Separating and Combining Fiscal Functions: The Musgravian Proposition Revisited

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The Prelude

Richard Musgrave’s proposition of separating fiscal functions into three independent branches, i.e. distribution, allocation and stabilization, has been widely accepted by public finance theorists and become almost the standard approach of text books world wide since he proposed it nearly a half century ago (Musgrave 1957; 1959). This paper attempts to show the limited applicability of his proposition, for example, if applied in the situation where the marginal cost of the public expenditure is increasing, separate pursuits of distribution and allocation functions limit the scope of improving the community’s welfare under the unanimity rule. Samuelson once criticized the three-branch approach assuming an omniscient referee (Samuelson, 1954). However, this paper views the problem from a different perspective in a democratic society because in this society, fiscal decisions must rest their justifications on the outcome of negotiation among the citizens (or their representatives) and, to have the minority protected, must seek the unanimous decisions as close as possible as Wicksell insisted (Wicksell, 1896).

This paper incorporates in the public good analysis the fact that the governmental production of a public good redistributes, via impacts on factor prices, the aggregate earned income of the community. The voters recognizing
such income redistributions entering from the back door, so to speak, fail the government to win the unanimous support for provision of a public good that yields to every citizen a utility larger than his tax in spite of the contrary assertions made by Wicksell, Musgrave and others. However, combining, instead of separating, the allocation and income redistribution programs it is possible for a government to formulate a proposal supplying the same public good in a way that let it win the unanimous support.

The paper develops and presents the argument in the following seven sections. Firstly the Wicksell-Musgrave proposition and secondly their critical conjecture are highlighted. In third section, earned income dependency on public goods production is discussed, which is followed by an interpretation of the voluntary exchange theory of a public finance. In section five, a geometric proof is given and in section six an attempt is made to combine distribution and allocation functions. Finally, the whole discussion and argument is summed up.

1. The Wicksell-Musgrave Proposition

Musgrave followed Kunt Wicksell’s then novel insistence that the amount of tax that a citizen bears should not be larger than the benefits that he receives from the expenditure of the government. In order to guarantee this value and counter value system, the amount of the benefits citizens receive must be ascertained. But they are subjective values. In Wicksell’s words, “each person can ultimately speak only for himself” (Musgrave and Peacock, 1958, p.90). To have their subjective evaluations revealed, the government must rely upon the citizens’ (or their representatives’) voting, and to have the minority protected, the unanimity must be sought. Conceiving the practicability of what he called “the principle of unanimity and voluntary consent in taxation,” Wicksell reasoned that if the sum of utilities that all the members of a community receive from a given quantity of a public good is larger than its cost, it should be possible to find ways of distributing the cost so as to make each citizen’s share smaller than the utility that he receives. He conjectured that if a tax-expenditure proposal satisfying this condition is voted upon, it will be approved unanimously. He stated that:

“Provided the expenditure in question holds out any prospect at all of creating utility exceeding cost, it will always be theoretically possible and approximately so in practice, to find a distribution of costs such that all parties regard the expenditure as beneficial and therefore approve it unanimously”
Wicksell called the method of ascertaining by the unanimity a tax-expenditure program that is Pareto-preferred to the initial situation "just taxation" because no one is forced to pay more than the benefits he receives but he added that "[it] is clear that justice in taxation tacitly presupposes justice in the existing distribution of property and income" (Musgrave and Peacock, 1958, p.108). This remark led Musgrave to develop his three-branch model. Musgrave argued that while allocation of resources for provision of a public good, being a positive sum game, is amenable to the unanimity rule, income distribution, being a zero-sum game, is not. Therefore, the income redistribution function must be separated from the resource allocation function and be carried out by a (qualified) majority decision rule, while the allocation function should be carried out under the unanimity rule. He thus proposed creation of an independent fiscal unit, the distribution branch, assigned solely the role of redistributing the community’s pre-tax aggregate income so that the second independent unit, the allocation branch, can perform the function of producing public goods on the basis of the "just" income distribution thus established. The third independent branch, the stabilization branch, is in charge of macro-economic policies.

