On the Relationship between Bargaining and Tariffs

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Abstract

It is often argued that governments of developing countries set higher tariffs because their firms have low bargaining power. We consider a bilateral oligopoly model where upstream firms in a developed country (North) export inputs to downstream firms in a developing country (South). Bargaining occurs over the input price and the level of output. We show that, indeed in the short run with fixed number of firms, low bargaining power of Southern firms leads to high tariffs. Surprisingly, we also find that low bargaining power of South can hurt North as well by increasing the tariff rates. In the long run, however, the relationship between the bargaining power and tariffs is generally non-monotone. The particular nature of non-monotonicity depends on the curvature of the demand function.

Keywords: Tariffs; bargaining; upstream-downstream relationship

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1 Introduction

The argument of the role of tariffs has a long history in international economics. It is conventionally discussed that, if the country is large enough to affect the world price, small positive tariffs increase this country’s welfare, because terms of trade is improved by such tariffs; or if the country is a small developing one, positive tariffs could be justified because it protects from foreign competition until the country has enough experience to “stand on its own feet” (Krugman and Obstfeld, 2008). These arguments, however, ignore an important issue of the trading system in the global economy: bargaining.

Recent empirical studies on global sourcing have pointed out the fragmentation of production chains over the world. Especially, vertical specialization of production has led to rapid growth in intermediate input trade. For instance, Hummels et al. (2001), Yeats (2001) and Yi (2002) all showed that international trade has been growing faster in components than in final goods.1 Furthermore, this growth of input trade has taken place in large parts via contracts, i.e., outsourcing. In fact, several researchers have concluded that international outsourcing has already replaced foreign direct investment (FDI), which has been a major production device in international trade. Hanson et al. (2005), for example, convincingly demonstrated that the growth of foreign outsourcing by U.S. firms would have outpaced the growth of their foreign intrafirm sourcing via FDI.2

Because firms have to make use of contracts in international outsourcing, bargaining over contracts should be taken into account explicitly. Surprisingly, international transaction has been assumed away from contractual aspects in the mainstream of traditional trade theory. The recent theoretical and empirical development, however, has revealed that contracting institutions have a crucial impact on the structure of international trade flows. In particular, the following two points should be stressed from our standpoint. First of all, it is known that contract institutions are closely related to comparative advantage: for instance, Levchenko (2007) and Nunn (2007) empirically investigated the linkage between the quality of contract institutions and the content each country

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1There are a lot of empirical evidence to support this vertical specialization phenomenon in international trade. World Trade Organization (WTO) annual report in 1998, another example of this view, illustrated that in the production of an “American” car, 30 percent of the car’s value originates in Korea, 17.5 percent in Japan, 7.5 percent in Germany, 4 percent in Taiwan and Singapore, 2.5 percent in the United Kingdom, and 1.5 percent in Ireland and Barbados. That is, “only 37 percent of the production value … is generated in the United States” (pp. 36).

2Spencer (2005) demonstrated the enormous growth of manufacturing exports from China, decomposing it into either processing exports (input trade) or ordinary exports (final-good trade). She showed that a large part of processing exports occurs through international outsourcing between foreign buyers and independent Chinese firms.
exports, and showed that countries with better legal systems export goods that are more intensive in contract-dependent input. Secondly, there is a definite feedback from contracting institutions to technology adaptation: better contracting institutions lead to the choice of technologies that are more sensitive to contractual frictions (see Acemoglu et al. [2007]). These facts imply that firms in developing countries might find it difficult to negotiate contracts with foreigners, because their weak contracting institutions result in biased comparative advantage and technology adaptation.

Facing these stylized facts, it is natural to suppose that local governments in developing countries have an incentive to intervene this contractual relationship. Our perspective is that, when firms engage in international outsourcing, the trade policy (i.e., tariff) plays a key role in contract negotiation between countries, and, therefore, the discussion of tariffs should reflect different bargaining power between countries. What is the relationship between bargaining power and tariffs? Does the traditional argument on tariffs still hold from the point of view of our interest? These are the central research questions that we address in the current paper.

The main findings of the current paper are twofold. First, bargaining power between two countries has a great influence on the formation of the optimum tariff. In particular, when a developing country’s bargaining power is weaker, it is more likely for this country to impose a positive tariff on imports. Second, a developed country can benefit by reducing its bargaining power in an oligopolistic market. In such a market, biased bargaining power always deteriorates gains from trade.

This paper is organized as follows. The rest of Section 1 briefly reviews the relationship to the literature. Section 2 describes the model structure briefly. In section 3, we consider the short-run equilibrium model in which downstream firms outsource input to upstream firms. In this setup, the number of firms is fixed and each firm in the market can find its partner certainly. In section 4, we examine the long-run equilibrium model in which free entry drives all firms’ profit to zero. Once free entry is allowed and the number of firms fluctuates, some of firms may fail to find their partner and the matching becomes stochastic. Finally, Section 5 offers a short summary along with further extension. All mathematical derivations of the main results are delegated to the Appendix.

Related Literature

Several papers have developed models that are closely related to the current paper. This literature is often referred as “vertical market structure” models in which upstream firms produce input for
downstream firms. There are three branches in the literature. The first one is related to strategic trade policy. For instance, Ishikawa and Lee (1997), Ishikawa and Spencer (1999) and Chen et al. (2003) analyze the strategic interaction between upstream firms and downstream firms under international oligopoly. They point out that, in the situation where a developing country imports input from a developed country, export subsidies on imported input could be beneficial for the developing country. These papers, however, do not take into account bargaining between countries explicitly; that is, their analysis lacks contractual aspects between firms, which is a key aspect in the global sourcing.

Another branch is to investigate this vertical oligopoly from the viewpoint of free entry. Ghosh and Morita (2007a, 2007b) are good examples for this category. In contrast to the papers mentioned above, their models apply the Nash bargaining between upstream and downstream firms to study the relationship between free entry and bargaining power. Their main focus is quite different from ours, however; they confine their attention to the closed economy and abstract from the interaction with foreign countries. As many empirical studies have documented, this channel plays a crucial role in shaping the production chains. In addition, they do not explore the policy implication under free entry. As easily expected, the government’s policy has a great influence both on the vertical market structure and on excessive/insufficient free entry. The current paper tackles this challenge and derive some results that are not pointed out in the previous literature.

Finally, there is a rapidly growing strand that analyzes international organization of production. Antràs (2003, 2005), Antràs and Helpman (2004), and Grossman and Helpman (2003, 2004, 2005) all construct international bargaining models in which upstream firms and downstream firms bargain over contracts. For example, Antràs (2005) explores a new product life cycle model in which weak bargaining power of a developing country generates the emergence of international outsourcing. The noticeable shortcomings of these papers, however, are that these papers do not relate bargaining to the policy issue. To our best knowledge, this is the first paper to investigate the normative analysis of outsourcing, thereby incorporating explicitly international bargaining.

3Ara (2008) develops a simple dynamic model in which the organization of heterogeneous firms changes over product life cycles.

4For instance, Antràs and Rossi-Hansberg (2009), one of surveys of the recent literature, note that “although the literature on organizations and trade has been extremely concerned with matching positive features of reality, ··· it has been much less concerned with the normative and more policy implications of changes in the international organization of production” (p.61).
2 Model

Consider a setting with two countries, North and South. North has a large number of upstream firms, \( U_i \) \((i = 1, 2, ..., n)\), each of whom exports a component to South. A large number of downstream firms, \( D_i \) \((i = 1, 2, ..., m)\), operate in South, each of whom assembles a component to produce a final good. A downstream firm in South, \( D_i \), does not have the technology to produce a high-tech component (imagine semiconductors or IC tips). Thus, \( D_i \) relies entirely on the import of components manufactured by an upstream firm \( U_i \) in North.

There are two methods to procure a component: intrafirm trade (FDI) and arm’s length trade (outsourcing). In this paper, we restrict our attention to the latter method of procurement, because (i) outsourcing now has grown faster in input trade relative to FDI, and (ii) input trade through FDI has no effect on the optimum tariff. In this paper, we assume that \( D_i \) asks \( U_i \) to produce a component via international contracts. After procuring components, \( D_i \) assembles it by himself to produce final goods and sells to Southern consumers.

