A model of inequality aversion and private provision of public goods

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Abstract

This study develops a model of inequality aversion and public goods that allows the case of the marginal rate of substitution to be variable. The utility function of the standard public goods model is nested in the Fehr-Schmidt model. An individual's contribution function for a public good is derived by solving the problem of a utility function that has a kink and examining both interior and corner solutions. The derived contribution function is not monotonic with respect to the other's provision. As a result, the model can be used to explain the empirical evidence of the effect of social comparison on public-good provision.

Keywords: Inequality Aversion, Kinky Preferences, Private Provision of Public Goods, Social Comparison, Variable Marginal Rate of Substitution

JEL Codes: D63, D91, H41

1. Introduction

The positive effects of social comparisons on charitable giving have been shown by several field experimental studies (e.g., Frey and Meier, 2004; Shang and Croson, 2009). Similarly, their positive effects on energy and water conservation have also been demonstrated by various studies (e.g., Allcott, 2011; Ferraro and Price, 2013; Allcott and Kessler, 2018). However, Croson and Shang (2013) find that when social comparison is too extreme, it ceases to influence charitable giving. This result implies a natural limitation of the effect of

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social comparisons, which cannot be explained by existing models (e.g., Ferraro and Price, 2013; Brandon et al., 2017; Allcott and Kessler 2018).

This study develops a model of the private provision of public goods that can fully explain the evidence on social comparisons obtained from field experiments. The theoretical foundation of the model is provided by nesting the utility function of the standard public goods model (e.g., Warr, 1983; Bergstrom et al., 1986) into the model of inequality aversion developed by Fehr and Schmidt (1999). A maximization problem that has a utility function with a kink is presented. To obtain the intuition of the model, a Cobb-Douglas function is adopted, and the problem is solved to derive an optimal response. The derived optimal contribution exhibits an inverted N-shape relationship, suggesting that an individual increases her contribution in response to others' increase over a specific range, but decreases it in other ranges.

This study contributes to the theoretical literature on both inequality aversion (e.g., Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000) and social comparisons (e.g., Ferraro and Price, 2013; Brandon et al., 2017; Allcott and Kessler 2018). Several laboratory experiments have examined the association between inequality aversion and voluntary provision of public goods (Buckley and Croson, 2006; Blanco et al., 2011; Teyssier, 2012). These studies used a linear public goods game by assuming the marginal rate of substitution (MRS) between the private and public good constant. The original Fehr-Schmidt model sufficiently explains decision making in a standard linear public goods game; however, the assumption these previous studies make (namely, constant MRS) may be too strong for the demand of public goods outside the laboratory.[1] Thus, this study extends the model of Fehr and Schmidt

[1]Derin-Güre and Uler's (2010) study is an exception as it takes a novel approach to model inequality aversion and public goods. In addition to standard private and public good terms, their utility function includes a term of the concave function of inequality in private good consumption. Their utility function exhibits a variable MRS, which is assumed to be differentiable. Consequently, their study focuses on an interior solution. By contrast, the current study proposes a utility function that is non-differentiable by nature and examines both interior and corner solutions.

(1999) to incorporate the case wherein the MRS is not constant.[2] As a result, the model can explain the non-monotonic effect of others' contribution on one's own contribution, which cannot be explained by the existing models on social comparison.

2. Model

Consider a model in which there is one private good, one public good, and two individuals, A and B. Each individual i consumes an amount x_i of the private good and donates an amount g_i to the supply of the public good. Let $G = g_A + g_B$ be the total private contributions to the public good. Both individuals i are endowed with wealth $w > 0$, which they allocate between private good x_i and contribution g_i. Let $\pi_i = \pi(x_i, G)$ be individual i's sub-utility, which corresponds to monetary payoffs in the Fehr-Schmidt model. Assume that $\frac{\partial \pi}{\partial x} > 0$, $\frac{\partial^2 \pi}{\partial x^2} < 0$, $\frac{\partial \pi}{\partial G} > 0$, and $\frac{\partial^2 \pi}{\partial G^2} < 0$.

Following Fehr and Schmidt (1999), consider individual B's preferences as follows:

$$U_B = \pi_B - \alpha \max\{\pi_A - \pi_B, 0\} - \beta \max\{\pi_B - \pi_A, 0\}.$$

For guilt parameter β, Fehr and Schmidt (1999) assume $0 \leq \beta < 1$. This study also adopts this assumption. For the envy parameter α, assume that $\alpha \geq 0$.[3] Previous studies have examined the case when $\frac{\partial \pi}{\partial G} > 0$, and $\frac{\partial^2 \pi}{\partial G^2} = 0$ (e.g., Buckley and Croson, 2006; Blanco et al., 2011). In this study, by contrast, the Cobb-Douglas function of $\pi(x_i, G) = \gamma \log x_i + (1-\gamma) \log G$, where $\gamma \in \left(\frac{1}{2}, 1\right)$ is examined for the case when MRS is not constant.

