Productive Consumption and Population Dynamics in an Endogenous Growth Model:

Why Are Population Growth Rates Declining in Poor Countries?[†]

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This paper shows that an endogenous growth model under productive consumption hypothesis (PCH) can be more tractable than we have considered so far by endogenizing population growth rate, and further explores dynamic implications of PCH. In contrast to Steger (2000), we focus on a BGP with a constant *level* of per-capita income. We find that the model may have a unique or multiple saddle-point stable BGPs in both no- and positive-saving phases. In the no-saving phase more relevant to poor economies, population growth rate may rise or decline monotonically along a transition path. The theoretical results are realistically relevant; the recent trend of declining population growth rates in modern developing countries could be explained. Furthermore, we find that "human development" aid promoting the accumulation of knowledge about nutrition, health and/or education may *reduce* per-capita GDP and does not always improve welfare.

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1.Introduction

For the last decade "human development" has received considerable attention as an important concept and strategy for poverty reduction on a world-wide basis. The UNDP (United Nations Development Programme), for example, has published Human Development Report every year since 1990. In the Millennium Development Goals, six out of eight goals are concerned with the concept of "human development", especially with an improvement in nutrition, health or education. The roles of international aid including Official Development Assistance from developed to developing countries are highlighted to attain these objectives.

In the traditional studies of development economics, on the other hand, the relationship between economic growth and human development (nutrition, health and/or education) has been regarded as important for growth of developing economies. Prominent economists have studied this idea as "productive- consumption" hypothesis (PCH): consumption improves productive potential of labor or enhances human capital (e.g.,Bliss and Stern (1978a,b), Gersovitz(1983), Dasgupta and Ray(1986), Ray and Streufert (1993)). Most of the previous papers have studied it in the form of static and dynamic efficiency wage hypothesis. An interesting exception must be Ray and Streufert (1993). They presented a dynamic analysis of the relation between land ownership and involuntary unemployment by focusing on a nutritional effect on rural workers' labor productivity. However, their model was of high originality and thus did not seem very tractable for growth economists. It is only recently that PCH was introduced in standard models of economic growth.[1]

[1] Recently growth economics and health economics have begun to collaborate. See, e.g., Lopez-Casasnovas, Rivera and Currais (2005).

PCH has been introduced into the neoclassical growth model with exogenous population growth rate by Steger (2000, 2002) and Gupta (2003)). Steger (2002) distinguishes two possibilities for modeling PCH. First, current consumption raises labor productivity of workers.[2] Second, it enhances the stock of human capital (disembodied knowledge) that improves nutrition, health, public sanitation or education. Using the latter setting, Steger (2000) shows, as a distinguished feature derived from PCH, that the model has a zero-saving phase as well as a positive-saving phase. He considers only a balanced growth path (BGP) with a constant *growth rate* of per-capita income as the steady-state equilibrium. However, taking into account that PCH is more relevant to developing economies, it should be no less important to examine properties of a BGP with a constant *level* of per-capita income. Gupta (2003) has derived this type of BGP in his endogenous growth model in which productive consumption improves labor productivity.[3] At the same time, he has shown that this BGP is unstable. In a simple AK model by Steger (2000), on the other hand, while a BGP with a constant *growth rate* of per-capita income exists in the no-saving phase, it does not exist in the positive-saving phase and, instead, only an *asymptotic* BGP exist. These results so far do not fully make clear dynamic implications of PCH, e.g., how many BGPs may exist, whether a BGP can be (saddle-point) stable, what properties a transition path may have in a phase diagram, etc.. The above properties of the model may also seem troublesome. Growth and development economists thus might possibly have an impression that introducing PCH into the standard growth model does not work very well and thus may

[2] This kind of endogenous growth model has been analyzed by Steger (2002) and Gupta(2003).

[3] In Gupta's formulation, productive consumption ($(1-\lambda)c$) only improves labor productivity but does not affect utility function. This fails to capture an importance aspect of PCH: consumption improves both productivity and utility at the same time.

not be a productive task.

This paper shows that an endogenous growth model under PCH can be more tractable by endogenizing population growth rate and further explores dynamic implications of PCH. In contrast to Steger (2000), we focus on a BGP with constant *level* of per-capita income. We find that the model may have a unique or multiple BGPs that is saddle-point stable in both no- and positive-saving phases. In the no-saving phase that is more relevant to poor economies, population growth rate may rise or decline monotonically along a transition path. Next, we explain based on data from *World development Indicators 2004* that the theoretical results are realistically relevant; in particular, the recent trend of declining population growth rates in modern developing countries could be explained. Furthermore, we explore a role of foreign aid to a developing country from some developed countries or international institutions. We find that "human development" aid promoting the accumulation of knowledge about nutrition, health and/or education may *reduce* per-capita GDP and does not always improve welfare.

In section 2 we present a basic model. Growth process in the positive-saving range is examined in section 3 and section 4 investigates income growth and population dynamics in the no-saving range. In section 5, we show the theoretical results are realistically relevant. Section 6 shows how "human development" aid may affect per capita income, population dynamics and welfare. Section 7 gives concluding remarks.

2. The Model

The aggregate production function $Y = F(K, L)$ exhibits constant returns to scale in capital K and labor L and satisfies the standard properties. Capital is composed of physical capital K_p and human capital K_h. In this paper K_h is intangible and disembodied to labor: it can be interpreted as knowledge capital. Following Steger (2000), we assume that physical and human capitals are perfect substitutes, that is, $K=K_p+K_h$.[4] Labor input is equal to population, which grows at a rate $n(t)$ at a point in time t,

$$\dot{L}(t)/L(t) = n(t) \tag{1}$$

We rewrite the production function into an intensive form $y = f(k)$ with $f'(k) > 0$ and $f''(k) < 0$, where $y=Y/L$ and $k=K/L$. The physical capital is accumulated by saving part of income. Then $k_P = K_P/L$ evolves according to

$$\dot{k}_P(t) = f(k(t)) - n(t)k_P(t) - c(t) \tag{2}$$

where $c(t)$ is per capita consumption. On the other hand, under PCH, the human capital K_h is accumulated by consumption activities. We suppose that per capita consumption leads to human capital accumulation through exchanging and creating consumption-based information and new knowledge. For example, if people find drinking water with salt and sugar stops dehydration of their children, they will exchange this information in the society and/or create new knowledge about, e.g., how much salt and sugar should be put into water or how often they should give it to their children. This leads to an increase in human capital at social level. By dividing this increment $\dot{K}_h(t)$ by the number of people $L(t)$, we will get the human-capital enhancement function $\phi(c)$ by Steger (2000). Therefore, $\dot{K}_h(t) = \phi[c(t)]L(t)$ holds. Function $\phi(c)$ is increasing and concave, i.e., $\phi'(c) > 0$, $\phi''(c) < 0$ with

4 In Steger's (2000) analysis the dynamic properties would remain unchanged if one treated these two capitals separately.

$\lim_{c \to 0} \phi'(c) = \infty$ and $\lim_{c \to \infty} \phi'(c) = 0$. The per capita human capital $k_h = K_h/L$ evolves according to

$$\dot{k}_h(t) = \phi[c(t)] - n(t)k_h(t) \qquad (3)$$

Therefore, the economy's capital per capita evolves according to

$$\dot{k}(t) = f(k(t)) - n(t)k(t) - \psi(c(t)) \qquad (4)$$

where $\psi(c(t)) = c(t) - \phi(c(t))$ is the net cost of consumption (NCC). Per capita consumption cannot exceed per capita income:

$$0 \le c(t) \le f(k(t)) \qquad (5)$$

The representative agent maximizes the intertemporal utility function

$$\int_0^\infty \left[\ln c(t) + \frac{n(t)^{1-\varepsilon} - 1}{1-\varepsilon} \right] \exp(-\rho t) dt \qquad (6)$$

where $\rho > 0$ is a constant time preference rate and $\varepsilon > 0$ ($\varepsilon \ne 1$) represents the intertemporal elasticity of substitution.[5] Following the standard practice of the literature on endogenous fertility, we assume that the instantaneous utility depends on the population growth rate n, as in Yip and Zhang (1997). One may claim that it should depend on the number of children. However, this formulation could be justified as follows. In Razin and Ben-Zion (1975) model in which each generation lives for one period, the gross population growth rate (L_{t+1}/L_t) may also be regarded as the per capita number of children of generation t. Thus in our model $1+n$ could be interpreted as the number of children. Furthermore, if we replace the *gross* population growth rate with the *net* population growth rate n, the dynamic properties of the equilibrium will remain unchanged. Therefore we make use of this instantaneous utility function.

