International Market Rivalry in a Developing Country and the Optimal Export Policy: A General Equilibrium Analysis of Strategic Trade Policy and Urban Unemployment

(Tentative Title)

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Abstract

The purpose of this paper is to investigate what is the optimal export policy for a developed country (DC) in a general equilibrium model of a developing country (LDC) with urban unemployment a la Harris and Todaro (HT: 1970). Even if DC firm competes a la Cournot against LDC firm (strategic substitute) in a urban manufactured good market, the optimal export policy can be an export tax, not a subsidy, because of the presence of income effect on imperfectly competitive manufactured good. First, an export subsidy decreases the aggregate income of an LDC. If it shifts down the market demand curve for manufactured good, the DC firm’s output and profit will decrease in equilibrium. Second, the optimal export policy for DC government is an export tax if LDC firm’s marginal cost is not much lower than DC firm’s marginal cost, given the preference for urban manufactured good, rural technology and population. Third, given the preference for urban manufactured good in the LDC market with international rivalry, the range of parameter values for optimal export tax is larger when marginal product of rural labor and/or population are smaller.

Key Words: strategic trade policy, urban unemployment, export tax

JEL Classification Numbers: F12, F13, F16

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1. Introduction

One prominent feature of the world economy has been the presence of international market rivalry among oligopolistic firms. Studies on strategic trade policy since 1980s have focused on the situation where firms of developed countries compete in imperfectly competitive markets located in a developed country (DC). As is well known, the optimal export policy for foreign government is an export subsidy in a model where home and foreign firms compete \textit{a la} Cournot, exporting a homogeneous good to the foreign market. This is because foreign export subsidy shifts rent (profit) from home firm to foreign firm, enhancing foreign welfare (Brander and Spencer (1985)).

In recent years international market rivalry has been observed frequently in LDC markets as well (For example, DC automakers such as Japan compete against local automakers such as Tata in Indian domestic markets). This raises a new question of how international rivalry among DC and LDC firms in a local (LDC) manufactured good market can interact with the domestic economic structure of an LDC. When foreign (DC) government provides an export subsidy to foreign firms, it reduces home (LDC) firms’ market share and shifts their profits to foreign country (DC). This may decrease the aggregate income (GDP) of an LDC not only by the reduction in manufacturing production but also by an expansion of urban unemployment due to rural-urban migration. In turn, a decrease in aggregate income may shift down the LDC market demand curve for urban manufactured good. If this shift is large enough, foreign (DC) export subsidy will reduce foreign firm’s profit (net of subsidy) and foreign welfare. Then, the optimal export policy for DC government can be an export tax, not subsidy.

In the previous studies on strategic trade policy that focus on international market rivalry in a developed country, the economic logic mentioned above has been ignored, even in the studies that use a general equilibrium framework. Brander and Spencer (1985) engaged in a general equilibrium analysis of export subsidy. In section 3 which examines how the deterioration of terms of trade might affect the incentive for choosing export subsidy, they separated the cases of segmented and integrated markets. For the latter case, they pointed out the possibility that home export subsidy decreased foreign firm’s output, the aggregate income and thus the market demand curve of foreign country. However, in their model of integrated markets, the authors regarded the income effects on imperfectly competitive goods as small because the effects of a reduction in foreign demand curve and of a rise in home demand curve are partially offset. They assume a quasi-linear utility function with no income effects on imperfectly competitive
goods (in the second paragraph on p.92). They did not even comment on it for the model of segmented markets. Chao and Yu (1997) analyzed effects of trade liberalization (a relaxation of import quota) in a general equilibrium model of a dualistic economy *a la* Harris and Todaro (HT: 1970) with oligopolistic urban manufacturing sector. Here again, they assume a quasi-linear utility function, ignoring the income effects on the manufactured good. Thus they did not take into consideration the possibility of shifting down the market demand curve for manufactured good.

However, Jones (1987, 1995) and Tawada and Okawa (1995) investigated trade policies by explicitly considering income effects on demand for good supplied by a monopolist in a general equilibrium model. Although they recognized the importance of income effects and a shift in demand curve in general equilibrium, they did not derive optimal trade policies under international oligopoly and unemployment.

The purpose of this paper is to investigate what is the optimal export policy for a developed country (DC) in a general equilibrium model of a developing country (LDC) with urban unemployment *a la* Harris and Todaro (1970). First, we show that an export subsidy of DC government decreases the aggregate income of an LDC and that if it shifts down the market demand curve for the manufactured good, the DC firm’s output and profit may decrease in equilibrium. The setting of HT model with urban unemployment, one should note, is not necessary for analyzing this economic mechanism. We present an analysis of a full employment model in Appendix F, showing that the optimal policy can be either an export subsidy or an export tax. In other words, the conclusion that the optimal export policy is an export subsidy under Cournot competition (strategic substitute) is not robust theoretically. Second, we proceed to derive the optimal export policy in the HT model. The optimal export policy for DC government is an export tax if LDC firm’s marginal cost is not much lower than DC firm’s marginal cost, given the preference for urban manufactured good, rural technology and population. Third, we examine when the possibility of optimal export tax is large. Given the preference for urban manufactured good in the LDC market with international rivalry, the range of parameter values for optimal export tax is larger when marginal product of rural labor and/or population are smaller.

2. The Model

We consider a general equilibrium model of the dualistic economy *a la* Harris and Todaro. While rural sector is perfectly competitive, the market for urban manufactured
goods has a duopolistic structure: one firm of this developing country (LDC) competes \textit{a la} Cournot against one firm who produces a homogeneous product in a developed country (DC) and exports it to this developing country. Rural product $x$ is the numeraire. The price of the urban manufactured good is $p$. The urban wage rate $w$ is institutionally fixed. The DC government provides the DC firm with specific export subsidy $s$ while the LDC government implements no policy.