If \( D_i \) and \( U_i \) agree upon international contracts, \( U_i \) has to make a relationship-specific investment for a particular \( D_i \). As is often emphasized in the industrial organization literature, we assume throughout this paper that this investment specificity generates the greatest value to only one party and makes it less valuable to others (see Grossman and Helpman [2002] for related discussions). As a consequence, when \( D_i \) and \( U_i \) engage in international outsourcing, only one-to-one matching takes place.\(^5\) Moreover, we assume that firms are not always successful in their searches. Let \( s(m, n) \) denote the number of pairs that are formed in this matching process, where \( s(m, n) = \min \{m, n\} \) and \( s(\cdot) \) is increasing in both of its arguments.

The unit cost of a component (resp. final good) is \( c_U \) (resp. \( c_D \)). Prior to production, each \( U_i \) (resp. \( D_i \)) has to incur entry cost \( F_U \) (resp. \( F_D \)). In what follows, we normalize \( c_D = 0 \) to simplify the analysis, but this normalization does not affect our result. Production of one unit of the final product requires one unit of intermediate product. The aggregate supply is denoted by \( Q = \sum_i q_i \), where \( q_i \) is the quantity of a final good that \( U_i \) produces. Throughout the paper, we assume that the inverse demand \( P(Q) \) has the following properties:

\(^5\)Although we focus on this kind of matching just for simplicity, the vast literature on the matching theory has argued a theoretical advantage of one-to-one matching. Especially, it is well-known that there always exists a stable matching in any one-to-one matching market. See Roth and Sotomayor (1990) for a textbook treatment.
Assumption 1. $P(Q)$ is continuously twice differentiable and $P'(Q) < 0$ for all $Q \in (0, \infty)$.

Assumption 2. $P_0 > c > P_\infty = 0$ where $P_0 \equiv \lim_{Q \to 0} P(Q)$, $P_\infty \equiv \lim_{Q \to \infty} P(Q)$ and $c \equiv c_U$.

Southern government imposes a specific tariff, $t$, on imported input. The government sets the tariff so as to maximize the social surplus in South. This welfare is composed of Southern consumer surplus, aggregate profits of Southern firms, and tariff revenue.

The timing of the game is as follows. First, the government sets a tariff rate, $t$. After observing this tariff rate, $U_i$ and $D_i$ enter the market and search for their partners and the matching pairs $s$ are formed. Finally, bargaining over contracts takes place and Cournot competition occurs in Southern market.

3 Short-Run Equilibrium

We first consider short-run equilibrium where entry costs $F_D$ and $F_U$ are sunk and the number of firms is fixed. Because $m$ and $n$ are fixed, the matching pair $s(m, n)$ is also fixed in this arrangement. In this section, therefore, we use the abbreviated notation $s$ to denote the matching pairs.

3.1 Bargaining

Consider a third-stage bargaining game. Each pair $i$ of downstream/upstream firms bargains over $(r_i, q_i)$, where downstream firm $D_i$ purchases $q_i$ units of the intermediate product from upstream firm $U_i$ at the unit price of $r_i$, and then produces $q_i$ units of the final product.

We characterize the outcome of the bargaining, using the formula of a generalized Nash bargaining solution, where every downstream firm has the same bargaining power denoted by $\beta \in (0, 1)$. The outcome of the third-stage bargaining is $s$ pairs of input prices and quantities $(\hat{r}_i, \hat{q}_i)$ ($i = 1, 2, ..., s$) which satisfy the following condition: $(r_i, q_i) = (\hat{r}_i, \hat{q}_i)$ is Nash solution to the bargaining problem between $D_i$ and $U_i$, given that both expect $(\hat{r}_j, \hat{q}_j)$ ($j \neq i$) to be agreed upon between $D_j$ and $U_j$. The relevant utility functions for the analysis of the bargaining are $D_i$’s profit $\left[ P\left(q_i + \sum_{j \neq i}^s q_j \right) - r_i \right] q_i$ and $U_i$’s profit $(r_i - c - t)q_i$. Also, we assume that the disagreement point is zero for both parties.
Then, \((r_i, q_i) = (\hat{r}_i, \hat{q}_i)\) solves the following maximization problem:

\[
(\hat{r}_i, \hat{q}_i) = \arg\max_{r_i, q_i} \left\{ P \left( q_i + \sum_{j \neq i} \hat{q}_j \right) - r_i \right\} \left[ (r_i - c - t)q_i \right]^{1-\beta},
\]

subject to

\[
\begin{align*}
&\left\{ P \left( q_i + \sum_{j \neq i} \hat{q}_j \right) - r_i \right\} q_i \geq 0, \\
&(r_i - c - t)q_i \geq 0.
\end{align*}
\]

(1)

In addition, we make the following assumptions for the uniqueness of the Cournot equilibrium, which are standard in the oligopoly literature:

**Assumption 3.** (i) \(\lim_{Q \to 0}[P(Q) + Q P'(Q)] = P_0\), and (ii) \((s + 1)P'(Q) + Q P''(Q) < 0\) for all \(Q > 0\) and \(s \geq 1\).

Under Assumptions 1-3, we find that, given any \(s \geq 1\), there exists a unique bargaining outcome characterized by \(\hat{r}_1 = ... = \hat{r}_s \equiv \hat{r}\) and \(\hat{q}_1 = ... = \hat{q}_s \equiv \hat{q}\), where \(\hat{r} (> 0)\) and \(\hat{q} (> 0)\) are uniquely determined by (2) and (3) below:

\[
\hat{q} = -\frac{P(s\hat{q}) - c - t}{P'(s\hat{q})},
\]

(2)

\[
\hat{r} = (1 - \beta)P(s\hat{q}) + \beta(c + t).
\]

(3)

Observe, due to the property of Nash bargaining, that (3) can be written as

\[
\frac{\beta}{1 - \beta} = \frac{P(s\hat{q}) - \hat{r}}{\hat{r} - c - t}.
\]

That is, the bargaining power ratio is exactly same as the ratio of price-cost margins in the downstream and upstream markets.

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6Assumption 3 (i) says that the marginal revenue is well-defined for all \(Q \geq 0\), while Assumption 3 (ii) holds if the demand function is not too convex.
Rewriting condition (2) as $P(s\hat{q}) + P'(s\hat{q})\hat{q} = c + t$ and summing over all $s$, we get

$$sP(\hat{Q}) + P'(\hat{Q})\hat{Q} = s(c + t),$$

(4)

where $\hat{Q} \equiv s\hat{q}$. From the characterization of the third-stage subgame equilibrium, we obtain the following lemma:

Lemma 1. An increase in the tariff rate leads to an increase in the input price $\hat{r}$ and a decrease in the level of output $\hat{Q}$. That is, $d\hat{Q}/dt < 0$ and $d\hat{r}/dt > 0$.

Proof. Totally differentiating (3) and (4) yields

$$\frac{d\hat{Q}}{dt} = \frac{s/P'(\hat{Q})}{s + 1 + \epsilon},$$

$$\frac{d\hat{r}}{dt} = \frac{s + \beta(1 + \epsilon)}{s + 1 + \epsilon},$$

where $\epsilon \equiv \frac{P''(\hat{Q})\hat{Q}}{P'(\hat{Q})}$ represents the elasticity of the slope of the demand. By applying Assumption 3 (ii), we get $\epsilon > -2$, implying that $s + 1 + \epsilon > 0$. Then, the result directly follows since $P'(.) < 0$ and $s + \beta(1 + \epsilon) > 0$.