[5] The representative agent here is an individual who is atomistic in the whole economy. Thus we do not incorporate the population growth rate (n) in the exponential term.

We assume away the cost for increasing population growth rate to reveal basic properties of the system. In the literature on endogenous growth with endogenous fertility, it is often assumed that the cost for increasing a fertility rate involves child-rearing cost (using time or goods in the present period). Introduction of child-rearing cost induces a possibility of rising fertility rate along transition path (Barro and Sala-i-Martin (1995)). We will show that this model under PCH may induce a possibility of rising population growth rate even if child-rearing cost is ruled out.

The representative individual chooses time paths of per capita consumption *c(t)* and population growth rate *n(t)* to maximizes (6) subject to (4) and the inequality constraint (5).[6] Following Leonard and Long (1992), the present-value Hamiltonian is defined as

$$H(c,n,k,\pi,t) = \left[\ln c + \frac{n^{1-\varepsilon}-1}{1-\varepsilon} \right] \exp(-\rho t) + \pi \quad [f(k) - nk - \psi(c)] \tag{7}$$

The Lagrangean function is $L(c,n,k,t,\pi,\lambda) = H(c,n,k,\pi,t) + \lambda[f(k)-c]$. The first-order conditions (FOC) for this problem with inequality constraint are

(i) c(t)* maximizes $H(c,n,k,\pi,t)$ subject to $f(k)-c \geq 0$.

$$\frac{\partial L}{\partial c} = \frac{\partial H}{\partial c} - \lambda = e^{-\rho t}\frac{1}{c} - \pi\psi'(c) - \lambda = 0 \tag{8-1}$$

with $\lambda \geq 0$, $f(k)-c \geq 0$, $\lambda[f(k)-c] = 0$

(ii) *n*(t)* maximizes $H(c,n,k,\pi,t)$

$$\frac{\partial L}{\partial n} = e^{-\rho t} n^{-\varepsilon} - \pi k = 0 \tag{8-2}$$

(iii) $\dot{\pi}(t) = -\frac{\partial L}{\partial k} = -[\pi(t) + \lambda(t)] f'(k(t)) + \pi(t) n(t)$ (8-3)

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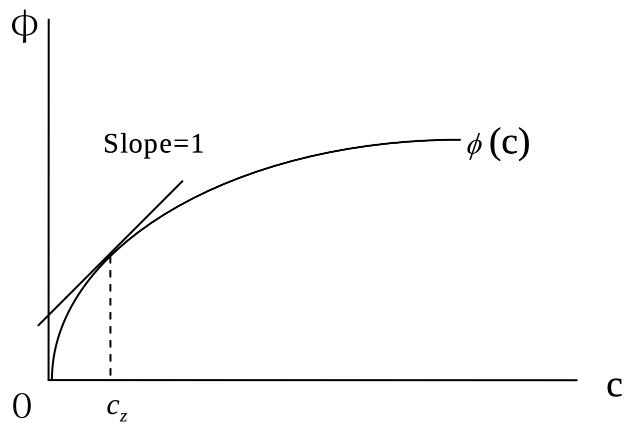
6 One could distinguish the equilibrium and optimal growth paths by assuming that an individual does not take into account the effect of $\phi(c)$. However, since the no-saving phase would not occur on the equilibrium path, we will focus on the optimal path along which each individual recognize this effect correctly.

(iv) $\dot{k}(t) = \frac{\partial L}{\partial \pi} = f(k(t)) - n(t)k(t) - \psi(c(t))$ (8-4)

As in Steger (2000), this model has two phases: *no*-saving ($c = f(k)$) and *positive*-saving ($c < f(k)$) phases.[7] If the marginal NCC ($\psi'(c) = 1 - \phi'(c)$) is positive, renunciation of current consumption will promote capital accumulation. Thus saving will be positive. Conversely, if it is negative, an *increase* in current consumption will promote capital accumulation. Thus there is no incentive for saving. In the no-saving phase where $c = f(k)$ lies in $[0, c_z]$, the economy moves along the production function $f(k)$. When the value of k exceeds the critical value k_Z with $\phi'(f(k_z)) = 1$, the economy switches into the positive-saving phase, in which per capita consumption diverges from $f(k)$. We will first consider the positive-saving phase and then proceed to the no-saving phase.

Figure 1. Human-capital Enhancement Function



[7] Steger (2000) discusses transition and asymptotic ranges separately for the positive-saving phase.

3. Growth Process with Physical Capital Accumulation

In this section we consider the positive-saving phase ($\lambda(t)=0$) in which physical capital accumulation takes place. Per capita consumption and population growth rate evolve according to

$$\dot{c}(t)=\frac{c(t)}{1+\eta(c(t))}[f'(k(t))-n(t)-\rho] \tag{9}$$

$$\dot{n}(t)=\frac{n(t)}{\varepsilon}\left[f'(k(t))-\rho-\frac{f(k(t))}{k(t)}+\frac{\psi(c(t))}{k(t)}\right] \tag{10}$$

where $\eta(c)=c\psi''(c)/\psi'(c)$. Since population growth rate is endogenous, (9) is a little different from the *Modified Keynes-Ramsey Rule* shown by Steger (2000).

The dynamic system is given by (4), (9) and (10). The steady-state equilibrium (c^*,k^*,n^*) is defined as the balanced growth path (BGP) in which $\dot{c}=\dot{k}=\dot{n}=0$ holds.

As shown in Appendix, there may exist at most two BGPs. We can examine the stability of BGP and transition dynamics: how population growth rate may be related to per capita income along the transition path. Although the transition path is in the (c,n,k) space, we will examine the correlation between population growth rate and per capita income by taking a projection onto the (n,k) plain. Setting $\dot{c}=0$, we obtain

$$\frac{\dot{n}(t)}{n(t)}=\frac{1}{\varepsilon}\left[n(t)-\frac{f(k(t))}{k(t)}+\frac{\psi(c(t))}{k(t)}\right] \tag{11}$$

Thus we can immediately obtain $\dot{n}/n=-(1/\varepsilon)(\dot{k}/k)$. From an arbitrary initial value $k(0)<k^*$, population growth rate will decline as per capita income grows along the projection of the transition path on the (n,k) plain.

Proposition 1 (BGPs in Positive-saving Phase)

In the positive-saving phase, (i) there may exist at most two BGPs. (ii) A BGP can be either saddle-point stable or unstable. (iii) Along the transition path, population growth rate will decline as per capita income increases.

Let us mention that the average saving rate rise during this growth process. Steger (2000) claims that the average saving rate needs to rise as a least requisite for the growth model, showing it by a simulation of AK model. If we assume the AK- type production function $y = Ak$, we can show it analytically.

In Steger (2000), population growth rate is given, Setting $\dot{n} = 0$ yields $f'(k) - \rho = [f(k) - \psi(k)]/k$. Using it, we get $\dot{c}/c = [1/\{1+\eta(c)\}](\dot{k}/k)$. A change in average propensity to consume is

$$\frac{d(c/y)/dt}{(c/y)} = \frac{-\eta(c)}{1+\eta(c)}\left(\frac{\dot{k}}{k}\right) \tag{12}$$

When $\dot{k}/k > 0$, the average saving rate (c/y) declines as per capital income increases. Therefore the average saving rate $(1-(c/y))$ rises as per capital income increases.

Result 1 (Saving Rate along Transition Path in Positive-saving Phase)

Under the AK production function, the average saving rate $(1-(c/y))$ rises as per capital income increases along the transition path.

4. Population Dynamics with Human Capital Accumulation

4.1 Equilibrium Conditions

Now let us move on to the no-saving phase ($\lambda(t) > 0$) in which only human capital is accumulated through productive consumption. The FOCs are

$$\frac{\partial L}{\partial c} = \frac{\partial H}{\partial c} - \lambda(t) = e^{-\rho t}\frac{1}{c} - \pi\psi'(c) - \lambda = 0 \tag{13-1}$$

$$\left(\frac{1}{n^{\varepsilon}}\right)e^{-\rho t} = \pi k \tag{13-2}$$

$$\dot{\pi}(t) = -[\pi(t) + \lambda(t)]f'(k(t)) + \pi(t)n(t) \tag{13-3}$$

$$\dot{k}(t) = \phi(f(k(t))) - n(t)k(t) \tag{14}$$

Since $\partial H / \partial c = \lambda(t) > 0$ leads to $c = f(k)$, (8-4) takes the form of (14). Differentiating (13-2) with respect to time yields $\varepsilon(\dot{n}/n) + \dot{k}/k + \dot{\pi}/\pi + \rho = 0$. We transform (13-3) by eliminating $\lambda(t)$ using (13-1) and (13-2)

$$\frac{\dot{\pi}}{\pi} = n - f'(k)\left[1 + \frac{n^{\varepsilon}}{\{f(k)/k\}} - \psi'(f(k))\right] \tag{15}$$

Substituting and rearranging the terms, we obtain

$$\dot{n}(t) = [n(t)/\varepsilon]\Gamma(k(t), n(t)) \tag{16}$$

where

$$\Gamma(k,n) = f'(k)\left[\phi'(f(k)) + \frac{n^{\varepsilon}}{\{f(k)/k\}}\right] - \frac{\phi(f(k))}{k} - \rho$$

The dynamic system for this phase is given by (14) and (16). The steady-state equilibrium (k^*, n^*) is defined as the BGP on which $\dot{k}(t) = \dot{n}(t) = 0$ holds. It is characterized by

$$\phi[f(k^*)] = n^* k^* \tag{17}$$

$$f'(k^*)\left[\phi'[f(k^*)] + \frac{n^{*\varepsilon}}{a(k^*)}\right] = b(k^*) + \rho \tag{18}$$

where $a(k) = f(k)/k$ and $b(k) = \phi[f(k)]/k$.