2. The Critical Conjecture

In the Wicksellian model the voting procedure is the key to make his "just taxation" operative in practice. The government and political parties table for vote alternative fiscal packages each of which consists of a quantity of a public good in question and allocations of its cost among the citizens until a package receives the unanimous votes or the largest number of (qualified) votes. Wicksell conjectured that if one of the tax-expenditure programs that promises an outcome Pareto-preferred to the initial situation was tabled for vote, a unanimous approval would be guaranteed. (Here I take it granted that he as well as Musgrave later assumed, as I do hereafter, the absence of strategic behaviors in the voting process.)

This conjecture plays the critical role in making what Wicksell considered his major contribution a viable proposition, that is, making his idea of "unanimity and fully voluntary consent in making of decisions" not a mere theoretical construct but a practically operative procedure.

Musgrave accepted this Wicksellian conjecture and extended it further. He
envisioned that once the initial income distribution is adjusted to be “just”, with the Lindahl pricing of public goods (Lindahl, 1919) along with the perfectly competitive private goods markets, the self-interested actions of individuals will produce the normatively correct outcome (Musgrave, 2000, p. 88).²

3. Earned Income Dependency on Public Goods Production

This paper argues that this Wicksell-Musgrave conjecture does not always hold good unless their implicit assumption holds. Their implicit assumption is that the voters’ earned incomes are independent of the size of the public good produced. But the assumption unlikely prevails in practice.

Wicksell assumed, as his statement above indicates, and Musgrave apparently agreed, that an individual voter compares the utility that he receives from the public good in question with his tax, that is his share of the cost of the public good, and attempts to maximize the difference between the two, namely the net utility, such that

\[(1) \quad U_i = U_i(x) - T_i \quad (i = 1, 2, ..., n)\]

and the community’s resource constraint is such that

\[(2) \quad \Sigma T_i = H(x)\]

where, x and T_i are the quantity of the public good and the amount of the tax individual i pays, respectively.

However, this paper argues that what an individual voter attempts to maximize is his utility function consisting of the public good and his disposable income such that

\[(3) \quad U_i = U_i(x, Y_i - T_i)\]

and the community’s resource constraint is such that

\[(4) \quad F(x, \Sigma(Y_i - T_i))=0\]

where, Y_i is individual i’s initial earned income.

Comparing expressions (1) and (3), one can observe that only when the earned income (before tax), Y_i, is constant, the two expressions yield an identical result with respect to a marginal change in x. But an individual’s earned income is generally a function of the public good produced. This is so because first, the governmental purchase of factors of production for production of a public good alters factor prices when factor intensities are different between the public and the private goods. If the public good production is more capital and less labor
intensive as compared with the private good, production of a new public good or expansion of the exiting one increases the relative market price of capital and reduces that of labor, and thereby increases earned income of individuals who own relatively more capital and less labor and reduces that of individuals who own relatively less capital and more labor. Only when factor intensities are the same between the public and private goods and/or all factors are owned in the same proportion by every individual, the public good production has neutral effect on the individual’s earned income.

Second, when the public good in question is a substitute (complement) of a private good, new or expansion of the public good production reduces (increases) the price of that private good and thereby reduces (increases) incomes of individuals engaging in production of that private good, only when the public good supply has no effects on the prices of private goods the public good production has neutral effects on individuals’ earned income. When $Y_i$ is function of $x$ the utility function expressed by (3) implies that in evaluating the consequence of his vote on a tax-expenditure proposal, each individual will take into account not only the net benefits as the difference between the utility arising from the consumption of the public good and the tax he pays but also the change in his earned income that the public good production will cause to happen. It is a widely recognized fact that the rent-seeking motive tends to dominate or at least influence behaviors of individual voters. For example, an employee in a defense industry will be more interested in the impact on his earned income than on his utility of “defense” that a marginal change in the defense expenditures generates. Another example may be such that captains and crews of ferries usually oppose to construction of a bridge crossing over the channel across which their ferries are providing transport services even the benefits they personally receive from the bridge are larger than their shares of the cost of constructing the bridge. Consequently, an individual whose net utility gain, namely, the difference between the utility he gains from the public good and the tax he pays, falls short of the amount of reduction in his earned income due to falling market prices of the factors he owns will not approve production of the public good even though the aggregate utility gains arising in the community as a whole is greater than the cost of the public good. In other words, in spite of the Wicksellian conjecture, the mere assurance that a tax expenditure program proposed promises every citizen benefits greater than his tax does not guarantee
its unanimous approval. Indeed, if the number of individuals in the group loosing their incomes is large, which is not unlikely, even the simple majority will not approve a proposal that promises the community as a whole the aggregate utility even if significantly exceeding the cost.

