# Love of Quality and Quantity

-A Non-Unit Demand Model of Trade on Vertically Differentiated Goods-

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## 1 Introduction

Nowadays, we sometimes hear the news that well-known companies of producing high quality products go out of business. These companies are often said to exit the market because of low quality but inexpensive imports from developing countries.(the Wal-mart effect) This anecdote suggests that a vertical product cycle - high quality goods in developed countries are replaced by low quality imports from developing countries - may be happening now. However, existing theoretical models do not provide satisfactory explanation for this vertical product cycle. A typical result is that trade liberalization should result in quality upgrading in equilibrium: Gabszewicz,Shaked, Sutton and Thisse (1981) predict that high quality goods will survive, while low quality goods will exit after the opening of trade,

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which is the opposite to the vertical product cycle. For another example, Flam and Helpman (1987) predict that low quality goods produced in the developed country will be replaced by the same low quality imports from developing countries, which is not the vertical product cycle but the horizontal product cycle (the catch-up effect). One reason why the vertical product cycle has not been shown by previous studies is because most of these studies assume the unit demand: each consumer buys at most one unit of a product regardless of his income. This assumption presumes that consumers love quality but not quantity and limits the favorable effect of low price on low quality goods. Alternatively, this paper presents a new non-unit demand model, where consumers love quantity as well as quality. Adding consumers' love of quality into the model, this paper shows that the firm producing the high quality goods will always lose profits after trade, while the firm producing the low quality goods will steal consumers from the firm producing the high quality goods and gain profits if the income gap between two country is large enough to make enough room for the low quality goods to attract low income consumers in the developed country. Numerical examples show that every consumer will gain from trade, yet trade will bring consumers heterogeneous gain from trade. If the income gap between two countries is not large enough, no consumer in the developed country changes to buy the low quality goods, the gain from trade increase with income. However, if the income gap between two countries is large enough, and low income consumers in the developed countries change to buy the low quality goods, the gain from trade and the income has a v-shape relationship: for those low-income consumers who switch to the low quality goods, the gain from trade decrease with income, while the gain from trade increases with income for those high-income consumers who do not switch to low-quality goods.

The gain from trade of marginal consumers hits the lowest because marginal consumers will suffer the negative effect of giving up the high quality goods the most.

The non-unit demand model with endogenous quality choice is developed in section 2, the open economy non-unit model is discussed in section 3, and the comparison between results of the non-unit demand model with that of the traditional unit demand model is shown in section 4.

# 2 A Non-unit Demand Model with Endogenous Quality Choice

## 2.1 Autarky

### 2.1.1 Consumers

Consumers love quantity as well as quality. The utility of consumers are assumed to be measured by the consumer surplus (CS).

$$U_x \equiv CS = s_j \theta_i q_j - \frac{q_j^2}{2} - p_j q_j \tag{1}$$

where $s_j$ is the quality of the good $j$, $q_j$ is the quantity of good $j$, and $\theta_i$ is the reciprocal of marginal utility of income of the consumer $i$. The marginal utility of income is assumed to decrease with income. Thus, $\theta_i$ increases with income. This paper misuses language and calls $\theta_i$ as income of the consumer $i$.

Maximization of the above utility function (1) gives the following individual demand:

$$q_i = s_j \theta_i - p_j \qquad if \ s\theta_i \geq p \tag{2}$$

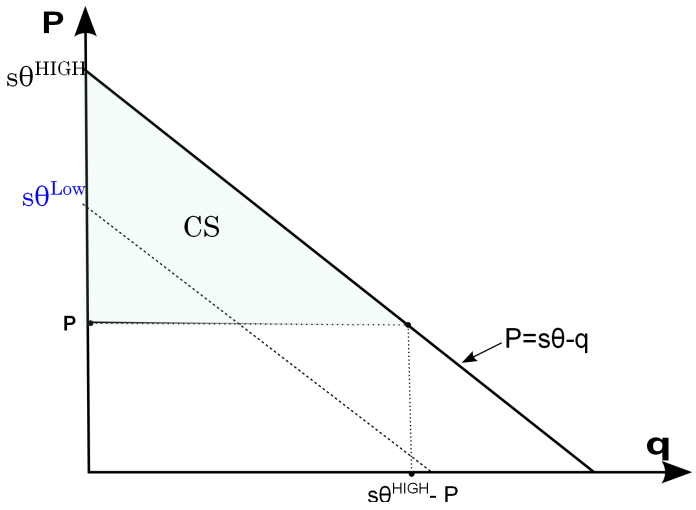



Figure 1: The Consumer Surplus

The individual demand $q_i$ increases with quality ($s_j$), income ($\theta_i$) but decreases with price.[1][2]

The consumer surplus achieved by the individual $i$ is

$$U^i = CS^i = \frac{(s_j\theta_i - p_j)^2}{2} \tag{3}$$

### 2.1.2 The Firm-Monopoly-

Assume that in each country, only one firm, the monopolist, produces a vertically differentiated good under autarky. This monopolist faces a **two-stage game**. At the first stage, the monopolist chooses the quality of goods without knowing that it will face free trade in the future. The quality choice is assumed to be **irreversible**; once the quality is determined, the monopolist cannot not change it.

[1] $q_i = 0$ if the price is greater than the reservation price $s\theta_i < p$

[2] The reservation price $s_j\theta_i$ implies that given the income $\theta$, consumers are willing to pay more to higher quality goods.

At the second stage, given the quality of products, the monopolist chooses the price of the product. Unlike the quality, price can be changed freely.

It is assumed that the income of consumers $\theta$ are uniformly distributed over the range $\theta \in [\underline{\Theta}, \bar{\Theta}]$. [3] Under the uniform distribution, the aggregate demand $Q$ the monopolist faces is given as follows.

$$Q = \int_{\underline{\theta}}^{\bar{\Theta}} (s\theta - p) n d\theta \tag{4}$$

where $\underline{\theta} = \max\left\{\underline{\Theta}, \frac{p}{\theta}\right\}$

$\frac{p}{\theta}$ is the income, at which the CS is zero, and

$n = \frac{L}{\bar{\Theta} - \underline{\Theta}}$ the number of consumers at each point of income distribution($L$ is the population size)

**The Price Choice-The Second Stage-** Solving backward, in the second stage, the monopolist sets the price. The marginal cost is normalized to be 0. The monopolist chooses the price to maximize the following profits by taking the lower limit of the market as endogenous $\underline{\theta} = \frac{p}{s}$.

$$\max \pi = \int_{\underline{\theta}}^{\bar{\Theta}} p(s\theta - p) n d\theta - F(s) \tag{5}$$

where $F(s)$ is the cost of quality, which depends only on the quality and is independent of quantity the monopolist produces.[4] The cost of quality $F(s)$ is assumed to be incurred every period in this model.[5]

[3] Similarly, in a foreign country, the income is uniformly distributed over the range $\theta^* \in [\underline{\Theta^*}, \bar{\Theta}^*]$.

[4] $F(s)$ is discussed more in the next paragraph.

[5] The cost of customer centers is an example of such cost.

The FOC with respect to price is given by

$$\frac{\left(\bar{\Theta}-\underline{\theta}\right)}{2}\left\{s\left(\bar{\Theta}+\underline{\theta}\right)-4p\right\}=0$$

Using the endogenous lower limit, $\underline{\theta}=\frac{p}{s}$, the above FOC gives the optimal price given the quality chosen in the first stage.

$$p=\frac{s\bar{\Theta}}{3} \tag{6}$$

This equation (6) implies that the monopolist chooses the combination of quality and price ($\frac{p}{s}=\underline{\theta}=\frac{\bar{\Theta}}{3}$) so as to make consumers with income $\frac{\bar{\Theta}}{3}$ can buy its product.

In other expressions, the market is fully covered if the lower limit ($\underline{\theta}=\frac{p}{s}=\frac{\bar{\Theta}}{3}$)is less than or equal to the lowest income. ($\underline{\Theta}$)

$$\textit{The market is}\begin{cases}\textit{fully covered} & \text{if } \bar{\Theta}\leq 3\underline{\Theta}\\ \textit{partially covered} & \text{if } \bar{\Theta}>3\underline{\Theta}\end{cases}$$

It implies that if the income distribution is so diverse, the monopolist chooses not to cover the whole market and sell only to rich consumers.

**The Irreversible Quality Choice-The First Stage-** In the first stage, the monopolist chooses the quality of products without knowing it will face free trade in the future. As mentioned before, the quality choice is assumed to be irreversible. The monopolist chooses the quality $s$ so as to maximizes profits in eq (5).

The cost of quality $F(s)$ in eq (5) increases with quality,$F'(s)$, and the curvature of the quality-cost function is assumed to be greater than 1 [6]:

$$F'(s) \ > \ 0 \ \ and \ \frac{s \cdot F''(s)}{F'(s)} \ > \ 1$$

In this paper, the following special functional form is used for $F(s)$.[7]

$$F(s) = \frac{\alpha}{2}s^2 - \beta s + \frac{\beta^2}{2\alpha} \tag{7}$$

$\alpha$ in the above equation is assumed to satisfy the following condition.[8]

$$\alpha \ > \ \max\left\{ \frac{4L}{\bar{\Theta}-\underline{\Theta}}\left(\frac{\bar{\Theta}}{3}\right)^3, \frac{\bar{\Theta}(\bar{\Theta}+\underline{\Theta})L}{6} \right\}$$

With this cost function, the FOC with respect to quality (s) is given by[9]

$$\frac{p}{2}n\left(\bar{\Theta}^2 - \underline{\theta}^2\right) + p(s\underline{\theta} - p)n\left(\frac{\partial\underline{\theta}}{\partial s}\right) - \alpha s + \beta = 0 \tag{8}$$

**The Autarky Equilibrium under the Fully Covered Market** If $\bar{\Theta} \leq 3\underline{\Theta}$, the market is fully covered and the lower limit of the market is equal to the lowest income in the country $\underline{\theta} = \underline{\Theta}$. Substituting $\underline{\theta} = \underline{\Theta}$ into eq (8), we have the optimal quality under fully covered market.

[6] This is a necessary and sufficient condition to satisfy the SOC w.r.t. quality

[7] $F'(s) = \alpha s - \beta$ and $\frac{s \cdot F''(s)}{F'(s)} = \frac{\alpha s}{\alpha s - \beta} \ > \ 1$ under this special functional form

[8] This condition is a necessary and sufficient condition for the SOC w.r.t quality.

[9] The second term in eq (8) is zero either because the CS of the lower limit consumers is zero (i.e. $s\underline{\theta} - p = 0$ ) under the fully covered market ($\bar{\Theta} \leq 3\underline{\Theta}$) or because the lower limit becomes exogenous ($\underline{\theta} = \underline{\Theta}$ and $\frac{\partial\underline{\theta}}{\partial s} = 0$ ) under the partially covered market ($\bar{\Theta} > 3\underline{\Theta}$).

$$s = \frac{\beta}{\left\{\alpha - \frac{\bar{\Theta}(\bar{\Theta}+\underline{\Theta})L}{6}\right\}} \tag{9}$$

Substituting the above eq (9) back to eq (6), we have the optimal price under fully covered market.

$$p = \frac{2\beta\bar{\Theta}}{\left\{\alpha - \frac{\bar{\Theta}(\bar{\Theta}+\underline{\Theta})L}{6}\right\}} \tag{10}$$

Once the optimal price and quality are determined, the equilibrium aggregate demand and profits can be calculated as follows.

