, \tau(s); \theta)}{\partial s} > 0 \). Therefore, the home government with externality type \( \theta \) selects \( s = \theta \) under the policy set in which \( \overline{p}^w \) is constant. ■
Proof of Lemma 3. We here show that, if \( \tilde{\tau} \) rises from zero under the policy set (10), then the global welfare \( W^G(s(\theta), \tau(\theta); \theta) \) decreases for all \( \theta \): the welfare gain by the home government is less than the welfare loss by the foreign government. Under the set (10), as \( \tilde{\tau} \) rises from zero, the subsidy choice remains the same at \( s(\theta) = \theta \) and \( \tau(\theta) \) rises for all \( \theta \). Thus, if \( \tilde{\tau} \) rises from zero under the set (10), then \( \tilde{p}^\omega(s(\theta), \tau(\theta)) \) falls and thus \( W^*(s(\theta), \tau(\theta)) \) decreases for all \( \theta \). The decrease of \( W^*(s(\theta), \tau(\theta)) \) is given by \( \frac{dW^*(s, \tau)}{d\tilde{p}^\omega} = -D^*(\tilde{p}^\omega) + Q^*(\tilde{p}^\omega) = E^*(\tilde{p}^\omega) > 0 \). This last strict inequality holds because of the assumption, \( M(s = \overline{\theta}, \tau = 0) > 0 \). Finally, we can show that, given \( s(\theta) = \theta \), \( W^G(s(\theta), \tau; \theta) \) decreases in \( \tau \): for \( s(\theta) = \theta \), \( \frac{\partial W^G(s(\theta), \tau; \theta)}{\partial \tau} = \tau \frac{\partial M}{\partial \tau} < 0 \) for any \( \tau > 0 \).

Proof of Proposition 1. We consider an alternative agreement that has the policy set:

\[ \{(s, \tau) : \tilde{p}^\omega(s, \tau) = \tilde{p}^\omega(s = \theta^c, \tau = 0)\} \quad \text{where} \quad \theta^c < \overline{\theta}. \]  

(A10)

The world price is constant on the set; thus, the set can be represented by a strictly decreasing function, \( \tau = \tau(s) \). This agreement entails pooling for \( \theta \geq \theta^c \): for all \( \theta \in [\theta^c, \overline{\theta}] \), \( s(\theta) = \theta^c \) and \( \tau(\theta) = 0 \). It also involves sorting for \( \theta < \theta^c \): for all \( \theta \in [0, \theta^c) \), \( s(\theta) = \theta \) and the choice of \( \tau(\theta) \) is determined by the function \( \tau = \tau(s) \). With such policy choices, we may write the expected global welfare under the alternative agreement as

\[ \int_0^{\theta^c} W^G(s(\theta), \tau(\theta); \theta)dF(\theta) + \int_{\theta^c}^{\overline{\theta}} W^G(s = \theta^c, \tau = 0; \theta)dF(\theta). \]  

(A11)

As \( \theta^c \to \overline{\theta} \), the alternative agreement approaches the separating agreement. Differentiation of (A11) with respect to \( \theta^c \) is reduced to the two terms:

\[ \int_0^{\theta^c} \frac{\partial W^G(s(\theta), \tau(\theta); \theta)}{\partial \theta^c}dF(\theta) + \int_{\theta^c}^{\overline{\theta}} \frac{\partial W^G(s = \theta^c, \tau = 0; \theta)}{\partial \theta^c}dF(\theta). \]  

(A12)

For types \( \theta \in [0, \theta^c) \) in the separating interval, if \( \theta^c \) rises, then \( \tau(\theta) \) rises: the choice \( \tau(\theta) \) is given by

\[ \tau(\theta) = \frac{d\tau}{ds}[s(\theta) - \theta^c] = \frac{Q'}{D' - Q'}\theta^c \]  

(A13)

where the slope, \( \frac{d\tau}{ds} = \frac{Q'}{D' - Q'} \), is measured at the policy choices for \( \theta < \theta^c \). We can then find that the first term in (A12) is negative:

\[ \frac{\partial W^G(s(\theta), \tau(\theta); \theta)}{\partial \theta^c} = \frac{\partial W^G(s(\theta), \tau(\theta); \theta)}{\partial \tau} \frac{d\tau(\theta)}{d\theta^c} = \tau(\theta) \frac{\partial M}{\partial \tau} \frac{d\tau(\theta)}{d\theta^c} < 0 \]  

for \( \theta < \theta^c \).