4. An Interpretation of the Voluntary Exchange Theory of a Public Finance

In the successive theorizations of the so called “voluntary exchange theories of public finance,” Wicksell (1896), Lindhal (1919), Musgrave (1939) and Bowen (1943) commonly assumed implicitly or explicitly the constant marginal cost of the public good in question. It is the critical assumption. Indeed, without it their theories, which intended to affirm achievement of a Pareto-optimum solution through voluntary consent and unanimity, largely collapse. This is shown below assuming a community consists of two groups, named groups A and B, each of which has a consistent group utility function satisfying the usual convexity conditions.

Let each tax-expenditure program be represented by a vector such that

\[ \mathbf{v}(x, y_a, y_b) \]

where \( x \) is quantity of the public good and \( y_i (=Y_i - T_i) \) is group \( i \)'s disposable income \((i= a, b)\).

There exist vectors which are elements of the set defined below:

**Set N** (The feasible set): All feasible vectors are elements of Set N such that

\[ N = \{v(x, y_a, y_b): f(x)+ y_a+ y_b- Y_a- Y_b=0 \} \]

Where \( f(x) \) is \( x \)'s cost function.

Geometrically, Set N is represented by Triangle \( O_aO_bT \) in Figure 1. The Figure is constructed in the following manner: the horizontal and vertical axes measure the quantities of the public good \( (x) \) and the numeraier private good \( (Y) \), respectively. The aggregate income of the community prior production of \( X \) is \( O_aO_b \), divided by point D between \( O_aD \) and \( O_bD \) representing initial incomes of group A \( (Y_a) \) and B \( (Y_b) \), respectively. Line \( O_bT \) is the production possibility frontier, the slope of which is the marginal rate of transformation of \( Y \) for \( X \), MRT. A point in the triangle represents a feasible vector, whose components \( x, y_a \) and \( y_b \) are measured by, respectively, the distance from the point to the vertical axis, that to the horizontal axis and the vertical distance to Line \( O_bT \). For example, the quantities of components of the vector that point H represents, \( x, y_a \) and \( y_b \) are
represented by the lengths of HL, HE and HF, respectively.

**Set S** (The cost-sharing set): The set of vectors attainable by sharing the cost of X between A and B is Set S which is the intersection of two sets, $S_a$ and $S_b$:

$$S = S_a \cap S_b$$

where $S_i \subset \{ \mathbf{v} \in \mathbb{N} : y_i > Y_i - f(x) \} \quad (i = a, b)$

Triangle DSU in Figure 2, which duplicates Figure 1, represents Set $S$. The lower boundary of $S$ is Line DS, which is the production possibility frontier of A when A bears the entire cost of X, that is

$$Y = Y_a - f(x)$$

The upper boundary of $S$ is the horizontal line containing point D, which is the production possibility frontier of B when B bears the entire cost of X, that is

$$Y = [Y_a + Y_b - f(x)] - [Y_b - f(x)] = Y_a$$

Any point in $S$ indicates how the cost of X is distributed between A and B. Point H, for example, indicates that A bears HK and B, HG of the total cost, GK, necessary to produce the quantity of x equal to OₐE.