Let us call a locus of (n,k) on which $\dot{k} = 0$ holds "kk curve". First, taking into account that (17) leads to $\phi(f(k))/k = n$ and that $\phi(f(k))/k$ is decreasing in k, the slope of kk curve is always negative:

$$\frac{dn}{dk} = \frac{\phi'(f)f'(k) - n}{k} < 0 \tag{19}$$

Second, a locus of (n,k) on which $\dot{n} = 0$ holds is called "nn curve". The slope

$$\frac{dn}{dk} = -\frac{\Gamma_k(k,n)}{\Gamma_n(k,n)} \tag{20}$$

can be either positive or negative, where

$$\Gamma_n(k,n) = \frac{kf'(k)}{f(k)}\varepsilon n^{\varepsilon-1} > 0 \tag{21}$$

$$\Gamma_k(k,n) = f''(k)\left[\phi' + \frac{n^\varepsilon}{a(k)}\right] + \phi''(f(k))[f'(k)]^2 - f'(k)\frac{n^\varepsilon}{a(k)}\frac{a'(k)}{a(k)} - b'(k) \tag{22}$$

4.2 Balanced-growth Equilibrium and Stability: Case of Increasing nn Curve

We will first consider the case where nn curve is increasing. This case will happen when the production function $f(k)$ and the human-capital enhancement function $\phi(c)$ are *strongly* concave. To see this, we should look at (22). While the third and fourth terms on the right-hand side of (22) are positive, $\Gamma_k(k,n) < 0$ holds when the sum of the first and second terms (negative) is dominant, that is, $|f''(k)|$ and $|\phi''(c)|$ are large enough.

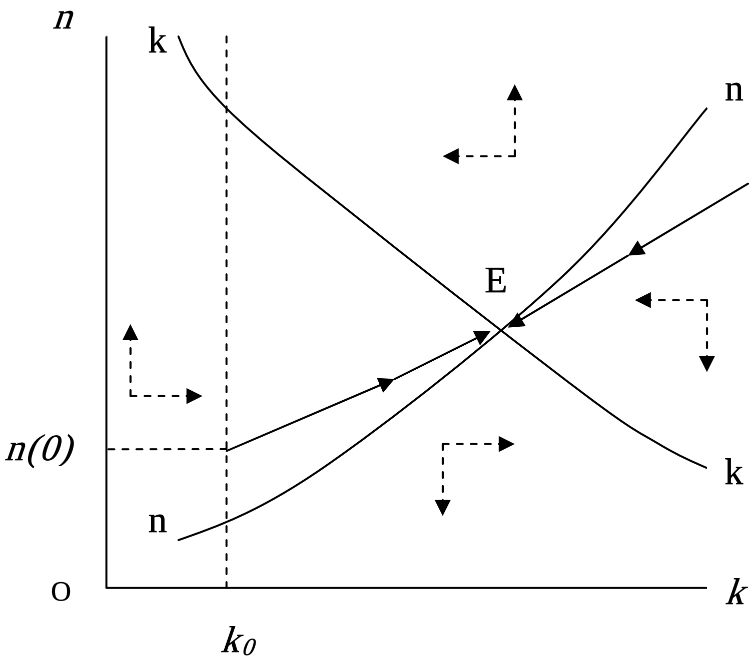
Proposition 3 (A Unique Saddle-point Stable BGP in No-saving Phase)

In the no-saving phase, there exists a unique BGP (k^*, n^*) *that is saddle-point stable if and only if nn curve is increasing (*$\Gamma_k(k,n) < 0$*).*

(Proof) When nn curve is increasing, it intersects with kk curve at one point. Thus a BGP uniquely exists. If the slope of kk curve is smaller than the slope of nn curve, $\frac{\phi'(f)f'(k)-n}{k} < -\frac{\Gamma_k(k,n)}{\Gamma_n(k,n)}$ holds. This is equivalent to Det $J^* < 0$, where J^* is the coefficient matrix of the linearlized system of (14) and (16), evaluated at a BGP. Det $J^* < 0$ means that only one of the two eigen values of J^* is negative. Thus the BGP is saddle-point stable. (Q.E.D.)

Figure 2 shows transition dynamics for this case: population growth rate *n* is *positively* related to per capita capital k. From a low initial level of k_0, the population growth rate rises as per capita income $y = f(k)$ increases along the transition path (An intuitive explanation will be given later).

Figure 2: Rising Population Growth Rate

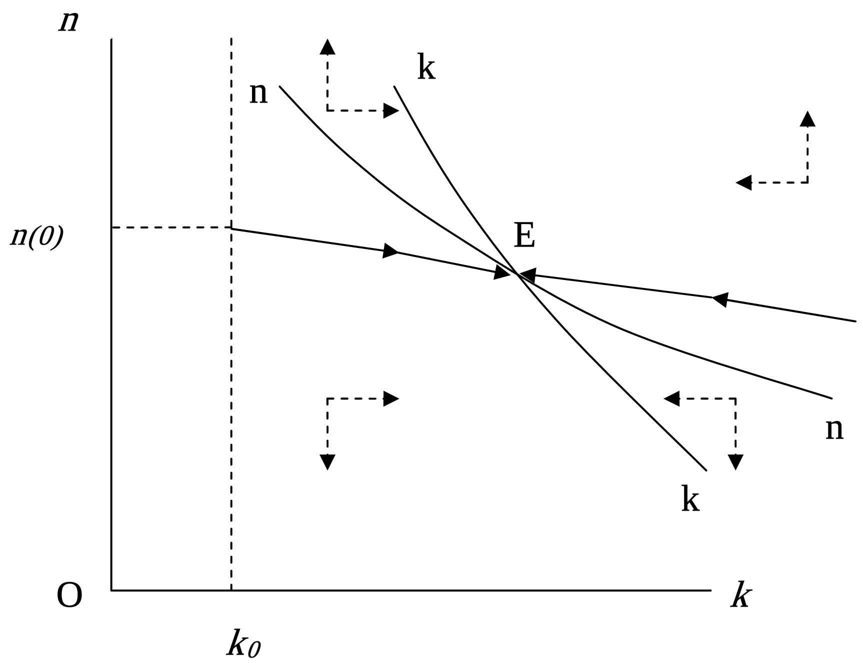


4.3 Case of Decreasing nn Curve

Next we consider the case where nn curve is decreasing ($\Gamma_k(k,n) > 0$). From what I have just explained above, this case will happen when the production function and the human-capital enhancement function are *weakly* concave: $|f''(k)|$ and $|\phi''(c)|$ are relatively small.