(A14)

The inequality in (A14) is immediate from (A13). We can show that the second term in (A12) is positive. For types \( \theta \in [\theta^c, \overline{\theta}] \) in the pooling interval, if \( \theta^c \) rises, then \( s(\theta) \equiv \theta^c \) also
rises with $\tau(\theta) \equiv 0$. Following the same step as in (A8), we find that
\[
\frac{\partial W^G(s = \theta^c, \tau = 0; \theta)}{\partial \theta^c} = [\theta - \theta^c] \frac{\partial Q}{\partial \theta^c} > 0 \text{ for } \theta > \theta^c.
\] (A15)

If $\theta^c \to \bar{\theta}$, then the term in (A15) approaches zero while the term in (A14) remains negative. For $\theta^c$ sufficiently close to $\bar{\theta}$, if $\theta^c$ decreases from $\bar{\theta}$, then the expected global welfare in (A13) increases: hence, the separating agreement can be improved upon by the alternative agreement that entails pooling for some $\theta$ at the top, $[\theta^c, \bar{\theta}]$. ■

**Proof of Proposition 3.** We show that an agreement is not optimal when its policy set is $\{(s, \tau) : (s_1, 0), (s_2, 0)\}$ where $s_1 = 0$. We first define a positive subsidy $s_0 > 0$ such that the home government with the lowest type 0 is indifferent between $(0, 0)$ and $(s_0, 0)$:
\[W(s = 0, \tau = 0; \theta = 0) = W(s = s_0, \tau = 0; \theta = 0).\]

It then follows that $s_2 \geq s_0$. If $s_2 < s_0$, then none of types will choose $(s_1, 0)$ and the policy set is reduced to a singleton. Since an agreement with a singleton is not optimal, we focus on an agreement in which the policy set is $\{(s, \tau) : (0, 0), (s_2, 0)\}$ where $s_2 \geq s_0$. The policy set has two possibilities: (i) $s_2 = s_0$ and (ii) $s_2 > s_0$.

Consider first the case (i) in which the policy set is $\{(s, \tau) : (0, 0), (s_0, 0)\}$. In this agreement, for $\theta = 0$, the home government selects $(0, 0)$ and for types $\theta > 0$, it selects $(s_0, 0)$. We construct an alternative agreement that has the policy set:
\[\{(s, \tau) : \bar{\rho}^w(s, \tau) = \bar{\rho}^w(s = s_0, \tau = 0)\}.\] (A16)

This alternative set can be uniquely represented by a decreasing function $\tau = \tau^A(s)$ for $s \leq s_0$. Observe that, for all types $\theta > 0$, this alternative set preserves the original world price $\bar{\rho}^w(s = s_0, \tau = 0)$. Because of this adjustment, for $\theta \in (0, s_0)$, the home welfare increases and thus the global welfare increases:
\[W^G(s^A(\theta), \tau^A(\theta); \theta) > W^G(s = s_0, \tau = 0; \theta) \text{ for } \theta \in (0, s_0).\]

Note that $s^A(\theta) = \theta$ and $\tau^A(\theta) > 0$. For $\theta = 0$, however, the global welfare is lower under the alternative agreement than under the original policy mix $(0, 0)$:
\[W^G(s^A(\theta), \tau^A(\theta); \theta) < W^G(s = 0, \tau = 0; \theta) \text{ for } \theta = 0.\]

For types $\theta \geq s_0$, the home welfare does not decrease and thus the global welfare does not decrease; the alternative set (A16) expands the choice set while it preserves the original world price $\bar{\rho}^w(s = s_0, \tau = 0)$. We can then compare the expected global welfare under the two agreements. Under the alternative agreement, the expected global welfare is
\[
\int_{0}^{\theta} W^G(s^A(\theta), \tau^A(\theta); \theta) dF(\theta) + \text{prob}(\theta = 0) \cdot W^G(s^A(\theta), \tau^A(\theta); \theta = 0).
\]
Under the case (i), the expected global welfare is

\[ \int_{\theta} W^G(s = s_0, \tau = 0; \theta)dF(\theta) + \text{prob}(\theta = 0) \cdot W^G(s = 0, \tau = 0; \theta = 0). \]

Since \( \text{prob}(\theta = 0) = 0 \) under the continuous distribution \( F \), we conclude that the expected global welfare is higher under the alternative agreement than under the case (i). Note lastly that we may restrict attention to \( s_0 < \theta \); if \( s_0 \geq \theta \), then the world price is at least as high as the one in the best separating agreement and thus the alternative agreement is not optimal.