**Set P** (The Pareto-preferred set): The set of vectors that are Pareto-preferred to the initial vector is P, which is the intersection of two sets, $P_a$ and $P_b$:

$$P = P_a \cap P_b$$

where $P_i \subset \{ \mathbf{v} \in \mathbb{N} : U(x, y_i) > U(0, Y_i) \} \quad (i = a, b)$; and $U_i(x, y_i)$ is the utility function of group i.

Geometrically, Set P is represented in Figure 2 by a lens-shaped area demarcated by A’s and B’s indifference curves both of which contain point D.¹

The lower boundary of P is A’s indifference curve ($\alpha_1$), representing A’s utility level equal to $U_a = U_a(0, Y_a)$. Its slope is

$$\frac{dY}{dx} = -\frac{MU_a}{MU_y} (= -MRS_a)$$

The upper boundary is B’s indifference curve representing B’s utility level equal to $U_b = U_b(0, Y_b)$. Since the distance to a point on the curve from the horizontal axis is $[Y_a + Y_b - f(x)] - y_b(x, U_b^0)$, where $U_b^0 = U_b(0, Y_b)$. The slope of the boundary curve is

$$\frac{dY}{dx} = -\frac{df(x)}{dx} + \frac{MU_y}{MU_b} (= MRS_b - MRT)$$

All authors mentioned above conjectured that if the sum of marginal utilities of all voters exceeds the marginal cost of the public good, there exists a
set of tax sharing methods that will be approved unanimously. In our term their conjecture amounts to say that there are vectors that are simultaneously elements of the Pareto-preferred set \((P)\) and cost-sharing set \((S)\), or that the intersection of the two sets is not empty:

\[
W = S \cap P \neq \emptyset
\]

Their conjecture is correct as long as \(Y_i (i=a, b)\) are independent of \(x\). But we argue that if it is a function of \(x\), the set of intersection could be empty. In that event no vector exists to which the voters agree unanimously even though the set of vectors Pareto-preferred to the initial vector does exist. In other words the existence of Pareto-preferred set does not assure the validity of the Wicksellian conclusion on the voting outcome. This will be proven below.

5. Geometrical Proof

Arguing for the feasibility of the voluntary consent and the unanimity in supplying a public good Wicksell made two explicit and one implicit assumptions. They are

(1) each party is unable to obtain net utility by supplying the public good alone, implying \(\text{MRS}_a < \text{MRT}\) and \(\text{MRS}_b < \text{MRT}\),

(2) the sum of the marginal utilities accruing to all parties is greater than the marginal cost of the public good or \(\text{MRS}_a + \text{MRS}_b > \text{MRT}\), and

(3) the marginal cost the public good is constant.

The first two are explicit and the third is implicit. When these three assumptions hold the Wicksellian model can be interpreted in terms of Figure 2 as follows:

The third assumption specifies the condition that the lower boundary of \(S\) is a straight line with the slope equal to \(\text{MRT}\) and the upper boundary of the set is a horizontal line both of which contain Point D. The first assumption sets the conditions that the absolute value of the slope of the lower boundary of \(P\) is smaller than that of \(S\), and that of the upper boundary of \(P\) is larger than that of \(S\). The second assumption implies that the absolute value of the slope of the lower boundary of \(P\) is greater than that of the upper boundary of \(P\). Because the lower boundary of \(P\) is A’s indifference curve \((\alpha_1)\), whose slope is \(-\text{MRS}_a\), and the upper boundary is B’s indifference curve \((\beta_1)\), whose slope is \(-\text{MRT} - \text{MRS}_b\), and the Wicksell’s second assumption can be restated as \(\text{MRS}_a > \text{MRT} - \text{MRS}_b\).

The second assumption guarantees that Set \(P\) represented by the lens
shaped area demarcated by the two indifference curves crossing each other at point D is not empty. The first and the third assumptions together guarantee that intersection of P and S (=W) is a non-empty set. The existence of non-empty intersection is critical. It makes the Wicksellian assertion viable that there exist vectors simultaneously elements of the cost-sharing and Pareto-preferred sets. The tax-expenditure programs represented by the vectors which are elements of the intersection will be approved unanimously.