In what follows, we make the final assumption on the demand for simplicity of the analysis:

Assumption 4. $\epsilon \equiv \frac{P''(\hat{Q})\hat{Q}}{P'(\hat{Q})}$ is constant.\footnote{This assumption holds for a variety of standard demand functions, including linear, constant elasticity and semi-log among others.}

Recall that the matching function $s = s(m, n)$ assigns the equilibrium matching pairs. Because the number of downstream/upstream firms is fixed in the short-run equilibrium, the number of successful matching pairs is also fixed in the second-stage subgame. The matching between $D$ and $U$, however, plays an important role in analyzing the long-run equilibrium.
3.2 Tariffs, Bargaining and Southern Welfare

In the first stage, Southern government chooses a tariff rate to maximize Southern welfare:

\[ W \equiv \int_0^{\hat{Q}} P(y)dy - P(\hat{Q})\hat{Q} + \{P(\hat{Q}) - c_D - \hat{r}\}\hat{Q} + t\hat{Q} \]

\[ \text{Consumer surplus} \quad \text{Southern profits} \quad \text{Tariiff revenue} \]

Differentiating \( W \) with respect to \( t \) and using Lemma 1 gives

\[ \frac{dW}{dt} \bigg|_{t=0} = \hat{Q} \left[ \frac{(1 + \epsilon) - \beta(2 + \epsilon)}{s + 1 + \epsilon} \right]. \]  

(5)

Thus, we have the following lemma:

**Lemma 2.** Starting from free trade, a small increase in tariff rates raises Southern welfare if bargaining strength of Southern firms is lower than a critical threshold. More formally, there exists a unique threshold \( \hat{\beta}(\epsilon) \) such that

\[ \frac{dW}{dt} \bigg|_{t=0} \geq 0 \iff \beta \leq \frac{1 + \epsilon}{2 + \epsilon} \equiv \hat{\beta}(\epsilon). \]

**Proof.** The result follows from rearranging (5). \( \square \)

Suppose the demand function is linear, i.e., \( \epsilon = 0 \). Then, a tariff improves Southern welfare if and only if bargaining power of Northern firms is greater than that of firms. In general, however, the elasticity of the the slope of the demand plays an important role to determine whether or not tariffs improve Southern welfare. If the elasticity \( \epsilon \) is constant, we get

\[ \hat{\beta}'(\epsilon) > 0, \quad \hat{\beta}''(\epsilon) < 0, \quad \hat{\beta}(-1) = 0, \quad \text{and} \quad \lim_{\epsilon \to \infty} \hat{\beta}(\epsilon) = 1. \]

This means that, given bargaining power, a less (more) \( \epsilon \) makes a subsidy (tariff) more possibly to be optimal. Surprisingly, when the elasticity of the the slope of the demand is small so that \( \epsilon < -1 \), the government should always impose a negative tariff (subsidy), irrespective of bargaining power.

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8Lemma 2 holds without Assumption 4. The following analysis, however, depends on Assumption 4.
We then analyze the optimum tariff in the short-run equilibrium. Letting $\hat{t}$ denote this tariff rate, it directly follows from $\frac{dW}{dt}|_{t=\hat{t}} = 0$ that

$$\hat{t} = \{ - P'(\hat{Q})\hat{q} \cdot (2 + \epsilon) \{ \hat{\beta}(\epsilon) - \beta \}. \quad (6)$$

Equation (6) reflects the relationship between bargaining power and optimum tariff. Note that the value of the first braces is always positive. Therefore, whether the optimum tariff is positive or not depends on the relative strength of bargaining power of Southern firms.

**Proposition 1.** The optimum tariff is closely related to bargaining power of Southern firms. That is,

(i) The optimum tariff is positive if and only if bargaining power of Southern firms is lower than the threshold $\hat{\beta}(\epsilon)$. More formally,

$$\hat{t} \succ 0 \iff \beta \preceq \hat{\beta}(\epsilon).$$

(ii) Furthermore, the optimum tariff is decreasing in bargaining power of Southern firms, i.e.,

$$\frac{\partial \hat{t}}{\partial \beta} < 0.$$ 

*Proof.* See the Appendix.

This proposition suggests that bargaining power plays a key role in shaping the optimum tariff when outsourcing is the main channel of input trade. Many developing countries typically impose heavy tariffs on imported inputs from developed countries. Traditionally, it is argued that this is because the governments have to protect domestic producers from fierce competition from foreign rivals.\(^9\)

On the contrary, Proposition 1 suggests that these protections stem from *weak bargaining power* in developing countries. Indeed, several papers have reported that producers in developing countries have little bargaining power due to their low technology of production.\(^10\)

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\(^9\)This is known as the *infant industry argument*. See Caves et al. (2006) and Krugman and Obstfeld (2008) among others.

\(^10\)See, for example, Wes (2000) and the papers cited therein.
3.3 Bargaining and Northern Profits

If tariffs are exogenous, higher bargaining power of South always hurts Northern firms. In other words, with exogenous tariffs, $\beta$ does not affect the total profits and hence higher $\beta$ implies a less share for Northern firms. When tariffs are set endogenously, however, an increase in $\beta$ leads to lower tariffs and higher overall profits. Thus, as $\beta$ increases, though Northern firms get a smaller share of total profits, Northern profit itself can be higher than before. This creates the possibility that an increase in $\beta$ is actually good for North. We formalize this argument in the below.

Let us go back to the third-stage bargaining and examine a small change in $D$’s bargaining power. Denote $U$’s profits and joint profits be $\hat{\pi}_U$ and $\hat{\Pi}$, respectively, where $\hat{\pi}_U \equiv (1 - \beta)\hat{\Pi}$ and $\hat{\Pi} \equiv [P(\hat{Q}) - c - \ell]\hat{q} = -P'(\hat{Q})\hat{q}^2$. Differentiating $\hat{\pi}_U$ with respect to $\beta$ yields

$$\frac{\partial \hat{\pi}_U}{\partial \beta} = -\hat{\Pi} + (1 - \beta) \frac{\partial \hat{\Pi}}{\partial t} \cdot \frac{\partial t}{\partial \beta}. \quad (7)$$

This equation illustrates that the rise in $D$’s bargaining power has two effects on $\hat{\pi}_U$. First, it directly reduces $\hat{\pi}_U$ by lowering the share of the joint profit. This direct effect is represented by the first term in (7) and exists even if tariffs are exogenous. Second, in the presence of endogenous tariffs, there is the indirect effect through the change in the optimum tariff, which is captured by the second term in (7).

In the first stage, we have to evaluate Equation (7) under the condition $t = \hat{t}$. It is evident that the first effect is negative and the second effect is positive.$^{11}$ Our interest is in, therefore, which effect dominates the other in equilibrium. After manipulating (7), we can find a threshold $\tilde{\beta}$ such that

$$\left. \frac{\partial \hat{\pi}_U}{\partial \beta} \right|_{\beta=0} \geq 0 \iff \beta \lesssim \tilde{\beta},$$

and its maximum is attained at

$$\beta = \begin{cases} \tilde{\beta} & \text{if } \hat{s} \in \left[1, \hat{s}(\beta, \epsilon)\right]; \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

$^{11}$To prove formally that the second effect is positive, we have to show that the joint profit $\hat{\Pi}$ is decreasing in $t$, i.e., $\frac{\partial \hat{\Pi}}{\partial \ell}|_{\ell=t} < 0$. See the Appendix for details.
where \( \tilde{\beta} \equiv (2 + \epsilon - \hat{s})/(2 + \epsilon) \) and \( \hat{s}(\beta, \epsilon) \equiv (2 + \epsilon)(1 - \beta) \). Thus, as long as the number of matching pairs is in the range of (8), a rise in bargaining power of Southern firms increases Northern profits. The next proposition formally states our second result.

**Proposition 2.** An increase in bargaining power in Southern firms can lead to an increase in Northern profits. More formally, there exists \( \hat{s}(\beta, \epsilon) \) such that \( \hat{\pi}_U \) is increasing in \( \beta \) if and only if \( \hat{s} \in [1, \hat{s}(\beta, \epsilon)] \).\(^{12}\)

*Proof.* See the Appendix. □

To understand this proposition, suppose that Southern firms face weak bargaining power \( (\beta < 1/2) \) and the demand is linear \( (\epsilon = 0) \), implying that \( \hat{s}(\epsilon) > 1 \). Then, Proposition 2 suggests that Northern firms might be better off by giving up their bargaining power. Note that this result holds for the small number of firms.\(^{13}\)

Proposition 2 offers an intriguing suggestion when international outsourcing takes place. If the number of firms is small, Northern firms can obtain more profits by reducing their bargaining power. The intuition behind the result is as follows. (The intuition behind Proposition 2 will be added soon.) Our conclusion in short-run equilibrium is therefore that biased bargaining power deteriorates gains from trade in an oligopolistic market.

### 3.4 Discussions

In this subsection, we check the robustness of the previous results. For this purpose, we develop alternative models that investigate the relationship between bargaining and tariffs. The first model is sequential bargaining and the second one is implicit bargaining through market competition. To save spaces, we only discuss the main results of these settings.

\(^{12}\)Strictly speaking, \( \hat{s}(\epsilon) \) have to be greater than 1.