\textbf{2.1 Consumers}

The representative consumer has a homothetic (homogeneous of degree one) utility function $U(D_x,D_y)$, where $D_x$ and $D_y$ are the consumption of rural good and urban manufactured good, respectively. We can thereby capture the income effect on the market demand for urban manufactured good. The first-order condition for utility maximization implies that the relative demand is a function of the relative price:

$$\frac{D_x}{D_y} = \phi(p)$$

with $\phi'(p) > 0$. The budget constraint $D_x + pD_y = I$ can be rewritten as

$$\left[p + \phi(p)\right]D_y = I,$$

where $I$ is the aggregate income. Thus the demand function for the urban manufactured good is

$$D_y = \frac{I}{p + \phi(p)}.$$

By solving this with respect to $p$, we obtain the inverse demand function $p = P(D_y,I)$.

Let us notice two properties of this inverse demand function. First, given the aggregate income $I$, a higher value of $p$ implies a higher value of $p + \phi(p)$ and thus a lower value of $D_y$:

$$P_1(y + Y, I) = \frac{\partial p}{\partial D_y} = -\frac{p + \phi(p)}{[1 + \phi'(p)]D_y} = -\frac{I}{[1 + \phi'(p)]D_y} \leq 0$$

Partially differentiating this with respect to $I$ leads to

$$P_{1i}(y + Y, I) = \frac{\partial P_1}{\partial I} = -\frac{1}{[1 + \phi'(p)]D_y} < 0$$

Second, given $D_y$, a higher value of $I$ implies a higher value of $p$, i.e., $P_2(D_y,I) =$...
\[ \frac{\partial P}{\partial I} = 1/D_y[1+\phi'(p)] > 0 \] holds. The aggregate income equals the real GDP:

\[ I = x + P(y + Y, I) \]

(1)

2.2 Urban Manufacturing Sector

The production function of the LDC manufacturing firm is \( y = L_y/m \) \((m > 0 \text{ is a constant})\). This firm chooses output \( y \) so that his profit \( \pi_y = p(y + Y, I) y - wmy \) is maximized, given the DC firm’s output \( Y \) and the aggregate income \( I \). Because the marginal cost of the LDC firm is \( mw \), his reaction function is

\[ P(y + Y, I) + yP_1(y + Y, I) = mw \]

(2)

Similarly, the profit of the DC firm is \( \pi_y = P(y + Y, I) + sY - CY \) and his reaction function is

\[ P(y + Y, I) + YP_1(y + Y, I) = C - s \]

(3)

where \( C \) is a constant marginal cost. We assume the strategic substitute relationship between firms, i.e., each firm’s marginal profit decreases as the rival firm’s output increases:

Assumption : \( P_y(y + Y, I) + yP_{11}(y + Y, I) < 0 \), \( P_1(y + Y, I) + YP_{11}(y + Y, I) < 0 \)

Note that under Assumption the second order condition for profit maximization is satisfied, i.e., \( 2P_1(y + Y, I) + yP_{11}(y = Y, I) < 0 \) and \( 2P_1(y + Y, I) + YP_{11}(y + Y, I) < 0 \) hold.

2.3 Rural Sector and labor Market

The production function of the representative rural firm is

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1 The conclusions remain essentially the same if we use a general function \( L_y = m(y) \) which is increasing and convex in output.

2 See Appendix F for the case in which firms correctly recognize a shift of the market demand curve caused by a change in the aggregate income \( I \) when choosing his output level. See also section 4.5 for a related explanation.
In the rural labor market equilibrium, the marginal product of rural labor equals the rural wage rate $w_r$:

$$w_r = f'(L_x)$$

(5)

where $L_x$ is rural labor employment.

The manufacturing labor employment is

$$L_y = my$$

(6)

Defining the urban unemployment ratio as $\mu = L_u / L_y$, the labor allocation in the entire economy is

$$L_x + (1 + \mu)L_y = L$$

(7)

As in Harris and Todaro (1970), the migration between rural and urban areas ceases when the expected urban wage rate $w_L/(L_y + L_u)$ equals the rural wage rate $w_r$:

$$w_r = \frac{w}{1 + \mu}$$

(8)

This is the “Harris-Todaro migration equilibrium condition (HT condition)”.

2.4 Solving the Model

In this model, given $L$, $w$, $C$, and $s$, eight equilibrium conditions (1) – (8) determine the equilibrium values of eight endogenous variables $(I, y, Y, x, w_r, L_x, L_y, \mu)$. Let us explain how to solve this model. From (4), (1) can be rewritten as

$$I = f(L_x) + p(y + Y, I)y$$

(9)

Next, we show that $L_x$ is a function of $y$. First, (7) is

$$L_x + (1 + \mu)my = L$$

(10)

Second, (8) can be transformed into

$$(1 + \mu)f'(L_x) = w$$

(11)
Solving (11) w.r.t. $1 + \mu$ and substituting it into (10), we obtain

$$ f'(L_x)[L - L_x] = mw y $$

(12)

Thus, we find $L_x = g(y)$, where $g'(y) = mw / \{f''(L_x)[L - L_x] - f'(L_x)\} < 0$. Therefore, (9) turns out to be

$$ I = f(g(y)) + p(y + Y, I) y $$

(13)

This model has a block recursive structure. Three equations (2), (3) and (13) determine equilibrium values of three endogenous variables $(y^*, Y^*, I^*)$. The equilibrium value of $L_x^*$ is obtained from (12). Substituting it into (11), we get the equilibrium value of $\mu^*$.

3. Comparative Statics of Export Subsidy

In this section we examine how a rise in export subsidy $s$ may affect the equilibrium by comparative statics. This is useful for deriving the optimal export policy of DC government in the next section.

