International Rent-shifting under Foreign Entry
through R&D and Licensing

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Abstract

We explore international rent-shifting when a domestic firm and a foreign rival compete in the domestic market. To serve the market, the foreign firm has to acquire a production technology through either R&D or licensing obtained from the domestic firm. In the presence of both R&D and licensing, the domestic firm deters the foreign firm from engaging itself in R&D. Then the foreign government can shift the rent from the domestic firm either directly or indirectly. However, such rent-shifting opportunities may be deterred by the domestic government. The shifted rent could exceed the amount paid by the foreign firm for licensing.

Key words: international oligopoly, R&D, licensing, rent-shifting

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1 Introduction

To acquire production technologies, firms typically engage in R&D. It is also widely observed that firms obtain licenses for production from other firms when some know-how is patented or unknown. The presence of R&D and licensing could lead to strategic interactions between firms and generate rent. In this paper, we explore the opportunities of international rent-shifting led by governments when both R&D and licensing are available in the framework of international duopoly.

We specifically consider a situation in which a domestic firm and a foreign rival produce slightly differentiated products and compete in the domestic market. To serve the domestic market, the foreign firm has to acquire a technology to produce a good through either R&D or licensing obtained from the domestic firm. It should be noted that the essence of technology is often embodied in sophisticated intermediate products that some firms are unable to produce. In this case, firms may purchase such key intermediate inputs from rival firms.\(^1\) We include this case as a kind of licensing in our analysis.

In our model, the domestic firm strategically licenses their technologies or supplies a key intermediate input to the foreign rival when the foreign firm is willing to engage in R&D to produce the good. That is, the domestic firm intends to deter the foreign firm from engaging itself in R&D. Such a strategy mitigates the loss of the domestic firm caused by the entry of the foreign firm to the domestic market. When the domestic firm sets the license fee or the input price, it tries to extract rent from the foreign firm as much as possible. However, the foreign government may intervene to manipulate rent-shifting through taxes and subsidies.\(^2\) We point out that the foreign government can shift the rent from the domestic firm to the foreign government either directly or indirectly. However, such a rent-shifting opportunity may be deterred by the domestic government through taxes. Interestingly, the shifted rent could exceed the payment from the foreign firm to the domestic firm. Moreover, the presence of rent-shifting could make the license fee negative.

Seminal work on international rent-shifting is a series of studies by Brander and Spencer.\(^3\) In particular, our analysis is related to Brander and Spencer (1981). In their

\(^1\)For example, Mitsubishi Motors Corporation (MMC) and PSA Peugeot Citroën have announced that MMC will provide PSA an MMC-made electric vehicle for Europe. The vehicle will be sold under Peugeot brand, in parallel to Mitsubishi’s own vehicle. Nissan Motor Co. is developing lithium-ion batteries for hybrid and electric vehicles. They are planning to provide it to other auto makers.

\(^2\)We focus on simple intervention through taxes and subsidies.

\(^3\)See Brander (1995).
model, there are two firms: a foreign firm and a domestic firm. The domestic market is monopolized by the foreign firm and the domestic firm is a potential entrant. The foreign monopolist deters the domestic firm from entering the market. In this situation, the domestic government can set a tariff without raising the consumer price, because the foreign monopolist keeps the consumer price constant to deter the entry. That is, the domestic government can shift the monopoly rent from the foreign firm without generating any burden on the domestic country. By contrast, in our model, the domestic firm deters the foreign firm not from entering the goods market but from engaging itself in R&D. This behavior results in room for rent-shifting by the governments.

Although strategic use of licensing has been studied in the literature of industrial organization, those studies basically deal with a closed economy.\(^4\) Thus, one cannot consider rent-shifting across countries. Only few studies analyze strategic use of licensing or input supply in the open economy framework. Chen et al. (2004) show that under international duopoly, trade liberalization leads to strategic outsourcing to the rival firm, which has a collusive effect. Horiuchi and Ishikawa (2009) explore the strategic relationship between tariffs and North–South technology transfer in an oligopoly model when technology is embodied in a key component that only North firms can produce.

The rest of the paper is organized as follows. In section 2, we present an international Cournot duopoly model. Specifically, we consider the foreign production of the good through either R&D or licensing. In section 3, we explore the rent-shifting by governments in the presence of R&D and licensing. Section 4 concludes.