\(^{13}\)For instance, suppose that Northern firms have strong bargaining power and propose take-it-or-leave-it offer to Southern firms \( (\beta = 0) \). Then, Proposition 2 realizes if and only if the market structure is monopoly. Although this seems unlikely to occur, this range becomes much broader in the presence of sequential bargaining (see the next subsection for related discussions).
3.4.1 Sequential Bargaining

So far, we have assumed that each pair of $D_i$ and $U_i$ bargains over the input price $r_i$ and the quantity of final goods $q_i$ at the same time. It might be more realistic, however, that each pair bargains over $r_i$ and $q_i$ separately. The supplementary notes of the current paper offers this alternative bargaining model in which $r_i$ and $q_i$ are negotiated sequentially.\textsuperscript{14}

In this sequential-bargaining setup, we find that there exist unique thresholds of bargaining power corresponding to $\hat{\beta}$ and $\tilde{\beta}$ in this simultaneous-bargaining model. These thresholds, however, are not exactly same among different bargaining models. The most interesting point is that the range of Proposition 2 is much greater than (8). The corresponding range of (8) in the alternative bargaining is given by

$$\beta = \begin{cases} 
\hat{\beta} & \text{if } \hat{s} \in \left[1, \frac{2+\epsilon}{\Delta}\right); \\
0 & \text{otherwise},
\end{cases}$$

(8')

where $\Delta \equiv \left\{ \beta + 1 - \frac{\hat{s}(1+\beta) + \epsilon}{\hat{s}(\epsilon+1+\epsilon)} \right\} / (1 - \beta) > 0$. As the supplementary note shows, if $\epsilon$ is not too large, the range of $\hat{s}$ that realizes $\partial \pi_U / \partial \beta > 0$ is greater than that of the basic model. For instance, consider a simple case where $\beta = 0$ and $\epsilon = 0$. Then, (8') satisfies when $\hat{s} \in [1, 1 + \sqrt{3})$ (i.e., monopoly and duopoly if the number of firms is discrete), whereas (8) holds only when it is monopoly. In general, the supplementary note demonstrates that the range of (9') is greater than that of (9) for any $\beta$, as long as $\epsilon < 2$. Furthermore, the two thresholds $\hat{\beta}$ and $\tilde{\beta}$ in the sequential bargaining model are closer than those of the simultaneous-bargaining model. This implies that, if contracts are subject to sequential bargaining, the welfare-maximizing point in South ($\hat{\beta}$) becomes much closer to the profit-maximizing point in North ($\tilde{\beta}$) and there are less conflicts between countries.

3.4.2 Implicit Bargaining

In the simultaneous-bargaining model, bargaining power is exogenously as given and it is not analyzed explicitly how bargaining power is formalized in the bargaining stage. Because bargaining power is not observable, however, this is not so appealing for policy implications. In order to make our model more realistic, therefore, we need to endogenize the determinant of bargaining power.

\textsuperscript{14}A supplementary note provides a detailed setting of this alternative bargaining model. In the model, each pair bargains over the input price $r_i$ first, and then each $D_i$ sells output taking $r \equiv (r_1, r_2, ..., r_s)$ as given. Therefore, there are four subgame stages in the sequential bargaining model.
In a separate paper, Ara and Ghosh (2010) build a different setup in which upstream firms supply their inputs to downstream firms without matching or bargaining between them. Instead, we assume that implicit bargaining takes place through market competition. In contrast to the previous model, there is an input market in North, and downstream firms buy inputs from the market. In this setup, we show that the following relationship holds:

\[
\frac{\beta}{1-\beta} = \frac{P(\hat{Q}) - c_D - \hat{r}}{\hat{r} - c_U - t} = \frac{n}{m + 1 + \epsilon_D},
\]

where \( \epsilon_D \) is the elasticity of the slope of the demand in South, i.e., output market. As we can see from equations (2) and (3), the markup ratio between downstream and upstream markets is given by \( \beta/(1-\beta) \) in the current setup. Ara and Ghosh (2010) demonstrates that this ratio is approximated by the relative number of firms in each market. Furthermore, we have

\[
\frac{\partial \hat{t}}{\partial m} = \frac{\partial \hat{t}}{\partial \beta} \cdot \frac{\partial \beta}{\partial m} > 0 \quad \text{and} \quad \frac{\partial \hat{t}}{\partial n} = \frac{\partial \hat{t}}{\partial \beta} \cdot \frac{\partial \beta}{\partial n} < 0,
\]

from Proposition 1 and the above proxy between bargaining power and relative market size. These conditions imply that the “thickness of markets” (McLaren, 2000) has a crucial impact on the optimum trade policy through implicit bargaining. It is important to note that, due to availability of the relevant data, we can test our prediction empirically by using the number of firms as a proxy of bargaining power.

4 Long-Run Equilibrium

In the previous section, we assume that one-to-one matching is deterministic: every firm in the market can find its partner with certainty. This assumption is not so crucial as long as the number of firms is fixed. Once free entry is allowed and the number of firms fluctuates, however, this assumption is no longer plausible. In this section, therefore, we extend our analysis to the case where free entry of \( D \) and \( U \) is permitted.
4.1 Welfare Maximization

In the last stage, each firm chooses its quantity, taking other firms’ behavior as given. The bargaining is formulated by (1) and the outcome is same as before. Especially, there exists a unique Nash equilibrium characterized by 
\[ \hat{q}_1 = \hat{q}_2 = \ldots = \hat{q}_s \equiv \hat{q} \text{ and } \hat{r}_1 = \hat{r}_2 = \ldots = \hat{r}_s \equiv \hat{r}, \] with the first-order condition satisfying.

In the second stage, the number of firms \( s = s(m, n) \) is endogenously determined by free entry condition. Letting \( \hat{s} \) denote the number of matching pairs satisfying the free entry condition, a tariff rate has a crucial impacts on \( \hat{s} \), because it is an endogenous variable in the long run. The following lemma shows that tariffs have a negative impact on the matching.

**Lemma 3.** The number of matching pairs is decreasing in a tariff rate. That is, \( d\hat{s}/dt < 0 \).

**Proof.** See the Appendix.

In the first stage, the government sets a tariff to maximize Southern welfare:

\[
W = \left[ \int_0^{\hat{Q}} P(y)dy - P(\hat{Q})\hat{Q} \right] + \frac{t\hat{Q}}{\text{Tariff revenue}}.
\]

Note, due to free entry, that Southern profits become zero in long-run equilibrium. Differentiating this with respect to \( t \),

\[
\frac{dW}{dt} \bigg|_{t=0} = \left( \frac{\hat{Q}}{2\hat{s} + \epsilon} \right) \epsilon.
\]

In long-run equilibrium, bargaining power has no effect on the optimum tariff and only the elasticity of the slope of the demand function \( \epsilon \) is important for the determinant of the optimum tariff. Therefore, whether a tariff rate is positive or negative is identical with the sign of \( \epsilon \).

**Lemma 4.** Starting from free trade, a small increase in tariff rates raises Southern welfare if the elasticity of the slope of the demand is positive. More formally,

\[
\frac{dW}{dt} \bigg|_{t=0} \gtrless 0 \iff \epsilon \gtrless 0.
\]

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Next, let us consider the optimum tariff under free entry. Defining this as $\hat{t}$, it follows directly from $\frac{dW}{dt} \bigg|_{t=\hat{t}} = 0$ that

$$\hat{t} = \left( -\frac{P'(\hat{Q}) \hat{q}}{2} \right) \cdot \epsilon. \quad (9)$$

Therefore:

**Proposition 3.** In long-run equilibrium, the optimum tariff $\hat{t}$ depends only on the elasticity of the slope of the demand $\epsilon$. That is,

$$\hat{t} \geq 0 \iff \epsilon \geq 0. \quad (15)$$

### 4.2 Change in Bargaining Power

In this subsection, we show that the optimum tariff has a non-monotone relationship in terms of $\beta$. To prove this, we specify the property of the matching function $s = s(m, n)$. Following the literature (e.g., Grossman and Helpman, 2002), we make three assumptions of this matching function, $s(m, n)$:

1. $s(\lambda m, \lambda n) = \lambda s(m, n); \quad (10)$
2. $\frac{\partial s(m, n)}{\partial m} > 0$ and $\frac{\partial s(m, n)}{\partial n} > 0; \quad (11)$
3. $\frac{\partial^2 s(m, n)}{\partial m^2} < 0$ and $\frac{\partial^2 s(m, n)}{\partial n^2} < 0. \quad (12)$