However, if his implicit (the third) assumption does not hold good the Wicksellian conjecture could loses its validity. If the marginal cost of x is increasing and consequently production of the public good increases the pre-tax income of B(A) and reduces that of A(B), public good production shifts the location of the cost-sharing set. Suppose that the income of B increases.

Then the cost-sharing set, denoted by S', becomes such that

\[ S' = S'_a \cap S'_b \]

where \( S' = \{ v \in N : y_i > Y_i(x) - f(x) \} \) (i= a, b)

Geometrically, as in Figure 3, the outer boundary of set N bows out and the cost-sharing set shifts bodily downward to the location of Set S', the area shaded by the vertical lines. The slope of the lower boundary of Set S' (curve DS) is \(-df(x)/dx -dY_a/dx\), and that of the upper boundary (curve DR) is \((0-dY_b/dx) = -dY_b/dx\). If the shift in the cost-sharing set is large enough so that the upper boundary of Set S' finds itself located below the lower boundary of the Pareto preferred set, P, the location of which remains unaffected, and, consequently, intersection of Set S' and Set P becomes empty. That is,

\[ \left| -dY_a/dx \right| > MRS_a, \]

\[ S' \cap P = \emptyset \]

The larger the change in earned incomes and the smaller A's marginal utility of the public good, the greater is the likelihood of this to happen. In such a case a Wicksellian type tax-expenditure proposal, which leaves changes in earned incomes as they happen, finds itself being represented by a vector not one of the elements of Set P. Consequently, it is unable to win the unanimous support. The Wicksellian conjecture turns out to be not always correct even when his two explicit assumptions hold if his third, implicit, does not.

If the public good is produced without A's payments, the apparent free-rider A turns out to be not actual free-rider at all. A pays the cost in the form
of a loss of income.

Note that the condition above assumes the situation where A pays no tax. If A is forced to pay a tax, say due to the pre-existing tax schedule, the condition for the empty intersection must be modified as below:

\[ \left| -\frac{dY_a}{dx} \right| > \text{MRS}_a - T_a \]

and the possibilities of the failure in reaching the unanimous agreement increase.

6. Combining Distribution and Allocation

However, there is a way to make a tax-expenditure proposal acceptable to all voters. It is to combine distribution with allocation proposals. For example, in Figure 3, if a tax-expenditure program represented by point H is proposed in isolation, it fails to receive the unanimous support because it is not one of the vectors in \( P \), which is the area demarcated by indifference curves of A and B both containing point D, \( (\alpha_1, \beta_1) \). Let this allocation proposal be combined with an income redistribution proposal that transfers a quantity of income equal to HJ from B to A. When Point J is located in the area demarcated by \( \triangle BJD \), this allocation-cum-distribution proposal places the proposed vector in Set \( P \) enabling vector represented by point J to be supported unanimously.

Note that it is essential to propose the both allocation and distribution propositions as a combined set. Suppose that they are separated, and an income equal to HJ is transferred to A without A’s commitment to approve the public good production to be proposed later. Once the transfer of income places A at point D’ (DD’ = HJ), A will no longer support the public good production as the sets S and P sharing Point D’ have no intersection except point D’. Since this income distribution makes A better off and B worse off, it will not be supported unanimously to begin with. However, if combined with the public good provision, the transfer is approved unanimously. The unanimity on an income transfer that Musgrave lamented as impossible becomes possible if the transfer is combined with provision of a public good.

Thus combining the allocation and distribution functions is a method superior to that of separating them as a fiscal means of enhancing the community’s welfare with sacrificing no one’s utility.

The target of the needed adjustment on the initial condition seems misdirected. What prevents realization of the “justice” under the Wicksellian
voting system is not unjust initial distribution of income but the unjust initial
distribution of the ownership of factors of production. If the initial ownerships are
evenly distributed, income redistribution would be unnecessary even when factor
prices are functions of the quantity of the public good produced and the
Musgravian distribution branch would become redundant. In fact, it appears that
the type of redistribution that Wicksell suggested is not year-by-year income
adjustment but once for all adjustment in the property ownerships. If once for all
readjustment in the factor ownerships is politically unacceptable, the combined
execution of the distribution and allocation functions is the second-best method
to improve the community’s welfare through the unanimous agreements.