First, totally differentiating (2), (3) and (13) yields

$$
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
dy \\
dY \\
dI
\end{bmatrix}
=
\begin{bmatrix}
0 \\
-1 \\
0
\end{bmatrix}
ds
$$

(14)

where each element is as follows (We show $a_{31} > 0$ in the Appendix A) :

- $a_{11} = 2P_1(y + Y, I) + yP_{11}(y + Y, I) < 0$
- $a_{12} = P_1(y + Y, I) + yP_{11}(y + Y, I) < 0$
- $a_{13} = P_1(y + Y, I) + yP_{12}(y + Y, I)$
- $a_{21} = P_1(y + Y, I) + yP_{11}(y + Y, I) < 0$
- $a_{22} = 2P_1(y + Y, I) + yP_{11}(y + Y, I) < 0$
- $a_{23} = P_1(y + Y, I) + yP_{12}(y + Y, I)$
- $a_{31} = P(y + Y, I) + yP_1(y + Y, I) + f'(L_x)g'(y) > 0$
- $a_{32} = yP_1(y + Y, I) < 0$
- $a_{33} = yP_2(y + Y, I) - 1$
Using the Cramer rule, we obtain the comparative static results.

\[
\frac{dy^*}{ds} = \frac{a_{13}a_{33} - a_{13}a_{32}}{\text{Det } A} \tag{15}
\]

\[
\frac{dY^*}{ds} = \frac{a_{13}a_{31} - a_{13}a_{33}}{\text{Det } A} \tag{16}
\]

\[
\frac{dl^*}{ds} = \frac{a_{13}a_{32} - a_{13}a_{31}}{\text{Det } A} < 0 \tag{17}
\]

Assuming that the equilibrium is perfectly stable, the determinant of the coefficient matrix A on the left-hand side of (14) has a positive sign, i.e., \( \text{Det } A < 0 \) (see Appendix B). As (17) shows, the export subsidy of DC government unambiguously decrease the aggregate income of LDC in this model. In order to elucidate the effect of a change in urban unemployment, we decompose the change in \( I \):

\[
I = x + py = (w_x L_x + \pi_x) + (w_L y + \pi_y)
\]

\[
= \frac{w}{1 + \mu} \left( L_x + w L_y + \pi_x + \pi_y \right)
\]

\[
= \frac{w}{1 + \mu} \left[ L_x + (1 + \mu)L_y \right] + (x - w_x L_x) + (py - w L_y)
\]

\[
= \frac{w}{1 + \mu} \left( L - w_x L_x + x + (py - wmy) \right)
\]

\[
= \frac{w[L - L_x]}{1 + \mu} + x + (p - mw)y \tag{18}
\]

The first term represents the effect of urban unemployment ratio \( \mu \). The second term is the value of rural product \( (x) \), which represents the effect of rural-urban migration. The third term represents the distortion due to duopolistic competition caused by the deviation of the price from the marginal cost.

Suppose that the income effect is absent (or very weak) on the market demand for urban manufactured good (because \( P_2 = 0 \) holds, \( a_{13} = 0 \) and \( a_{33} = -1 \)). Then we obtain \( \frac{dy^*}{ds} < 0 \) and \( \frac{dY^*}{ds} > 0 \). Base on this case, we describe the effect on the aggregate income in more details. A rise in export subsidy decreases LDC firm’s output \( y^* \) and thus manufacturing labor employment \( L_y \). The expected urban wage rate falls short of rural wage rate. The resulting urban-to-rural migration increases rural population \( L_x \) and lowers rural wage rate \( w_y \). Thus the urban unemployment ratio \( \mu \) necessarily rises. The first term on the right-hand side of (18) represents that a decrease in \( y \) (a decrease in \( L_y \) and an increase in \( L_x \)) and a rise in \( \mu \) lower the aggregate
income $I$. On the contrary, the second term represents that an increase in $x$ raises the aggregate income $I$. According to (17), we know the overall effects (including the third term) decrease the aggregate income $I$.

Furthermore, let us ask when an increase in urban unemployment enhances a decrease in the aggregate income $I$. The aggregate income $I$ decreases to a greater extent when a rise in $\mu$ is larger and thus the level of urban unemployment $L_u = \mu L_y$ declines more greatly. The more greatly the urban unemployment ratio $\mu$ rises, the more greatly $w_x$ declines. Intuitively, when the marginal product $f'(L_x)$ of rural labor curve has a steeper slope, rural wage rate declines faster. Thus the number of migrants from urban to rural area is smaller until the expected wage rates are equalized. In this situation, the level of urban unemployment increases more greatly. To sum up, when the rural technology exhibits stronger diminishing returns, the urban unemployment effect tends to be stronger and therefore the aggregate income decreases to a greater extent.

A change in the equilibrium output $Y^*$ of DC firm depends on the decrease in the aggregate income $I$ mentioned above and how much the demand for urban manufactured good in the LDC market respond to a change in the aggregate income $I$ (income elasticity of demand). If $P_z(y + Y, I) > 0$ is large enough, $a_{13} = P_z(y + Y, I)$ + $y P_{12}(y + Y, I) > 0$ and $a_{33} = y P_z(y + Y, I) - 1 > 0$ hold. Then, under $a_{31} > 0$, the nominator of (16) is positive and thus $Y^*$ decreases. Therefore, the equilibrium profit of DC firm decreases.

**Proposition 1**

*Suppose that a firm producing in a developed country (DC) exports a manufactured good and competes a la Cournot against a firm in a market of a developing country (LDC) with urban unemployment a la Harris and Todaro. Assume that the income effect on urban manufactured good is present. Suppose that the DC government subsidizes export of the DC firm. Then, (a) if rural technology exhibits sufficiently strong diminishing returns, and/or (b) if the market demand for urban manufactured good responds sensitively enough to a change in the aggregate income, the equilibrium output and profit of the DC firm decrease.*
One may criticize against this proposition because the impact of a change in the manufacturing income on the aggregate income (GDP) must probably be weak in a developing country where rural production is dominant. However, one should pay attention to the fact that the income elasticity of manufactured good tends to be high in developing countries. Therefore, even if a decrease in the aggregate income is small, the corresponding downward shift of the market demand curve for urban manufactured good can be large.

In the analysis above, we have focused on the general equilibrium effect of DC government’s export subsidy on the market demand curve in an LDC with urban unemployment. However, this economic logic clearly holds even if we assume a full employment economy. In other words, if we extend a partial equilibrium model of strategic trade policy into a general equilibrium model in which the income effect is present on imperfectly competitive good, the similar result can be derived. Furthermore, when export subsidy decreases the equilibrium output of DC firm, the optimal export subsidy for DC government can be export tax, not subsidy (Appendix F explicitly shows these results).

What differs between our model and the full employment model? In both models the DC government’s export subsidy decreases the LDC aggregate income. In our model, the expansion of urban unemployment enhances the decrease in the aggregate income, in addition to the effect of decreasing the manufacturing income which is also present in the full employment model.