Mathematically speaking, equation (10) means that the matching function is homogeneous of degree one. Economically speaking, this means a constant-returns-to-scale matching. The implication of (12) is equivalent with complementarity in matching (i.e., $\frac{\partial^2 s(m, n)}{\partial m \partial n} > 0$).\(^{16}\)

In what follows, we use a “normalized” matching function, which is defined as below. First of all, letting $\lambda = 1/m$ in (10), we have

$$s(m, n) = m \cdot s \left( 1, \frac{n}{m} \right) \equiv mS(z), \quad (10')$$

\(^{15}\)The intuition behind the above result should be clear. In long-run equilibrium, free entry drives Southern profits to zero. As a result, whether South has strong bargaining power or not becomes irrelevant to the optimum tariff.\(^{16}\)To see this, we get from (10),

$$m \frac{\partial s(m, n)}{\partial m} + n \frac{\partial s(m, n)}{\partial n} = s(m, n).$$

Differentiating this with respect to $m$ and $n$, we see that (12) holds if and only if $\frac{\partial^2 s(m, n)}{\partial m \partial n} > 0.$
where \( z \equiv \frac{n}{m} \). In long-run equilibrium, this ratio \( z \) has to satisfy the expected free entry conditions:

\[
\frac{s(m,n)}{m} \cdot \beta R = F_D; \quad \frac{s(m,n)}{n} \cdot (1 - \beta) R = F_U \quad \iff \quad z(\beta) = \left( \frac{1 - \beta}{\beta} \right) \left( \frac{F_D}{F_U} \right).
\]

Moreover, differentiating (10') with respect to \( n \), we get \( S'(z) > 0 \). Note that this reflects the free entry conditions in the both markets simultaneously. Finally, we get corresponding conditions with (11) and (12) in terms of \( S(z) \):

\[
S(z) > zS'(z); \quad (11')
\]
\[
S''(z) < 0. \quad (12')
\]

We summarize this observation in the following lemma:

**Lemma 5.** Suppose that the matching function \( s(m,n) \) has the properties of (10)-(12). Then, there exists a normalized matching function \( S(z) \) that has the corresponding properties of (10')-(12'), where \( z \) satisfies the free entry conditions in the both markets.

In addition, the following lemma shows that the optimum tariff is closely related with the matching function.

**Lemma 6.** If \( \epsilon > 0 \) (\( \epsilon < 0 \)), an increase in the number of successful matching pairs leads to a decrease (increase) in the optimum tariff. That is,

\[
\text{sgn} \{ \tilde{t}'(\beta) \} = \begin{cases} 
-\text{sgn} \{ \phi'(\beta) \} & \text{if } \epsilon > 0; \\
\text{sgn} \{ \phi'(\beta) \} & \text{otherwise},
\end{cases}
\]

where \( \phi(\beta) \equiv \beta S(\beta) \).\(^{17}\)

*Proof.* See the Appendix.

---

\(^{17}\)Recall, from the free entry condition, that \( z \) is a function of \( \beta \).
Using Lemma 6, we can explore the relationship between bargaining power and the optimum tariff. From the definition of $\phi(\beta)$, we have

$$\phi'(\beta) = S(z) + \beta S'(z)z' \left( \frac{z'}{\beta} \right) = S(z) - \frac{zS'(z)}{1-\beta}.$$ 

Then, it is straightforward to see that

$$\lim_{\beta \to 0} \phi'(\beta) > 0 \quad \text{and} \quad \lim_{\beta \to 1} \phi'(\beta) < 0;$$

$$\phi''(\beta) = \frac{1}{\beta^3} \left( \frac{F_D}{F_U} \right)^2 S''(\beta) < 0.$$

Figure 1 depicts the optimum tariff in terms of bargaining power. It directly follows from the above argument that the optimum tariff becomes a U-shaped (inversed U-shaped) function if $\epsilon$ is positive (negative), thereby taking the minimum (maximum) at $\beta$ where $\phi'(\beta) = 0$. Thus, even if we allow the possibility of free entry and stochastic matching between $D$ and $U$, free trade is more likely to occur around equal bargaining power.

**Proposition 4.** In long-run equilibrium, the optimum tariff has non-monotone relationship with bargaining power. Moreover, the optimum tariff is minimized around equal bargaining power.

The intuition for the U-shaped relationship above goes as follows. If bargaining power is equal, there exist a lot of matching pairs in the market. Because more matching pairs produce more aggregate products consumed and sold, the government has no incentive to set a high tariff rate. If bargaining power is biased, on the contrary, the number of firms is also biased due to free entry. This makes matching pairs fewer and consumer surplus and producer surplus become lower. The only way to enhance the welfare is to set a high tariff on imported input. As a result, the optimum tariff becomes high when bargaining power is biased.

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18For instance, suppose that the matching function is $s(m, n) = mn/(m + n)$. (Notice that this function satisfies all conditions of [10]-[12].) Then, $S(z) = z/(1 + z)$ and its cutoff point takes at $\beta = 1/2$ if $F_D = F_U$.  

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5 Concluding Remarks

In this paper, we have developed an international bargaining model in which South procures intermediate input from North by international outsourcing. We have focused on whether bargaining power can have an effect on forming the optimum tariff. Our main results are that (i) bargaining power is closely related to the optimum tariff; and (ii) equal bargaining power makes every country receive more gains from trade. Although there are some differences between short-run and long-run equilibria, we have shown that the main findings hold in the general demand and matching functions.

We would like to conclude this paper with brief mentions for future extension. First, it is interesting to consider an alternative matching in our model. In particular, the current paper adopts one-to-one matching, but sometimes firms may find it more profitable to match more than one firm. Secondly, the effect of firm heterogeneity should be examined. We have assumed symmetric firms for simplicity, but it is well known that more productive firms find more partners in global sourcing. This stylized fact can be formalized in our setup by using the Melitz (2003) model. We believe, however, that the central result of our model would not change even in the presence of firm heterogeneity: biased bargaining power always deteriorates gains from trade.

Appendix

Proof of Proposition 1. The optimum tariff is given by

\[ \hat{t} = \left\{ -P'(\hat{Q})\hat{q} \cdot (2 + \epsilon) \right\} \left\{ \hat{\beta}(\epsilon) - \beta \right\}. \]

Noting the functional form \( Q(t(\beta), \beta) \),

\[
\frac{\partial \hat{t}}{\partial \beta} = -\{ \hat{\beta}(\epsilon) - \beta \} (2 + \epsilon) \left[ P''(\hat{Q}) \frac{\hat{Q}}{s} + P'(\hat{Q}) \frac{1}{s} + \frac{d\hat{Q}}{d\beta} \right] + \left\{ -P'(\hat{Q})\hat{q}(2 + \epsilon) \right\} (-1). \]  \hspace{1cm} (A.1)

where \( \frac{d\hat{Q}}{d\beta} = \frac{d\hat{Q}}{dt} + \frac{\partial \hat{Q}}{\partial t} \cdot \frac{dt}{d\beta} \). From (2),

\[
\frac{d\hat{Q}}{d\beta} = \frac{\hat{s}\frac{dn}{s}}{\hat{s}+1} P'(\hat{Q}) + P''(\hat{Q})\hat{Q}. \]

\[19\text{See, in particular, Bernard et al. (2009) for detailed evidence.} \]
Substituting this into (A.1) and evaluating it at \( t = \hat{t} \) yields

\[
\frac{\partial \hat{t}}{\partial \beta} = -(2 + \epsilon) \left\{ \beta(\epsilon) - \beta \right\} \frac{\partial \hat{t}}{\partial \beta} \left( \frac{1 + \epsilon}{\hat{s} + 1 + \epsilon} \right) - P'(\hat{Q})\hat{q}.
\]

After solving for \( \frac{\partial \hat{t}}{\partial \beta} \), we obtain

\[
\frac{\partial \hat{t}}{\partial \beta} = \frac{P'(\hat{Q})\hat{q} \cdot (2 + \epsilon)}{1 + [(1 + \epsilon) - \beta(2 + \epsilon)] \left( \frac{\epsilon + 1}{\hat{s} + 1 + \epsilon} \right)} < 0.
\]

Thus, \( \hat{t} \) is necessarily decreasing in \( \beta \).