Consider next the case (ii) in which the policy set is \( \{(s, \tau) : (0, 0), (s_2, 0)\} \) where \( s_2 > s_0 \). Given that \( (0, 0) \) and \( (s_0, 0) \) are indifferent for the lowest type 0 and that \( s_2 > s_0 \), there exists \( \hat{\theta} > 0 \) such that \( (0, 0) \) is preferred to \( (s_2, 0) \) for \( \theta \in [0, \hat{\theta}] \) and \( (s_2, 0) \) is preferred to \( (0, 0) \) for \( \theta \in (\hat{\theta}, \overline{\theta}) \). Thus, \( \hat{\theta} > 0 \) is the critical type at which \( (0, 0) \) and \( (s_2, 0) \) are indifferent:

\[ W(s = 0, \tau = 0; \hat{\theta}) = W(s = s_2, \tau = 0; \hat{\theta}). \]

Selecting a subsidy \( \hat{s} \in (0, \hat{\theta}) \), we construct an alternative agreement in which the policy set is \( \{(s, \tau) : (\hat{s}, 0), (s_2, 0)\} \). Given that \( (0, 0) \) and \( (s_2, 0) \) are indifferent for type \( \hat{\theta} \), we find that \( (\hat{s}, 0) \) is preferred to \( (s_2, 0) \) for all types \( \theta \leq \hat{\theta} \). There also exist some types \( \theta \in (\hat{\theta}, \overline{\theta}) \) for which \( (\hat{s}, 0) \) is preferred to \( (s_2, 0) \). For all types \( \theta \in (\hat{\theta}, \overline{\theta}) \), the world price in the alternative set is at least as high as the one in the original set. Thus, in order to show that the expected global welfare increases with the new policy set, it suffices to compare the policies for \( \theta \in [0, \hat{\theta}] \) and show that

\[ \int_{\theta} W^G(s = \hat{s}, \tau = 0; \theta)dF(\theta) > \int_{\theta} W^G(s = 0, \tau = 0; \theta)dF(\theta). \]  \hspace{1cm} (A17)

To compare the two pooling policies for \( \theta \in [0, \hat{\theta}] \), we find \( (s = s^p, \tau = 0) \) that maximizes

\[ \int_{\theta} W^G(s = s^p, \tau = 0; \theta)dF(\theta). \]  \hspace{1cm} (A18)

We differentiate this term with respect to \( s^p \):

\[ \frac{\partial}{\partial s^p} \int_{\theta} W^G(s = s^p, \tau = 0; \theta)dF(\theta) = [\int_{\theta} \theta dF(\theta) - s^p] \frac{\partial Q}{\partial s^p}. \]

The term (A18) is maximized when \( s^p = \int_{\theta} \theta dF(\theta) > 0 \). Since we can always set \( \hat{s} = \int_{\theta} \theta dF(\theta) \), the inequality (A17) holds. \( \blacksquare \)

\textbf{Proof of Proposition 4.} (i) We first provide the proof for part (i). In three steps, we present the features that the policy set \( \{(s, \tau)\} \) in any optimal agreement must satisfy.
Step 1: We show that, in any optimal agreement, policies for type 0 satisfies $s(0) = 0$ and $\tau(0) > 0$. Assume that $s(0) > 0$ in an optimal agreement. We construct an alternative agreement in which the policy choice set expands to the subsidy $s \leq s(0)$:

$$\{(s, \tau) : \hat{p}^w(s, \tau) = \hat{p}^w(s(0), \tau(0)) \text{ for } s \in [0, s(0)]\}. \quad (A19)$$

The policy set for $s > s(0)$ remains the same. For some $\theta \in [0, s(0)]$, the home welfare then increases without lowering the world price for any type. This generates a contradiction; $s(0) = 0$ in an optimal agreement. Given $s(0) = 0$, assume next that $\tau(0) = 0$ in an optimal agreement. As in the proof of Proposition 3, we can consider two cases: the policy mix $(b, \tau)$ is chosen (i) only by type $\theta = 0$ or (ii) by types $\theta \in [0, \theta]$. A similar procedure confirms that an alternative agreement improves on the original agreement. This causes a contradiction; $\tau(0) > 0$ in an optimal agreement.