Substituting eq (9) and eq (10) into eq(4), we have the equilibrium aggregate demand.

$$Q = \frac{\beta\left(\bar{\Theta}+3\underline{\Theta}\right)L}{6\left\{\alpha - \frac{\bar{\Theta}(\bar{\Theta}+\underline{\Theta})L}{6}\right\}} \tag{11}$$

Substituting eq (9) and eq (10) into eq(5), we have the equilibrium profits.

$$\pi = \frac{\beta^2\left(9\alpha - 2\bar{\Theta}^2 L\right)}{18\left\{\alpha - \frac{\bar{\Theta}(\bar{\Theta}+\underline{\Theta})L}{6}\right\}^2} - \frac{\beta^2}{2\alpha} \tag{12}$$

**The Autarky Equilibrium under the Partially Covered Market** Similarly, if $\bar{\Theta} > 3\underline{\Theta}$, the market is not fully covered and the lower limit of the market is equal to a third of the highest income $\underline{\theta} = \frac{p}{s} = \frac{\bar{\Theta}}{3}$. Substituting $\underline{\theta} = \frac{\bar{\Theta}}{3}$ into eq (8), we have the optimal quality under partially covered market.

$$s = \frac{\beta}{\left\{\alpha - \left(\frac{4L}{\bar{\Theta}-\underline{\Theta}}\right)\left(\frac{\bar{\Theta}}{3}\right)^3\right\}} \tag{13}$$

Again substituting this optimal quality (13) into eq (6), we have the equilibrium price under the partially covered market.

$$p = \frac{\beta\bar{\Theta}}{3\left\{\alpha - \left(\frac{4L}{\bar{\Theta}-\underline{\Theta}}\right)\left(\frac{\bar{\Theta}}{3}\right)^3\right\}} \tag{14}$$

Substituting eq (13)and(14) back to eq (4) and (5), the equilibrium aggregate demand and the equilibrium profits under the partially covered market can be derived as follows.

$$Q = \frac{2\beta\bar{\Theta}^2 L}{9\left\{\alpha - \left(\frac{4L}{\bar{\Theta}-\underline{\Theta}}\right)\left(\frac{\bar{\Theta}}{3}\right)^3\right\}(\bar{\Theta}-\underline{\Theta})} \tag{15}$$

$$\pi = \frac{\beta^2}{2\left\{\alpha - \left(\frac{4L}{\bar{\Theta}-\underline{\Theta}}\right)\left(\frac{\bar{\Theta}}{3}\right)^3\right\}} - \frac{\beta^2}{2\alpha} \tag{16}$$

**The Closed Economy Equilibrium -Summary-** To summarize, the autarky equilibrium is characterized by the following equilibrium variables.

$$s = \begin{cases} \dfrac{6\beta}{\left\{6\alpha - \bar{\Theta}(\bar{\Theta}+\underline{\Theta})L\right\}} & (9) \text{ if } \bar{\Theta} \leq 3\underline{\Theta} \\ \dfrac{\beta}{\left\{\alpha - \left(\frac{4L}{\bar{\Theta}-\underline{\Theta}}\right)\left(\frac{\bar{\Theta}}{3}\right)^3\right\}} & (13) \text{ if } \bar{\Theta} > 3\underline{\Theta} \end{cases}$$

$$p^{au} = \begin{cases} \dfrac{2\beta\bar{\Theta}}{\left\{6\alpha - \bar{\Theta}(\bar{\Theta}+\underline{\Theta})L\right\}} & (10) \text{ if } \bar{\Theta} \leq 3\underline{\Theta} \\ \dfrac{\beta\bar{\Theta}}{3\left\{\alpha - \left(\frac{4L}{\bar{\Theta}-\underline{\Theta}}\right)\left(\frac{\bar{\Theta}}{3}\right)^3\right\}} & (14) \text{ if } \bar{\Theta} > 3\underline{\Theta} \end{cases}$$

$$Q^{au} = \begin{cases} \frac{\beta(\bar{\Theta}+3\underline{\Theta})L}{\left\{6\alpha-\bar{\Theta}(\bar{\Theta}+\underline{\Theta})L\right\}} & (11) \text{ if } \bar{\Theta} \leq 3\underline{\Theta} \\ \frac{2\beta\bar{\Theta}^2 L}{9\left\{\alpha-\left(\frac{4L}{\bar{\Theta}-\underline{\Theta}}\right)\left(\frac{\bar{\Theta}}{3}\right)^3\right\}(\bar{\Theta}-\underline{\Theta})} & (15) \text{ if } \bar{\Theta} > 3\underline{\Theta} \end{cases}$$

$$\pi^{au} = \begin{cases} \frac{2\beta^2\left(9\alpha-2\bar{\Theta}^2 L\right)}{\left\{6\alpha-\bar{\Theta}(\bar{\Theta}+\underline{\Theta})L\right\}^2} - \frac{\beta^2}{2\alpha} & (12) \text{ if } \bar{\Theta} \leq 3\underline{\Theta} \\ \frac{\beta^2}{2\left\{\alpha-\left(\frac{4L}{\bar{\Theta}-\underline{\Theta}}\right)\left(\frac{\bar{\Theta}}{3}\right)^3\right\}} - \frac{\beta^2}{2\alpha} & (16) \text{ if } \bar{\Theta} > 3\underline{\Theta} \end{cases}$$

All of the above variables-$s,\ p,\ Q,\ \pi$- increase with the highest income, the lowest income, and the population size ($\bar{\Theta}$,$\underline{\Theta}$,$L$): it implies that the monopolist in a country with higher income and larger population size produces higher quality goods, charges higher price, sells more products, and earns more profits. [10]

## 3 The Open Economy:Duopoly

Consider the situation where two countries-a developed country (home) and a developing country (foreign)- start free trade. In each country, as described in the previous section, the monopolist produces vertically differentiated goods. Two countries have the different income distribution and the different population size, yet they are assumed to share the same cost function and the same utility function.

To make the model manageable, this paper considers a special case with three assumptions: first, the income distribution of two countries **just overlaps** (i.e. $\bar{\Theta}^*=\underline{\Theta}$) [figure 2], second, **the**

[10] No variable faces the discontinuity problem:all variables are continuous at $\bar{\Theta}=3\underline{\Theta}$.

**number of consumers at each point of income** (=density) is the same in both countries,(i.e. $n=\frac{L}{\bar{\Theta}-\underline{\Theta}}=\frac{L^*}{\bar{\Theta}^*-\underline{\Theta^*}}=n^*$), and third,both markets were fully covered under autarky.($\bar{\Theta} \leq 3\underline{\Theta}$ in home and $\bar{\Theta}^* \leq 3\underline{\Theta^*}$ in foreign) [11]

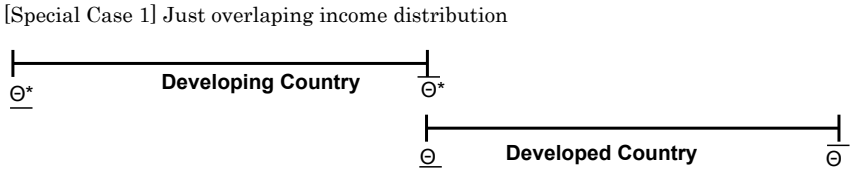



Figure 2: Income Distributions of Two Countries

**The Bertrand Competition after Trade** After the starting of free trade, two former monpolists under autarky now face the competition. Since the highest income (and the lowest income) in the developed country are assumed to be higher than those in the developing country, the quality of goods produced in the developed country is higher than that of goods produced in the developing country. ($s > s^*$) [12] As mentioned before, the quality of goods is assumed not to be changed, while the price can be changed freely. Thus,the high-quality goods' firm in the developed country and the low-quality goods' firm in the developing country start the price competition after trade. Specifically, these two firms are assumed to start the Bertrand competition:they set the new price simultaneously given the other firm's price.

Furthermore, it is assumed that the arbitrage prevents these two firms from exercising the price

[11] With the first two assumptions, free trade just brings the extension of income distribution from $[\underline{\Theta}, \bar{\Theta}]$ to $[\underline{\Theta^*}, \bar{\Theta}]$.

[12] $s=\frac{\beta}{\left\{\alpha-\frac{\bar{\Theta}(\bar{\Theta}+\underline{\Theta})L}{6}\right\}}$ when $\bar{\Theta} \leq 3\underline{\Theta}$ and $s^*=\frac{\beta}{\left\{\alpha-\frac{\bar{\Theta}^*(\bar{\Theta}^*+\underline{\Theta}^*)L^*}{6}\right\}}$ when $\bar{\Theta}^* \leq 3\underline{\Theta^*}$ Since the quality increases with the highest income, the lowest income, and the population size, $s > s^*$

discrimination in two markets. Hence, both **firms charge the same price in home and abroad**.

### 3.1 Consumers

After trade, consumers in both countries now have choice between the high quality goods and the low quality ones. Consumers in two countries assumed to share the same utility function, which is equal to the CS. With this assumption, marginal consumers achieve the same level of CS by buying either of two goods. That is,

$$\frac{1}{2}\left(s\tilde{\theta}-p\right)^2=\frac{1}{2}\left(s^*\tilde{\theta}-p^*\right)^2$$

Solving the above equation, the income of marginal consumers who are indifferent between the high quality goods and the low quality ones can be identified [13]:

$$\tilde{\theta}=\frac{p-p^*}{s-s^*} \tag{17}$$

If the income of marginal consumers is greater than the lowest income in the developed country($\tilde{\theta} \geq \underline{\Theta}$), The low-quality goods' firm in the developing country will export and steal consumers fromthe high-quality goods' firm, vice versa.

[13]The derivation of eq (17) is shown in the appendix B

### 3.2 Firms: The Bertrand Competition

After the beginning of free trade, two firms set price simultaneously. Given the income of marginal consumers eq(17), each firm faces the following aggregate demand and profit function.

Since $n = n^*$ by assumption, the aggregate demand and the profit function of The low-quality goods' firm are given by

$$Q^* = \int_{\underline{\theta}^*}^{\tilde{\theta}} (s^*\theta - p^*)\ nd\theta \tag{18}$$

$$\pi^* = \int_{\underline{\theta}^*}^{\tilde{\theta}} p^*(s^*\theta - p^*)\ nd\theta - F(s^*) \tag{19}$$

where $\underline{\theta}^*=\max\{\underline{\Theta}^*, \frac{p^*}{s^*}\}$

The cost of quality, which should be incurred in every period=$F(s^*) = \frac{\alpha}{2}s^{*2} - \beta s^* + \frac{\beta^2}{2\alpha}$

The FOC with respect to price ($p^*$) gives the reaction function of The low-quality goods' firm [14]:

$$p^* = \frac{s^*p}{3s} \tag{20}$$

The aggregate demand and the profit function ofthe high-quality goods' firm are given by

$$Q = \int_{\tilde{\theta}}^{\bar{\Theta}} (s\theta - p)\ nd\theta \tag{21}$$

[14]There are two solutions for $p^*$. However, the solution in eq(20) is the only solution that satisfies the second order condition. Appendix shows the derivaration of eq (20).

$$\pi = \int_{\tilde{\theta}}^{\bar{\Theta}} p(s\theta - p)\ nd\theta - F(s) \tag{22}$$

$$\text{where } F(s) = \frac{\alpha}{2}s^2 - \beta s + \frac{\beta^2}{2\alpha}$$

The FOC with respect to p gives the reaction function ofthe high-quality goods' firm:

$$p = \frac{2\left\{(s-s^*)^2\bar{\Theta} - s^*p^*\right\} \pm \sqrt{4\left\{\bar{\Theta}(s-s^*)^2 - s^*p^*\right\}^2 - 3s(s-2s^*)\left\{\bar{\Theta}^2(s-s^*)^2 - p^{*2}\right\}}}{3(s-2s^*)} \tag{23}$$