**Proof of Proposition 2.** Differentiating the joint profit \( \hat{\Pi} = -P'(\hat{Q})\hat{q}^2 \) with respect to \( t \) and evaluating it \( t = \hat{t} \), we have

\[
\left. \frac{\partial \hat{\Pi}}{\partial t} \right|_{t=\hat{t}} = -\hat{q} \cdot \left( \frac{2 + \epsilon}{\hat{s} + 1 + \epsilon} \right) < 0.
\]

Moreover, from Proposition 1, we know that \( \frac{\partial \hat{t}}{\partial \beta} < 0 \). Therefore, the sign of the second term is opposite to that of the first term. After manipulating (7), we can find a threshold \( \tilde{\beta} \) such that

\[
\frac{\partial \hat{\pi}_U}{\partial \beta} = \left( -\frac{P'(\hat{Q})\hat{q}^2}{\hat{s} + (1 - \beta)(\epsilon + 1)(\epsilon + 2)} \right) \cdot (\tilde{\beta} - \beta),
\]

where

\[
\tilde{\beta} \equiv \frac{2 + \epsilon - \hat{s}}{2 + \epsilon}.
\]

Since the value of the first parentheses is always positive, we have

\[
\frac{\partial \hat{\pi}_U}{\partial \beta} \leq 0 \iff \beta \leq \tilde{\beta}.
\]

Then, it is immediate to see that \( \hat{\pi}_U(\beta) \) is maximized at \( \tilde{\beta} \) if \( \hat{s} \in [1, \hat{s}(\beta, \epsilon)] \) and 0 otherwise, where \( \hat{s}(\beta, \epsilon) \equiv (2 + \epsilon)(1 - \beta) \). Thus, a rise in bargaining power of Southern firms increases Northern profit, as long as the number of matching pairs is \( \hat{s} \in [1, \hat{s}(\beta, \epsilon)] \).

**Proof of Lemma 3.** Using the definition of the matching function, \( D \)'s free entry condition can be written as

\[
-\frac{P'(\hat{Q})\hat{Q}^2}{\hat{s}^2} = \frac{F_D}{\phi(\beta)},
\]

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where $\phi(\beta) \equiv \beta S(\beta)$. Differentiating this free entry condition with respect to $t$,

$$\frac{d\hat{Q}}{dt} = \frac{2\hat{q}}{2 + \epsilon} \frac{d\hat{s}}{dt}. \quad \text{(A.2)}$$

Furthermore, notice that $\frac{d\hat{Q}}{dr} = \frac{d\hat{Q}}{\hat{s}} \frac{d\hat{s}}{dt} + \frac{d\hat{Q}}{\hat{q}} \frac{d\hat{q}}{dt}$ where

$$\frac{\partial \hat{Q}}{\partial \hat{t}} = \frac{-\hat{s}}{P'(\hat{Q})(\hat{s} + 1 + \epsilon)};$$

$$\frac{\partial \hat{Q}}{\partial \hat{s}} = \frac{-\hat{q}}{\hat{s} + 1 + \epsilon}.$$

Substituting these into the above equation and solving this yield for $\frac{d\hat{s}}{dt}$, we get

$$\frac{d\hat{s}}{dt} = \left(\frac{2 + \epsilon}{2\hat{s} + \epsilon}\right) \left(\frac{\hat{s}}{P'(\hat{Q})\hat{q}}\right) < 0.$$

Finally, plugging this solution into (A.2), we have $\frac{d\hat{Q}}{dt}$

$$\frac{d\hat{Q}}{dt} = \frac{2\hat{s}}{(2\hat{s} + \epsilon)P'(\hat{Q})} < 0.$$

This completes the proof.

**Proof of Lemma 6.** The optimum tariff under free entry can be expressed as

$$\hat{t}\hat{q}(t, \beta)\phi(\beta) = \frac{\epsilon D}{2}. \quad \text{(9')}$$

We first show that, for given $\beta$ and $\epsilon$, LHS of (9') is increasing in $t$. The derivative of LHS of (8') with respect to $t$ is

$$\frac{dLHS}{dt} = \phi(\beta) \left[\hat{q}(t, \beta) + \hat{t} \cdot \frac{d\hat{q}}{dt} \right]. \quad \text{(A.3)}$$

Thus, we need to know $\frac{d\hat{q}}{dt}$. The next lemma is useful for this proof:

**Lemma A.1.** The sign of $\frac{d\hat{q}}{dt}$ is equivalent with that of $\epsilon$.

**Proof.** Differentiating $D$'s free entry condition $-P'(\hat{Q})\hat{q}^2 = F_D/\phi(\beta)$ with respect to $t$,

$$\left[ -P''(\hat{Q})\hat{q}^2 \right] \frac{d\hat{Q}}{dt} + \left[ -2P'(\hat{Q})\hat{q} \right] \frac{d\hat{q}}{dt} = 0.$$
Solving this for \( \frac{d\tilde{q}}{dt} \), we get
\[
\frac{d\tilde{q}}{dt} = -\frac{\epsilon}{2\hat{s}} \cdot \frac{d\tilde{Q}}{dt}.
\]

Lemma A.1 directly follows from this equation, because \( \frac{d\tilde{q}}{dt} < 0 \).

Due to Lemma A.1, (A.3) is always positive, because both \( \tilde{t} \) and \( \frac{d\tilde{q}}{dt} \) are positive (negative) if \( \epsilon \) is positive (negative). Thus, we need to only check the derivative of LHS of
\[
\tilde{t} = \left( \frac{\epsilon F_D}{2} \right) \left( \frac{1}{\phi(\beta)\tilde{q}} \right)
\]
with respect to \( \beta \), taking \( t \) in \( \tilde{q}(t, \beta) \) as given:
\[
\text{sgn} \left( \frac{d\tilde{t}}{d\beta} \right) = -\text{sgn} (\epsilon) \cdot \text{sgn} \left( \frac{d[\phi(\beta)\tilde{q}]}{d\beta} \right). \tag{A.4}
\]

From the free entry condition, \( \phi(\beta)\tilde{q}[-P'(\tilde{Q})\tilde{q}] = F_D \) and since RHS of this equation is constant,
\[
\text{sgn} \left( \frac{d[\phi(\beta)\tilde{q}]}{d\beta} \right) = -\text{sgn} \left( \frac{d(-P'(\tilde{Q})\tilde{q})}{d\beta} \right).
\]

Thus, to see the sign of (A.4), it is enough to check the sign of \( \frac{d(-P'(\tilde{Q})\tilde{q})}{d\beta} \), which is given by
\[
\frac{d(-P'(\tilde{Q})\tilde{q})}{d\beta} \left( \frac{P'(\tilde{Q})\tilde{q}}{\hat{s} + \epsilon} \right) \frac{d\hat{s}}{d\beta},
\]
which means that the sign of \( \frac{d[\phi(\beta)\tilde{q}]}{d\beta} \) and the sign of \( \frac{d\hat{s}}{d\beta} \) are identical. Therefore, we get the following lemma, which illustrate the relationship between bargaining power and the number of successful matching firms:

**Lemma A.2.**
\[
\text{sgn} \left( \frac{d\tilde{t}}{d\beta} \right) = \begin{cases} 
-\text{sgn} \left( \frac{d\hat{s}}{d\beta} \right) & \text{if } \epsilon > 0; \\
\text{sgn} \left( \frac{d\hat{s}}{d\beta} \right) & \text{otherwise.}
\end{cases}
\]

This result is intuitive. It asserts that, in long-run equilibrium, bargaining power has an impact on the number of matching pairs. This, in turn, changes the optimum tariff against the imported input.

From Lemma A.2, we only have to check the sign of \( \frac{d\hat{s}}{d\beta} \). Differentiating \( D \)'s free entry condition $-\frac{P'(\tilde{Q})\tilde{q}^2}{\hat{s}^2}$ =
\[ \frac{F_n}{\phi(\beta)} \] with respect to \( \beta \), we obtain
\[ \frac{1}{\hat{s}} \left( \frac{2\hat{s} + \epsilon}{\hat{s} + 1 + \epsilon} \right) \frac{d\hat{s}}{d\beta} = \frac{\phi'(\beta)}{\phi(\beta)}. \]

Combining this equation with Lemma A.2 completes the proof of Lemma 6.

References


Figure 1: The optimum tariff in long-run equilibrium
Supplementary Note

In this supplementary note, we offer detailed procedures of our results that are omitted from the main text. For simplicity, we assume \( c_D = 0 \) and \( c_U = c \) in the following sections; however, the following results hold under a general setup.