Step 2: We establish that, given that $s(0) = 0$ and $\tau(0) > 0$, any optimal agreement entails a sorting interval at the bottom. Assume to the contrary that an optimal agreement involves pooling at $(s(0), \tau(0))$ for $\theta \in [0, \theta_c]$. Consider the policy set that is inferior to $(s(0), \tau(0))$ for type $\theta_c$:

$$\{(s, \tau) : W(s, \tau; \theta_c) \leq W(s(0), \tau(0); \theta_c)\}. \quad (A20)$$

This set is the region below the function $W(s, \tau; \theta_c)$ that crosses the point $(s(0), \tau(0))$. For the original agreement to be incentive compatible, policies for $\theta > \theta_c$ should be in this region. To find an alternative agreement, we consider the policy mix $(s_c, \tau_c)$ that maximizes $\hat{p}^w(s, \tau)$ subject to the iso-welfare function:

$$\{(s, \tau) : W(s, \tau; \theta_c) = W(s(0), \tau(0); \theta_c)\}. \quad (A21)$$

Since $\hat{p}^w(s, \tau)$ rises as the associated iso-world price function shifts down, we can find a point at which the iso-world price function is tangential to the function $(A21)$. This tangent point is the policy mix $(s_c, \tau_c)$ that maximizes $\hat{p}^w(s, \tau)$ within the set $(A21)$. The tangent point satisfies $s_c = \theta_c$ for the same reason shown in Lemma 1.

We now construct an alternative agreement for two cases: (a) $\hat{p}^w(s_c, \tau_c) \geq \hat{p}^w(s, \tau)$ for any $(s, \tau)$ in $(A20)$ and (b) $\hat{p}^w(s_c, \tau_c) < \hat{p}^w(s, \tau)$ for some $(s, \tau)$ in $(A20)$. For the case (a), we define an alternative agreement that has the policy set for $s \leq s_c = \theta_c$:

$$\{(s, \tau) : \hat{p}^w(s, \tau) = \hat{p}^w(s_c, \tau_c) \text{ for all } s \leq s_c = \theta_c\}. \quad (A22)$$

The policy set for $s > \theta_c$ remains the same. In this set, the world price is constant at $\hat{p}^w(s_c, \tau_c) > \hat{p}^w(s(0), \tau(0))$. It then follows that, for any policy mix $(s, \tau)$ in $(A20)$,

$$W(s_c, \tau_c; \theta_c) = W(s(0), \tau(0); \theta_c) \geq W(s, \tau; \theta_c).$$
This inequality means that, for all $\theta \leq \theta_c$, policies in the set (A22) are preferred to any policies in (A20). The alternative agreement then raises the world price for all $\theta \leq \theta_c$ when the subsidy choice for $\theta \leq \theta_c$ is made along the iso-world price set (A22): $s(\theta) = \theta$ for all $\theta \leq \theta_c$. Further, since $\hat{p}^w(s_c, \tau_c) \geq \hat{p}^w(s, \tau)$ for any $(s, \tau)$ in (A20), the alternative agreement does not lower the world price for any $\theta > \theta_c$ even when some types $\theta > \theta_c$ prefers to switch to policies in the alternative set (A22). In comparison with the original agreement, the alternative agreement entails sorting along with a higher world price for all $\theta \leq \theta_c$, and so achieves a higher global welfare for those types. This agreement neither decreases the home welfare for any $\theta > \theta_c$ nor lowers the world price for any $\theta$ even when types $\theta > \theta_c$ switch to policies in the set (A22). Hence, the expected global welfare is higher under the alternative agreement. This contradicts the assumption that an optimal agreement involves pooling at $(s(0), \tau(0))$ for $\theta \in [0, \theta_c]$.

For the case (b), suppose that there exists a policy mix $(s', \tau')$ in (A20) such that $\hat{p}^w(s_c, \tau_c) < \hat{p}^w(s', \tau')$. Without loss of generality, we pick a policy mix $(s', \tau')$ that satisfies $\hat{p}^w(s', \tau') \geq \hat{p}^w(s, \tau)$ for any $(s, \tau)$ in (A20). We first show that $s' > 0$. To see this, we recall $\hat{p}^w(s_c, \tau_c) > \hat{p}^w(s(0), \tau(0))$ and observe

$$\hat{p}^w(s', \tau') > \hat{p}^w(s_c, \tau_c) > \hat{p}^w(s(0), \tau(0)).$$

If $s' = 0$, then it must follow from these inequalities that $\tau' < \tau(0)$ and that $(s', \tau')$ is never chosen by any $\theta$: hence, $s' > 0$. Given $s' > 0$, we next define an alternative agreement that has the policy set for all $s \leq s'$:

$$\{(s, \tau) : \hat{p}^w(s, \tau) = \hat{p}^w(s', \tau') \text{ for all } s \leq s'\}.$$  \hspace{1cm} (A23)