4. Optimal Export Policy for Developed Country

In this section we proceed to derive the optimal export policy for the DC government. As is well known, it is an export subsidy in the traditional partial-equilibrium model of strategic trade policy and in the general-equilibrium model with no income effects on imperfectly competitive good. In contrast, in our general-equilibrium model with income effects on imperfectly competitive urban manufactured good, it can be either export subsidy or export tax even if the outputs of firms are strategic substitutes.

In the following analysis we investigate when the optimal export policy is export tax. However, if one assumes a homothetic utility function, the analyses are too complicated to derive clear implications. Instead, we make use of a simple linear demand function.

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3 The calculations turns out very complicated even if we use Cobb-Douglas utility function
with income effects.

**4.1 Formulation of Linear Model**

We specify the inverse demand function and the rural production function as follows:

\[ p = \theta I - (y + Y) \quad \theta > 0 \]
\[ f(L_r) = RL_r \quad R > 0 \]

The parameter \( \theta \) represents the strength of consumers’ preferences for urban manufactured goods.\(^4\) A high value of \( \theta \) means that, given the aggregate income \( I \), the market demand \( y + Y \) is larger for any price level \( p \). The parameter \( R \) represents the marginal product of rural labor.

First, we examine how a rise in export subsidy \( s \) may affect the duopoly equilibrium by comparative statics. In our linear model, (2), (3) and (13) are

\[ \theta I - 2y - Y = mw \]
\[ \theta I - y - 2Y = C - s \]
\[ I = [RL - wmy] + [\theta I - (y + Y)]y \]

(20)
(21)
(22)

Totally differentiating them yields

\[
\begin{bmatrix}
-2 & -1 & \theta \\
-1 & -2 & \theta \\
0 & -y & \theta y -1
\end{bmatrix}
\begin{bmatrix}
dy \\
dY \\
dI
\end{bmatrix}
= \begin{bmatrix}
0 \\
-ds \\
0
\end{bmatrix}
\]

(23)

The comparative static results are

\[
\frac{dy^*}{ds} = \frac{1}{Det.A}
\]
\[
\frac{dY^*}{ds} = \frac{2(\theta y - 1)}{Det.A}
\]
\[
\frac{dI^*}{ds} = \frac{2y}{Det.A}
\]

(24)
(25)
(26)

where \( Det.A = 2(\theta y - 1) - 1 \). We separate two cases depending on the sign of \( \theta y - 1 \).

In the first case where \( \theta y - 1 < 0 \) holds, \( Det.A < 0 \) necessarily holds. Thus we get \( dY^* / ds > 0 \) as well as \( dy^* / ds < 0 \) and \( dI^* / ds < 0 \). This is the typical situation of strategic trade policy: the export subsidy of DC government decreases LDC firm’s output and increases DC firm’s output in equilibrium. The aggregate income of LDC

\[ U = D^1 - y D^0 \] and use inverse demand function \( p = \theta I / D_y \).

\(^4\) A utility function that generates this demand function may depend on the aggregate income \( I \). This is not always a problem. I thank Professor Hiroshi Kurata for useful discussion about this point.
decreases because DC firm deprived of the rent from LDC firm. In this case, however, the decrease in \( I \) (the absolute value of (26)) is relatively small in the sense that \( y < (1/\theta) \) holds. Therefore, DC firm’s output increases.

In the second case where \( \theta y - 1 > 0 \) holds, under perfect stability of equilibrium, \( \text{Det}. A < 0 \) holds. While \( dy^*/ds < 0 \) and \( dI^*/ds < 0 \) hold as in the first case, \( dY^*/ds < 0 \) holds. In this case, the decrease in \( I \) (the absolute value of (26)) is relatively large in the sense that \( y > (1/\theta) \) holds. Therefore, DC firm’s output decreases.

In both cases, a decrease in \( y^* \) induces an increase in \( L_i^* \), and thus a rise in \( \mu^* \) by (11).

### 4.2 Conditions for Optimal Export Policy

Next we examine under what conditions the optimal export policy is export tax.

The economic welfare of DC is \( W^* = \pi_i^* - sY^* = P(y^* + Y^*, I^*)Y^* - CY^* \). The export subsidy rate \( s^o \) that maximizes welfare must satisfies the first order condition \( dW^*/ds = 0 \). That is,

\[
y^*P_1(y^* + Y^*, I^*)(dy^*/ds) + [P(y^* + Y^*, I^*) + Y^*P_2(y^* + Y^*, I^*) - C](dY^*/ds) + Y^*P_2(y^* + Y^*, I^*)(dI^*/ds) = 0
\]

(27)

By (3), we have

\[
s(dy^*/ds) = Y^*P_1(y^* + Y^*, I^*)(dy^*/ds) + Y^*P_2(y^* + Y^*, I^*)(dI^*/ds)
\]

In our linear model, it is

\[
2s[\theta y^*(s) - 1] = Y^*(s)[2\theta y^*(s) - 1]
\]

(28)

Let us derive the value of \( y^*(s) \). With (20), substituting \( \theta I - (y + Y) = y + mw \) into (22) we get \( I = RL + y^2 \). Substituting this into (20), solving it w.r.t. \( Y \), and substituting the resulting equation to (21), we obtain \( \theta y(s)^2 - 3y(s) + F(s) = 0 \), where

\[
F(s) = -s + [(\theta RL - mw) - (mw - C)]
\]

Therefore the duopoly equilibrium output levels are

\[
y^*(s) = \frac{3 \pm \sqrt{9 - 4\theta F(s)}}{2\theta}
\]

(29)

In order for them to be real numbers, we assume \( 4\theta F(s) \leq 9 \). The necessary and sufficient condition for it is
Appendix C shows that only the smaller solution of (29) satisfies the stability condition $\text{Det}.A = 2(\theta y - 1) - 1 < 0$. We thus focus on this solution in the following. The DC output in equilibrium can be obtained by subtracting (21) from (20):\[
Y^*(s) = y^*(s) + s + [mw - C]
\]Substituting (29) and (30) into (28), we obtain\[
2\theta s + [9 - 4\theta F(s)] + 2(2\theta M + 3) = (2\theta M + 5)\sqrt{9 - 4\theta F(s)}
\] (31)

4.3 Derivation of Optimal Export Subsidy

We derive the optimal export policy using figures. Let us denote the left-hand side of (31) as $\Delta(s)$ and the right-hand side of it as $\Gamma(s)$.