### 3.3 The Open Economy Equilibrium

The intersection of two reaction functions (20), (23) yields the Nash equilibrium in this special case.[15]

$$p = \frac{3s(s-s^*)\bar{\Theta}\left\{6s(s-s*) - \sqrt{9(s-s^*)^2 + 16s^{*2}}\right\}}{27(s-s^*)^2 - 16s^{*2}} \tag{24}$$

$$p^* = \frac{s^*\bar{\Theta}\left\{6s(s-s*) - \sqrt{9(s-s^*)^2 + 16s^{*2}}\right\}}{27(s-s^*)^2 - 16s^{*2}} \tag{25}$$

The income of marginal consumers is

$$\tilde{\theta} = \frac{(3s-s^*)\bar{\Theta}\left\{6s(s-s*) - \sqrt{9(s-s^*)^2 + 16s^{*2}}\right\}}{27(s-s^*)^2 - 16s^{*2}} \tag{26}$$

Eq (22)and(19) give the post-trade profits:

[15]The derivation of eq (24), (25), and (26) are shown in Appendix B.

$$\pi = np(\bar{\Theta} - \tilde{\theta}) \left\{ \frac{s}{2} \left( \bar{\Theta} + \tilde{\theta} \right) - p \right\} - \underbrace{\left\{ \frac{\alpha}{2} s^2 - \beta s + \frac{\beta^2}{2\alpha} \right\}}_{F(s)} \qquad (22)$$

$$\pi^* = np^*(\tilde{\theta} - \underline{\Theta^*}) \left\{ \frac{s^*}{2} \left( \tilde{\theta} + \underline{\Theta^*} \right) - p^* \right\} - \underbrace{\left\{ \frac{\alpha}{2} s^{*2} - \beta s^* + \frac{\beta^2}{2\alpha} \right\}}_{F(s^*)} \qquad (19)$$

# 4 The Importance of the Assumption of Non-Unit Demand

**-The Comparison of the Non-unit Demand Model with the Traditional Unit Demand Model-**

So far, based on the assumption that consumers love quantity as well as quality,this paper has discussed the Non-Unit Demand (Non-UD) model with endogenous once-and-for-all quality choice. This Non-UD model is quite different from traditional Unit-Demand (UD) models used in previous studies: the Non-UD model allows both the endogenous quality choice and the non-unit demand, while most traditional UD models allow neither of them. Thus, different results are attributed to mixed effects of the endogenous quality choice and the assumption of non-unit demand. In order to emphasize the importance of the assumption of non-unit demand, this section compares results of **the Non-UD model with the endogenous quality choice** [shown in the previous section] with results of **the UD model with the endogenous quality choice** [presented in Appendix A] when $2\underline{\Theta} < \bar{\Theta} \leq 3\underline{\Theta}$ and $2\underline{\Theta^*} < \bar{\Theta}^* \leq 3\underline{\Theta^*}$.[16] Specifically, this section compares quality chosen by

[16]If the income distribution is in these ranges, the quality chosen by monopolist depends on the income distribution in both models. As shown in Appendix A, under the UD model, the quality chosen by monopolist is independent of the

monopolists, the exporter of goods, prices, aggregate demand, profit of two firms under the Non-UD model with those variables under the UD model.

This section shows five main results. First, under both models, two countries produce more vertically differentiated goods as the income gap between two countries become wider. However, the extent of differentiation is always greater under the Non-UD model where higher quality is rewarded more by the increase in demand from existing consumers as well as the increase in new consumers. Second, this section shows that in both models,the firm of low-quality goods (high-quality goods) will be the exporter if the income gap between two countries is large (small). Third, if the income gap between two countries is large, two models show different consumer staling effects after trade: under the Non-UD model where consumers love quantity as well as quality, the quantity offered by the low-quality firm will attract consumers in the developed country and the low-quality goods' firm will steal consumers from the high-quality goods' firm. On the contrary, under the UD model where consumer loves quality only, no existing consumer will be stolen from the high-quality goods' firm.[17] Forth, in both models, as a result of the competition against low quality goods' firm followed by a largedecrease in price, high quality goods' firm lose profits at any income diversity level. Finally, when the income gap between two countries is large, two models reach the oppsite conclusions about profits of the firm producing low quality goods. Under the Non-UD model, where wealthier consumers in

income distribution, and countries with different income distributions produce exactly the same quality if the highest income is less than twice as large as the lowest income ($\bar{\Theta} > 2\underline{\Theta}$ and $\bar{\Theta}^* > 2\underline{\Theta}^*$)

[17]If the income gap beween two countries is small, the high-quality goods firm will steal consumers from the low-quality goods' firm in both models.

the developed country buy more goods, the firm producing low quality goods gain profits after trade because of export to the developed coutry. On the contrary, under the traditional UD model, where every consumer buys at most one unit, except a rare case, the firm of low qualty goods loses its profits after trade .[18]

### 4.1 The Non-UD Model *VS* the UD Model: the Choice of Quality

Figure 3 shows the quality of goods chosen under two models [$s=\frac{6\beta}{6\alpha-\bar{\Theta}(\bar{\Theta}+\underline{\Theta})L}$,$s^*=\frac{6\beta}{6\alpha-\bar{\Theta}^*(\bar{\Theta}^*+\underline{\Theta}^*)L^*}$ in the Non-UD model], [$s^{UD}=\frac{n\bar{\Theta}^2+4\beta}{4\alpha}$,$s^{*UD}=\frac{n^*\bar{\Theta}^{*2}+4\beta}{4\alpha}$ in the UD model] when $2\underline{\Theta}<\bar{\Theta}\leq 3\underline{\Theta}$ and $2\underline{\Theta}^*<\bar{\Theta}^*\leq 3\underline{\Theta}^*$.[19] [20] [21] As shown in figure 3,the difference in income distribution between two countries is reflected more in quality in the Non-UD model than in the UD model. That is because in the UD model, higher quality

[18]The firm producing low quality goods may gain profits if the marginal cost of improving quality is large, and the population size in the developed country is large. However, even if such a rare case happens, the increase in profits is not because the low quality goods' firm steals consumers from the high quality goods' firm but because the low qulity goods' firm covers consumers in the developed country who have not been covered by the high quality goods' firm under autarky.

[19]All graphs in this section are graphs for the special case where (1) the number of consumers at each point of income is the same in two countries $n=n^*=\frac{L}{\bar{\Theta}-\underline{\Theta}}=\frac{L^*}{\bar{\Theta}^*-\underline{\Theta}^*}$ (2) the income distribution of two countries **just overlaps**. ($\underline{\Theta}=\bar{\Theta}^*$)

[20]Changing the population size $(L, L^*)$ and the overlapping income ($\underline{\Theta}=\bar{\Theta}^*$) does not change qualitative results. The population size $L^*$ in the developing country with lower income is set to be small to satisfy $n = n^*$. $\alpha$ and $\beta$, both of which determine the cost of quality are set to be large enough to satisfy the SOC w.r.t. quality but small enough not to invalidate the effect of income distribution on the quality choice.

[21]The horizontal axis in all graph shows the income diversity in each country: $\frac{\bar{\Theta}}{\underline{\Theta}}=1+\frac{L}{L^*}\left(\frac{\bar{\Theta}^*}{\underline{\Theta}^*}-1\right)$

brings just the new consumers of lower income, while in the Non-UD model, higher quality brings not only new consumers of lower income but also the increase in demand from existing consumers of higher income. This reward system gives the firm of high-quality goods more incentive to choose even higher quality in the Non-UD model, and consequently, the quality difference between two goods becomes larger in the Non-UD model than in the UD model.

Figure 3 also shows that as the highest income in the developed country increases and the lowest income in the developing country in the developing country decreases without changing the lowest income in the developed country and the highest income in the developing country (i.e. two countries become dissimilar), the quality difference between the high quality goods produced in the developed country and the low quality ones produced in the developing country become larger in both models.[22] That is, as two countries become dissimilar, they produce more differentiated products. The following lemma summarizes the quality choice under two models.

**Lemma 1.** *As the difference in income between two countries becomes larger and two countries become dissimilar, two countries produce more differentiated products in both models.*[23] *However, the extent of differentiation is always greater under the Non-UD model where higher quality is rewarded more by the increase in demand from existing consumers as well as the increase in new consumers.*

[22] Of course, the dissimilarity is reflected in quality more in the Non-UD model.

[23] two countries become dissimilar if the highest income in the developed country increases and the lowest income in the developing country in the developing country decreases simmultaneously without changing the lowest income in the developed country and the highest income in the developing country.

## 4.2 The Non-UD Model *VS* the UD Model: Exporters of Products and the Consumer Stealing Effect

### 4.2.1 Exporters of Products and the Consumer Stealing Effect in the Non-UD Model

Figure 4, which displays the difference between the post-trade lower limit of the high-quality goods' firm ($\tilde{\theta}$) and its autarky lower limit (=the lowest income=$\underline{\Theta}$) shows which firm will export and steal consumers from the other firm after trade.[24] It shows that **if the income distribution in home and abroad are not so diverse and two countries are similar,the high-quality goods' firm will be the exporter and steal consumers from the low-quality goods' firm**. However, the completely opposite thing will happen if two countries are dissimilar: **if the income distribution in home and abroad are diverse(= two countries are dissimilar) enough, the low-quality goods' firm will be the exporter and steal consumers from the high-quality goods' firm** . It implies that if the high-quality goods' firm lose existing consumers after trade, the firm stealing these consumers must be a low-quality goods' firm from a dissimilar developing country rather than a low-quality goods' firm from a similar developed country.

### 4.2.2 Exporters of Products and the Consumer Stealing Effect in the UD model

The left graph of figure 5, a graph of the difference between "the post-trade lower limit of the high-quality goods' firm" ($\tilde{\theta}$) and "the lowest income" ($\neq$ the autarky lower limit), shows which firm will

[24] If $\tilde{\theta} > \underline{\Theta}$, the low-quality goods' firm will export and steal existing consumers from the high-quality goods' firm, and vice versa.

export after trade. [25] The left panel of figure 5 seems to tell the same story as figure 4: if the income distribution in home and abroad are diverse enough (two countries are dissimilar enough), the low-quality goods' firm will be the exporter. On the other hand, if two countries are similar, the high-quality goods' firm will be the exporter.

However, the right panel of figure 5 tells a different story. The right panel of figure 5, the graph of the difference between "the high-quality firm's autarky lower limit($\underline{\theta}=\frac{\bar{\Theta}}{2}$)" and "its post-trade lower limit ($\tilde{\theta}$)", shows whether or not the high-quality goods' firm will lose existing consumers after trade. As shown in the positive difference ($\underline{\theta}-\tilde{\theta} > 0$) at any income diversity level, the post-trade lower limit will not exceed the autarky lower limit at all. It means that the high-quality goods' firm will not lose any existing consumer after trade, it rather gain new (domestic or foreign) consumers: if there are domestic consumers of the low-quality goods, they are consumers who were not covered under autarky.