7. Conclusions

Richard Musgrave’s famous proposition of separating fiscal functions into
three independent branches is his attempt to make the Wicksellian just taxation
under the principle of voluntary consent and unanimity feasible. It required of
him to separate income redistribution which is of a zero-sum nature, not
amenable to the unanimity rule from the resource allocation which is a
positive-sum game, amenable to the unanimity rule. Both Wicksell and Musgrave
conjectured that voters would unanimously approve a tax-expenditure proposal
representing a package of a quantity of the public good and distributions of its
cost so long as the benefits accruing to each voter is greater than his tax.

However, this paper contended that a voter’s real interest would be not in
the amount of his tax but in his share of the community’s disposable income.
Therefore, even the benefits from the public good exceed his tax a voter would not
approve a tax-expenditure program if the expected loss in his earned income that
the public good production induces outweighs the benefits that exceed his tax.
This fact is proven in this paper by showing the emptiness of intersection of the
set of vectors Pareto-preferred to the initial vector and the set of vectors attainable
by sharing the cost of the public good. Here a vector represents a tax-expenditure
program whose components are the quantity of the public good and distributions
of the aggregate disposal income among the citizens.

Given (uneven) distribution of the ownerships of factors of production, the
greater the rate of increase in the marginal cost of the public good and the larger
complementary and substitute relationships between the public and private
goods, the greater are the chances of the Wicksellian model to collapse. Only
when the marginal cost of a public good is constant and/or production effects of a public good on market prices are neutral, the Wicksellian unanimity approach and consequently the Musgravian separation proposal are certain to perform their expected roles.

However, if redistribution and allocation programs are combined and made into a joint proposal, it would win unanimous approval in a situation where both the proposals would be rejected, had they been tabled independently. Thus, this paper concludes that combining allocation and distribution functions is superior approach over separating them in order to improve a community’s welfare beyond the initial situation with sacrificing no one’s welfare.

See Shibata (1971) for a hypothetical mechanism in which the adjusted initial income distribution yields through Lindahl pricing the desired welfare distribution.

The implications of such a rent-seeking voter's behavior are examined by (Shibata, 1973).

In this Figure A's indifference curve is expressed as usual. B's indifference curve is expressed by making the vertical axis and the Line \( O_bT \) as the coordinates and \( O_b \) as the origin. In other words, the quantities of \( x \) and \( y_b \) entering in B's utility function as arguments are expressed by the horizontal distance from a point on B's indifference curve to the vertical axis and the vertical distance from that point to the line \( O_bT \), respectively. The construction of this types of figures was presented and explained in Shibata (1971).

The vertical distance between curve DR and curve DS represents the cost of the public good being produced. For example, when the quantity of the public good produced is \( O\alpha E \), the vertical distance between the two curves at that output, \( GH \), is equal to the vertical difference between the initial income, \( O\alpha O_b \) and the aggregate disposable income \( EF \), or the distance \( MF \).

This is so because the upper boundary of the post-distribution cost share set is curve \( D'R' \). Obviously, B's new budget curve, \( D'R' \), is the vertical displacement of curve DR. And the lower boundary of the Pareto-preferred set is A's indifference curve containing point \( D' (\alpha_d) \) which is a vertical displacement of the indifference curve containing point \( D (\alpha_1) \), assuming that the income effect is absent in this range of A's utility function.
REFERENCES


Fig. 1 Set N

\[ N = \{ (x, y_a, y_b) : y_a + y_b + f(x) - Y_a - Y_b = 0 \} \]

All feasible fiscal packages \((y)\) are represented by elements of the production possibility set.

Fig. 2 \( W = P \cap S \neq \emptyset \)

Intersection of Pareto preferred and cost sharing sets (not-empty)
Constant marginal cost case.
Fig. 3  \( W' = P \cap S' = \emptyset \)

Pareto preferred and cost sharing sets are mutually exclusive

Increasing marginal cost case