In Section 3, each pair of \( D_i \) and \( U_i \) bargains over the input price \( r_i \) and the quantity of final goods \( q_i \) at the same time. In this section, we offer an alternative bargaining model in which each pair bargains over the output price \( r_i \) first, and then each \( D_i \) sells output taking \( r_i \) as given. Our purpose here is to show that the main results in the last section still hold in this sequential bargaining setup. Note that in this bargaining, the game is composed of four stages: (i) The government sets a tariff on imported input; (ii) the matching function assigns the number of successful pairs; (iii) Each \( U_i \) chooses the input price \( r_i \); and (iv) Each \( D_i \) chooses the quantity of final goods \( q_i \). Keeping this difference in mind, let us solve the alternative bargaining model by backward induction.

In the fourth stage, each \( D_i \) maximizes

\[
\hat{q}_i = \arg\max_{q_i} \left[ P \left( q_i + \sum_{j \neq i} q_j \right) - \tilde{r} q_i \right],
\]

where \( \tilde{r} \) is determined in the third-stage subgame. As in the main text, we can easily see that there exists a unique Nash equilibrium characterized by \( \hat{q}_1 = ... = \hat{q}_s = \hat{q} \). The first-order condition is

\[
\hat{q} = -\frac{P(\hat{Q}) - \tilde{r}}{P'(\hat{Q})},
\]

where \( \hat{Q} = s\hat{q} \). The aggregate first-order condition for \( D \) is

\[
sP(\hat{Q}) + P'(\hat{Q})\hat{Q} = s\tilde{r}. \tag{S.1}
\]

Observing \( \hat{q} \), each \( U_i \) chooses the input price so as to maximize his objective function in the third stage. Following Naylor (2002), we assume that \( U_i \)'s objective function is Cobb-Douglas weighted by bargaining power as follows:

\[
\hat{r}_i = \arg\max_{r_i} \left[ (P(\hat{Q}) - r_i)\hat{q}\beta_1 [(r_i - c - t)\hat{q}]^{1-\beta} \right].
\]

It is straightforward to find a unique Nash equilibrium \( \hat{r}_1 = ... = \hat{r}_s = \hat{r} \) such that

\[
\frac{P(\hat{Q}) - \hat{r}}{\hat{r} - c - t} = \Delta, \tag{S.2}
\]

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where
\[ \Delta \equiv \beta + 1 - s(1+\beta+\epsilon) \frac{1}{1-\beta} > 0. \]

Recall that RHS of (S.2) is expressed as \( \frac{\beta}{1-\beta} \) (i.e., the ratio of bargaining power between \( D \) and \( U \)) in the basic model. Rearranging (S.2),
\[ \hat{r} = \frac{P(\hat{Q}) + \Delta(c + t)}{1 + \Delta}. \]  
(S.3)

Differentiating (S.1) and (S.3) with respect to \( t \),
\[ \frac{d\hat{Q}}{dt} = \frac{s\frac{dr}{dt}}{(s+1)P'(\hat{Q}) + P''(\hat{Q})\hat{Q}}, \]
and
\[ \frac{d\hat{r}}{dt} = \frac{1}{1 + \Delta} \left\{ P'(\hat{Q}) \frac{d\hat{Q}}{dt} + \Delta \right\}. \]

Setting \( r = \hat{r} \) and solving for \( \frac{d\hat{Q}}{dt} \) and \( \frac{d\hat{r}}{dt} \),
\[ \frac{d\hat{Q}}{dt} = \frac{s\Delta}{P'(\hat{Q})(1 + \epsilon + \Delta(s + 1 + \epsilon))} < 0, \]
and
\[ \frac{d\hat{r}}{dt} = \frac{(s + 1 + \epsilon)\Delta}{1 + \epsilon + \Delta(s + 1 + \epsilon)} > 0. \]

Note that the signs of these directions are same as these in the basic model. Using these conditions, then, we derive the optimum tariff.

In the second stage, the matching function \( s = s(m, n) \) determines the number of successfully matched pairs \( \hat{s} \) in the subgame equilibrium. As in the main text, the matching function plays no role, because the number of firms is fixed in our setup.

In the first stage, Southern government sets a tariff so as to maximize the social welfare. Clearly, this welfare is identical with that in Section 3. Because of the sequential bargaining, however, the marginal effect of \( t \) on the welfare is different.

\[ \left. \frac{dW}{dt} \right|_{t=0} = \left( P(\hat{Q}) - \hat{r} \right) \frac{d\hat{Q}}{dt} - \hat{Q} \frac{d\hat{r}}{dt} + \hat{Q} = \hat{Q} \left[ -\Delta + 1 + \epsilon \right] \frac{1}{1 + \epsilon + \Delta(s + 1 + \epsilon)}. \]

Thus,
\[ \left. \frac{dW}{dt} \right|_{t=0} \leq 0 \iff \Delta \geq 1 + \epsilon. \]
To see this implication, the following observation is useful.

\[ \frac{\partial \Delta}{\partial \beta} > 0 \quad \text{for} \quad \forall \beta, \epsilon, n. \quad (S.4) \]

Then, we have the next proposition.

**Lemma S.1.** Starting from free trade, the optimal trade policy depends upon bargaining power.

\[
\left. \frac{dW}{dt} \right|_{t=0} \leq 0 \iff \beta \preceq \hat{\beta}.
\]

where \( \hat{\beta} \) is an implicit solution for \( \Delta = 1 + \epsilon \).

For instance, under a linear demand, \( \Delta(\beta) = \frac{1 + \beta}{1 - \beta} \cdot \frac{\gamma}{\pi + 1} \). Since \( \Delta(\beta) \) is increasing in \( \beta \) and \( \Delta(0) = \frac{\gamma}{\pi + 1} < 1 \), there is always a threshold \( \hat{\beta} \) at which the direction of the optimal policy determines. It is important to note that Lemma S.1 holds for non-constant elasticity \( \epsilon \); however, the argument below is applied only for a constant elasticity.

Letting \( \hat{t} \) be the optimum tariff, we have

\[
\left. \frac{dW}{dt} \right|_{t=\hat{t}} = \hat{Q} \left[ \frac{-\Delta + 1 + \epsilon}{1 + \epsilon + \Delta(s + 1 + \epsilon)} \right] + \hat{t} \left[ \frac{s\Delta}{P'(\hat{Q})(1 + \epsilon + \Delta(s + 1 + \epsilon))} \right] = 0,
\]

and thus

\[
\hat{t} = -\frac{P'(\hat{Q})\hat{Q}}{s\Delta} (-\Delta + 1 + \epsilon). \quad (S.5)
\]

Therefore,

**Proposition S.1.** The optimum tariff is closely related to \( D \)'s bargaining power. That is,

\[
\hat{t} \preceq 0 \iff \beta \preceq \hat{\beta}.
\]

Although the cutoff point \( \hat{\beta} \) in sequential bargaining is different from that in the simultaneous bargaining, the implication of this proposition is essentially equivalent.

Next, we examine the change of bargaining power on the foreign profits. Note that, because of the
sequential bargaining, the profit function is different from that in the basic model:

$$\pi = (\hat{r} - c - t)\hat{q} = -\frac{P'(\hat{Q})\hat{Q}^2}{\Delta s^2}.$$  

We want to show here that \(\frac{\partial \pi}{\partial \beta} > 0\) in some ranges of parameters: raising \(D\)'s bargaining power increases the foreign profits. It follows from

$$\frac{\partial \pi}{\partial \beta} = \frac{\partial \pi}{\partial \Delta} \cdot \frac{\partial \Delta}{\partial \beta}$$  

and (S.4) that the sign of \(\frac{\partial \pi}{\partial \beta}\) is identical with the sign of \(\frac{\partial \pi}{\partial \Delta}\). In what follows, therefore, we use \(\Delta\) as approximate bargaining power. Differentiating (S.5) with respect to \(\Delta\),

$$\frac{\partial \hat{t}}{\partial \Delta} = -\frac{P'(\hat{Q})\hat{Q}}{\hat{s}\Delta^2}[(1 + \epsilon - \Delta)\eta - 1](1 + \epsilon),$$

where

$$\eta \equiv \frac{d\hat{Q}}{d\Delta} \frac{d\hat{r}}{d\hat{Q}}$$

is the elasticity of output with respect to (proxy) bargaining power. From (S.2) and (S.3),

$$\frac{d\hat{Q}}{d\Delta} = \frac{\hat{s}}{P'(\hat{Q})\frac{dr}{d\Delta}}$$

and

$$\frac{d\hat{r}}{d\Delta} = \frac{P'(\hat{Q})}{1 + \Delta} \left( \frac{d\hat{Q}}{d\Delta} + \hat{Q} \frac{\Delta}{\hat{s}} + \frac{\Delta}{P'(\hat{Q})} \frac{\partial t}{\partial \Delta} \right).$$

Rearranging these equations and evaluating \(r = \hat{r}\) and \(t = \hat{t}\),

$$\eta[1 + \epsilon + \Delta(\hat{s} + 1 + \epsilon)] = 1 + \frac{\hat{s}\Delta^2}{P'(\hat{Q})\hat{Q}} \frac{\partial \hat{t}}{\partial \Delta}. \tag{S.7}$$

Substituting (S.6) into (S.7),

$$\eta = \frac{2 + \epsilon}{(\epsilon + 1)(\epsilon + 2) + \Delta \hat{s}} > 0.$$  

Note that this inequality holds even if \(\epsilon < -1\).