To sum up, the right and the left panel of figure 5 show that if the income gap between two countries is large enough, the low-quality goods' firm will export its product to the developed country. However, the low-quality goods' firm's new consumers in the developed country are consumers who did not buy anything under autarky. Therefore, no consumer will be stolen from the high-quality goods' firm in the UD model. On the other hand, if the income gap between two countries is small, the high-quality goods' firm will be the exporter and sell its product to foreign consumers who used to buy low-quality goods under autarky. Thus, if there is any firm which steals existing consumers

[25] When $2\underline{\Theta}<\bar{\Theta}\leq 3\underline{\Theta}$, autarky market was partially covered and the lower limit in home was $\underline{\theta}=\frac{\bar{\Theta}}{2}>\underline{\Theta}$

from the other firm, the firm must be the high-quality goods' firm in the UD model. [26] [27] In the UD model, the low-quality goods' firm will never steal consumers from the high-quality goods' firm even if it exports to the developed country. The following lemma summarizes the above discussion.

**Lemma 2.** *In both models, if the income gap between two countries are large (small) and two countries are dissimilar (similar) enough, the firm of low-quality goods (high-quality goods) will be the exporter.*

*Moreover, if the income gap between two countries is small, the firm of high-quality goods will steal consumers from the low-quality goods' firm in both models. However, if the income gap between two countries is large, two models reach different conclusions: under the Non-UD model where consumers love quantity as well as quality, the quantity offered by the low-quality firm will attract consumers in the developed country and the low-quality goods' firm will steal consumers from the high-quality goods' firm. On the contrary, under the UD model where consumer loves quality only, no existing consumer will be stolen from the high-quality goods' firm.*

[26] This prediction corresponds with the prediction of previous studies,but figure 5 shows this prediction is true only when two countries are similar.

[27] This prediction is different from the prediction by the Non-UD model; if two countries are dissimilar(similar) in income, the firm of low quality (the high quality) goods will steal consumers from the other firm.

## 4.3 The Non-UD Model *VS* the UD Model:High Quality Products' Firms

### 4.3.1 High Quality Products' Firms in the Non-UD Model: price, aggregate demand, and profit

After the opening of trade,the high-quality goods' firm, the former monopolist in the developed country, faces the competition against the low-quality goods' firm from the developed country. As shown in the right panel of figure 6, the high-quality goods' firm responds to the new foreign rival firm by discounting its products. Specifically, as the income gap between two countries decreases and goods produced in two countries become more similar, the high-quality goods' firm discounts its product more, and its aggregate demand increases monotonically (the left panel of figure 6). However, as shown in figure 7, the positive effect of an increase in the aggregate demand is not large enough to cover the negative effect of the discount, which decreases profits of the high-quality goods' firm at any level of income diversity.

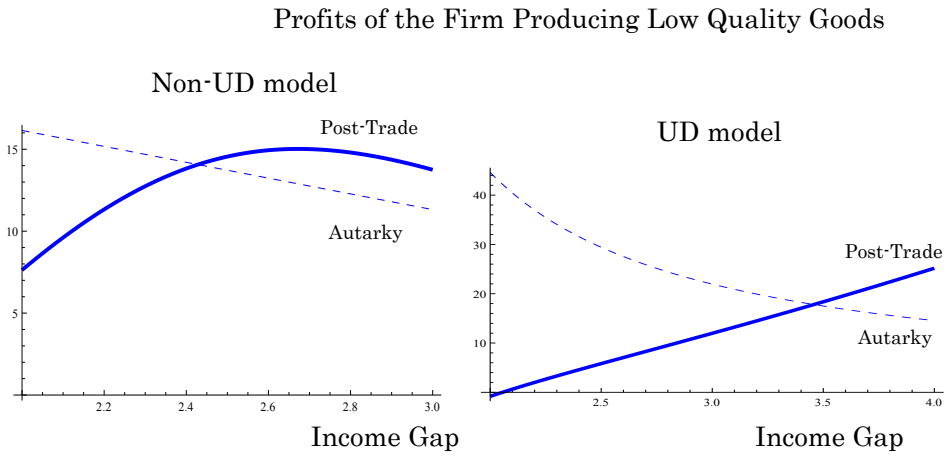
### 4.3.2 High Quality Products' Firms in the UD Model: price, aggregate demand, and profit

Like the high-quality goods' firm in the Non-UD Model,the high-quality goods' firm in the UD model also discounts its products after trade, which increases its aggregate demand monotonically. (figure 8). However, as shown in figure 9, the positive effect of an increase in sales does not exceed the negative effect of a decrease in price, and consequently the high-quality goods' firm will experience a decrease in profits after trade.

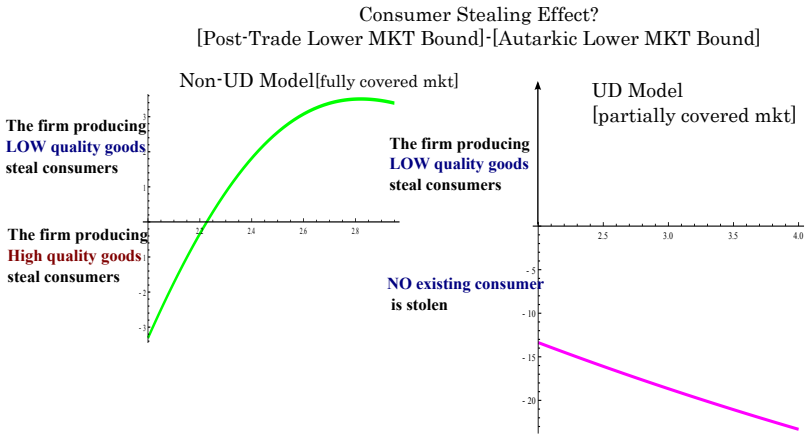
To summarize, in both models, as a result of the competition against low quality goods' firm followed by a large decrease in price, high quality goods' firm lose profits at any condition.

### 4.4 The Non-UD Model *VS* the UD Model: the Firm Producing Low Quality Goods

As figure 11 shows, in both models, the firm producing low quality goods will gain profits if the income between two countries is large enough and the firm exports its product to the developed country.




However, two models reach different conclusions about the consumer stealing effect. Figure 4.4 shows that if the income gap between two countries is large enough, the firm producing the low quality goods will **steal consumers from the other firm** and gain profits, while no consumer is stolen in the UD model.




# 5 Numerical Examples

This section presents numerical examples corresponding to the discussion in the previous section.

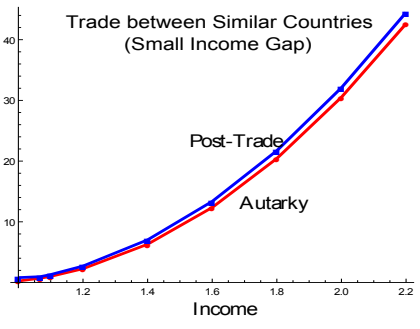
Table 1 shows the case where the income gap between two countries are large and two countries are dissimilar. As discussed in the previous section, two models reach different conclusions about the firm of low quality goods: the firm producing low-quality goods increases profits by stealing consumers from the other firm, while in the UD model, its post-trade profit gain without staling any consumer. With or without the consumer stealing effect has different implications from trade. As shown in figure 5, 5, 5, every consumer will be better off after trade. In percentage, low-income consumers in the developed country will get more gain from trade than high-income consumers in the same country. In value, trade has different implications. If the income gap between two countries is small and no consumer is stolen from the firm producing the high quality goods, the gain from trade monotonically increases with income. However, if the income gap between two countries is large enough, and the firm producing low quality goods steal some consumers from the other firm, the gain from trade and

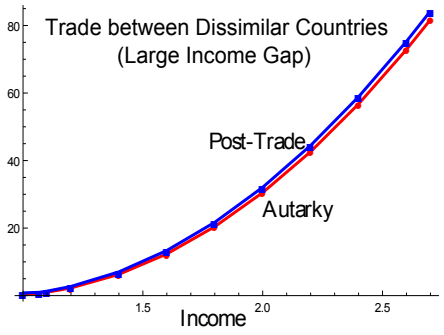
income have a v-shape relationship. Marginal consumers, who are suffering the most from downgrade of their consumption will get the lowest gain from trade.

| Non-UD model: $\bar{\Theta}$=135 $\underline{\Theta}$=$\bar{\Theta}^*$=50 $\underline{\Theta}^*$=24.5 $L$=100 $L^*$=30 $n$=$n^*$=1.176 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $s$ | $s^*$ | $p$ | $p^*$ | $Q$ | $Q^*$ | $\pi$ | $\pi^*$ | $w$ | $w^*$ | Exporter | consumer stealing effect |
| Autarky | 0.142 | 0.036 | 6.4 | 0.6 | 675 | 22 | 1135 | 13 | 4019 | 22 | | |
| Trade | 0.142 | 0.036 | 6.2 | 0.5 | 690 | 30 | 1096 | 15 | 4116 | 26 | **Low** quality | **YES** |
| | | | | | | | | | | | | |
| UD model: $\bar{\Theta}$=135 $\underline{\Theta}$=$\bar{\Theta}^*$=50 $\underline{\Theta}^*$=24.5 $L$=100 $L^*$=30 $n$=$n^*$=1.176 | | | | | | | | | | | | |
| | $s$ | $s^*$ | $p$ | $p^*$ | $Q$ | $Q^*$ | $\pi$ | $\pi^*$ | $w$ | $w^*$ | Exporter | consumer stealing effect |
| Autarky | 0.044 | 0.035 | 3 | 0.9 | 79 | 29 | 211 | 26 | | | | |
| Trade | 0.044 | 0.035 | 0.7 | 0.3 | 100 | 30 | 45 | 8 | | | **Low** quality | **NONE** |

Table 1: Non-UD model VS UD model: a large income gap

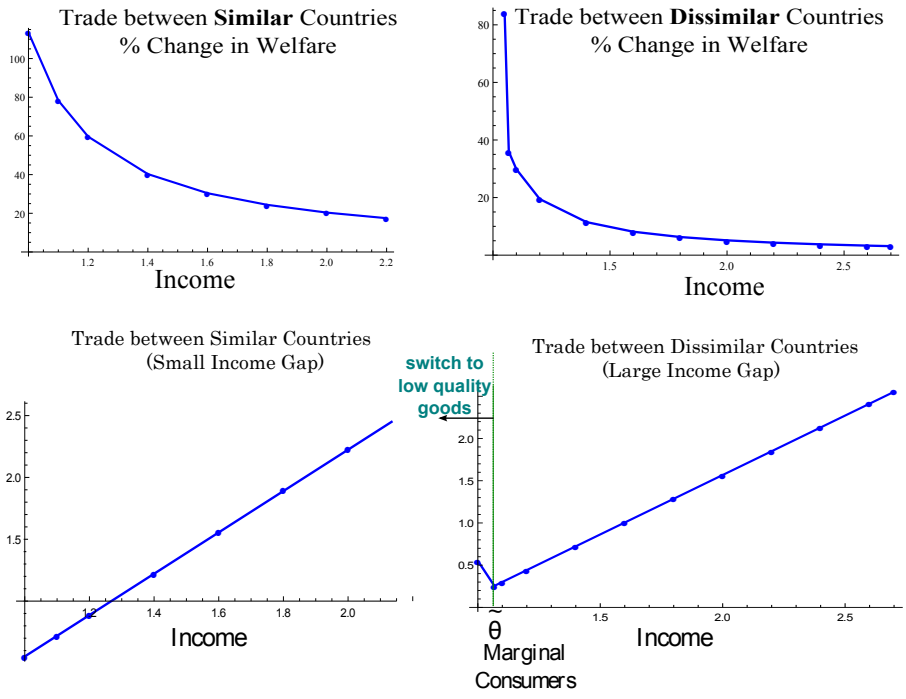
Table 2 shows the case where two countries are similar. As discussed in proposition **??**, two models reach the same conclusion in this case.