First, the graph of $\Delta(s)$ is increasing ($\Delta'(s) = 6\theta > 0$). Setting $s = 0$, we get $F(0) = (\theta RL - mw) - (mw - C) = N - M$, where $N = \theta RL - mw$ and $M = mw - C$ (both have an ambiguous sign). Using this, we have $\Delta(0) = [9 - 4\theta F(0)] + 2(2\theta M + 3) = 15 - 4\theta [N - 2M]$, whose sign is ambiguous even under $4\theta F(s) \leq 9$. The intersection with the horizontal axis can be derived by solving
\[ \Delta(s) = 0, \]
\[ s^D = \frac{2}{3}(N - 2M) - \frac{5}{2\theta} \]  \hspace{1cm} (32)

Second, the graph of \( \Gamma(s) \) has a slope of
\[ \Gamma'(s) = \frac{2\theta(2\theta M + 5)}{\sqrt{9 - 4\theta F(s)}} \]  \hspace{1cm} (33)

with \( \Gamma(0) = (2\theta M + 5)\sqrt{9 - 4\theta F(0)} \). This graph can be either increasing or decreasing, depending on the sign of \( 2\theta M + 5 \).

We focus on the case of \( 2\theta M + 5 < 0 \) (\( M = mw - C < -(5/2\theta) < 0 \)), where DC firm’s marginal cost (\( C \)) is substantially higher than LDC firm’s marginal cost (\( mw \)). This situation must probably be more important and appropriate at least in two respects from the viewpoint of the setting of our model. First, the fixed urban wage rate (\( w \)) is substantially lower than DC wage rate. Second, because we assume the duopoly between DC and LDC firms the marginal product of manufacturing labor (\( 1/m \)) is so high that LDC firm can compete against (coexist with) DC firm.\(^5\)

In this case, the graph of \( \Gamma(s) \) is a decreasing curve which intersects at \( \Gamma(0) < 0 \) with the vertical axis. The intersection \( s^G \) with the horizontal axis can be derived by solving \( \Gamma(s) = 0 \):
\[ s^G = (N - M) - \frac{9}{4\theta} \]  \hspace{1cm} (34)

Under \( 2\theta M + 5 < 0 \), \( \Gamma(s) \) takes a negative values. Thus effective part of the \( \Gamma(s) \) curve is southeastern part of \( s^G \). The optimal export subsidy \( s^O \) is determined by \( \Delta(s^O) = \Gamma(s^O) \). When the \( \Delta(s) \) curve intersects with the \( \Gamma(s) \) curve in the third orthant, the optimal policy is export tax, not subsidy. That is, if (a) \( 2\theta M + 5 < 0 \), (b) \( s^G < s^O \) and (c) \( \Delta(0) > \Gamma(0) \) all hold, the optimal policy is export tax (\( s^O < 0 \)).

Condition (a) is rewritten as
\[ mw + \frac{5}{2\theta} < C \]  \hspace{1cm} (35)

Condition (b) is
\[ \theta RL + \frac{3}{4\theta} < C \]  \hspace{1cm} (36)

The range of parameter values that satisfies condition (c) under condition(a) is

\(^5\) In the case of \( 2\theta M + 5 > 0 \), conditions for optimal export tax will be more complicated. However, this case should be less important for the reason mentioned above.
\((5/4\theta) > N - M\) (See Appendix D). Rearranging the terms, we have
\[
2mw + \left[\frac{5}{4\theta} - \theta RL\right] > C
\]

(37)

Thus we obtain the next lemma.

**Lemma 1.** In the duopoly model in which the demand function for urban manufactured good and rural production function are linear, the optimal export policy for DC government is determined by (31). If (35), (36) and (37) all hold, the optimal policy is an export tax.

### 4.4 Economic Situation for Optimal Export Tax

Let us investigate in what economic situation the optimal policy is an export tax. We denote the combination of \((mw, C)\) that satisfy (35), (36) and (37) with equality as aa line, bb line and cc line, respectively. We show the area that satisfies all the conditions (a), (b) and (c) in Figure 3 and 4.

First, aa line is a line with slope=1 and intersects with the C axis at \(5/2\theta > 0\). bb line is a horizontal line whose high is \(C = 0\theta RL + (3/4\theta) > 0\). cc line has a slope=2 and intersects with the C axis at \((5/4\theta) - \theta RL\) which can be either positive or negative. Note that \(5/2\theta > (5/4\theta) - \theta RL\). However, the other relations concerning the intersection with the C axis are ambiguous. We thus focus on two benchmark cases, the case of \(5/2\theta = 0\theta RL + (3/4\theta)\) and the case of \(5/2\theta = (5/4\theta) - \theta RL\).

In the first benchmark case, because \(5/2\theta = 0\theta RL + (3/4\theta)\) holds, \(\theta RL = 7/4\theta \in \mathbb{C}\) holds. The vertical intercept is \((5/4\theta) - \theta RL = -(1/2\theta) < 0\). Thus the area of \((mw, C)\) that satisfies (a), (b) and (c) can be shown as the shaded area (with downward-sloping stripes) in Figure 3. The shaded area with horizontal lines represents the combinations of \((mw, C)\) that induces the optimal export subsidy.
Here we note that the area for optimal export tax can be compatible with condition (S*). First, because (37) holds, we have \( (5/4 \theta) > C - 2mw + \theta RL \) and thus \( (9/4 \theta) > C - 2mw + \theta RL \) holds. The values of \( s \) that satisfy (S*) can be negative. Next, in the case of Figure 3, if (35) holds then (36) necessarily holds. In other words, \( mw + (5/2 \theta) > \theta RL + (3/4 \theta) \) and thus \( \theta RL - mw < (7/4 \theta) \) must hold. Under (35), the second term on the right-hand side of (S*) has a positive sign while the first term of it can have negative sign. Thus the values \( s \) of that satisfy (S*) can be negative.