[2] Engelmann (2012) demonstrates that extending the Fehr-Schmidt model by adding a term for efficiency concerns is misguided, since it is equivalent to a much simpler change. This argument also applies when adding a term for public goods if MRS is constant. Instead of an attempt such as the one that Engelmann (2012) criticizes, the current study adds a term that is a concave function of the total amount of the public good provision.

[3] Fehr and Schmidt (1999) further assume $\beta \leq \alpha$.

Then, individual B's contribution g_B can be found by solving

$$\max_{x_B, g_B} U_B = \begin{cases} -\alpha\left[\gamma \log\ x_A + (1-\gamma)\log G\right] + (1+\alpha)\left[\gamma \log x_B + (1-\gamma)\log G\right] \\ \text{if } \pi_B \leq \pi_A, \\ \beta\left[\gamma \log\ x_A + (1-\gamma)\log G\right] + (1-\beta)\left[\gamma \log x_B + (1-\gamma)\log G\right] \\ \text{if } \pi_B > \pi_A. \end{cases}$$

$$\text{s.t. } x_A + g_A = w,\ x_B + g_B = w,\ g_A + g_B = G.$$

Following Bergstrom et al. (1986), it is assumed that individual B takes the contribution of A as exogenously given (the Nash assumption). By substituting $g_B = G - g_A$ into the above and the budget constraints into the utility function, the optimization problem is equivalent to

$$\max_{G} U_B = \begin{cases} -\alpha\gamma \log\ (w - g_A) + (1+\alpha)\gamma \log(w - G + g_A) + (1-\gamma)\log G \\ \text{and } 2g_A \leq G, \\ \beta\gamma \log\ (w - g_A) + (1-\beta)\gamma \log(w - G + g_A) + (1-\gamma)\log G \\ \text{and } 2g_A > G. \end{cases}$$

This utility function is not differentiable if $\alpha \neq 0$ or $\beta \neq 0$. Therefore, to solve this maximization problem, it is split into two by adding the conditions $G - 2g_A \geq 0$ and $2g_A - G > 0$ as constraints. This makes U_B differentiable within each sub-problem. Then, each sub-problem can be solved by applying the Kuhn-Tucker conditions, and both interior and corner solutions to the original problem can be obtained by comparing the utility levels of the solutions to the two sub-problems. Using this procedure, individual B's optimal response

g_B^* is shown and studied in the next section.

Note that

$$MRS = \frac{\partial U_B/\partial x_B}{\partial U_B/\partial G} = \frac{(1-k)\,\gamma}{1-\gamma}.\frac{G}{w+g_A-G},$$

where $k = -\alpha, \beta$, which means the MRS between the private and public goods is not constant. Further, the MRS when approximating from the left and right to $G = 2g_A$ differs if $-\alpha \neq \beta$.

3. Comparative statics of the optimal response

Consider a case wherein $\alpha > 0$ and $0 < \beta < 1$. Moreover, the study examines the case wherein $g_A \in (0, w)$. To derive optimal response $g_B^*(g_A)$, the study considers the problem with constraint $2g_A \leq G$. This is the case wherein individual B's sub-utility is relatively low or equal to that of A. By solving this sub-problem, the solution can be written as

$$g_B = \begin{cases} \frac{1-\gamma}{1+\alpha\gamma}w - \frac{(1+\alpha)\gamma}{1+\alpha\gamma}g_A & \text{for } 0 < g_A < g_A^\alpha, \\ g_A & \text{for } g_A^\alpha \leq g_A < w, \end{cases}$$

where $g_A^\alpha = \frac{1-\gamma}{1+\gamma+2\alpha\gamma}w$.

Next, the study considers the problem with the constraint of $g_A \leq G \leq 2g_A$, which is the case wherein individual B's sub-utility is relatively high. This study examines the case wherein $0 < \beta < 1 - \frac{1-\gamma}{\gamma}$.[4] By solving this sub-problem, the solution can be written as

[4]See the Online Appendix for the case wherein $1 - \frac{1-\gamma}{\gamma} \leq \beta < 1$. Note that if $1 - \frac{1-\gamma}{\gamma} \leq \beta$, then $w \leq \overline{g_A^\beta}$.

$$g_B = \begin{cases} g_A & \text{for } 0 < g_A \leq \underline{g_A^\beta}, \\ \frac{1-\gamma}{1-\beta\gamma} w - \frac{(1-\beta)\gamma}{1-\beta\gamma} g_A & \text{for } \underline{g_A^\beta} < g_A < \overline{g_A^\beta}, \\ 0 & \text{for } \overline{g_A^\beta} \leq g_A < w, \end{cases}$$

where $\underline{g_A^\beta} = \frac{1-\gamma}{1+\gamma-2\beta\gamma} w$ and $\overline{g_A^\beta} = \frac{1-\gamma}{(1-\beta)\gamma} w$.