| The Non-UD model: $\bar{\Theta}$=110 $\underline{\Theta}$=$\bar{\Theta}^*$=50 $\underline{\Theta}^*$=32 $L$=100 $L^*$=30 $n$=$n^*$=1.67 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $s$ | $s^*$ | $p$ | $p^*$ | $Q$ | $Q^*$ | $\pi$ | $\pi^*$ | $\tilde{\theta}$ | *Exporter* | Losers of consumers |
| Autarky | 0.074 | 0.036 | 2.7 | 0.6 | 321 | 26 | 442 | 15 | | | |
| Trade | 0.074 | 0.036 | 2.3 | 0.4 | 367 | 32 | 400 | 11 | 49.7 | high quality | low quality |
| | | | | | | | | | | | |
| The UD model: $\bar{\Theta}$=110 $\underline{\Theta}$=$\bar{\Theta}^*$=50 $\underline{\Theta}^*$=32 $L$=100 $L^*$=30 $n$=$n^*$=1.667 | | | | | | | | | | | |
| | $s$ | $s^*$ | $p$ | $p^*$ | $Q$ | $Q^*$ | $\pi$ | $\pi^*$ | $\tilde{\theta}$ | *Exporter* | Losers of consumers |
| Autarky | 0.043 | 0.036 | 2.4 | 0.9 | 92 | 42 | 197 | 37 | | | |
| Trade | 0.043 | 0.036 | 0.5 | 0.2 | 116 | 30 | 35 | 2 | 40.52 | high quality | low quality |

Table 2: The Non-UD model VS the UD model: Two Similar Countries



# 6 Conclusion

Observing that the unit demand model used by existing studies do not provide satisfactory explanation for the vertical product cycle, this paper presents a new alternative non-unit demand model, where consumers love quantity as well as quality. Adding consumers' love of quality into the model, this paper shows that the firm producing the high quality goods will always lose profits after trade, while the firm producing the low quality goods will steal consumers from the firm producing the high quality goods and gain profits if the income distribution of two countries is dissimilar enough to make room for the low-quality good's firm to sell its products to wealthier consumers. As shown above, the contribution of this paper is to propose a new non-unit demand model that can consider consumers' love of quantity and to show that the firm producing low quality goods steal consumers from the other firm and gains profit after trade if the income gap between two countries is large enough. Numerical examples show that every consumer will gain from trade, yet trade will bring consumers heterogeneous gain from trade. If the income gap between two countries is not large enough, no consumer in the developed country changes to buy the low quality goods, the gain from trade increase with income. However, if the income gap between two countries is large enough, and low income consumers in the developed countries change to buy the low quality goods, the gain from trade and the income has a v-shape relationship: for those low-income consumers who switch to the low quality goods, the gain from trade decrease with income, while the gain from trade increases with income for those high-income consumers who do not switch to low-quality goods. The gain from trade of marginal consumers hits the lowest because marginal consumers will suffer the negative effect of giving up the high quality

goods the most..

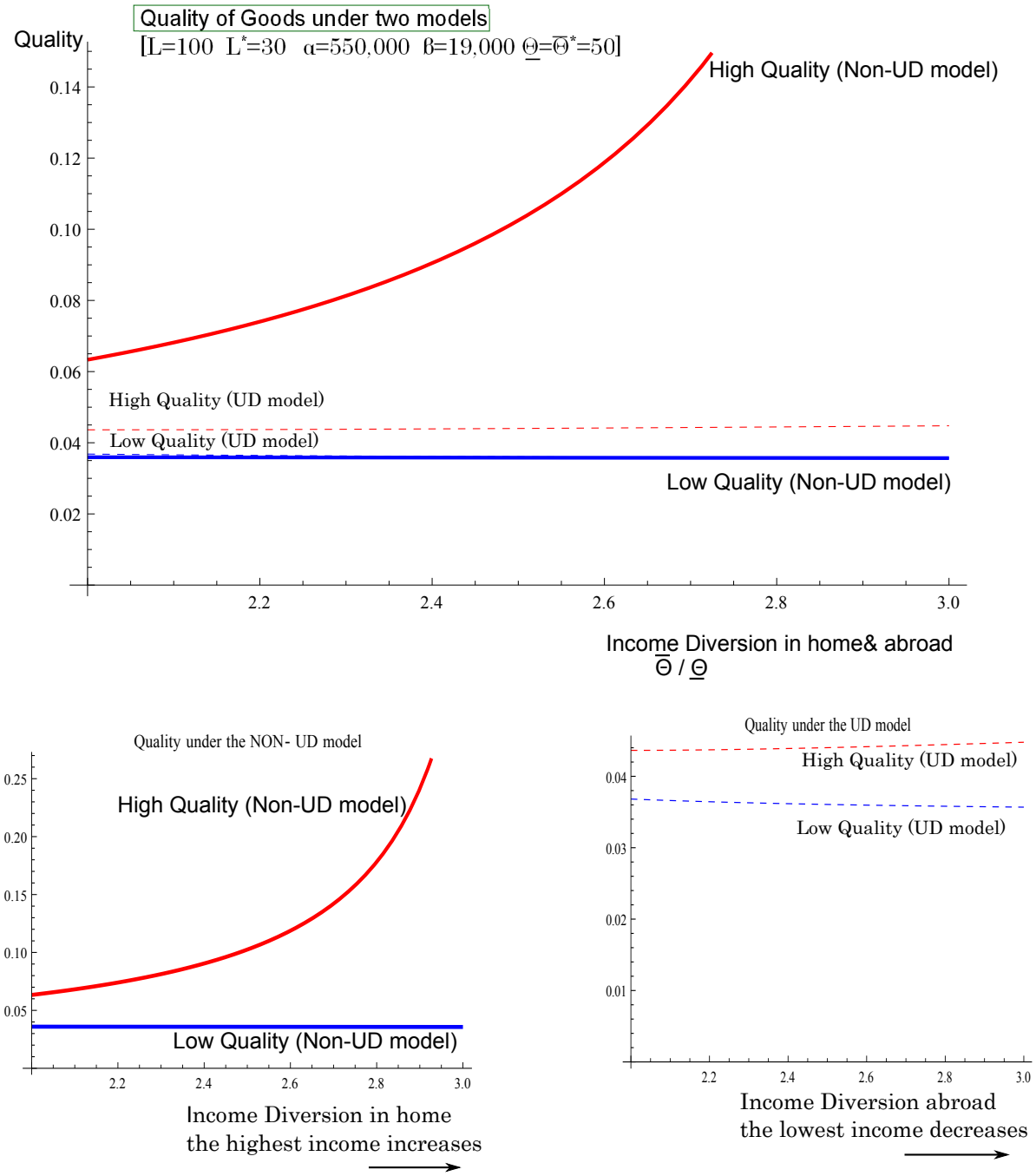



Figure 3: The Non-UD model VS the UD model: the Quality Choice

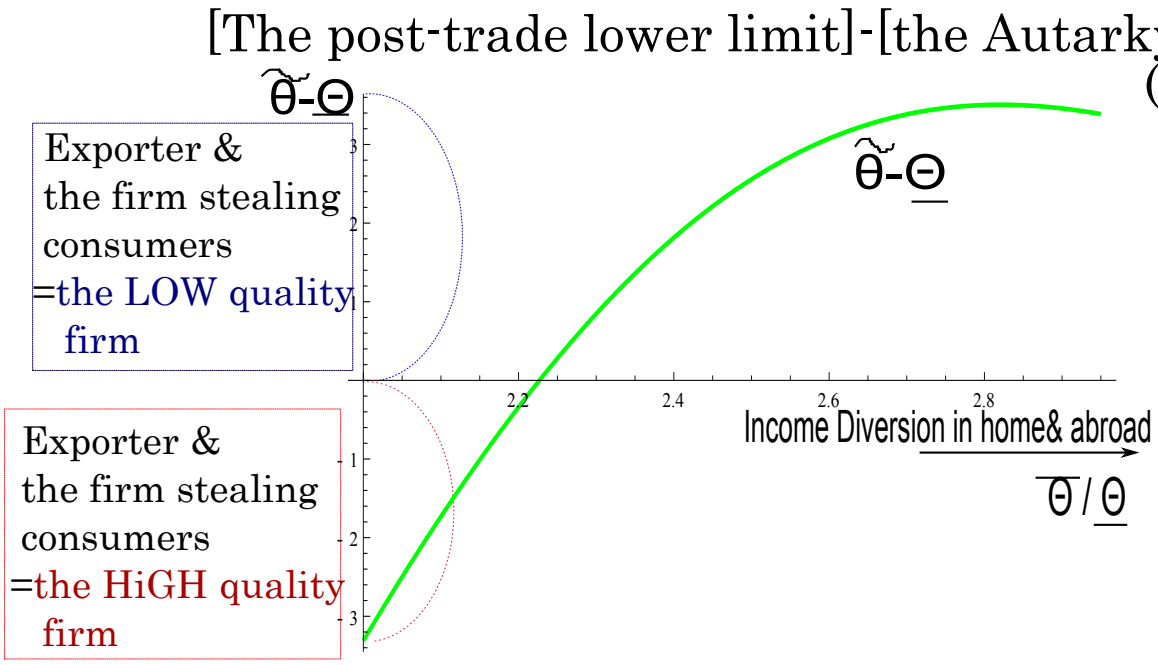



Figure 4: Which firm will export? [the Non-UD model]

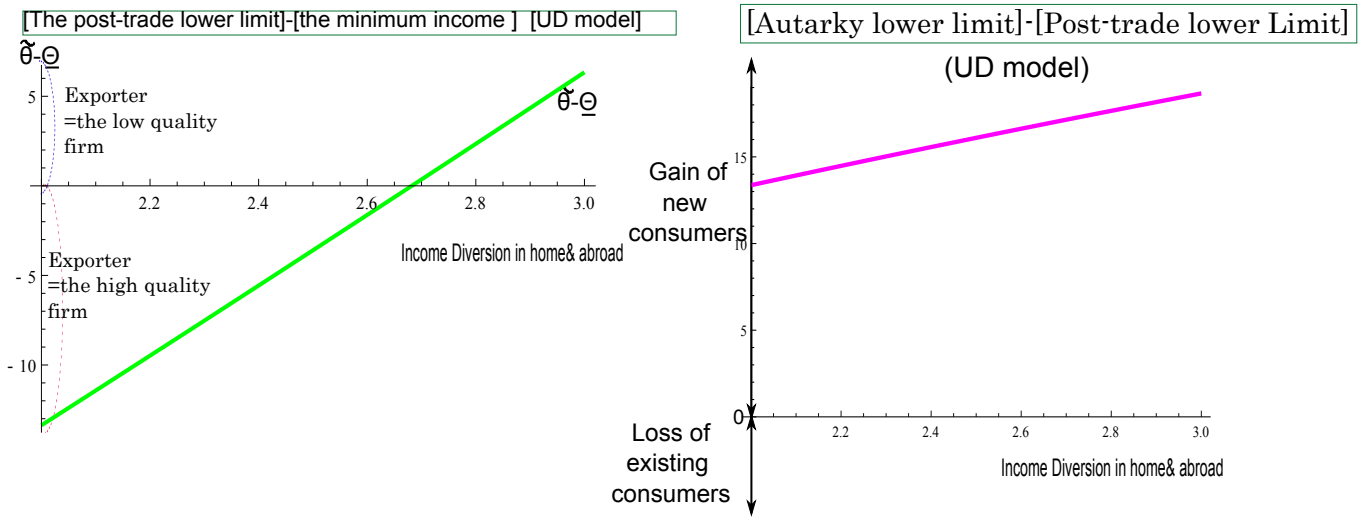



Figure 5: Which firm will export? Which firm will steal consumers from the other firm? [the UD model]