The policy set for $s > s'$ remains the same as in the original agreement. We recall the original agreement in which $(s(0), \tau(0))$ is selected by $\theta \in [0, \theta_c]$; thus, there exists a type $\hat{\theta} > \theta_c$ for which $(s', \tau')$ is at least weekly preferred to any other choice $(s, \tau)$ with $s > s'$ in the set (A20). This means that, for all $\theta \leq \hat{\theta}$, the policy choice is made along the iso-world price set (A23): $s(\theta) = \theta$ for all $\theta \leq \hat{\theta}$. In comparison with the original agreement, the alternative agreement entails sorting along with a higher world price for some $\theta \leq \hat{\theta}$ and yet it does not lower the world price for any $\theta$. Hence, the expected global welfare is higher under the alternative agreement. This contradicts the assumption that an optimal agreement involves pooling at $(s(0), \tau(0))$ for $\theta \in [0, \theta_c]$. Lastly, note that we restrict attention to the case of $s' < \hat{\theta}$. The reason is that the alternative agreement with $s' \geq \hat{\theta}$ is not optimal, since the best separating agreement is not optimal.

**Step 3:** We lastly show that, having a sorting interval for types $\theta \in [0, \theta_c]$, any optimal agreement involves at least as high world price for $\theta \in [0, \theta_c]$ as for $\theta \in (\theta_c, \bar{\theta}]$. Assume to
the contrary that there exists a policy mix \((s', \tau')\) such that \(\hat{p}^w(s', \tau') > \hat{p}^w(s(\theta_c), \tau(\theta_c))\) in the set \((A20)\). We first show that, if such a policy mix \((s', \tau')\) exists, then \(s' > s(\theta_c) = \theta_c\). If \(s' \leq \theta_c\), then there exists a policy mix in the sorting interval such that \(s(\theta) = s'\) and \(\tau(\theta) > \tau'\) for \(\theta \in [0, \theta_c]\). The policy mix \((s', \tau')\) is then never selected by any \(\theta\); hence, \(s' > \theta_c\). The remaining proof is analogous to the case (b) in Step 2.

(ii) We next provide the proof for part (ii). In three steps, we present the features that the policy set \(\{(s, \tau)\}\) in any optimal agreement must satisfy.

**Step 1:** We show that, if any optimal agreement involves a jump from a policy mix \((s_1, \tau_1)\) to \((s_2, \tau_2)\) where \(s_2 > s_1\), then (a) \(\hat{p}^w(s_1, \tau_1) > \hat{p}^w(s_2, \tau_2)\) and (b) when the jump is made above zero tariff \((\tau_1 > 0)\), the level of jump is made such that \((s_1, \tau_1)\) and \((s_2, \tau_2)\) are indifferent for the type \(\theta = s_1\) and the agreement entails pooling at \((s_2, \tau_2)\) for all \(\theta \in (s_1, s_2)\):

\[
W(s_1, \tau_1; \theta) = W(s_2, \tau_2; \theta) \quad \text{for} \quad \theta = s_1 \\
W(s_1, \tau_1; \theta) < W(s_2, \tau_2; \theta) \quad \text{for all} \quad \theta \in (s_1, s_2].
\]

Without loss of generality, we consider the case in which a jump is made from the sorting interval at the bottom, \([0, \theta_c]\), and so \((s_1, \tau_1)\) is an endpoint of the sorting interval: \(s_1 = \theta_c\).

For the proof for part (a), we refer to Step 3 in the proof (i), which implies \(\hat{p}^w(s_1, \tau_1) \geq \hat{p}^w(s_2, \tau_2)\). Further, if \(\hat{p}^w(s_1, \tau_1) = \hat{p}^w(s_2, \tau_2)\), then the sorting interval at the bottom can be extended to a new endpoint \((s_2, \tau_2)\). The home government can then be better off for \(\theta \in (s_1, s_2)\) without lowering the world price for any \(\theta\). This result contradicts the optimality of the original agreement: hence, if an optimal agreement involves a jump from \((s_1, \tau_1)\) to \((s_2, \tau_2)\), then \(\hat{p}^w(s_1, \tau_1) > \hat{p}^w(s_2, \tau_2)\). We next provide the proof for part (b). Because of part (a) and incentive compatibility for \(\theta_c\), the policy mix \((s_2, \tau_2)\) must be located within the set:

\[
\{(s, \tau) : \hat{p}^w(s_1, \tau_1) > \hat{p}^w(s, \tau) \quad \text{and} \quad W(s_1, \tau_1; \theta_c) \geq W(s, \tau; \theta_c)\}.
\]