If $g_B = g_A$, then the level of U_B is independent of α and β. Based on this observation, solutions of the two sub-problems are compared to derive the optimal response of the inequality aversion ($\alpha > 0$ and $\beta > 0$):

$$g_B^* = \begin{cases} \frac{1-\gamma}{1+\alpha\gamma} w - \frac{(1+\alpha)\gamma}{1+\alpha\gamma} g_A & \text{for } 0 < g_A < g_A^\alpha, \\ g_A & \text{for } g_A^\alpha \leq g_A \leq \underline{g_A^\beta}, \\ \frac{1-\gamma}{1-\beta\gamma} w - \frac{(1-\beta)\gamma}{1-\beta\gamma} g_A & \text{for } \underline{g_A^\beta} < g_A < \overline{g_A^\beta}, \\ 0 & \text{for } \overline{g_A^\beta} \leq g_A < w. \end{cases}$$

This contribution function is non-monotonic in that

$$\frac{\partial g_B^*}{\partial g_A} \begin{cases} < 0 & \text{for } 0 < g_A < g_A^\alpha, \\ > 0 & \text{for } g_A^\alpha \leq g_A \leq \underline{g_A^\beta}, \\ < 0 & \text{for } \underline{g_A^\beta} < g_A < \overline{g_A^\beta}, \\ = 0 & \text{for } \overline{g_A^\beta} \leq g_A < w. \end{cases}$$

Figure 1 illustrates the contribution function when $\alpha > 0$ and $0 < \beta < 1 - \frac{1-\gamma}{\gamma}$. If individual B believes that the other's contribution is very low ($0 < g_A < g_A^\alpha$), B decreases her contribution with the other's increase. If the contribution is in the range of $g_A^\alpha \leq g_A \leq \underline{g_A^\beta}$, individual B increases her contribution along with an increase in A's contribution to stay at the kink point of the utility function. However, if individual B believes the other's contribution is high ($\underline{g_A^\beta} < g_A < \overline{g_A^\beta}$), B decreases her contribution again with the other's increase. Finally, if individual B believes that the other's contribution is in the range of

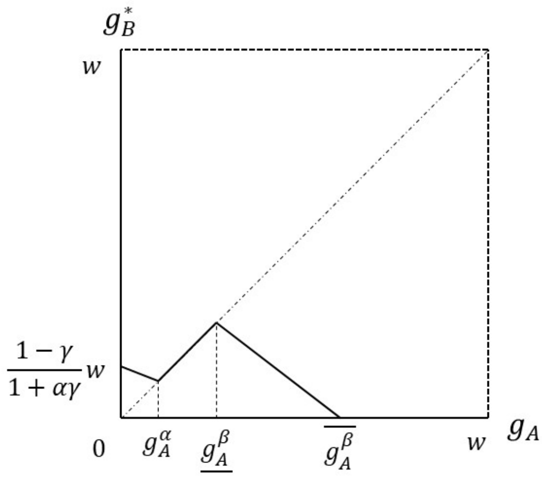

Figure 1: The optimal response ($\alpha > 0$ and $0 < \beta < 1 - \frac{1-\gamma}{\gamma}$)

$\overline{g_A^\beta} \leq g_A < w$, B does not contribute at all and becomes a free rider. As a result, there exist multiple Nash equilibria in this model: $g_A^* = g_B^* \in \left[g_A^\alpha, \underline{g_A^\beta}\right]$.

Note that when $\alpha = \beta = 0$, $U_B = \pi_B$ and the optimal response is

$$g_B^* = \begin{cases} (1-\gamma)\,w - \gamma g_A & \text{for } 0 < g_A < \frac{1-\gamma}{\gamma}w, \\ 0 & \text{for } \frac{1-\gamma}{\gamma}w \leq g_A < w. \end{cases}$$

This implies that $\frac{\partial g_B^*}{\partial g_A} \leq 0$ for $0 < g_A < w$.

4. Discussion and conclusions

The previous literature on the voluntary provision of public goods presented a general model that includes variable MRS (e.g., Warr, 1983; Bergstrom et al., 1986); most existing studies on inequality aversion and public goods have examined cases with constant MRS. To fill this gap in the literature, this study proposes a model of voluntary provision of public goods with variable MRS and *kinky* inequality aversion, as developed by Fehr and Schmidt (1999). An interesting insight from analyzing the comparative statics is that an individual increases her contribution with the same amount as the other individual over a specific range (conditional cooperation), while the individual decreases it in other ranges. This is consistent with the findings of Shang and Croson (2009) and Croson and Shang (2013).

There are several directions for future research. For example, empirical testing of the model by using laboratory experiments is required. Several studies have adopted sequential public good games to test models of social preferences (e.g., Teyssier, 2012). In addition, Uler (2011) provides an experimental design to study public goods provision when the MRS is not constant. Combining these approaches may enable testing of the model.

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