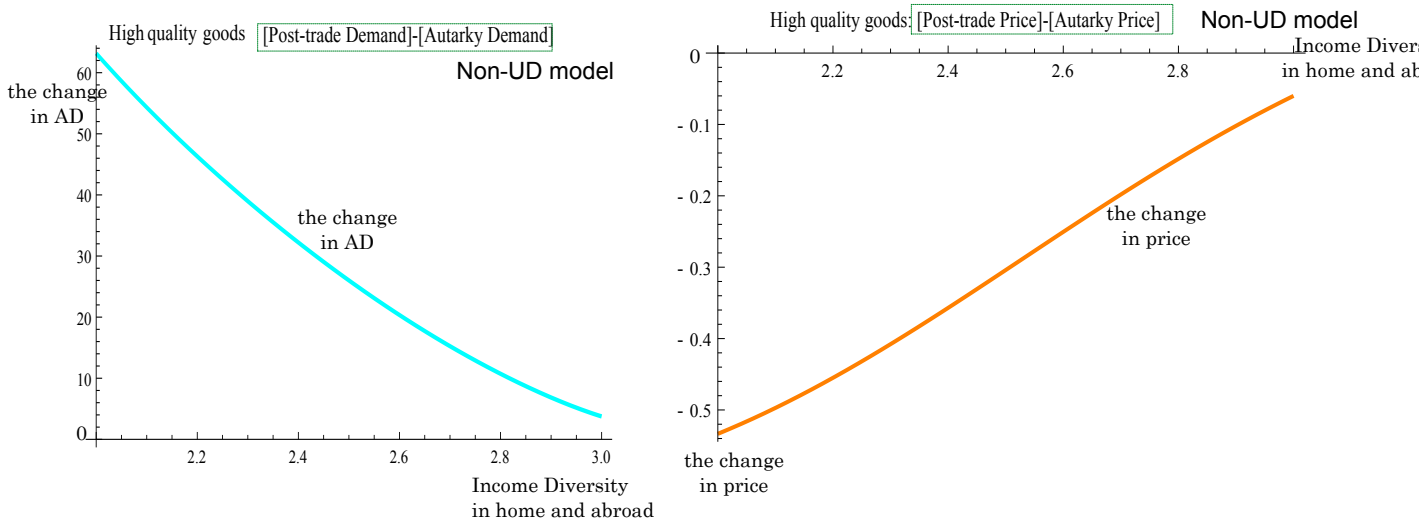



Figure 6: The Change in the Aggregate Demand and Price of High-quality goods (Non-UD model)

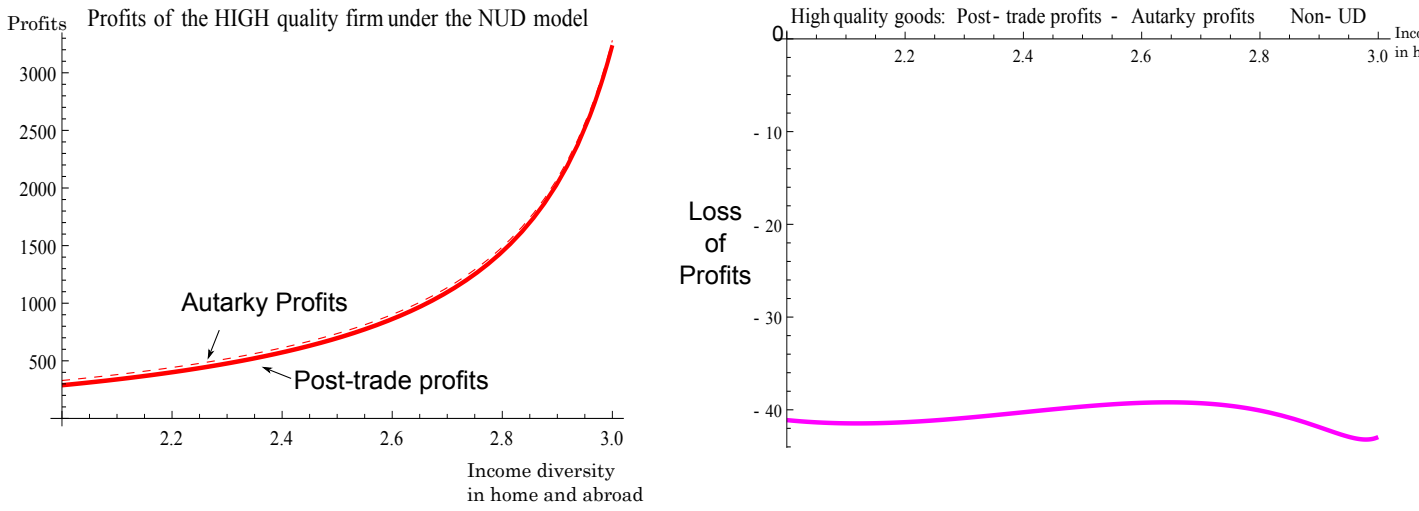



Figure 7: Profits of the High-Quality Goods' Firm in the Non-UD Model


Figure 8: The Change in the Aggregate Demand and Price of High Quality Goods (the UD model)

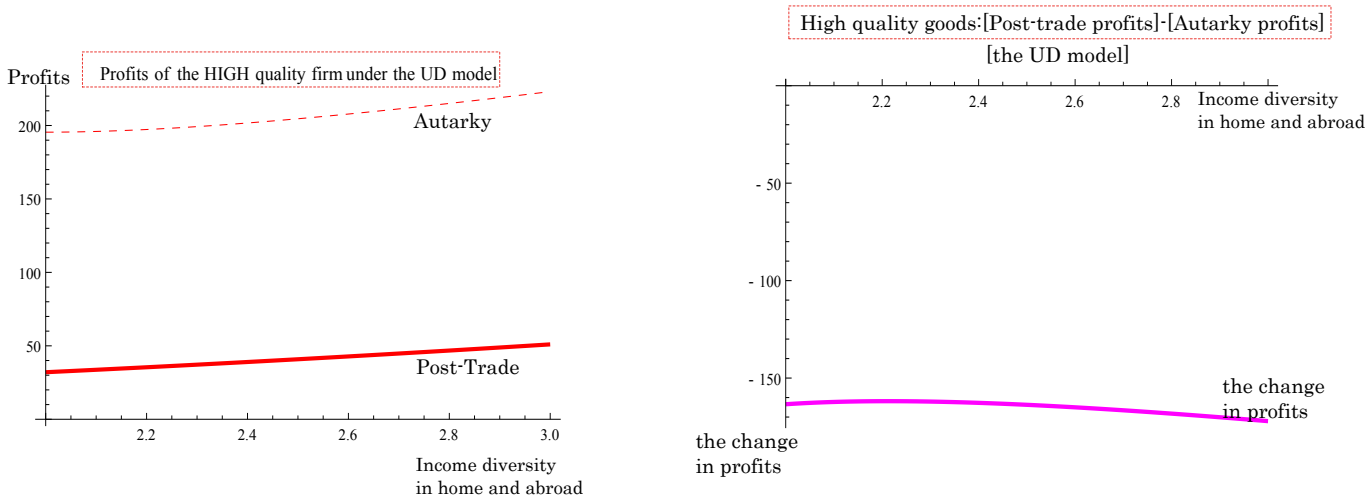



Figure 9: Profits of the High Quality Firm (the UD model)

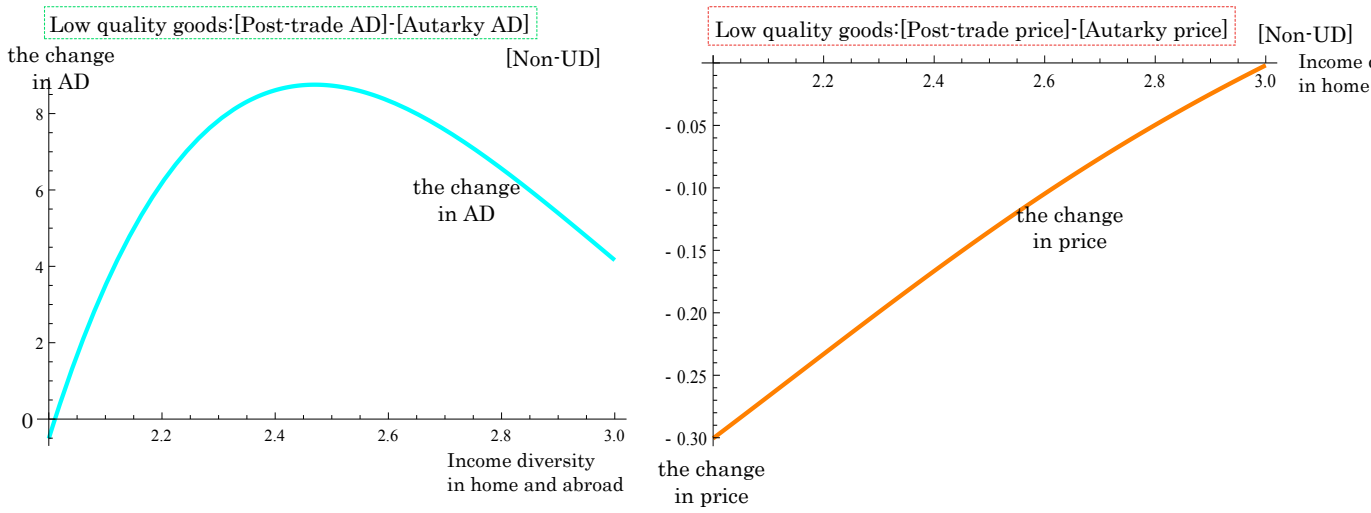



Figure 10: The Change in Aggregate Demand and Price of the Low-quality Goods' Firm (the Non-UD Model)

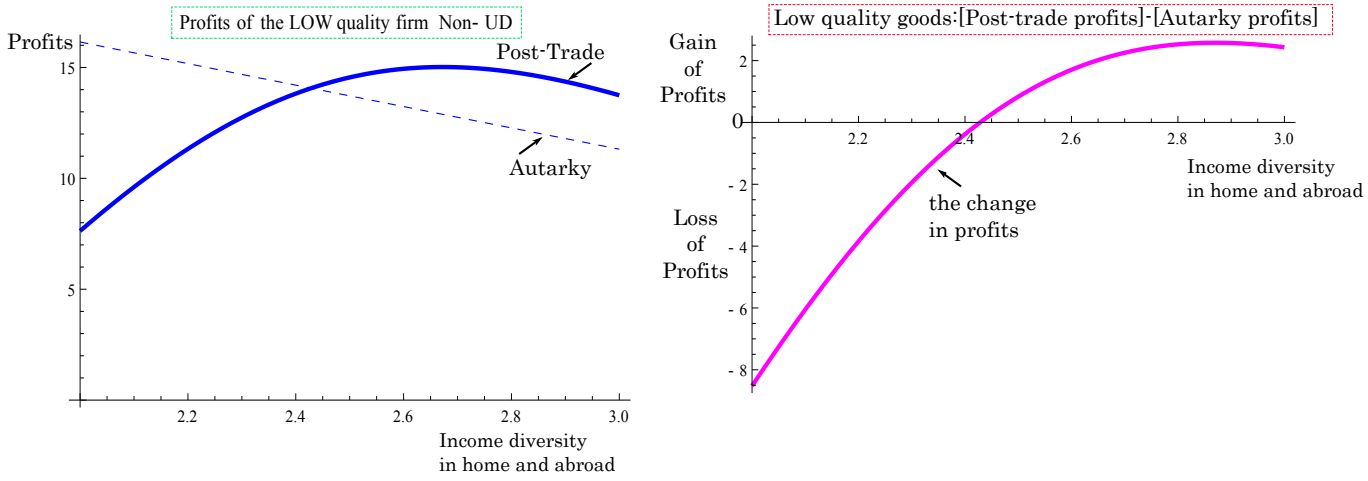



Figure 11: Profits of the Low Quality Firm (the Non-UD Model)