Since the slope of nn curve may be either larger or smaller than that of kk curve, they may intersect with each other at more than one point. Thus there may exist multiple BGPs. If nn curve is flatter than kk curve, $\frac{\phi'(f)f'(k)-n}{k} < -\frac{\Gamma_k(k,n)}{\Gamma_n(k,n)}$ holds. This is equivalent to $\mathrm{DetJ}^* < 0$, implying that the BGP is saddle-point stable. Figure 3 shows the case of a unique saddle-point stable BGP. Along a transition path, population growth rate will decline as per capita income grows.[8]

Figure 3: Declining Population Growth Rate



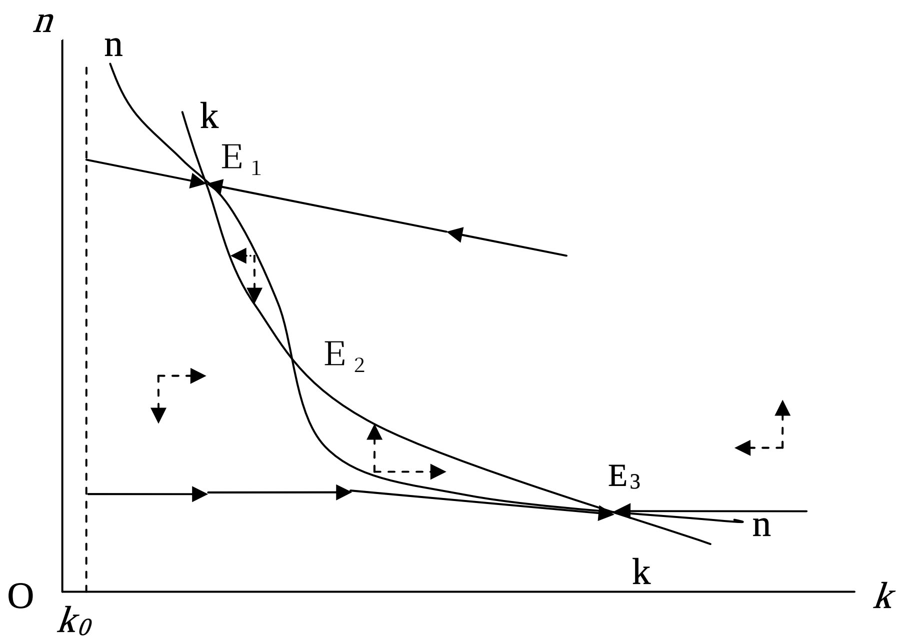
Intuitively, the change in population growth rate along the transition path is linked to the growth of per capita income by (8-2): The discounted present value of the marginal utility from an increase in population growth rate $(1/n^{\varepsilon})e^{-\rho t}$ is equal to the imputed value of capital πk. When the imputed price $\pi e^{\rho t}$ declines at a rate higher than the growth rate of k, the population growth rate n will rise, and vice versa.

Let us now consider the case of multiple BGPs. First, consider the case where there exist only two BGPs. Since they are saddle-point stable or unstable, only the stable one is economically meaningful. Next, even if there are more than two BGPs that are

8 The BGP is unstable when $\Gamma_k(k,n) > 0$ and nn curve is steeper than kk curve. This case will happen when $|f''(k)|$ and $|\phi''(c)|$ are very small.

saddle-point stable (Figure 4), the representative agent can find a growth path along which the maximized intertemporal utility is higher than the other growth paths, at least in principle. Therefore, the BGP and transition path will be uniquely chosen. If the maximized utilities should be equal among them, an optimal path will be indeterminate. In this case, since kk curve is downward-sloping, one BGP involves a high population growth rate and low per capita income, while the other a low population growth rate and high per capita income.

Figure 4. Multiple Saddle-point Stable BGPs



The transitional dynamics is the same qualitatively both when BGP is unique and when it is multiple. From a low initial level of k_0, population growth rate *n* will decline as per capita income increases.

Proposition 4 (Multiple Saddle-point Stable BGPs in No-saving Range)

Suppose that nn curve is decreasing in the no-saving phase. Then (i) there may exist multiple BGPs (k^*, n^*). *(ii) A BGP is saddle-point stable if and only if the slope of kk curve is smaller than the slope of nn curve. (iii) When there are more than one saddle-point stable BGPs, a path will be uniquely chosen that maximizes the intertemporal utility along transition path and at BGP. (iv) Along a transition path, population growth rate declines as per capita income grows.*

Finally, let us elucidate how the no-saving phase will switch to the positive-saving phase. When per capita income exceeds the critical value $f(\hat{k})$ in the no-saving phase, the economy moves onto the saddle-point path toward a BGP for the positive-saving phase. Thus population growth rate may rise or decline first and then will decline monotonically as per capita income increases.

5. Discussion: Empirical Evidence

We have shown how transition dynamics in this model could explain the relationship between per capita income and population growth rate in poor developing economies. In this section we will discuss whether these theoretical results are realistically relevant, based on data from *World Development Indicators (2004).*

The recent casual observations and empirical studies have often shown that population growth rates have been declining not only in the world as a whole but also in developing countries (see e.g. Table 6.1 on p.105 in Tietenberg (2006)). A decline in population growth rates in some developing countries could be explained by the theory of demographic transition: as nations develop, they eventually reach a point where birth rates fall. However, one should note, this applies to economies that have succeeded in income growth in the last several decades (e.g., Mexico, Brazil, Indonesia etc.). For relatively poor developing countries in South-East Asia, Latin America or Africa, the income growth has not been so smooth that the theory of demographic transition can apply. Thus it will be important to explore a possibility of a different explanation for the declining population growth rates that are widely observed mainly in these areas. We can do it by focusing on the no-saving phase of the present model.

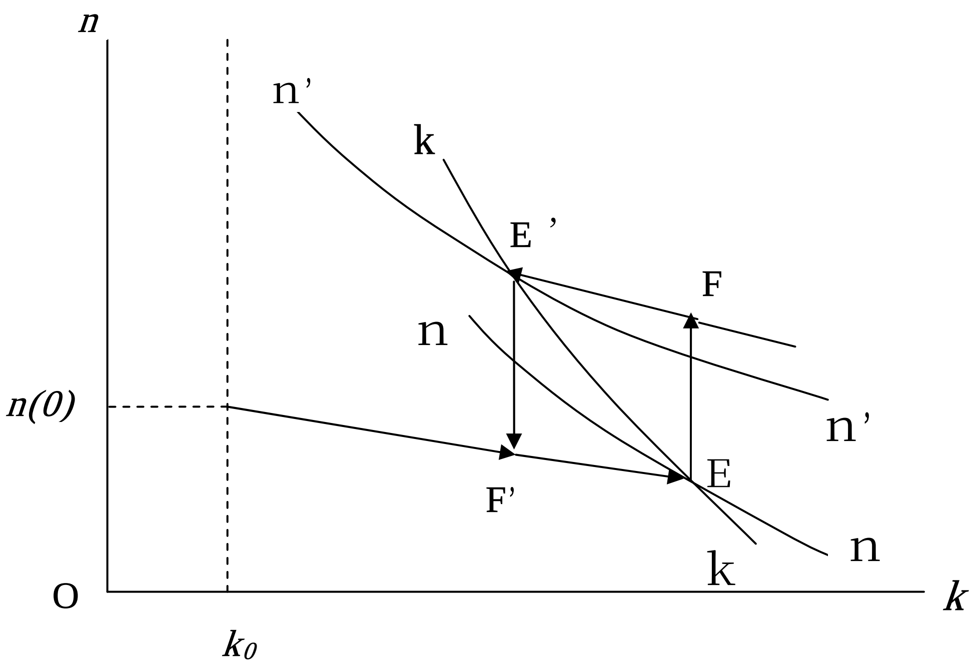
First, we will look at the rather exceptional case for positive correlation between per capita income and population growth rate (Figure 2). This positive correlation is consistent with the data of Nepal on Table 1. Second, a negative correlation between population growth rate and per capita GDP (Figure 3 and 4) is consistent with the data of India and Columbia on Table 2 and 3. The negative correlation has recently been observed very frequently in data of modern developing economies.

However, looking more carefully into the data of *WDI*, one can also find the data from African countries such as Ghana and Sudan (on Table 4 and 5) that exhibit more complicated, or scattered, relations. Taking into count that African countries have often experienced exogenous shocks, we will explore a possible explanation for these data by comparative statics and dynamics.

We will suppose here, as one of the possible explanations, that the time preference rate ρ changes exogenously. For example, when military conflicts or a domestic wars occur, people in African countries may become more myopic. When the war ends, they will come to think their lives on a long-run basis again. Let us present an explanation for the case of decreasing nn curve (one can easily make a similar discussion for the case of increasing nn curve).

In Figure 5, the economy moves from the initial point $(k_0, n(0))$ to E (BGP). Suppose that ρ rises exogenously. Then nn curve shifts upward while kk curve remains unchanged. Thus the economy will jump from E to F and then moves along the new transition path toward E': per capita income $f(k^*)$ is lower while population growth rate n^* is higher. When the wars end and ρ declines to the initial value, the economy will jump from E' to F' and moves toward E. If the economy experiences this kind of movements, the data will probably be scattered. The present model does not always contradict the data exhibiting non-monotonic relations between population growth rate and per capita income.