Assume to the contrary that \((s_2, \tau_2)\) satisfies \(W(s_1, \tau_1; \theta_c) > W(s_2, \tau_2; \theta_c)\) in an optimal agreement. This means that \((s_2, \tau_2)\) is located below the set \(\{(s, \tau) : W(s, \tau; \theta_c) = W(s_1, \tau_1; \theta_c)\}\) in which the home welfare is constant at \(W(s_1, \tau_1; \theta_c)\). Then there exists \(\hat{\theta} > \theta_c\) such that, for \(\theta \in [\theta_c, \hat{\theta}]\), \((s_1, \tau_1)\) is preferred to \((s_2, \tau_2)\). We now construct an alternative agreement by lengthening the original sorting interval at the bottom up to the type \(\hat{\theta} > s_1 = \theta_c\) while preserving the original world price \(\hat{p}^w(s_1, \tau_1)\):

\[
\{(s, \tau) : \hat{p}^w(s, \tau) = \hat{p}^w(s_1, \tau_1) \quad \text{for all} \quad s \leq \hat{\theta}\}.
\]

The policy set for \(s > \hat{\theta}\) remains the same. Since the sorting interval at the bottom is lengthened while the associated lowest world price is preserved, this change increases the
global welfare for the extended range \((\theta_c, \bar{\theta})\) without lowering the world price for any other \(\theta\). It increases the expected global welfare and generates a contradiction: hence, if an optimal agreement involves a jump from \((s_1, \tau_1)\) to \((s_2, \tau_2)\) when \(\tau_1 > 0\), then \(W(s_1, \tau_1; \theta_c) = W(s_2, \tau_2; \theta_c)\). It also directly follows that \((s_2, \tau_2)\) is preferred to \((s_1, \tau_1)\) for all \(\theta \in (s_1, s_2)\) in an optimal agreement.

Step 2: We show that, in any optimal agreement, there exists a type \(\theta^c \geq \theta_c\) such that the policy set entails a pooling interval at the top, \([\theta^c, \bar{\theta}]\). Assume to the contrary that, in an optimal agreement, there exists a type \(\theta_2 \geq \theta_c\) such that the policy set involves a sorting interval at the top for \(\theta \in [\theta_2, \bar{\theta}]\). Since the full sorting scheme for all \(\theta \in [0, \bar{\theta}]\) is not optimal, we restrict attention to the case in which an optimal agreement involves pooling at \((s_2, \tau_2)\) for \(\theta \in (\theta_1, \theta_2]\). This means that the agreement involves a jump by the type \(\theta_1 = s_1\) from a policy mix \((s_1, \tau_1)\) to \((s_2, \tau_2)\) such that \((s_1, \tau_1)\) and \((s_2, \tau_2)\) are indifferent for \(\theta_1\): \(W(s_1, \tau_1; \theta_1) = W(s_2, \tau_2; \theta_1)\).

We first show that, if the policy set involves a sorting interval at the top, \([\theta_2, \bar{\theta}]\), then it takes the form for any \(s \geq \theta_2\):

\[
\{(s, \tau) : \tilde{p}^w(s, \tau) \geq \bar{p}^w(s = \bar{\theta}, \tau = 0) \text{ for all } s \geq \theta_2\}.
\]  

(A24)

The reason is that a sorting interval at the top cannot contain any policy mix at which the world price is lower than \(\bar{p}^w(s = \bar{\theta}, \tau = 0)\). By shifting down the iso-world price set (A24), we next construct an alternative policy set for all \(s \geq \theta_2\):

\[
\{(s, \tau) : \tilde{p}^w(s, \tau) = \bar{p}^w(s = \theta^c, \tau = 0) \text{ for all } s \geq \theta_2\}.
\]  

(A25)