\section{Basic Model}

There are two firms, a domestic firm (firm \(d\)) and a foreign firm (firm \(f\)). Firm \(d\) produces good \(Y\) in the domestic country and serves the domestic market. Firm \(f\) is entering the domestic market by producing good \(X\), which is a close substitute of good \(Y\). To produce good \(X\), however, firm \(f\) needs to acquire its production technology. We specifically consider two possibilities. In the first case, firm \(f\) engages in R&D to develop good \(X\) by itself. In this case, firm \(f\) incurs fixed costs (FCs) of R&D, \(F\). In the second, firm \(f\) obtains a license for a technology to produce a key intermediate input such as a hybrid engine to produce good \(X\) from firm \(d\). In this case, firm \(f\) pays license fees to firm \(d\). We should mention that firm \(d\) may provide firm \(f\) with a key input itself instead.

\(^4\)In the industrial organization literature, licensing is strategically used to (partially) deter entry. See Gallini (1984) and Yi (1999), for example.
of a technology to produce it. When firm $f$ enters the domestic market, the two firms engage in Cournot competition.

Demands are characterized by a representative consumer that consumes goods $X$ and $Y$ as well as a numéraire good. The numéraire good is competitively produced and freely traded between countries, and generates no externalities. We assume the following utility function:

$$U = \alpha x + \beta y - \frac{x^2 + y^2}{2} - \phi xy + m,$$

where $x$, $y$ and $m$ are, respectively, the consumption of goods $X$ and $Y$ and the numéraire good, $\alpha$ and $\beta$ are parameters, and $0 < \phi < 1$ is a parameter indicating the degree of substitutability between goods $X$ and $Y$.

Then the inverse demands for the imperfectly substitutable goods $X$ and $Y$ are, respectively, given by

$$p_x = \alpha - x - \phi y,$$  \hspace{1cm} (1a)

$$p_y = \beta - y - \phi x,$$  \hspace{1cm} (1b)

where $p_x$ and $p_y$ are the consumer prices of goods $X$ and $Y$. Consumer surplus (CS) is given by

$$CS = \alpha x + \beta y - \frac{x^2 + y^2}{2} - \phi xy - (p_x x + p_y y) = \frac{x^2 + y^2}{2} + \phi xy$$

We first consider the R&D case. The profits of firms $f$ and $d$ can be written respectively as

$$\pi_f = (p_x - c_x)x - F,$$

$$\pi_d = (p_y - c_y)y,$$

where $c_i$ ($i = x, y$) is the constant marginal cost (MC) to produce good $i$. Then the first order conditions (FOCs) for profit maximization are:

$$\frac{d\pi_f}{dx} = -x + p_x - c_x = 0,$$

$$\frac{d\pi_d}{dy} = -y + p_y - c_y = 0.$$

In equilibrium, we have

$$x_R = \frac{2A - \phi B}{4 - \phi^2}, y_R = \frac{2B - \phi A}{4 - \phi^2}.$$
where $A \equiv \alpha - c_x$ and $B \equiv \beta - c_y$. As the market size, $\alpha(\beta)$, becomes larger and the MC, $c_x (c_y)$, becomes smaller, $A \ (B)$ becomes larger. We call $A \ (B)$ the effective market size for good $X \ (Y)$. We assume that both firms serve the domestic markets in equilibrium, that is,

$$2B - \phi A > 0, 2A - \phi B > 0. \quad (2)$$

By using the FOCs, the profits of firms $f$ and $d$ are

$$\pi_f^0 = (x_f^0)^2, \pi_d^0 = (y_d^0)^2. \quad (3)$$

Thus, the following lemma is immediate:

**Lemma 1** *The profits increase if and only if the output rises.*

We next examine firm $d$’s technology licensing to firm $f$ and compare this licensing equilibrium with the R&D equilibrium. We assume that before firm $f$ decides whether or not to engage in R&D, firm $d$ decides whether or not to offer a take-it-or-leave-it licensing offer to firm $f$.5 When firm $f$ will not develop good $X$, firm $d$ has no incentive for licensing as long as $\phi$ is relatively high. This is because firm $d$ can enjoy the monopoly situation without licensing. If $\delta$ is sufficiently low, however, firm $d$ may have an incentive for licensing.6 In this case, however, firm $d$ will design license fees to extract all the rent from firm $f$. Therefore, we consider the case where in the absence of licensing, firm $f$ is willing to develop good $X$ through R&D, which harms firm $d$. In this situation, firm $d$ has an incentive to grant firm $f$ permission to use its technology to produce good $X$ in return for license fees, because licensing generates revenue for firm $d$ and mitigates its loss. Thus, firm $d$ designs a licensing contract so that firm $f$ is willing to accept it. It should be noted that firm $d$ cannot extract all the rent from firm $f$ because of firm $f$’s outside option, i.e., R&D.

In the presence of licensing, profits of the two firms are given by

$$\pi_f = (p_x - c_x)x - (R + rx) = (p_x - c_x - r)x - R$$
$$\pi_d = (p_y - c_y)y + (R + rx),$$

where $R$ and $r$ are, respectively, a fixed fee and a per-unit royalty. For simplicity, we assume that firm $f$’s MC under licensing and that under R&D are the same. Since the

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5 The qualitative nature of our results would remain unchanged even if licensing fees are determined by bargaining between the two firms.

6 Firm $d$ has the incentive if $\phi = 0$. 

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outside option for firm $f$ is R&D, firm $d$ faces the following maximization problem:

$$\pi^d_L \equiv \max_{r,R} \pi^d; \text{ s.t. } \pi^f \geq \pi^f_R,$$

(4)

where $\pi^f_R$ is firm $f$’s profits with R&D. In the equilibrium, firm $f$ is indifferent between R&D and licensing.

The appendix proves the following lemma.

**Lemma 2** When $4B\delta + B\delta^3 - 4C\delta^2 \leq 0$, firm $d$ sets $r = 0$ and $R = F$. When $4B\delta + B\delta^3 - 4C\delta^2 > 0$, firm $d$ sets $r = \tau$ and $R = 0$ if $r^* \geq \tau$ and sets $r = r^*$ and $R = R^*$ if $r^* < \tau$, where

$$r^* \equiv \frac{- (4B\delta + B\delta^3 - 4C\delta^2)}{2 (3\delta^2 - 4)},$$

$$\tau \equiv C - \frac{1}{2}B\delta - \frac{1}{2}\sqrt{-16F - 4BC\delta + 4C^2 + 8F\delta^2 - F\delta^4 + B^2\delta^2},$$

$$R^* \equiv \left(\frac{2(C - r^*) - \delta B}{4 - \delta^2}\right)^2 - \left(\frac{2C - \delta B}{4 - \delta^2}\right)^2 + F.$$

This lemma implies that depending on the parameter values, there are three kinds of licensing contract.

### 3 Rent-shifting in the Presence of R&D and Licensing

In the presence of licensing, firm $d$ sets license fees to extract rent from firm $f$ as much as possible. However, there are opportunities for the foreign government to shift the rent back from firm $d$ through simple taxes and subsidies. By facing such opportunities, the domestic government may try to shift the rent from firm $d$ to the domestic government before the foreign government moves. In this section, we explore the rent-shifting by the foreign government as well as the domestic government.

#### 3.1 Rent-shifting by the Foreign Government

In this subsection, we consider three measures to shift the rent from firm $d$ to the foreign government. One is direct measures and the other two are indirect ones. First, we explore direct measures when firm $d$ supplies a key intermediate input to firm $f$. Suppose that the foreign government imposes a tariff on the input. This usually causes tariff shifting. That is, the tariff increases the input price which firm $f$ faces. In our model, however, firm $d$ cannot directly shift the tariff to firm $f$, because the tariff shifting leads firm $f$
to engage itself in R&D. In fact, the foreign government could generate more than full rent-shifting through tariffs, that is, the shifted rent could exceed the payment from the foreign firm to the domestic firm.

Suppose that the foreign government imposes a specific tariff, $t$, on the import of the key input. Even if a tariff is imposed, firm $d$ cannot raise the price beyond $\tau$. When $t = \tau$, therefore, the payment by firm $f$ are fully shifted from firm $d$ to the foreign government. However, we should note that the profits of firm $d$ are still larger under licensing than under R&D even with $t = \tau$, because firm $f$’s effective MC under licensing, $c_x + \tau$, is higher than that under R&D, $c_x$. This implies that the foreign government can increase the tariff rate beyond $\tau$ without affecting the price firm $f$ faces. We should mention that a foreign tariff on the input does not affect the output of each firm.