Next, we consider how the area may change when marginal productivity \( R \) of rural labor (technology) and population \( L \) change, given preference parameter \( \theta \). The horizontal bb line lowers while cc line shifts upward. In this process the second benchmark case is reached as shown by Figure 4. The vertical intercepts of bb line and of cc line coincide at a positive value.
From above analyses, we obtain two results. First, we conclude that given $\theta$, $R$, and $L$, the optimal export policy for DC government is an export tax if $mw$ is not much lower than $C$. Second, given $\theta$, the area for optimal export tax is larger when $R$ and/or $L$ are smaller. (The dotted line in Figure 4 shows cc line before $R$ or $L$ change.) This establishes the next proposition.

**Proposition 2:**

In the duopoly model in which the demand function for urban manufactured good and rural production function are linear,

1. The optimal export policy for DC government is an export tax, not subsidy, if LDC firm’s marginal cost $mw$ is not much lower than DC firm’s marginal cost $C$, given the parameters for the preference for urban manufactured good $\theta$, rural technology $R$ and population $L$.

2. Given preference for urban manufactured good ($\theta$) in the LDC market with international rivalry, the possibility of optimal export tax is larger when marginal product of rural labor $R$ and/or population $L$ are smaller.

An intuitive explanation for (1) is as follows. If LDC firm’s marginal cost $mw$ is substantially lower than DC firm’s marginal cost $C$, the DC firm’s market share is very small in the initial equilibrium. Then an impact of his profit shifting on the aggregate
income of an LDC will be so weak that the downward shift of the market demand curve for urban manufactured good is moderate. Thus the optimal export policy for DC government is an export subsidy. In contrast, if LDC firm’s marginal cost \( mw \) is not much lower than DC firm’s marginal cost \( C \), the DC firm’s market share is relatively large in the initial equilibrium. Then the profit shifting effects of a DC export subsidy substantially decrease the LDC aggregate income and thus lower the market demand curve for the urban manufactured good. Thus the optimal export policy for DC government is an export tax.

Next, in (2), when the marginal productivity of rural labor \( R \) decreases, an increase in rural production due to urban-to-rural migration tends to be smaller. Furthermore, a reduction in \( R \) lowers rural wage rate \( w \) and thus raises the urban unemployment ratio \( \mu \). This unemployment effect enhances a decrease in the aggregate income and thus a downward shift of the market demand curve for urban manufactured good tend to be large. Thus the optimal export policy for DC government is more likely to be an export tax.

Finally, a reduction in labor endowment \( L \) enhances a decrease in the aggregate income. To see this, combining (5), (7) and (8) and totally differentiating the resulting equation, we get

\[
\frac{d}{dL} \left( f'(L) - f''(L)(L-L_c)\right) = f'(L)\frac{dL}{dL}.
\]

Thus a reduction in \( L \) tends to decrease rural population \( L_c \). The decrease in the aggregate income is thus enhanced, implying that the optimal export policy is more likely to be an export tax.

4.5 Discussion
Let us discuss the theoretical implication and realistic relevance for the conclusion that the optimal export policy for DC government can be an export tax.

First, from the theoretical viewpoint, whether the optimal export policy is a subsidy or a tax depends on whether relations between competing firms are strategic substitute (subsidy) or complement (tax), as is well known in the traditional literature on strategic trade policy. In our model, although the relations between firms are strategic substitute, the downward shift of market demand curve due to a decrease in the aggregate income \( I \) could be interpreted as inducing an economic situation similar to that where firms’ relations are strategic complement. Actually, in Appendix F, we show an analysis of a firm’s reaction curve on the presumption that he correctly takes into consideration a change in aggregate income and the induced shift of market demand curve. Then, when the induced shift \( (P_2(y+Y,I) = \partial P / \partial I > 0) \) is large, the (outputs of) firms are strategic complement (i.e., his reaction curve is decreasing). It is clear that the optimal export policy is an export tax. In this paper, we focus not on this trivial case but on a less trivial
case where each firm does not take into consideration a change in the shift of market demand curve induced by a change in aggregate income.

Second, the conclusion that an export tax can be optimal for DC government seems consistent with some realistic situation in LDC markets under international rivalry. While in the 1980s DC oligopolistic firms kept exporting to each other under trade policies conducted by their governments, one can recently observe some situations where DC oligopolistic firms choose foreign direct investment (FDI), not exporting, when they supply their goods to the LDC market (e.g., Japanese automakers invest and produce in India). If DC government chooses an export tax, not subsidy, the DC firms may choose FDI (local production) in order to escape from being taxed if FDI cost is low enough. In this respect, our result may be consistent with (at least, does not contradict) a realistic feature of recent LDC markets under international rivalry.

5. Concluding Remarks

In this paper we have investigated what is the optimal export policy for a developed country (DC) in a general equilibrium model of a developing country (LDC) with urban unemployment a la Harris and Todaro (HT: 1970). Even if DC firm competes a la Cournot against LDC firm (strategic substitute) in a urban manufactured good market, the optimal export policy can be an export tax, not a subsidy, because of the presence of income effect on imperfectly competitive manufactured good. First, an export subsidy decreases the aggregate income of an LDC. If it shifts down the market demand curve for the manufactured good, the DC firm’s output and profit may decrease in equilibrium. Second, the optimal export policy for DC government is an export tax if LDC firm’s marginal cost is not much lower than DC firm’s marginal cost, given the preference for urban manufactured good, rural technology and population. Third, given the preference for urban manufactured good in the LDC market with international rivalry, the range of parameter values for optimal export tax is larger when marginal product of rural labor and/or population are smaller.