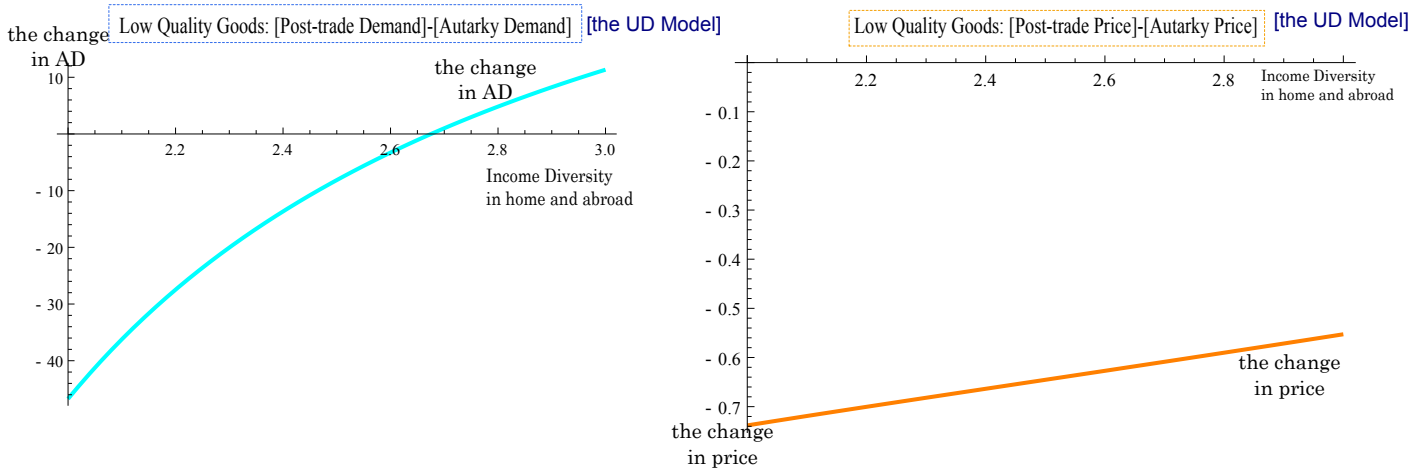



Figure 12: The Change in the Aggregate Demand and Price of Low Quality Goods (the UD model)

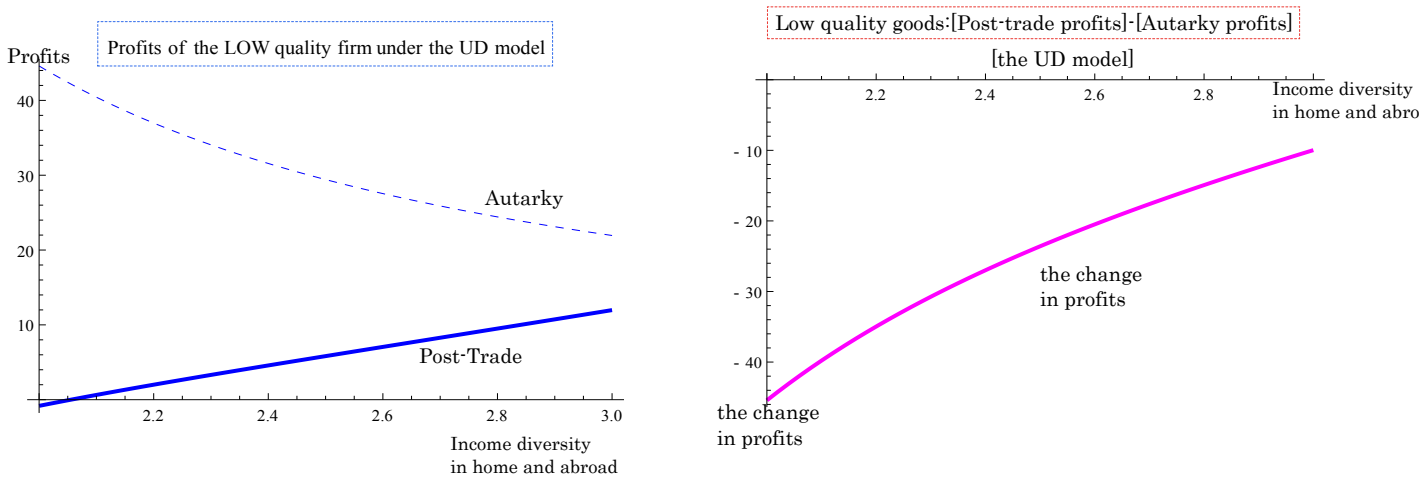



Figure 13: The Profits of the Low Quality Firm (the UD model)

# Appendix A: The Traditional Unit Demand Model with the Endogenous Quality Choice

Most models in previous studies allow neither the endogenous quality choice nor the non-unit demand, while my model allows both the non-unit demand and the endogenous once-and-all quality choice. Therefore, different results between my model and models in previous studies are explained by mixed effects of the non-unit demand and the endogenous quality choice.

To show how important the non-unit demand assumption plays in my model, this section presents the unit demand version of my model: the model presented in this section allows the once-and-for-all endogenous quality choice but imposes the unit demand assumption on consumers.

**Consumers-The Unit Demand-** First, the utility of consumers assumed in many previous models is given by

$$U_i = s_j \theta_i - p$$

where $\theta_i$ is the income of the consumer $i$.

The individual demand is simply given by

$$q_i = \begin{cases} 1 & \text{if } s\theta_i \geq p \\ 0 & \text{if } s\theta_i < p \end{cases}$$

**The Firm-Monopoly under Autarky-** Given the above individual demand, the aggregate demand the monopolist faces is equal to the number of consumers whose income is greater than or equal to $\underline{\theta} = \frac{p}{s}$:

$$Q^{UD} = \int_{\underline{\theta}}^{\bar{\Theta}} n \, d\theta = n\left(\bar{\Theta} - \frac{p}{s}\right) \tag{27}$$

The monopolist's profits is given by

$$\pi^{UD} = pQ - \underbrace{\left\{\frac{\alpha}{2}s^s - \beta s + \frac{\beta^2}{2\alpha}\right\}}_{F(s)} \tag{28}$$

The monopolist faces the two-stage game: in the first stage, it chooses the quality without knowing that it will face free trade in the future, and then it sets the price in the second stage. As mentioned in previous sections, the quality choice is irreversible, while the price choice is reversible.

**The Second Stage:the Price Choice** In the second stage, given the quality chosen in the first stage, the monopolist sets price. The FOC with respect to price gives the optimal combination of price and quality:

$$p^{UD} = \frac{s\bar{\Theta}}{2} \tag{29}$$

eq (29) implies that under the unit demand assumtipion, the firm chooses price and quality to make consumers with income $\underline{\theta} = \frac{p}{s} = \frac{\bar{\Theta}}{2}$ purchase its products. [28]

[28]Compared eq(29) with (6), the monopolist sell its products less consumers under the unit demand model than it does under the non-unit demand model.

In other expressions, eq (29) implies that

$$The\ market\ is \begin{cases} fully\ covered & \text{if } \bar{\Theta} \leq 2\underline{\Theta} \\ partially\ covered & \text{if } \bar{\Theta} > 2\underline{\Theta} \end{cases}$$

It becomes harder for the market to be fully covered under the unit demand model.

**The First Stage:the Irreversible Quality Choice** In the first stage, the monopolist chooses the quality of its product once and for all without knowing that free trade will start in the future.

**The Irreversible Quality Choice under the Fully Covered Market** If the income distribution is not so diverse ($\bar{\Theta} \leq 2\underline{\Theta}$),as shown above, the monopolist covers the whole market. In such a case, the lower limit becomes **exogenous** variable, the lowest income ($\underline{\theta} = \underline{\Theta}$).

Substituting $\underline{\theta} = \underline{\Theta}$ into the profit function (28), the FOC with respect to s gives the optimal quality.

$$s^{UD} = \frac{\beta}{\alpha} \tag{30}$$

Eq(30) shows that the optimal quality under the exogenous lower limit is the quality, which simply minimizes the cost of quality. This optimal quality does not depends on the income distribution at all. Hence, if $\bar{\Theta} \leq 2\underline{\Theta}$ in all countries, all country produce the same quality of goods.

**The Irreversible Quality Choice under the Partially Covered Market** If the income distribution is diverse enough ($\bar{\Theta} > 2\underline{\Theta}$),the monopolist choose the combination of price and quality

so as to make consumers with income $\frac{\bar{\Theta}}{2}$ or above buy its products. In this a case, given the quality, the monopolist treats the lower limit as **endogenous** variable, ($\underline{\theta} = \frac{p}{s}$).

Substituting $\underline{\theta} = \frac{p}{s}$ into eq (28), we get the FOC with respect to s.

$$s^{UD} = \frac{\left(\frac{\bar{\Theta}^2 L}{\bar{\Theta}-\underline{\Theta}}\right) + 4\beta}{4\alpha} \tag{31}$$

The optimal quality under the partially covered market shows the same characteristics as the optimal quality under the non-unit demand model: the quality increases with the highest income, the lowest income, and the population size but decreases with the cost of quality.[29]

**The Closed Economy Equilibrium under the Unit Demand Model** Substituting the optimal quality and price back to eq(27),(28), and (29), we get the closed economy equilibrium under the unit demand model.

$$s^{UD} = \begin{cases} \frac{\beta}{\alpha} & (30) \text{ if } \bar{\Theta} \leq 2\underline{\Theta} \\ \frac{n\bar{\Theta}^2+4\beta}{4\alpha} & (31) \text{ if } \bar{\Theta} > 2\underline{\Theta} \end{cases}$$

$$p^{au_{UD}} = \begin{cases} \frac{\beta\bar{\Theta}}{2\alpha} & \text{if } \bar{\Theta} \leq 2\underline{\Theta} \\ \frac{\bar{\Theta}\left(n\bar{\Theta}^2+4\beta\right)}{8\alpha} & \text{if } \bar{\Theta} > 2\underline{\Theta} \end{cases}$$

[29]The optimal quality under the partially covered market is higher than the optimal quality under the fully covered market. $s = \frac{\left(\frac{\bar{\Theta}^2 L}{\bar{\Theta}-\underline{\Theta}}\right)+4\beta}{4\alpha} > \frac{\beta}{\alpha}$

$$Q^{au_{UD}} = \begin{cases} L & \text{if } \bar{\Theta} \leq 2\underline{\Theta} \\ \frac{n\bar{\Theta}}{2} & \text{if } \bar{\Theta} > 2\underline{\Theta} \end{cases}$$

$$\pi^{au_{UD}} = \begin{cases} \frac{\beta\bar{\Theta}L}{2\alpha} & \text{if } \bar{\Theta} \leq 2\underline{\Theta} \\ \frac{n\bar{\Theta}^2\left(n\bar{\Theta}^2+8\beta\right)}{32\alpha} & \text{if } \bar{\Theta} > 2\underline{\Theta} \end{cases}$$

note: $n \equiv \frac{L}{\bar{\Theta}-\underline{\Theta}}$ (the number of consumers at each point of income)

## The Open Economy under the UD model

### The Post-trade Bertrand Competition

**Case 1: Both markets were fully covered under autarky** ($\bar{\Theta} \leq 2\underline{\Theta}$ and $\bar{\Theta} \leq 2\underline{\Theta}$)

If the income distribution is not diverse in both countries ($\bar{\Theta} \leq 2\underline{\Theta}$ and $\bar{\Theta} \leq 2\underline{\Theta}$), and markets in both countries were fully covered under autarky, as shown in the previous section, the exactly same quality of goods are produced in two countries. ($s = s^* = \frac{\beta}{\alpha}$)[30] If this is a case, the post trade price competition between two firms is just the Bertrand competition of homogeneous goods: the price will be reduced to the marginal cost.