Formally, in the presence of tariffs on the key intermediate input, (4) is modified as follows:

$$\pi^d_L = \max_{\pi^R} \pi^d - ty^f_L; \text{ s.t. } \pi^f \geq \pi^R.$$  

Then, the foreign government faces the following maximization problem:

$$\max_t ty^f_L; \text{ s.t. } \pi^d_L \geq \pi^d_R,$$  

where $\pi^d_R$ is the profits of firm $d$ with R&D. In the equilibrium, $\pi^d_L = \pi^d_R$ holds, that is, firm $d$ is indifferent between R&D and licensing.

Next we investigate indirect measures to shift the rent. Firm $d$ can charge license fees, because firm $f$’s profits become larger under licensing without any fees than under R&D. That is, the licensing results in room for arbitrage for firm $d$. The foreign government can indirectly shift the rent by reducing the room. Suppose that a production tax is collected from firm $f$ under only licensing. If a production tax, $\tau$, is introduced before firm $d$ makes a licensing offer, we have $r + \tau = \tau$. This is because firm $f$ will engage itself in R&D if the effective MC (which equals $c_x + r + \tau$) exceeds $c_x + \tau$. By increasing $\tau$, therefore, the license fee, $r$, falls. That is, a production tax reduces room for arbitrage. In particular, by setting $\tau = \tau$, firm $f$ gets technologies without any payment to firm $d$.

As in the case of tariffs on a key input, the foreign government can raise the tax rate further, because firm $f$’s effective MC (which equals $c_x + \tau$ in the case of production taxes) is higher under licensing than under R&D. Therefore, we have $r < 0$ with $\tau > \tau$ in the equilibrium. Moreover, a production tax does not affect the total emissions, because both firm $f$’s effective MC, $c_x + r + \tau = c^f + \tau$, and firm $d$’s MC are constant.

\footnote{It is assumed that no production tax is imposed when firm $f$ is engaged in R&D.}
Formally, in the presence of production taxes, (4) becomes

$$\pi^d_{Lr} \equiv \max_{r, R} \pi^d; \text{ s.t. } \pi^f - \tau y^f_L \geq \pi^f_R.$$  

By affecting the constraint, production taxes make indirect rent-shifting possible. The maximization problem for the foreign government is

$$\max_{\tau} \tau y^f_L; \text{ s.t. } \pi^d_{Lr} \geq \pi^d_R.$$  

The foreign government can also indirectly shift the rent from firm $d$ through a lump-sum R&D subsidy, $S$, to firm $f$, which is committed before firm $d$ moves. We first consider a case in which $y^f_R > 0$ but $\pi^f_R = (y^f_R)^2 - F < 0$ hold and hence firm $f$ does not engage in R&D. In this case, by setting the subsidy $S \geq -\pi^f_R$, the foreign government can induce R&D. Since firm $d$ prefers licensing to R&D, however, firm $d$ offers a licensing contract whenever the subsidy leads firm $f$ to engage itself in R&D. This implies that the foreign government does not pay the subsidy in equilibrium. Even if firm $f$ engages in R&D without the subsidy, the foreign government could use R&D subsidies as a device to shift the rent from firm $d$. With R&D subsidies, (4) is modified as

$$\pi^d_{Ls} \equiv \max_{r, R} \pi^d; \text{ s.t. } \pi^f \geq \pi^f_R + S.$$  

Thus, the license fees become lower as the subsidy rises. To maximize the shifted rent, the foreign government sets $S = F$ so that $\pi^d_{Ls} = \pi^d_R$ holds. With $S = F$, firm $f$ obtains the license without any payment.

Thus, we obtain:

**Proposition 1** The foreign government can shift the rent associated with licensing from firm $d$ to the foreign country directly by levying a tariff on the key intermediate input and indirectly by imposing a production tax on firm $f$ under licensing or by committing itself to an R&D subsidy. The shifted rent could exceed the payment by firm $f$ with the tariff on the key input. The license fees could be negative with the production tax on firm $f$. The rent-shifting does not affect CS.