Figure 5. Change in Time Preference



6. Human Development and Welfare

We will examine effects of “human development” aid by focusing on the role of the human-capital-enhancement function. Let us replace $\phi(c)$ with $\theta\phi(c)$, where $\theta > 0$ is an exogenous parameter. Since we assume away the cost for a rise in θ, we could interpret it as an introduction of foreign aid for “human development”. Then the definition of BGP changes into

$$\theta\phi[f(k^*)] = n^* k^* \tag{23}$$

$$f'(k^*)\left[\theta\phi'[f(k^*)] + \frac{n^{*\varepsilon}}{a(k^*)}\right] = \frac{\theta\phi(f(k^*))}{k^*} + \rho \tag{24}$$

6.1 Comparative Statics and Dynamics

A rise in θ shifts up both kk and nn curves. A new BGP can be located either northeast or northwest of the initial BGP. To see this, let us check how much a rise in θ will shift kk and nn curves upward (how much n needs to rise with k fixed) respectively, by using (23) and (24). From (23), one unit increase in θ raises n by $\phi(f(k^*))/k^*$. From (24), it raises $(f'(k^*)/a(k^*))n^{\varepsilon}$ by less than $\phi(f(k^*))/k^*$. Since $f'(k^*)/a(k^*)$ is smaller than unity, n may have to rise by either more or less than $\phi(f(k^*))/k^*$. Therefore, a shift of kk curve, in general, may be either larger or smaller than the shift of nn curve.

Let us examine movements along a transition path, focusing on the more frequently observed case when population growth rate declines as per capita income increases (nn curve is downward sloping). When E’ lies northeast of E (Figure 6), per capita income keeps rising and population growth rate continues to decline (n jumps up to F at the time when θ rises). The case where E' lies northwest of E is more interesting (Figure 7): at first, population growth rate will decline as per capita income increases, and then it jumps up to F. Thereafter per capita income will begin to decrease and the population growth rate begins to rise along the transition path toward E'.

Figure 6. Human Development Aid (Declining Population Growth Rate)

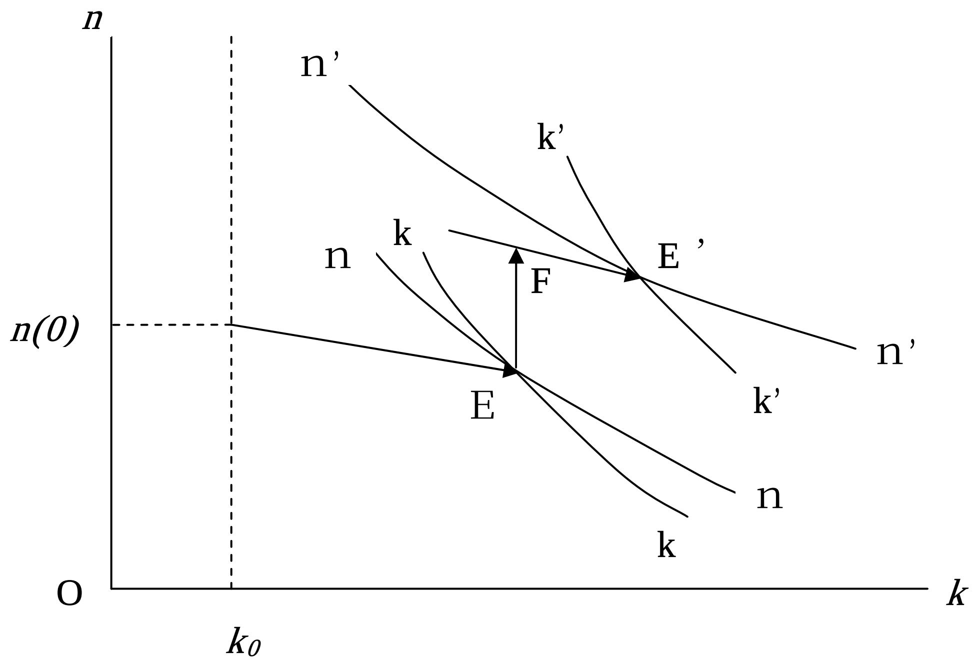
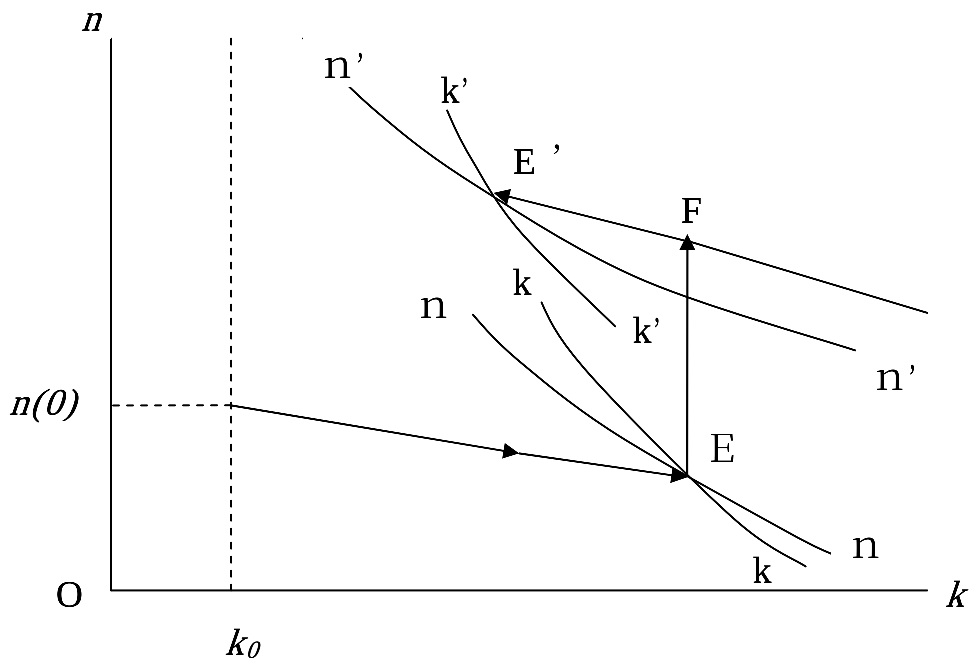


Figure 7. Human Development Aid (Per-capital income rises and then declines)



6.2 Welfare on Balanced-growth Equilibrium

We will finally examine welfare on BGPs, though welfare along transition path can hardly be evaluated. Since $c^* = f(k^*)$ holds in the BGP, (6) leads to

$$U^* = \frac{1}{\rho}\left[\ln f(k^*) + \frac{(n^*)^{1-\varepsilon} - 1}{1-\varepsilon}\right] \tag{25}$$

Welfare on BGP is higher when k^* and n^* are larger. When a new BGP is located northeast of the initial BGP, welfare on the new BGP is higher than in the initial BGP. Thus in the case of Figure 6, human development aid will improve welfare in the long-run. However, in the case of Figure 7, one cannot say anything definite about whether human development aid will improve welfare.

7. Concluding Remarks

We have shown that an endogenous growth model under PCH can be more tractable than we have considered so far by endogenizing population growth rate and further investigated dynamic implications of PCH. In contrast to Steger (2000), we focus on a BGP with a constant *level* of per capita income. We have found that the model may have a unique or multiple saddle-point stable BGPs in both no- and positive-saving phases. In the positive-saving phase there may be one saddle-point stable BGPs. Along a transitional path, population growth rate declines as per capita income increases. In the no-saving phase more relevant to poor economies, population growth rate may rise or decline monotonically along a transition path. The theoretical results turn out to be realistically relevant in reference to data from *World Development Indicators (2004)*: in particular, the recent trend of declining population growth rates in modern developing countries could be explained, and exogenous changes in time preference rate could explain complicated relations between population growth rate and per capita GDP in some African countries. Furthermore, we find that "human development" aid enhancing human capital accumulation may *reduce* per-capita GDP and does not always improve welfare.

Let us elucidate qualifications of this paper. First, we have assumed away child-rearing cost. It is important to examine whether or not the qualitative results or properties of BGPs will change if this cost is explicitly incorporated. Second, the present model is of a one-sector closed economy. Extension to open economy may be useful for obtaining further implications of PCH. The present paper will only be a starting point toward future research.