**Case 2: Both markets were partially covered under autarky** ($\bar{\Theta} > 2\underline{\Theta}$ and $\bar{\Theta} > 2\underline{\Theta}$) If the income distribution is diverse enough in both countries,as shown in the previous section, the quality

[30] As noted before,two countries choose the same quality whose cost is the lowest.

chosen under autarky depends on the income distribution: the quality of goods produced in the developed country is higher than that produced in the developing country.[31]

$$s = \frac{n\bar{\Theta}^2+4\beta}{4\alpha} \qquad s^* = \frac{n^*\bar{\Theta^*}^2+4\beta}{4\alpha} \qquad \Rightarrow s > s^* \;\; if \;\; \bar{\Theta} > \bar{\Theta^*}$$

After the opening of trade, consumers have three choices: they buy one unit of the high quality goods, buy one unit of the low quality good, or do not buy anything. The marginal consumers, who are indifferent between the low quality good and the high quality one achieve the same utility if they buy either of two goods by one unit:

$$s\tilde{\theta} - p = s^*\tilde{\theta} - p^*$$

The income of such marginal consumers is

$$\tilde{\theta} = \frac{p - p^*}{s - s^*}$$

Given the above marginal consumers, aggregate demand of the high quality goods and that of the low quality ones are derived as follows.[32]

$$Q^{UD} = \int_{\tilde{\theta}}^{\bar{\Theta}} n d\theta \qquad Q^{*UD} = \int_{\underline{\theta^*}}^{\tilde{\theta}} n d\theta$$

$$\text{where } \underline{\theta^*} = \max\left\{\underline{\Theta^*}, \tfrac{p^*}{s^*}\right\}$$

[31] $n = n^*$ by assumption

[32] The definition of marginal consumer's income under the UD model happens to be the same as the definition under the Non-UD model.

Facing the above aggregate demand, The low-quality goods' firm maximizes the following profit function.

$$\pi^{*UD} = \int_{\underline{\theta^*}}^{\tilde{\theta}} p\ n\ d\theta - F(s^*) \tag{32}$$

The FOC with respect to $p^*$ gives the reaction function of The low-quality goods' firm.[33]

$$p^* = \frac{s^* p}{2s} \tag{33}$$

Similarly,the high-quality goods' firm maximizes the following profit function.

$$\pi^{UD} = \int_{\tilde{\theta}}^{\bar{\Theta}} p\ n\ d\theta - F(s) \tag{34}$$

The reaction function ofthe high-quality goods' firm is given by the FOC with respect to p:

$$p = \frac{p^*}{2} + \frac{(s - s^*)\bar{\Theta}}{2} \tag{35}$$

THe intersection of two reaction functions (35), (33) gives the equilibrium price after trade:

$$p^{UD} = \frac{2s(s - s^*)\bar{\Theta}}{(4s - s^*)} \tag{36}$$

$$p^{*UD} = \frac{s^*(s - s^*)\bar{\Theta}}{(4s - s^*)} \tag{37}$$

Substituting the equilibrium prices (36), (37) back into $\tilde{\theta}$,eq (34) and (32), we have the open economy equilibrium under the UD model:

[33]Again, The low-quality goods' firm maximizes its profits by treating the lower limit $\underline{\theta^*} = \frac{p^*}{s^*}$ endogenous variable.

The income of marginal consumer is given by

$$\tilde{\theta}^{UD} = \frac{(2s - s^*)\bar{\Theta}}{(4s - s^*)} \tag{38}$$

Post-trade profits of two firms are given by

$$\pi^{UD} = \frac{4s^2(s - s^*)n\bar{\Theta}^2}{(4s - s^*)^2} - \frac{n^2\bar{\Theta}^4}{32\alpha} \tag{39}$$

If the quality difference between the high quality and the low quality is large enough, $[\ \frac{p^*}{s^*} = \frac{(s-s^*)\bar{\Theta}}{(4s-s^*)} \leq \underline{\Theta}^*$ $\Leftrightarrow\ (4s-s^*)\underline{\Theta}^* - (s-s^*)\bar{\Theta} \geq 0]$ the market in the developing country will be fully covered after trade, and profits of The low-quality goods' firm will be

$$\pi^{*UD} = \frac{ns^*(s - s^*)\bar{\Theta}\left\{s\bar{\Theta} - \{(4s - s^*)\underline{\Theta}^* - (s - s^*)\bar{\Theta}\}\right\}}{(4s - s^*)^2} - \frac{n^2\bar{\Theta^*}^4}{32\alpha} \quad if\ (4s-s^*)\underline{\Theta}^* - (s-s^*)\bar{\Theta} \geq 0 \tag{40}$$

If the difference between the high quality and the low quality is not large enough,$[\ \frac{p^*}{s^*} = \frac{(s-s^*)\bar{\Theta}}{(4s-s^*)} > \underline{\Theta}^*$ $\Leftrightarrow (4s-s^*)\underline{\Theta}^* - (s-s^*)\bar{\Theta} < 0]$,the market in the developing country will continue to be partially covered after trade, and profits of The low-quality goods' firm will be

$$\pi^{*UD} = \frac{nss^*(s - s^*)\bar{\Theta}^2}{(4s - s^*)^2} - \frac{n^2\bar{\Theta^*}^4}{32\alpha} \quad if\ (4s-s^*)\underline{\Theta}^* - (s-s^*)\bar{\Theta} < 0 \tag{41}$$

# Appendix B

## The Derivation of eq (17)

Marginal consumers achieve the same level of CS whether they buy the high quality goods ($s$) or the low quality ones ($s^*$):

$$\frac{1}{2}\left(s\tilde{\theta}-p\right)^2=\frac{1}{2}\left(s^*\tilde{\theta}-p^*\right)^2$$

Expanding the above equation, we get

$$\left(s^2-s^{*2}\right)\tilde{\theta}^2-2(sp-s^*p^*)\tilde{\theta}+\left(p^2-p^{*2}\right)=0$$

Solving this quadratic equation, we have two solutions:

$$\tilde{\theta}=\frac{p+p^*}{s+s^*},\quad\frac{p-p^*}{s-s^*}$$

Under the first solution, The low-quality goods' firm faces the upward demand [$\frac{\partial\tilde{\theta}}{\partial p^*}>0$ (larger the market share)], while both firms face the downward demand under the second solutuion. [$\frac{\partial\tilde{\theta}}{\partial p^*}<0$ and $\frac{\partial\tilde{\theta}}{\partial p}>0$ (The increase in price results in the loww of the market share)] Thus, the only economically reasonable solution is the second solution [eq (17)].

$$\tilde{\theta}=\frac{p-p^*}{s-s^*}\qquad(17)$$

**The Derivation of the reaction function of the low-quality goods' firm-eq(20)-**

After trade, The low-quality goods' firm maximizes the following reaction function by treating the lower limit as an endogenous variable $\underline{\theta}^*=\frac{p^*}{s^*}$:

$$\pi^* = \int_{\underline{\theta}^*}^{\tilde{\theta}} p^*(s^*\theta - p^*)\ nd\theta - F(s^*) \quad (19)$$

$$\text{where } \tilde{\theta} = \frac{p-p^*}{s-s^*} \text{ the upper limit}$$

The FOC with respect to $p^*$ yields the following quadratic equation:

$$3s^2p^{*2} - 4ss^*p^* + s^{*2}p^2 = 0$$

This quadratic equation gives two roots:

$$p^* = \frac{s^*p}{3s}, \quad \frac{s^*p}{s}$$

However, the Second Order Condition(SOC) requires that

$$\frac{ns}{s^*(s-s^*)^2}\,(3sp^* - 2s^*p\ ) < 0$$

$$\Leftrightarrow\ 3sp^* - 2s^*p < 0$$

$$\Leftrightarrow\ p^* < \frac{2s^*p}{3s} \quad \text{[SOC]}$$

The only solution that satisfies the above SOC is the first root.

$$p^* = \frac{s^*p}{3s} \quad (20)$$

## The Derivaration of the Open Economy Equilibrium

Substitute the reaction function of The low-quality goods' firm $p^* = \frac{s^* p}{3s}$ [eq(20)] into the reaction function ofthe high-quality goods' firm eq (23), we get the following quadratic equation.

$$(s-2s^*)\left\{27(s-s^*)^2-16s^{*2}\right\}p^2-36s\bar{\Theta}(s-2s^*)(s-s^*)^2p+9s^2(s-2s^*)\bar{\Theta}^2(s-s^*)^2=0$$

If $s \neq 2s^*$, the above quadratic equation becomes as follows.

$$\left\{27(s-s^*)^2-16s^{*2}\right\}p^2-36s\bar{\Theta}(s-s^*)^2p+9s^2\bar{\Theta}^2(s-s^*)^2=0$$

Solving this quadratic equation, we have two solutions.

$$p=\frac{3s(s-s^*)\bar{\Theta}\left\{6s(s-s*)^2\pm\sqrt{9(s-s^*)^2+16s^{*2}}\right\}}{27(s-s^*)^2-16s^{*2}}$$

Numerical examples show that the positive root solution ($p=\frac{3s(s-s^*)\bar{\Theta}\left\{6s(s-s*)^2+\sqrt{9(s-s^*)^2+16s^{*2}}\right\}}{27(s-s^*)^2-16s^{*2}}$) is so high thatthe high-quality goods' firm face no demand, and the other negative root solution always bring higher profits to the firm. Therefore, the solution, which maximizes profits ofthe high-quality goods' firm is the negative root solution:

$$p=\frac{3s(s-s^*)\bar{\Theta}\left\{6s(s-s*)^2-\sqrt{9(s-s^*)^2+16s^{*2}}\right\}}{27(s-s^*)^2-16s^{*2}} \tag{24}$$

The Nash equilibrium price of the low quality products is

$$p^* = \frac{s^* p}{3s} = \frac{s^* \bar{\Theta} \left\{ 6s(s - s*)^2 - \sqrt{9(s - s^*)^2 + 16 s^{*2}} \right\}}{27(s - s^*)^2 - 16 s^{*2}} \tag{25}$$

The income of marginal consumers is

$$\tilde{\theta} = \frac{p - p^*}{s - s^*} = \frac{(3s - s^*) \bar{\Theta} \left\{ 6s(s - s*)^2 - \sqrt{9(s - s^*)^2 + 16 s^{*2}} \right\}}{27(s - s^*)^2 - 16 s^{*2}} \tag{26}$$

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