The policy set for \(\theta < \theta_2\) remains the same. If \(\theta^c < \bar{\theta}\), then this set involves pooling at \((\theta^c, 0)\) for \(\theta \in [\theta^c, \bar{\theta}]\) and sorting for \(\theta \in [\theta_2, \theta^c]\). If \(\theta^c \rightarrow \bar{\theta}\), the alternative set (A25) approaches the original set (A24). We differentiate the expected global welfare under the alternative set with respect to \(\theta^c\):

\[
\int_{\theta_2}^{\theta^c} \frac{\partial W^G(s(\theta), \tau(\theta); \theta)}{\partial \theta_c} dF(\theta) + \int_{\theta^c}^{\bar{\theta}} \frac{\partial W^G(s = \theta^c, \tau = 0; \theta)}{\partial \theta_c} dF(\theta).
\]  

(A26)

We directly follow the proof of Proposition 1 to show that, if \(\theta_c \rightarrow \bar{\theta}\), then the first term in (A26) remains negative while the second term approaches zero. For \(\theta_c\) sufficiently close to \(\bar{\theta}\), if \(\theta_c\) decreases from \(\bar{\theta}\), then the expected global welfare increases. For \(\theta_c < \bar{\theta}\), however, the endpoint of the new sorting interval for \(\theta \in [\theta_2, \theta^c]\) is located below the set \(\{(s, \tau) : W(s, \tau; \theta_1) = W(s_1, \tau_1; \theta_1)\}\). From this set, we can select \((s_2', \tau_2')\) that is indifferent to \((s_1, \tau_1)\) for type \(\theta_1\) and preserves the world price:

\[
W(s_2', \tau_2'; \theta_1) = W(s_1, \tau_1; \theta_1) \text{ and } \tilde{p}^w(s_2', \tau_2') = \tilde{p}^w(s = \theta^c, \tau = 0).
\]
Observe that $s_2' < s_2 = \theta_2$. We next construct another policy set for all $s \geq s_2'$:

$$\{(s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(s = \theta^c, \tau = 0) \text{ for all } s \geq s_2'\}. \tag{A27}$$

The policy set for $s < s_2'$ remains the same. Consistent with the result in Step 1 (b), an endpoint of (A27) is now indifferent to $(s_1, \tau_1)$ for type $\theta_1$. This set now has a longer sorting interval, $[s_2', \theta^c)$, with the world price unaffected. The expected global welfare is at least as high in (A27) as in (A25). We thus conclude that an optimal agreement cannot entail a sorting interval at the top.

**Step 3:** We prove that any optimal agreement entails pooling at the top with zero tariffs.

Assume that an optimal agreement entails pooling at $(s^p, \tau^p)$ with $\tau^p > 0$ for $\theta \in [\hat{\theta}, \overline{\theta}]$. We select the policy mix $(\tilde{s}, \tilde{\tau})$ that maximizes $\tilde{p}^w(s, \tau)$ within the set: $\{(s, \tau) : W(s, \tau; \hat{\theta}) = W(s^p, \tau^p; \hat{\theta})\}$. The policy mix $(\tilde{s}, \tilde{\tau})$ is the point at which the iso-world price function is tangential to the set, which means that $\tilde{s} = \hat{\theta}$. We next construct an alternative policy set for all $s \geq \tilde{s} = \hat{\theta}$:

$$\{(s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(\tilde{s}, \tilde{\tau}) \text{ for all } s \geq \tilde{s} = \hat{\theta}\}.$$

The policies for $s < \tilde{s}$ remain the same. The alternative agreement involves a sorting interval for $\theta \in [\hat{\theta}, \overline{\theta}]$ while it keeps the world price $\tilde{p}^w(\tilde{s}, \tilde{\tau}) \geq \tilde{p}^w(s^p, \tau^p)$ for those types. The expected global welfare thus increases, which causes a contradiction: hence, any optimal agreement entails pooling at the top with zero tariffs. ■

**Proof of Proposition 5.** (i) We extend the finding from the main text: $\tilde{p}^w(s(\theta_0), \tau(\theta_0)) \leq \tilde{p}^w(s(\theta_c), \tau(\theta_c))$ for any $\theta_0 > \theta_c$. We here show that, in an optimal agreement, $\tilde{p}^w(s(\theta_2), \tau(\theta_2)) \leq \tilde{p}^w(s(\theta_1), \tau(\theta_1))$ for any $\theta_2 > \theta_1$. Suppose that an optimal agreement satisfies $\tilde{p}^w(s(\theta_2), \tau(\theta_2)) > \tilde{p}^w(s(\theta_1), \tau(\theta_1))$ for some $\theta_2 > \theta_1$. Incentive compatibility then implies that the assumed agreement has the policy set in the region:

$$\{(s, \tau) : W(s, \tau; \theta_c) \leq W(s(\theta_c), \tau(\theta_c); \theta_c) \text{ and } W(s, \tau; \theta_2) \leq W(s(\theta_2), \tau(\theta_2); \theta_2)\}. \tag{A28}$$