From (S.7), the sign of \(\frac{\partial \hat{t}}{\partial \Delta}\) depends on \((1 + \epsilon - \Delta)\eta - 1\) and \((1 + \epsilon)\). It is easy to check that the former is always negative. Therefore,

**Lemma S.2.** Raising \(D\)'s bargaining power reduces the optimum tariff if the demand is not too convex:

$$\frac{\partial \hat{t}}{\partial \Delta} \leq 0 \iff \epsilon \geq -1.$$  

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Differentiating $\pi$ with respect to $\Delta$, 

$$\frac{\partial \pi}{\partial \Delta} = - \frac{P'(Q) \hat{Q}^2}{\Delta^2 \hat{s}} \{ (2 + \epsilon) \eta - 1 \}.$$ 

The sign of $\frac{\partial \pi}{\partial \Delta}$ is the same as $(2 + \epsilon) \eta - 1$. It is straightforward to see that $\frac{\partial \pi}{\partial \Delta} > 0$ if 

$$\hat{s} \in \left[ 1, \frac{2 + \epsilon}{\Delta} \right]. \quad (S.8)$$

This observation proves the robustness of the central result of the paper.

**Proposition S.2.** If the number of matching firms is limited to the range of $(S.8)$, the rise in $D$’s bargaining power increases $U$’s profits.

In the alternative bargaining model, the range of $\hat{s}$ that realizes $\frac{\partial \pi}{\partial \beta} > 0$ is greater than that of the basic model, if $\epsilon$ is not too large. For instance, let us consider a simple case where $\beta = 0$ and $\epsilon = 0$. Then, $(S.8)$ satisfies when $\hat{s} \in [1, 1 + \sqrt{3})$ (i.e., monopoly and duopoly if the number of firms is discrete), whereas $(10)$ holds only when it is monopoly. In general,

**Claim S.1.** In the alternative bargaining model, if $\epsilon < 2$, the range of $(S.8)$ is greater than that of $(10)$ for any $\beta$.

It is important to note that, under sequential bargaining with small $\epsilon$, the two thresholds $\hat{\beta}$ and $\tilde{\beta}$ are closer than those of the simultaneous model. This implies that if contracts are subject to renegotiation or long-term relationship, these two thresholds becomes much similar and there are less conflicts between countries. Therefore,

**Claim S.2.** If $\epsilon < 2$, the thresholds $\hat{\beta}$ and $\tilde{\beta}$ becomes closer under sequential bargaining.

The conclusion of this supplementary note is to confirm that all results hold in the presence of sequential bargaining. Minor differences are:

(i) The threshold of the optimal policy depends on $n$ as well as $\epsilon$;

(ii) The rise in $D$’s bargaining power might not reduce the optimum tariff;

(iii) The range of $\hat{s}$ that satisfies $\frac{\partial \pi}{\partial \Delta} > 0$ becomes wider if $\epsilon$ is not too large.

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Appendix

Proof of Equation (S.4). From the text, $\Delta(\beta)$ is

$$\Delta(\beta) \equiv \beta + 1 - \frac{\beta(1+\beta)\epsilon}{\beta + 1 + \epsilon - b s}.$$  

Rewriting this form,

$$\Delta(\beta) = (1 + \beta)\tilde{\beta}(\tilde{\beta} + \epsilon) - \epsilon.$$  

Since $\tilde{\beta}(\tilde{\beta} + 1 + \epsilon)$ in the denominator is always positive, this term has no effect on the sign of $\Delta(\beta)$. Letting

$$\Omega(\beta) \equiv (1 + \beta)\tilde{\beta}(\tilde{\beta} + \epsilon) - \epsilon(1 - \beta),$$

and differentiating $\Omega(\beta)$ with respect to $\beta$,

$$\frac{\partial \Omega(\beta)}{\partial \beta} = 2\tilde{\beta}(\tilde{\beta} + \epsilon) - \frac{\epsilon(1 - \beta)}{(1 - \beta)^2}.$$  \hfill (S.12)

It is clear that the numerator of (S.12) is increasing in $\beta$, because $2(2\tilde{\beta} + \epsilon) > 0$. Therefore, $\Delta(\beta)$ is increasing in $\beta$ if (B.2) is positive evaluated at $\tilde{\beta} = 1$:

$$\left.\frac{\partial \Omega(\beta)}{\partial \beta}\right|_{\tilde{\beta}=1} = \frac{2 + \epsilon}{(1 - \beta)^2} > 0.$$  

This completes the proof. \hfill \Box

Proof of Claim S.1. To prove this, it is enough to show that $\tilde{\beta} > \beta^*$ for all $\tilde{\beta}$, where $\beta^*$ is defined as the threshold at which $\frac{\partial \pi}{\partial \Delta} = 0$. From Section 3, we know that $\tilde{\beta}$ is

$$\tilde{\beta} = \frac{2 + \epsilon - \tilde{\beta}}{2 + \epsilon},$$

whereas $\beta^*$ is

$$\beta^* = \frac{-\tilde{\beta}^2 + 2\tilde{\beta} + 2 + \epsilon^2 + 4\epsilon}{\tilde{\beta}^2 + 2\tilde{\beta} + 2 + \epsilon^2 + 3\epsilon}.$$  

After some manipulations, $\tilde{\beta} > \beta^*$ can be written as

$$-\frac{(\tilde{\beta} + \epsilon)(\tilde{\beta} - \epsilon - 2 + 2\tilde{\beta} - \tilde{\beta}^2)}{(\tilde{\beta}^2 + 2\tilde{\beta} + 2 + \epsilon^2 + 3\epsilon)(2 + \epsilon)} > 0.$$  

This inequality hold if and only if $\tilde{\beta} - \epsilon - 2 + 2\tilde{\beta} - \tilde{\beta}^2 < 0$. Noting that this is a quadratic form of $\tilde{\beta}$, this condition is equivalent to

$$\tilde{\beta}^2 - (2 + \epsilon)\tilde{\beta} + (2 + \epsilon) > 0,$$

for all $\tilde{\beta}$ which holds if and only if $\epsilon < 2$. \hfill \Box

Proof of Claim S.2. Here, we want to prove that

$$\tilde{\beta} < \tilde{\beta}^* < \beta^* < \hat{\beta},$$  \hfill (S.13)
where \( \beta^* \) is an implicit solution for \( \Delta = 1 + \epsilon \) as shown in the above. From Claim S.1, we know that \( \beta > \beta^* \) if \( \epsilon < 2 \). It suffices thus to show that \( \beta < \beta^* \) to prove Claim S.2. By definition, \( \beta^* \) is such that

\[
\beta^* + 1 - \frac{2(1+\beta^*)+\epsilon}{(1+\beta^*)} = 1 + \epsilon.
\]

Solving this explicitly for \( \beta^* \), we have

\[
\beta^* = \frac{1 + \beta + \epsilon + \epsilon^2}{2\beta + 1 + 3\epsilon + \epsilon^2}.
\]

Since \( \beta = \frac{1 + \epsilon}{2\epsilon} \), \( \beta < \beta^* \) if

\[
(2 + \epsilon) + \epsilon^2 + \epsilon - 1 > 0, \quad \text{for } \forall \beta.
\]

This is a linear function of \( \beta \) with a positive slope. From this, we only have to check whether or not the above inequality holds at \( \beta = 1 \). This yields \( (\epsilon + 1)^2 > 0 \) and thus (S.13) necessarily follows.

Reference