Appendix

A.1 Proof of Proposition1:

Defining $\mu(t) = \pi(t)e^{\rho t}$, FOCs (8) leads to

$$\frac{1}{c} = \mu\psi'(c) \tag{A1-1}$$

$$n(t)^{\varepsilon}\mu(t)k(t) = 1 \tag{A1-2}$$

$$\dot{\mu}(t) = \rho\mu(t) - \mu(t)[f'(k(t)) - n(t)] \tag{A1-3}$$

and $\lim_{t\to 0}\pi(t)k(t)\exp(-\rho t) = 0$. First, differentiating (A1-1) and eliminating $\dot{\mu}/\mu$ by using (A1-3), we get (9). Next, differentiating (A1-2) yields $\varepsilon(\dot{n}/n) + \dot{\mu}/\mu + \dot{k}/k = 0$. Using (4) and (A1-2), (A1-3) leads to (10). The dynamic system for positive-saving phase is

$$\dot{c}(t) = \frac{c(t)}{1+\eta(c(t))}[f'(k(t)) - n(t) - \rho]$$

$$\dot{n}(t) = \frac{n(t)}{\varepsilon}\left[f'(k(t)) - \rho - \frac{f(k(t))}{k(t)} + \frac{\psi(c(t))}{k(t)}\right]$$

$$\dot{k}(t) = f(k(t)) - n(t)k(t) - \psi(c(t))$$

First, we will examine the existence of BGP. Eliminating μ using (A1-1) and (A1-2), we get $kn^{\varepsilon} = c\psi'(c)$. From $\dot{c}$=0, we get $f'(k) = n + \rho$ holds. Combining them leads to

$$k[f'(k) - \rho]^{\varepsilon} = c\psi'(c) \tag{A1-4}$$

The slope of this curve is

$$\frac{dc}{dk} = \frac{[f'(k) - \rho]^{\varepsilon-1}\{f'(k) - \rho + k\varepsilon f''(k)\}}{\psi'(c) + c\psi''(c)} \tag{A1-5}$$

where $\psi'(c) + c\psi''(c) = [1 - \phi'(c)] - c\phi''(c) > 0$. The locus of (c,k) that satisfies (A1-4) takes an inverse U-shape. Next, using $\dot{n}$=0, we get

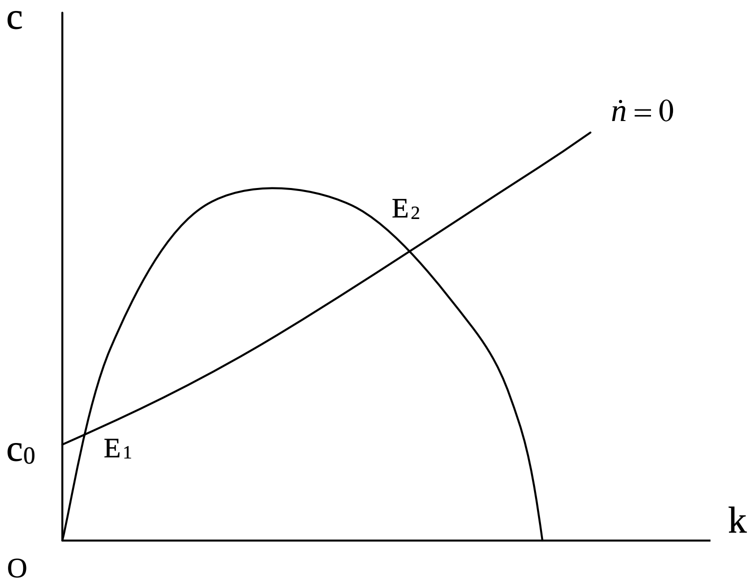
$$f(k) - kf'(k) + \rho k - \psi(c) = 0 \tag{A1-6}$$

The slope of this curve is

$$\frac{dc}{dk} = \frac{\rho - kf''(k)}{\psi'(c)} > 0$$

In addition, the locus of (c,k) that satisfies (A1-6) starts from a positive value c_0 on the vertical axis.[9] Therefore the two curves typically intersect at point E_1 and E_2.

Figure A1. BGPs in Positive-saving Phase



Second, let us examine the stability of BGPs. The linearlized system around the BGP is

$$\begin{bmatrix} \dot{c} \\ \dot{n} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{c}{1+\eta(c)} & \frac{c}{1+\eta(c)} f''(k) \\ \frac{n\psi'(c)}{\varepsilon k} & 0 & \frac{n}{\varepsilon k}[kf''(k)-\rho] \\ -\psi'(c) & -k & f'(k)-n \end{bmatrix} \begin{bmatrix} c-c^* \\ n-n^* \\ k-k^* \end{bmatrix} \tag{A1-7}$$

The characteristic equation for the coefficient matrix *J** evaluated at BGP is

---

[9] Setting $k=0$ we get $c=0$ and $c=c_o>0$ with $c_o=\phi(c_o)$. In the positive-saving range only the latter is valid

$\lambda^3 - TraceJ^*\lambda^2 + BJ^*\lambda - DetJ^* = 0$. Three characteristic roots λ_1, λ_2, λ_3 satisfy the following relations.

$$TraceJ^* = \lambda_1 + \lambda_2 + \lambda_3 = f'(k) - n = \rho > 0$$

$$BJ^* = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = \frac{c\psi'(c)}{1+\eta(c)}\left[\frac{n}{\varepsilon k} + f''(k)\right] + \frac{n}{\varepsilon}[kf''(k) - \rho]$$

$$DetJ^* = \lambda_1\lambda_2\lambda_3 = 0$$

In the positive-saving phase, $\psi'(c) > 0$ and thus $\eta(c) > 0$ hold.

Since $DetJ^*$=0 holds, at least one of the three eigen values is zero. However, $TraceJ^* > 0$ means that $\lambda_1 = \lambda_2 = \lambda_3 = 0$ is impossible. Suppose that $BJ^* = 0$ holds. If $\lambda_3 = 0$, then $BJ^* = \lambda_1\lambda_2 = 0$. Thus one of the other two eigen values is zero. If $\lambda_2 = \lambda_3 = 0$, then $\lambda_1 > 0$. Therefore, there are no negative eigen values. Suppose that $BJ^* > 0$ holds. Clearly $\lambda_2 = \lambda_3 = 0$ is impossible. If $\lambda_3 = 0$ holds, we get $\lambda_1 > 0$ and $\lambda_2 > 0$. Therefore, there are no negative eigen values. Suppose that $BJ^* < 0$ holds. If $\lambda_3 = 0$ holds, we get $\lambda_1 < 0$ and $\lambda_2 > 0$. The number of negative eigen values equals the number of state variables =1. Therefore, the BGP is saddle-point stable. In all three cases above, there is no possibility of two negative eigen values. Thus a BGP cannot be perfectly stable. (Q.E.D.)

A.2 Local Stability of BGP in the No-saving Phase

We will now examine the stability of a steady-state equilibrium. The properties of transitional dynamics can be investigated, focusing on the relation between the slopes of $\dot{k}(t)=0$ and $\dot{n}(t)=0$ curves. The linearlized system around the steady-state equilibrium is

$$\begin{bmatrix} \dot{k} \\ \dot{n} \end{bmatrix} = \begin{bmatrix} \phi'(f)f'(k^*)-n & -k^* \\ (n^*/\varepsilon)\Gamma_k(k^*,n^*) & (n^*/\varepsilon)\Gamma_n(k^*,n^*) \end{bmatrix} \begin{bmatrix} k-k^* \\ n-n^* \end{bmatrix} \tag{A2-1}$$

We denote the coefficient matrix by J^*.

$$\text{Trace } J^* = [\phi'(f)f'(k^*)-n^*]+(n^*/\varepsilon)\Gamma_n(k^*,n^*)=\rho>0 \tag{A2-2}$$

$$\text{Det } J^* = [\phi'(f)f'(k^*)-n^*][(n^*/\varepsilon)\Gamma_n(k^*,n^*)]+k^*[(n^*/\varepsilon)\Gamma_k(k^*,n^*)] \tag{A2-3}$$

From (A2-2), the BGP cannot be perfectly stable. The BGP may be either saddle-point stable or unstable, depending on $DetJ^*$ is negative or positive.