Finally let us point out some qualifications of this paper. First, we assume that only DC government implements trade policy. An LDC government may respond to it by some policy such as trade or labor market policies. Second, DC firm can possibly switch from FDI to exporting facing the DC government’s export tax. These may be interesting themes for future research.
Appendix A: Proof of \( a_{31} > 0 \)

The third term of \( a_{31} \) can be rewritten as

\[
f'(L_x)g'(y) = w_x \left\{ \frac{mw}{f''(L_x)[L - L_x] - f'(L_x)} \right\} = mw \left\{ \frac{w_x}{f''(L_x)[L - L_x] - f'(L_x)} \right\}
\]

\[
= mw \left\{ \frac{w_x}{f''(L_x)L_c - w_x} \right\} = mw \left[ \frac{L_c}{L_x} \left( \frac{L_c}{w_x} f''(L_x) - 1 \right) \right]^{-1}
\]

\[
= mw \left[ \left( \frac{L_c}{L_x} \left( \frac{L_c}{w_x} d w_x dL_x \right) \right)^{-1} \right]^{-1} = mw \left\{ \frac{1}{\left[ -L_c / L_x \eta(L_x) - 1 \right]} \right\}
\]

where \( \eta(L_x) = -(L_c / w_x)(dw_x / dL_x) > 0 \) is wage elasticity of rural labor demand. Using the above relation and (2), we get

\[
a_{31} = P(y + Y, I) + y P_t(y + Y, I) + f'(L_x)g'(y) = mw + f'(L_x)g'(y)
\]

\[
= mw \left\{ 1 - \frac{1}{1 + [L_c / L_x] \eta(L_x)} \right\} > 0
\]

Appendix B. Stability of Equilibrium

The adjustment process for the subsystem (2), (3) and (13) is assumed to be the following:

\[
\dot{y} = \alpha \left[ P(y + Y, I) + y P_t(y + Y, I) - mw_m \right] \quad \alpha > 0
\]

\[
\dot{Y} = \beta \left[ P(y + Y, I) + y P_t(y + Y, I) - (C - s) \right] \quad \beta > 0
\]

\[
\dot{I} = \gamma \left[ f(g(y)) + P(y + Y, I) y - 1 \right] \quad \gamma > 0
\]

Linearizing the system around the equilibrium point \( \dot{y} = \dot{Y} = \dot{I} = 0 \) and denoting the coefficient matrix as \( A \), the stability condition is that its three eigen values \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are all negative. Thus we obtain \( \det A = \lambda_1 \lambda_2 \lambda_3 < 0 \).

Appendix C: Stable Solution of Linear Model

The stability condition \( \det A = 2(\theta y - 1) - 1 < 0 \) is equivalent to \( 2\theta y - 1 < 2 \). Substituting the smaller solution of (29) yields
This solution thus satisfies the stability condition. Clearly, the larger solution does not satisfy it.

**Appendix D : Representation for Condition (c)**

Condition (c) $\Delta(0) > \Gamma(0)$ is $[9 - 4\theta F(0)] + 2(2\theta M + 3) > (2\theta M + 5)\sqrt{9 - 4\theta F(0)}$. First, setting $f = \sqrt{9 - 4\theta F(0)}$, we have $f^2 - (2\theta M + 5)f + 2(2\theta M + 3) > 0$. We draw the graph of $G(f) = f^2 - (2\theta M + 5)f + 2(2\theta M + 3)$.

![Graph of G(f)](image_url)

Under condition (a), $G(0) = 2(2\theta M + 3) < 0$ holds. We also have

$$G(f) = \left[f - \frac{2\theta M + 5}{2}\right]^2 + 2(2\theta M + 3) - \left(\frac{2\theta M + 5}{2}\right)^2$$

Thus the graph of $G(f)$ takes the minimum value (negative) at $f = (2\theta M + 5)/2 < 0$. By solving $G(f) = 0$, the horizontal intercept is derived, i.e., $f = 2$ and $2\theta M + 3 < 0$. Therefore, under condition (a), condition (c) is equivalent to $f = \sqrt{9 - 4\theta F(0)} > 2$. Rewriting it yields
Appendix E: Analysis of Full Employment Model

In order to formulate a full employment model, we only need to regard the fixed urban wage rate $w$ as a flexible wage rate in an integrated labor market. Then the marginal product of rural labor equals this wage rate $w$. We implicitly assume that an urban manufacturing firm regards $w$ as given because rural sector is dominant in the whole economy. The general equilibrium system is composed of seven independent equations (1)-(4), (6), (5’) and (7’).

\[
I = x + P_y(y + Y, I) y \\
P_y(y + Y, I) + yP_y(y + Y, I) = mw \\
P_y(y + Y, I) + YP_y(y + Y, I) = C - s \\
x = f(L_x) \\
w = f'(L_x) \\
L_y = my \\
L_x + L_y = L
\]

Given $L$, $C$ and $s$, the system determines the equilibrium values of seven endogenous variables ($I, y, Y, x, w, L_x, L_y$). To solve this model, we modify (2) and (13) in the text into (2’) and (13’) below. The three equations below determines $(y^*, Y^*, I^*)$:

\[
P_y(y + Y, I) + yP_y(y + Y, I) = mf^*(L - my) \\
P_y(y + Y, I) + YP_y(y + Y, I) = C - s \\
I = f(L - my) + P(y + Y, I) y
\]

Totally differentiating them, we obtain

\[
\begin{bmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23} \\
b_{31} & b_{32} & b_{33}
\end{bmatrix}
\begin{bmatrix}
dy \\
dY \\
dI
\end{bmatrix}
= \begin{bmatrix}
0 \\
-1 \\
0
\end{bmatrix}
ds
\]

Each element of the coefficient matrix B on the right-hand side is
\[ b_1 = 2P_y (y + Y, I) + yP_y (y + Y, I) + mf''(L - my) < 0 \]
\[ b_{12} = a_{12} < 0 \]
\[ b_{13} = a_{13} \]
\[ b_{21} = a_{21} < 0 \]
\[ b_{22} = a_{22} < 0 \]
\[ b_{23} = a_{23} > 0 \]
\[ b_{31} = P(y + Y, I) + yP_y (y + Y, I) - mf'(L, I) = 0 \]
\[ b_{32} = a_{32} < 0 \]
\[ b_{33} = a_{33} \]

Using the Cramer rule, the comparative static results are

\[ \frac{dy^*}{ds} = \frac{b_{23}b_{32} - b_{23}b_{33}}{\text{Det.}B} \quad (15') \]

\[ \frac{dY^*}{ds} = \frac{b_{13}b_{31} - b_{13}b_{33}}{\text{Det.}B} = \frac{-b_{11}b_{33}}{\text{Det.}B} \quad (16') \]

\[ \frac{dT^*}{ds} = \frac{b_{12}b_{32} - b_{12}b_{33}}{\text{Det.}B} = \frac{b_{11}b_{32}}{\text{Det.}B} < 0 \quad (17') \]

We assume the stability of equilibrium and thus the determinant of B is negative, i.e., \( \text{Det.}B < 0 \). As in the HT model, an export subsidy of DC government necessarily decreases the aggregate income \( I^* \) in an LDC in the full employment model. The equilibrium output \( Y^* \) of DC firm may either increase or decrease. When \( b_{33} < 0 \) holds (\( P_y > 0 \) is small), \( y^* \) decreases but \( Y^* \) increases. When \( b_{33} > 0 \) holds (\( P_y > 0 \) is large), \( Y^* \) decreases and \( y^* \) may either increase or decrease. If LDC firm’s output \( y^* \) decreases, manufacturing employment \( L_y^* \) decreases and rural employment \( L_z^* \) increases.