Observe that the policy point $(s(\theta_1), \tau(\theta_1))$ is located in between the iso-welfare function for type $\theta_2$ and the iso-world price function:

$$\{(s, \tau) : W(s, \tau; \theta_2) = W(s(\theta_2), \tau(\theta_2); \theta_2)\} \tag{A29}$$

$$\{(s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(s(\theta_2), \tau(\theta_2))\}. \tag{A30}$$

We first find that the inequality, $s(\theta_1) \leq s(\theta_2)$, holds; if $s(\theta_1) > s(\theta_2)$ holds, then $(s(\theta_2), \tau(\theta_2))$ is preferred to $(s(\theta_1), \tau(\theta_1))$ for type $\theta_1$, which is a contradiction. We next find the policy point that maximizes $\tilde{p}^w(s, \tau)$ within the iso-welfare function for type $\theta_2$ shown in (A29). The world price is maximized at the point where the iso-world price function, $\{(s, \tau) : \tilde{p}^w(s, \tau) = k$
for a constant \( k > 0 \), is tangential to the set \((A29)\) if the tangent point is above zero tariff or at zero tariff. Otherwise, the world price is maximized at the point where the iso-world price function crosses the set \((A29)\) at zero tariff. Let the point be denoted by \((s_2, \tau_2)\). It then follows that \( \tilde{p}^w(s_2, \tau_2) \geq \tilde{p}^w(s(\theta_2), \tau(\theta_2)) \). We now explore two possibilities: (a) \( \tilde{p}^w(s(\theta_c), \tau(\theta_c)) \leq \tilde{p}^w(s_2, \tau_2) \) and (b) \( \tilde{p}^w(s(\theta_c), \tau(\theta_c)) > \tilde{p}^w(s_2, \tau_2) \).

For the case (a), we consider an alternative agreement in which the sorting interval at the bottom can be lengthened up to \((s_2, \tau_2)\) at the world price \( \tilde{p}^w(s_2, \tau_2) \geq \tilde{p}^w(s(\theta_c), \tau(\theta_c)) \):

\[
\{ (s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(s_2, \tau_2) \text{ for all } s \leq s_2 \}.
\]

The policy set for \( s > s_2 \) remains the same. For all \( \theta \leq s_2 \), the policy choice is made along this sorting interval: \( s(\theta) = \theta \) for all \( \theta \leq s_2 \). Since the sorting interval at the bottom is lengthened at the world price \( \tilde{p}^w(s_2, \tau_2) > \tilde{p}^w(s(\theta_2), \tau(\theta_2)) \), the expected global welfare increases. For the case (b), we use the iso-world price function \( \{(s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(s_2, \tau_2)\} \). Since \( \tilde{p}^w(s(\theta_c), \tau(\theta_c)) > \tilde{p}^w(s_2, \tau_2) \), there exists a point at which the iso-world price function crosses the iso-welfare function for type \( \theta_c \):

\[
\{ (s, \tau) : W(s, \tau; \theta_c) = W(s(\theta_c), \tau(\theta_c); \theta_c) \}.
\]

Let the point be \((s_0, \tau_0)\). We then consider an alternative agreement in which the policy set for \( s \in [s_0, s_2] \) has a constant world price \( \tilde{p}^w(s_2, \tau_2) \geq \tilde{p}^w(s(\theta_2), \tau(\theta_2)) \):

\[
\{ (s, \tau) : \tilde{p}^w(s, \tau) = \tilde{p}^w(s_2, \tau_2) \text{ for all } s \in [s_0, s_2] \}.
\]

The policy set for \( s \notin [s_0, s_2] \) remains the same. For all \( \theta \in [s_0, s_2] \), the policy choice is made along this sorting interval: \( s(\theta) = \theta \) for all \( \theta \in [s_0, s_2] \). Located in the region \((A28)\), the new sorting interval, \([s_0, s_2]\), increases the global welfare in that interval without affecting the global welfare in the other interval. Hence, in both cases, there exists an alternative agreement that improves on the original agreement. ■

7 References