In the licensing equilibrium where the foreign government completely shifts the rent from firm $d$, the profits of firms $f$ and $d$ are the same with those in the R&D equilibrium. When the rent-shifting is caused by taxes, CS is less under R&D, because the rent-shifting does not affect the total consumption. In this case, therefore, domestic welfare is lower in the licensing equilibrium with complete rent-shifting than in the R&D equilibrium.
3.2 Rent-shifting by the Domestic Government

In this subsection, we point out that the domestic government can prevent the foreign government from shifting rent from firm \( d \) by using simple taxes if the domestic government can move before the foreign government. We explore both direct and indirect measures.

First, we consider direct measures: a tax on the license fees or an export tax on the key input. When a specific tax on the license fees or a specific export tax on the input, \( \rho \), is imposed, (4) becomes

\[
\pi_{Ld}^d \equiv \max_{r,R} \pi^d - \rho y^f_L; \quad \text{s.t. } \pi^f \geq \pi_R^f.
\]

Then, the maximization problem for the domestic government is

\[
\max_{\rho} \rho y^f_L; \quad \text{s.t. } \pi_{Ld}^d \geq \pi_R^d.
\]

Even if the tax is imposed, firm \( d \) cannot shift the tax to firm \( f \). This is because the tax shifting leads firm \( f \) to engage itself in R&D instead of licensing. Thus, the license fees faced by firm \( f \) remain to be \( \pi^f \). Under the optimal tax, the tax rate is greater than \( \pi^f \).

The above case is similar to the case in which the foreign government sets a tariff on the input to shift the rent from firm \( d \).

Next we consider tariffs on the final good as indirect measures. A tariff on the final good usually decreases the profits of firm \( f \). When \( r = \pi^f \), a tariff induces firm \( f \) to switch from licensing to R&D. Thus, firm \( d \) lowers the license fee to prevent such a switch. In the presence of a specific tariff, \( T \), on the final good, (4) is modified as follows:

\[
\pi_{LT}^d \equiv \max_{r,R} \pi^d; \quad \text{s.t. } \pi^f - Ty^f_L \geq \pi_R^f.
\]

Then, the domestic government faces the following maximization problem:

\[
\max_T Ty^f_L; \quad \text{s.t. } \pi_{LT}^d \geq \pi_R^d.
\]

This is similar to the case in which the foreign government shifts the rent from firm \( d \) through a production tax. By imposing a tariff on the final good, the domestic government can indirectly shift the rent from firm \( d \) to the domestic government. In particular, the domestic government can eliminate the room for the rent-shifting by the foreign government.

Thus, the following proposition is established:

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8 If an export tax on the input and an import tariff on the input are imposed simultaneously, then any combination \((t, \rho)\) satisfying \( t + \rho = \rho^* \) (where \( \rho^* \equiv \arg \max_{\rho} \rho y^f_L; \quad \text{s.t. } \pi_{Ld}^d \geq \pi_R^d \)) is a Nash equilibrium.
Proposition 2 If the domestic government can move before the foreign government, then the domestic government can prevent the foreign government from shifting rent from firm d by imposing either a tax on the license fees (an export tax on the key input) or a tariff on the final good.

4 Concluding Remarks

Using an international duopoly model, we have pointed out possible strategic interactions between the domestic firm and the foreign government in the presence of R&D and licensing. The foreign government can shift rent from the domestic firm by levying a tariff on a key intermediate input, by imposing a production tax on the foreign firm under licensing, or by committing itself to R&D subsidies. In particular, the shifted rent could exceed the payment by the foreign firm with the tariff and the license fees could be negative with the production tax on the foreign firm. However, such rent-shifting can be deterred by the domestic government. The domestic government may have an incentive to shift the rent from the domestic firm by imposing a tax on the license fees or an export tax on the key input or by levying a tariff on the final good. The presence of an outside option (i.e. R&D) plays a crucial role to derive the results.