Next we investigate under what conditions the optimal export policy is an export tax, using the linear model presented in section 4. For this model, since (14’) turns out to be (23) which is the same in the HT model, (24), (25) and (26) are the same.

The first-order condition (FOC) for welfare maximization is obtained by modifying (31) replacing \( F(s) \) with \( H(s) = -s + [(\theta RL - mR) - (mR - C)] \) and \( M \) with \( M_R = mR - C \):

\[ 2\theta s + [9 - 4\theta H(s)] + 2(2\theta M_R + 3) = (2\theta M_R + 5)\sqrt{9 - 4\theta H(s)} \]

The analysis in the following can be done in a similar fashion to that for the HT model.
Condition (a) is \( 2\theta M_R + 5 < 0 \), i.e.,

\[
mR + \frac{5}{2\theta} < C \quad (35')
\]

Denoting \( N_R = \theta RL - mR \), condition (b) is \( N_R + M_R < -(3/4\theta) \). i.e.,

\[
\theta RL + \frac{3}{4\theta} < C \quad (36)
\]

Under condition (a), condition (c) is \( (5/4\theta) > N_R - M_R \), i.e.,

\[
[2m - \theta L]R + \frac{5}{4\theta} > C \quad (37')
\]

The range of parameter values \((R,C)\) that the optimal export policy is an export tax \((x^0 < 0)\) is the area that satisfies (a), (b) and (c). We denote the combination of \((R,C)\) that satisfies (35’), (36) and (37’) as a line, bb line and cc line, respectively.

First, aa line is an increasing line with the slope=\(m\) and intersect with the C axis at \((5/2\theta) > 0\). bb line is an increasing line with the slope=\(\theta L\) and intersect with the C axis at \((3/4\theta) > 0\). cc line is a line with the slope=\(2m - \theta L\), which can be positive or negative. Its intercept on the C axis is \((5/4\theta) > 0\). One can consider various cases depending on the slopes of the three lines. Among them, when \(m > \theta L\) holds, all the three conditions are satisfied. In this case, the slope of aa line > the slope of bb line holds. At the same time, \(2m - \theta L > m\) i.e., the slope of cc line > the slope of aa line also holds. Therefore the combinations of \((R,C)\) that satisfy three conditions (a), (b) and (c) are shown by the shaded area with horizontal lines.
Note that there exists no combinations of \((R,C)\) that satisfy (a), (b) and (c) when \(\theta L > m > (\theta L / 2)\) and when \((\theta L / 2) > m\) (figures are abbreviated).

We can examine a change in the area (shaded with horizontal lines) for optimal export tax by comparative statics. When population \(L\) decreases, aa line remains the same. bb line becomes flatter with the same intercept on the C axis. cc line becomes steeper with the same intercept \((5/4\theta)\) on the C axis. The shaded area expands. Therefore the range for optimal export tax is larger when LDC population is smaller.

Appendix F: Derivation of Reaction Curve when Firms Recognize the Effect on Aggregate Income \(I\)

In this appendix we show an analysis of a firm’s reaction curve on the presumption that he correctly takes into consideration a change in aggregate income and the induced shift of market demand curve. First, \(I = f(g(y)) + P(y + Y, I)\) implicitly determines \(I = I(y, Y)\). The reaction functions of LDC and DC firms are

\[
\begin{align*}
(2) \quad P(y + Y, I(y, Y)) + y \cdot P_1(y + Y, I(y, Y)) &= m \cdot w \\
(3) \quad P(y + Y, I(y, Y)) + Y \cdot P_1(y + Y, I(y, Y)) &= C - s
\end{align*}
\]

Totally differentiating (2), the slope of LDC firm’s reaction curve is
\[
\{2P_1(y + Y, I(\cdot)) + yP_{11}(y + Y, I(\cdot))\} dy + \{P_1(y + Y, I(\cdot)) + yP_{11}(y + Y, I(\cdot))\} dy + \{P_2(\cdot) + yP_{12}(\cdot)\} dI = 0
\]

where \(dI = I_y(y, Y) dy + I_y(y, Y) dY\). Totally differentiating (1) yields

\[
dI = [f'(\cdot)g'(y) + P(y + Y, I) + yP_1(y + Y, I)] dy + P_1(y + Y, I)ydY
\]

Thus we have

\[
I_y = f'(L_y)g'(y) + P(y + Y, I) + yP_1(y + Y, I) = f'(L_y)g'(y) + m \cdot w > 0
\]

\[
I_y = yP_1(y + Y, I) < 0
\]

Using them, the slope of LDC firm’s reaction curve is

\[
\{2P_1(\cdot) + yP_{11}(\cdot) + [P_2(\cdot) + yP_{12}(\cdot)]I_y\} dy + \{P_1(\cdot) + yP_{11}(\cdot) + [P_2(\cdot) + yP_{12}(\cdot)]I_y\} dY = 0
\]

Therefore we obtain

\[
\frac{dY}{dy} = -\frac{2P_1(\cdot) + yP_{11}(\cdot) + [P_2(\cdot) + yP_{12}(\cdot)]I_y}{P_1(\cdot) + yP_{11}(\cdot) + [P_2(\cdot) + yP_{12}(\cdot)]I_y} < 0
\]

If \(P_2(y + Y, I) > 0\) is so large that \(P_2 + yP_{12} > 0\) holds, LDC firm’s reaction curve is increasing, implying that firms are strategic complement.
References