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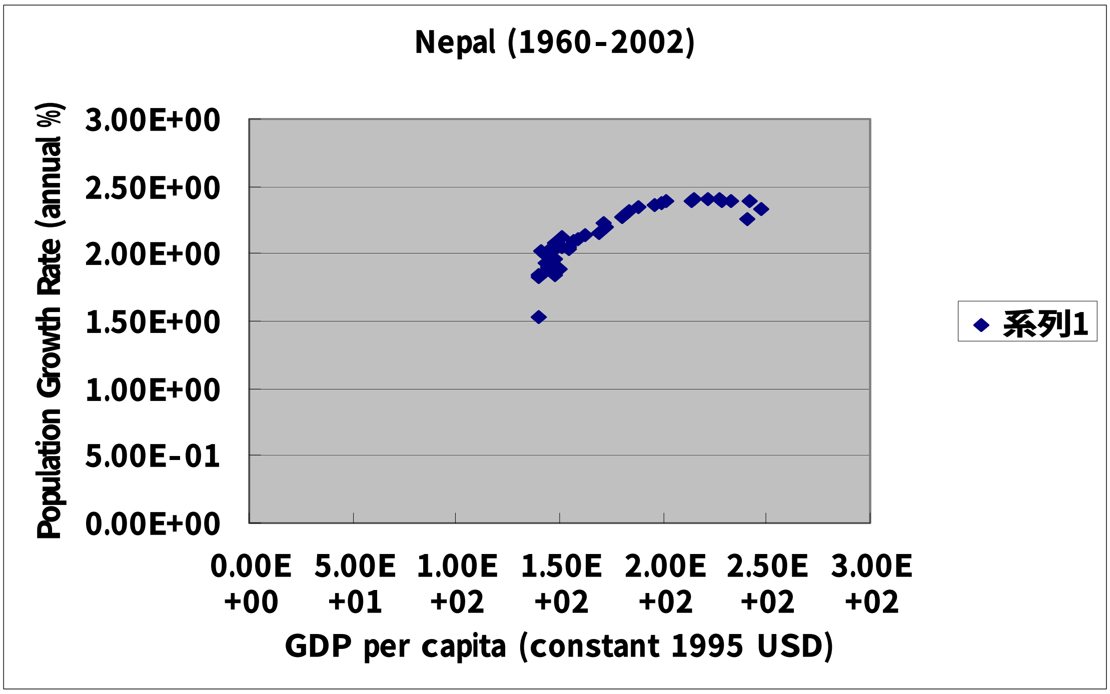
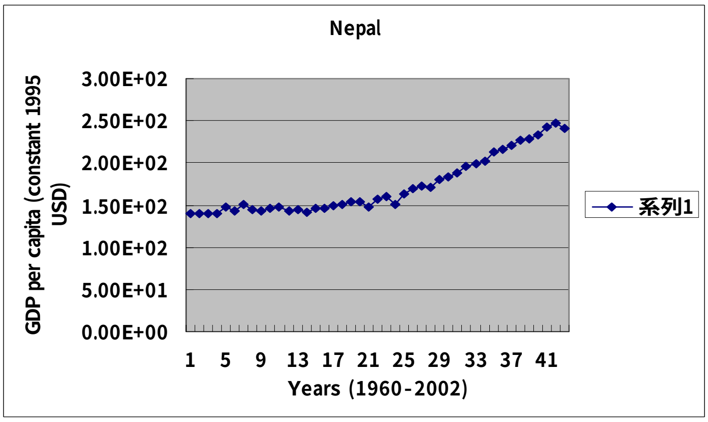
三野和雄、「経済成長モデルにおける不決定性」、西村・福田編　前掲書.

Table 1. GDP per capita and Population Growth Rate (Nepal)





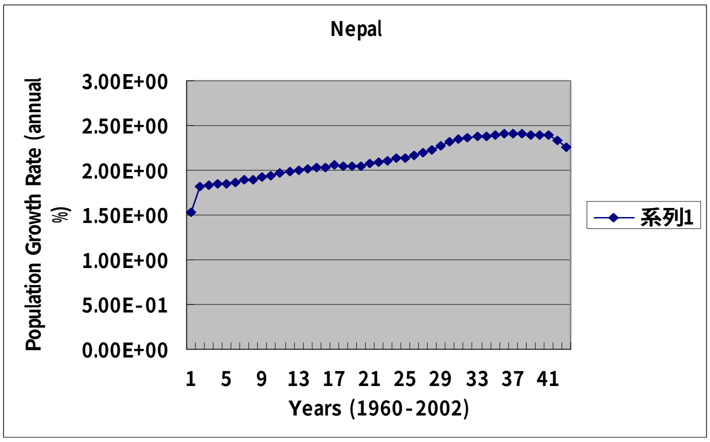
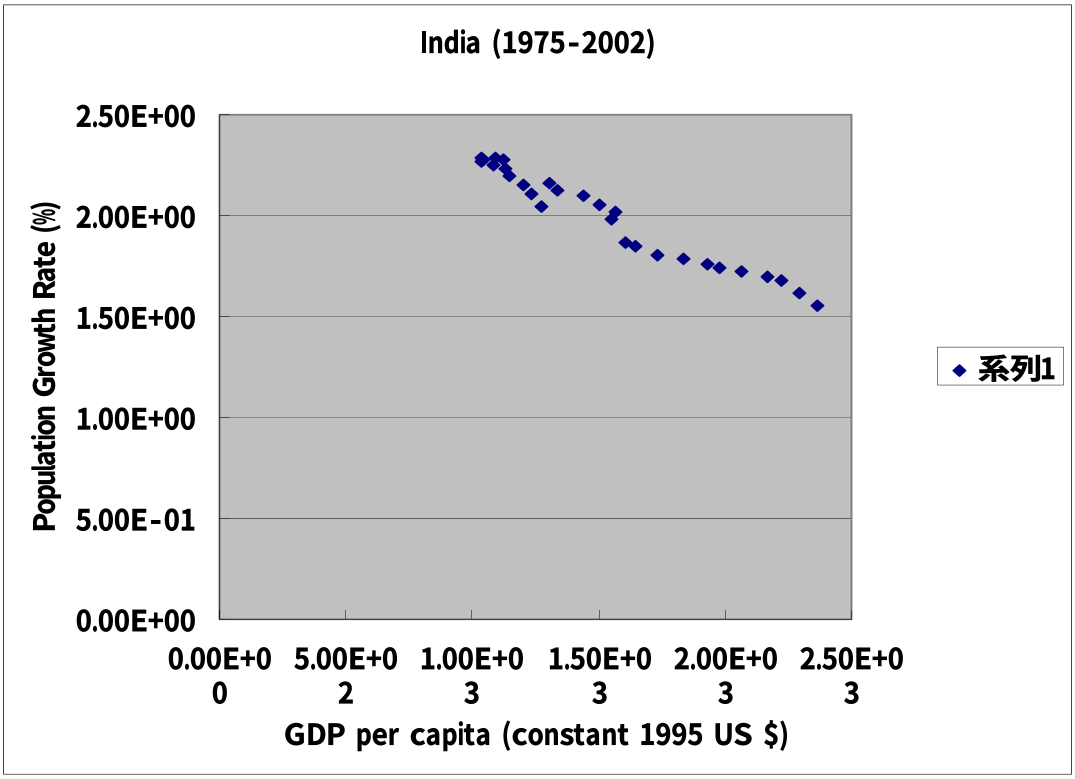
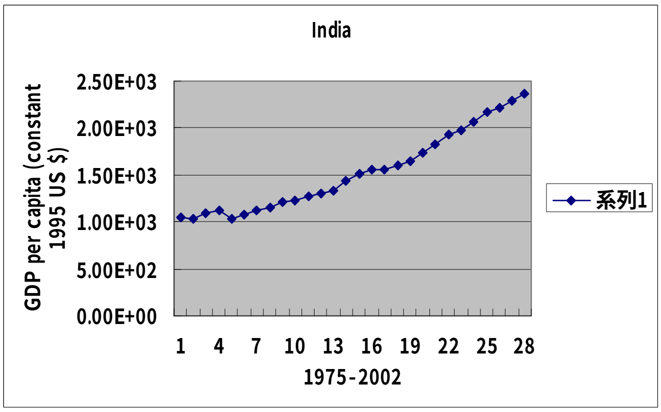


Table 2. GDP per capita and Population Growth Rate (India)