Appendix

Proof of Lemma 2. Given $r$, the equilibrium outputs are

$$x = \frac{2(C - r) - \delta B}{4 - \delta^2}, y = \frac{2B - \delta(C - r)}{4 - \delta^2}.$$  

Thus, we have

$$\pi^d = \left(\frac{2B - \delta(C - r)}{4 - \delta^2}\right)^2 + R + r\frac{2(C - r) - \delta B}{4 - \delta^2}.$$  

Noting

$$R = \left(\frac{2(C - r) - \delta B}{4 - \delta^2}\right)^2 - \pi^f_R,$$

we obtain

$$\pi^d = \frac{(2B - \delta(C - r))^2}{4 - \delta^2} + \frac{(2(C - r) - \delta B)^2}{4 - \delta^2} - \frac{(2C - \delta B)^2}{4 - \delta^2} + F,$$

or

$$\pi^d = \frac{r^2 (3\delta^2 - 4) + r (4B\delta + B\delta^3 - 4C\delta^3) + (4B^2 - 4BC\delta + C^2\delta^2)}{(\delta + 2)^2 (\delta - 2)^2} + F,$$
which takes the maximum value at \( r = - \left( 4B\delta + B\delta^3 - 4C\delta^2 \right) / 2 (3\delta^2 - 4) \equiv r^* \). Since 
\[ 3\delta^2 - 4 < 0, r^* < 0 \] if \( 4B\delta + B\delta^3 - 4C\delta^2 < 0 \) and \( r^* > 0 \) if \( 4B\delta + B\delta^3 - 4C\delta^2 > 0 \). Thus, when \( 4B\delta + B\delta^3 - 4C\delta^2 \leq 0 \), firm \( d \) sets \( r = 0 \) and \( R = F \). When \( 4B\delta + B\delta^3 - 4C\delta^2 > 0 \), we have two cases depending on the size of the maximum royalty firm \( d \) can charge, \( \tau \), which satisfies
\[ \left( \frac{2(C - r) - \delta B}{4 - \delta^2} \right)^2 = \left( \frac{2C - \delta B}{4 - \delta^2} \right)^2 - F. \]

Thus,
\[ \tau = C - \frac{1}{2} B\delta - \frac{1}{2} \sqrt{-16F - 4BC\delta + 4C^2 + 8F\delta^2 - F\delta^4 + B^2\delta^2}. \]

If \( r^* \geq \tau \), then firm \( d \) sets \( r = \tau \) and \( R = 0 \). If \( r^* < \tau \), on the other hand, firm \( d \) sets \( r = r^* \) and
\[ R = \left( \frac{2(C - r^*) - \delta B}{4 - \delta^2} \right)^2 - \left( \frac{2C - \delta B}{4 - \delta^2} \right)^2 + F \equiv R^*. \]

Given \( r \), the equilibrium outputs are
\[ y^f = \frac{B - 2(\delta + r)}{3}, \quad y^d = \frac{B + (\delta + r)}{3}. \]

Thus, we have
\[ \pi^d = \left( \frac{B + (\delta + r)}{3} \right)^2 + R + r \frac{B - 2(\delta + r)}{3}. \]

Noting
\[ R = \left( \frac{B - 2(\delta + r)}{3} \right)^2 - \pi^f_R \]
\[ = \left( \frac{B - 2(\delta + r)}{3} \right)^2 - \left( \frac{B - 2\delta}{3} \right)^2 + F, \]

We obtain
\[ \pi^d = \left( \frac{B + (\delta + r)}{3} \right)^2 + \left( \frac{B - 2(\delta + r)}{3} \right)^2 - \left( \frac{B - 2\delta}{3} \right)^2 + F + r \frac{B - 2(\delta + r)}{3} \]
\[ = \frac{2B\delta + B^2 + \delta^2 - r^2 + r(B + 4\delta)}{9} + F, \]

which takes the maximum value at \( r = (B + 4\delta)/2 \equiv r^* \). If firm \( d \) sets \( r = r^* \), then the output of firm \( f \) becomes zero. Thus, the maximum royalty firm \( d \) can charge, \( \tau \), satisfies
\[ \left( \frac{B - 2(\delta + r)}{3} \right)^2 = \left( \frac{B - 2\delta}{3} \right)^2 - F. \]

Thus,
\[ \tau = \frac{1}{2} B - \delta - \frac{3}{2} \sqrt{\left( \frac{B - 2\delta}{3} \right)^2 - F}. \]
References