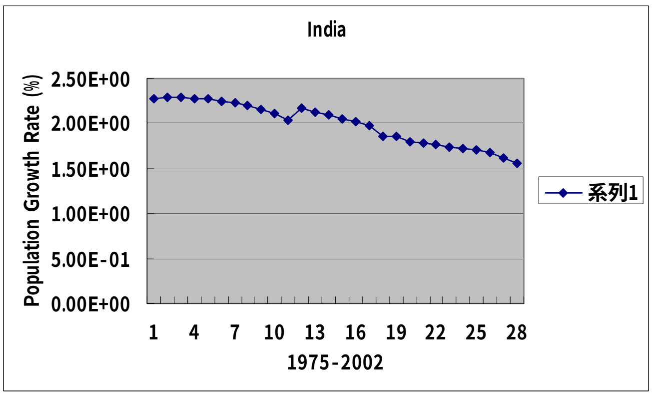
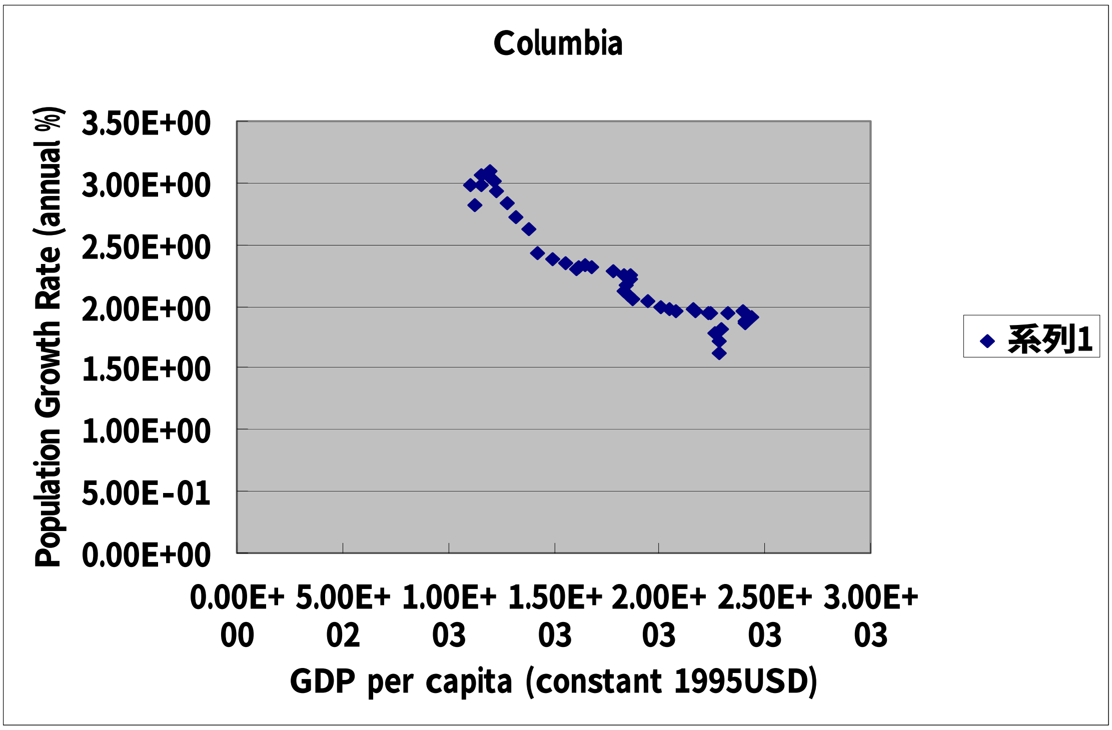
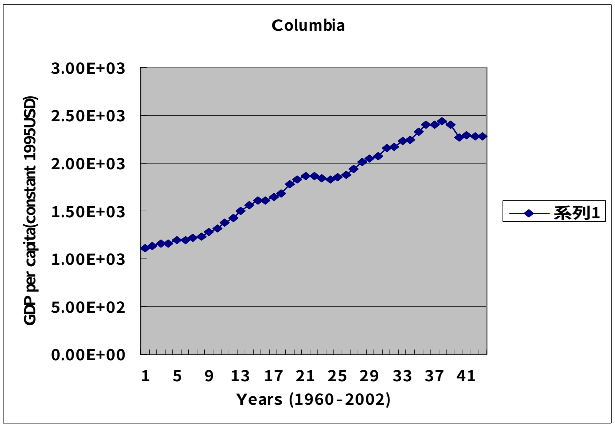


Table 3. GDP per capita and Population Growth Rate (Columbia)





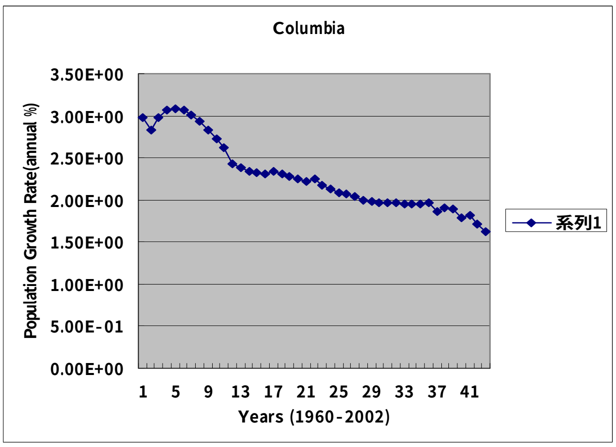
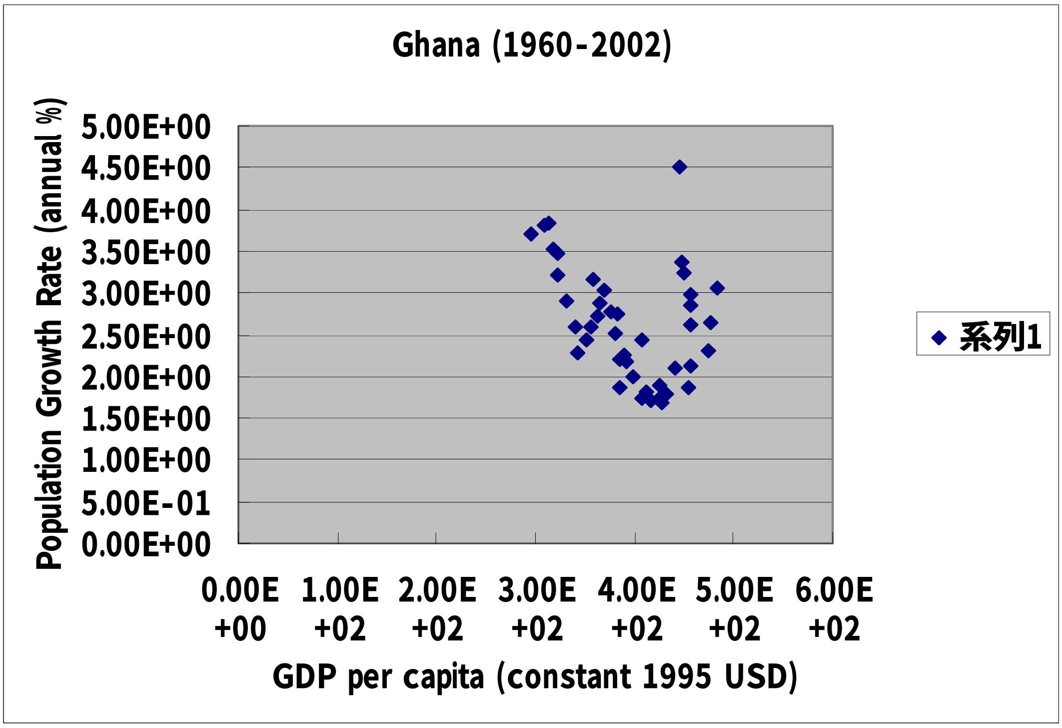
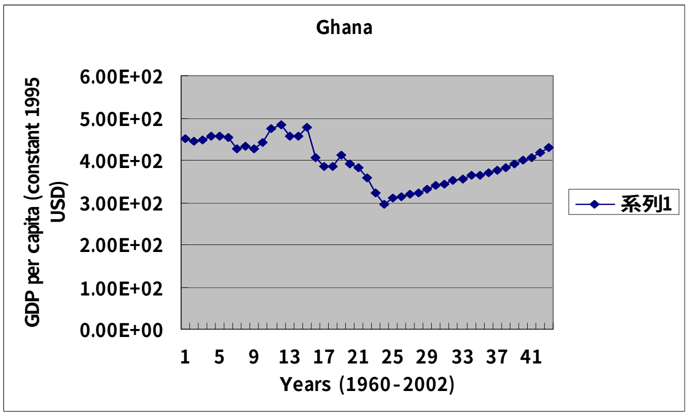


Table 4. GDP per capita and Population Growth Rate (Ghana)





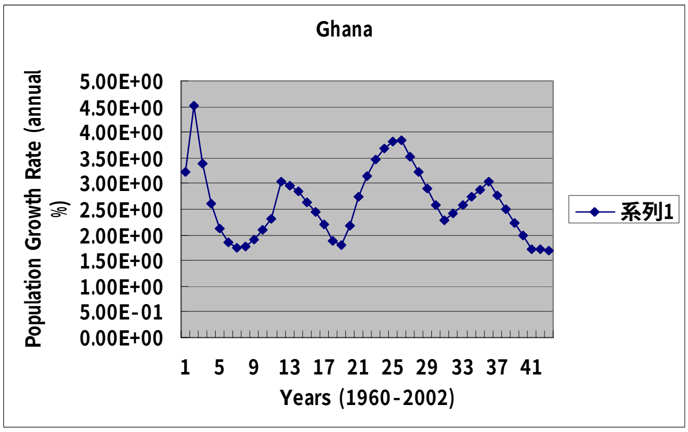


Table 5. GDP per capita and Population Growth Rate (Sudan)